Aircraft Flight Control

ECE 5115 Controls Systems Lab 2

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12/02/22

1. Executive Summary

In this lab two state space representation controllers were designed for an aircraft. The aircraft system is first introduced, and the open loop analysis is performed. The open loop analysis showed that the aircraft system was unstable.

The first controller designed was a state feedback controller, and the second was a linear quadratic regulator controller. After simulating the aircraft with both controllers, the linear quadratic regulator was determined to be the better controller. The linear quadratic regulator controller eliminated overshoot and increased the aircrafts system response time.

2. Introduction

The movement of an aircraft can be modeled using nine state variables. The inputs to the movement of the aircraft are the elevator for the pitch, the rubber for the yaw, and the ailerons for the roll. The outputs are the pitch, yaw, and roll. Figures 1 and 2 illustrate the aircraft system.

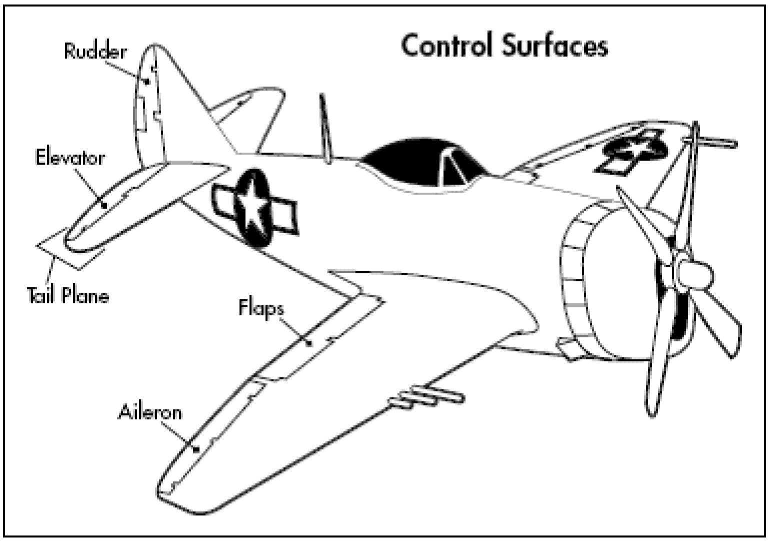

Figure 2.1: Control surfaces of an aircraft.

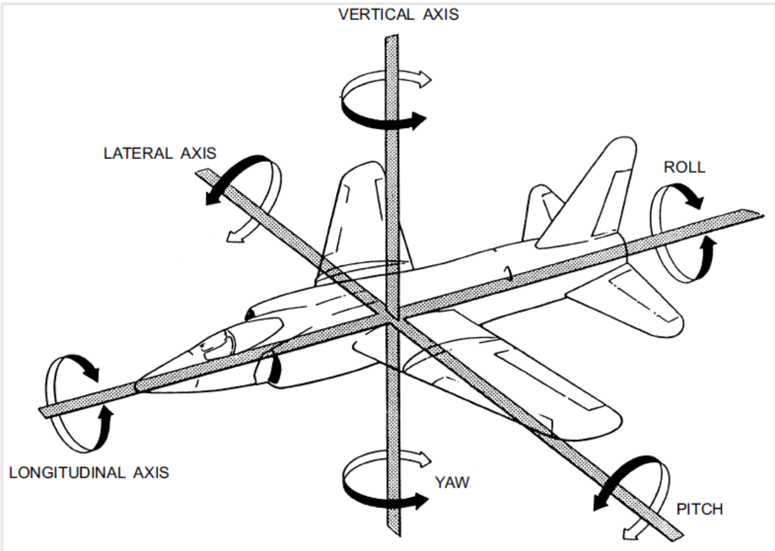

Figure 2.2: Aircraft axes pitch, yaw, and roll.

Using the coefficients from table 1 and the aerodynamic variables from table 3, the longitudinal dynamic equations are as follows,

$$\Delta\dot{u} = X_u \Delta u + X_\alpha \alpha - g\theta + X_E \delta_E \tag{2.1}$$

$$\dot{\alpha} = \frac{Z_u}{V}\Delta u + \frac{Z_\alpha}{V}\alpha + q + \frac{Z_E}{V}\delta_E \tag{2.2}$$

$$\dot{q} = M_u \Delta u + M_\alpha \alpha + M_q q + M_E \delta_E \tag{2.3}$$

$$\dot{\theta} = q \tag{2.4}$$

Table 1. Longitudinal aerodynamic coefficients.

X_u	-0.1
g	9.81
$\frac{Z_u}{V}$	-0.1
$\frac{Z_\alpha}{V}$	-0.1
M_q	-0.5
M_u	-0.05
M_α	-5
X_E	-1
M_E	-10
X_α	-14
$\frac{Z_E}{V}$	-0.1
Z_α	-1

Using the coefficients from table 2 and the aerodynamic variables from table 3, the lateral dynamic equations are,

$$\dot{\beta} = \frac{Y_\beta}{V}\beta + \frac{Y_p}{V}p + \left(\frac{Y_r}{V} - 1\right)r + \frac{g}{V}\emptyset + \frac{Y_A}{V}\delta_A + \frac{Y_R}{V}\delta_R \tag{2.5}$$

$$\dot{p} = L_\beta\beta + L_p p + L_r r + L_A\delta_A + L_R\delta_R \tag{2.6}$$

$$\dot{r} = N_\beta\beta + N_p p + N_r r + N_A\delta_A + N_R\delta_R \tag{2.7}$$

$$\dot{\emptyset} = p \tag{2.8}$$

$$\dot{\varphi} = r \tag{2.9}$$

Table 2: Lateral aerodynamic coefficients.

$\frac{Y_\beta}{V}$	-1
$\frac{Y_p}{V}$	0.75
$\frac{Y_r}{V}$	-0.50
g	9.81
$\frac{g}{V}$	0.05
$\frac{Y_A}{V}$	0.50
$\frac{Y_R}{V}$	-1
L_β	0.1
L_p	0.1
L_r	0.1
L_A	0.25
L_R	1
N_β	0.25
N_p	0.25
N_r	-0.25
N_A	0.25
N_R	0.75

Table 3: Aerodynamic variables.

	Longitudinal	Lateral
Rates	α: angle of attack q: pitch rate Δu: change in speed	β: side slip angle p: roll rate r: yaw rate
Positions	z: altitude θ: pitch	ψ: roll angle ϕ: yaw angle x: forward displacement y: cross track displacement
Controls	δ_E: elevator deflection	δ_A: aileron deflection δ_R: rudder deflection

Putting the aircraft system into state space representation gives,

$$\begin{bmatrix} \dot{\Delta u} \\ \propto \\ q \\ \theta \\ \beta \\ p \\ r \\ \emptyset \\ \varphi \end{bmatrix} = \begin{bmatrix} X_u & X_\propto & 0 & -g & 0 & 0 & 0 & 0 & 0 \\ \frac{Z_u}{V} & \frac{Z_\propto}{V} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_u & M_\propto & M_q & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{Y_\beta}{V} & \frac{Y_p}{V} & \left(\frac{Y_r}{V} - 1\right) & \frac{g}{V} & 0 \\ 0 & 0 & 0 & 0 & L_\beta & L_p & L_r & 0 & 0 \\ 0 & 0 & 0 & 0 & N_\beta & N_p & N_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \propto \\ q \\ \theta \\ \beta \\ p \\ r \\ \emptyset \\ \varphi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ M_E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{Y_A}{V} & \frac{Y_R}{V} \\ 0 & L_A & L_R \\ 0 & N_A & N_R \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_A \\ \delta_R \end{bmatrix} \quad (2.10)$$

$$\begin{bmatrix} \dot{\theta} \\ \emptyset \\ \varphi \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta u \\ \propto \\ q \\ \theta \\ \beta \\ p \\ r \\ \emptyset \\ \varphi \end{bmatrix} \quad (2.11)$$

3. Background

Finding the eigen values of the system and simulating it highlights the need of designing a controller for the system. The eigen values are,

$$\begin{gathered} 0, -1.1439 + 1.9025i, -1.1439 - 1.9025i, \\ 0.3439 + 0.8818i, 0.3439 - 0.8818i, -0.6483 + 0.4170i \\ -0.6483 - 0.4170i, 0.1711, \text{and } -0.0246 + 0.0000i \end{gathered} \quad (3.1)$$

Automatically we can see the system is unstable as it has two poles with positive real terms at 0.3439 + 0.8818i, 0.3439 - 0.8818i. Simulating the open system further illustrates the unstableness. Figure 3.1 shows the impulse, step, and frequency response for the pitch with respect to the elevator input.

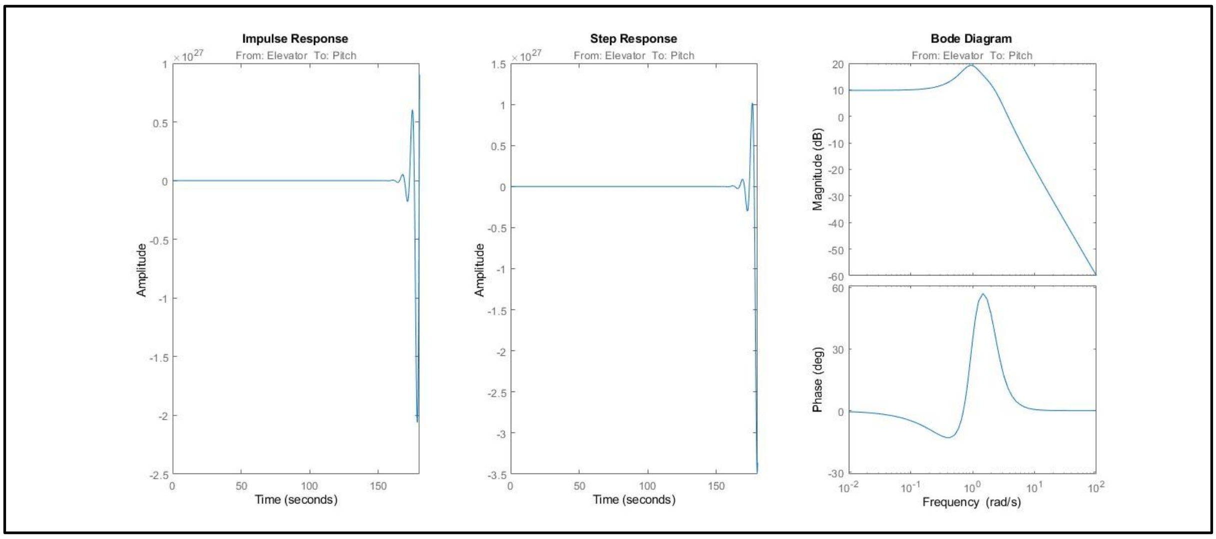

Figure 3.1: Impulse, step, and frequency response of the pitch with respect to the elevator input.

For both the impulse and the step response the system goes to infinity. The gain and phase margins for the pitch are, infinite and -168.9997 respectively.

Figure 3.2 displays the impulse, step, and frequency response of the yaw with respect to the aileron and rubber inputs.

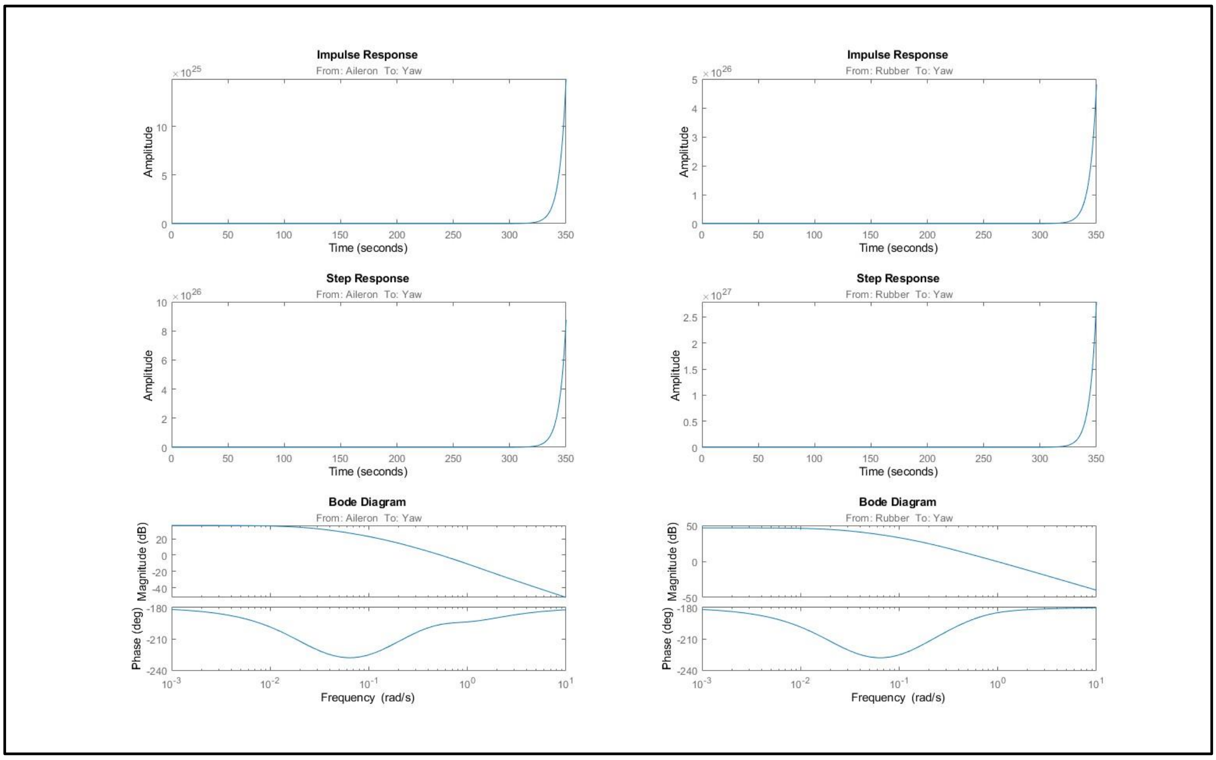

Figure 3.2: Impulse, step, and frequency response for the yaw with respect to the aileron and rubber inputs.

The yaw is unstable, both the impulse and step response go to infinity. The gain and phase margins for the yaw, with respect to the aileron are, 0.0148 and -16.2712 respectively, and with respect to the rubber are, 0.0047 and -5.1893 respectively.

Figure 3.3 displays the impulse, step, and frequency response of the roll with respect to the aileron and rubber inputs.

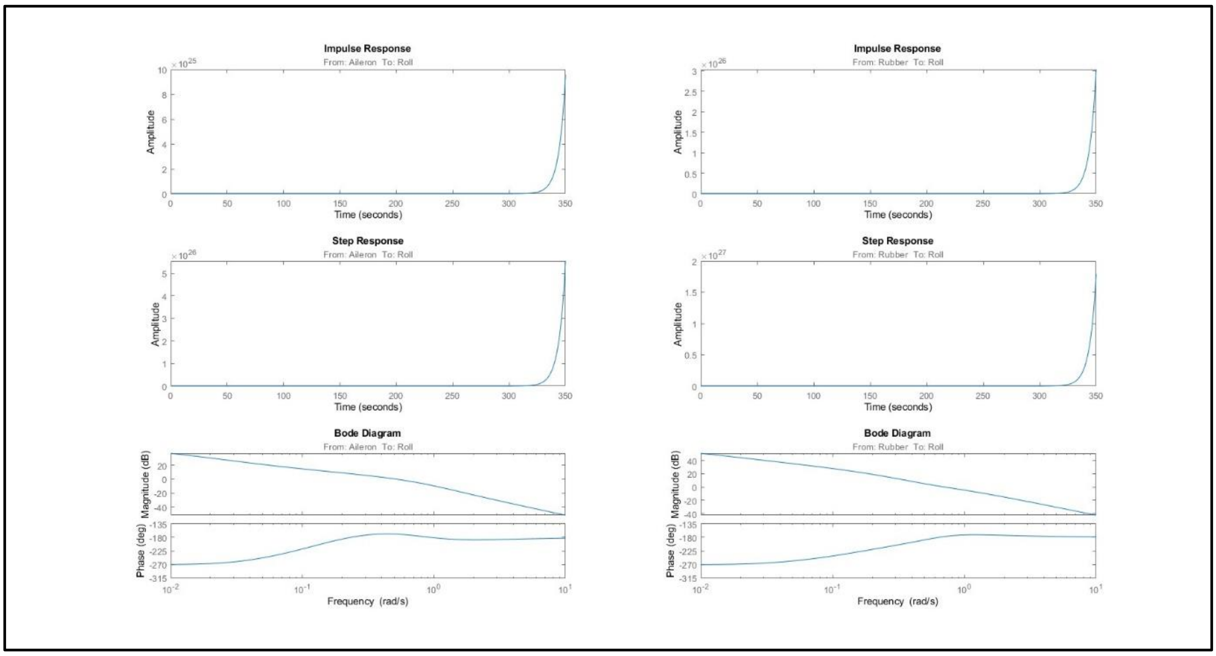

Figure 3.3: Impulse, step, and frequency response for the roll with respect to the aileron and rubber inputs.

Like the two other outputs, the roll is unstable, both the impulse and step response go to infinity. The gain and phase margins for the roll, with respect to the aileron are, 0.4105 and 10.7523 respectively, and with respect to the rubber are, 0.8668 and 2.5141 respectively.

In this lab we will design two state space controllers for an aircraft. The goal of the controllers is to stabilize the aircraft such there is level flight, stability is maintained, and oscillations are kept to a minimum magnitude and frequency. The first controller designed will be a state feedback controller, and the second will be a linear quadratic regulator.

4. Solution

The two aircraft controllers where designed and simulated using MATLAB[1].

4.1 Controller 1: State Feedback

For this controller the poles of the open loop system were examined. There are three poles that cause instability in the system. To make the system stable, the poles will be moved. The pole at zero will now be at -0.5, and the conjugate pair of poles at 0.3439 + 0.8818i, 0.3439 - 0.8818i were changed to -0.3439 + 0.8818i, -0.3439 - 0.8818i. With this change in pole placement, the system is now stable. The feedback gain matrix K is,

$$K = \begin{bmatrix} 0.0145 & 0.2098 & -0.1266 & -0.2630 & -0.0011 & -0.0113 & -0.0007 & -0.003 & 0.0011 \\ 0.5274 & 1.8691 & -4.5318 & -2.0639 & 0.1802 & -1.0818 & 0.9596 & -0.2455 & 0.0819 \\ 0.4535 & -0.2300 & -2.2130 & -1.7955 & 0.3415 & 0.7017 & 0.4101 & 0.387 & 0.1461 \end{bmatrix} \quad (4.1)$$

Figure 4.1 displays the impulse, step, and frequency response of the pitch with respect to the elevator.

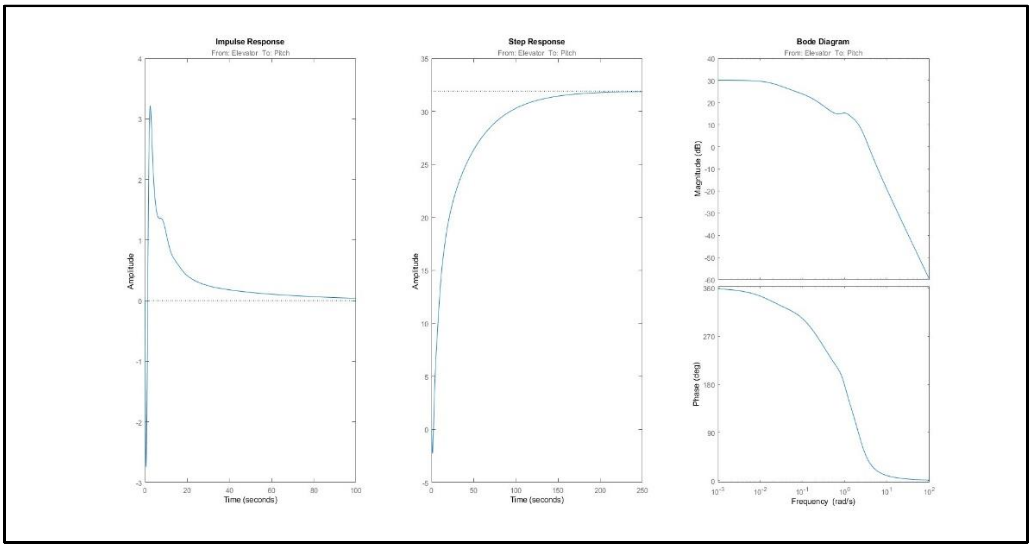

Figure 4.1: Impulse, step, and frequency response of the pitch with state feedback.

With the step response we can see that the system vastly overshoot and takes 137[s] to stabilize. The gain and phase margins for the pitch with respect to the elevator are, 0.1718 and -144.2847 respectively.

Figure 4.2 displays the impulse, step, and frequency response of the yaw with respect to the aileron and rubber.

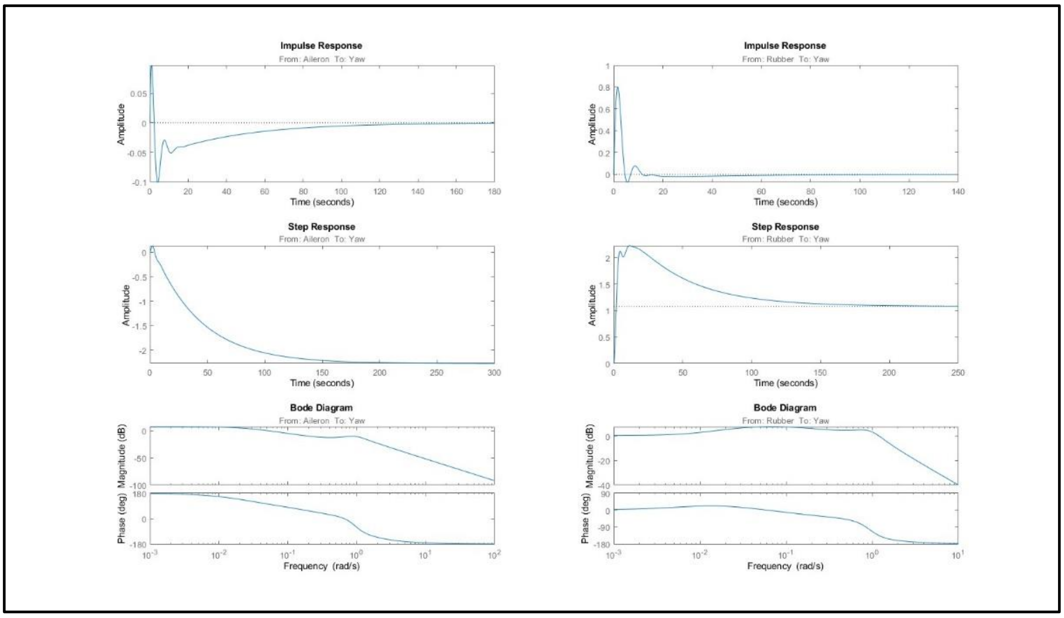

Figure 4.2: Impulse, step, and frequency response of the yaw with state feedback.

While the yaw is stable, it is not ideal as it overshoots and it has a stabilizing time of 180[s]. The gain and phase margins for the yaw with respect to the aileron are, 0.4392 and -76.9126, and infinite and 43.5942 respectively with respect to the rubber.

Figure 4.3 display the impulse, step, and frequency response of the roll with respect to the aileron and rubber.

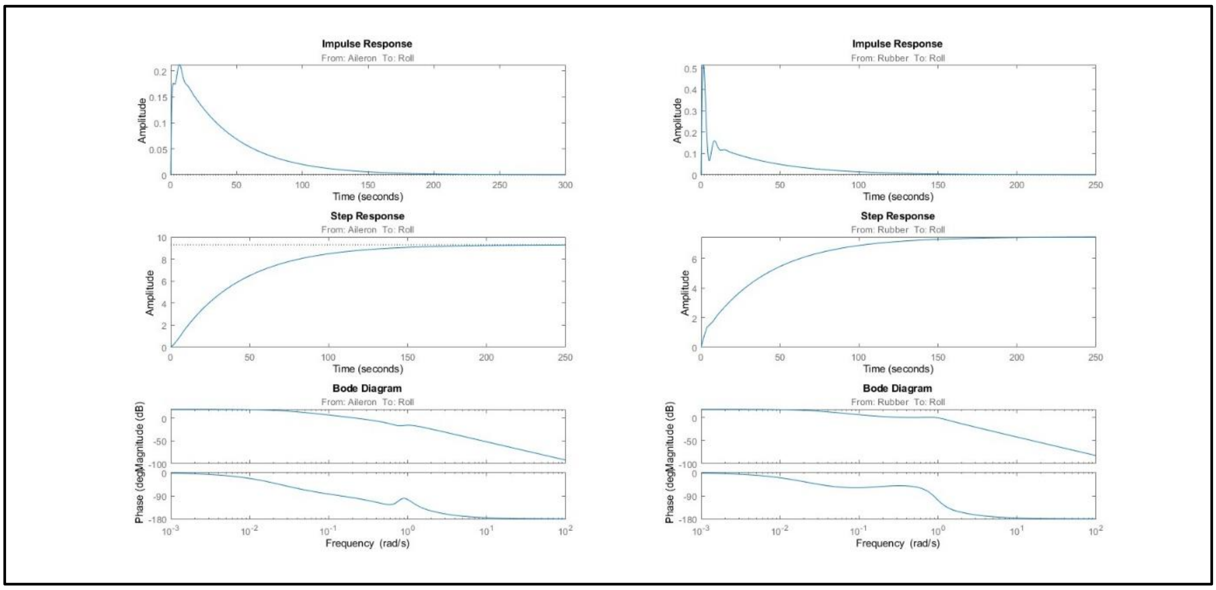

Figure 4.3: Impulse, step, and frequency response of the roll with state feedback.

While the roll is stable, it is not ideal as it overshoots and it has a stabilizing time of 160[s]. The gain and phase margins for the roll with respect to the aileron are, infinite and 80.8575, and infinite and 89.4127 respectively with respect to the rubber.

4.2 Controller 2: Linear Quadratic Regulator

To design the linear quadratic regulator, we need to select the matrix values for Q_L and R_L to be used to solve the Riccati equation. The values that where decided are,

$$Q_L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 135 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.2}$$

$$R_L = \begin{bmatrix} 125 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.3}$$

The metric for deciding which values got higher values for the matrix Q_L was that for all the state variables in which the outputs depend on should be higher to assure higher settling speeds[2]. For the matrix R_L, the pitch value was penalized more than the others to get the system to have a smaller tracking error. These values where found by tuning various set of parameters. The system poles are,

$$\begin{gathered} -0.0946, -0.1000, -0.7993 + 0.0483i \\ -0.7993 - 0.0483i, -1.0240 + 0.6902i, -1.0240 - 0.6902i \\ -2.2249, -9.7904, -13.0138 \end{gathered} \tag{4.4}$$

The feedback gain matrix K for the linear quadratic regulator is,

$$K = \begin{bmatrix} 0.1581 & -0.5511 & -1.2407 & -1.4209 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8362 & -1.5295 & 7.3645 & -0.2451 & 0.9607 \\ 0 & 0 & 0 & 0 & -0.1509 & 9.5175 & 3.4149 & 0.9545 & 0.2774 \end{bmatrix} \quad (4.5)$$

Figure 4.4 displays the impulse, step, and frequency response of the pitch with respect to the elevator.

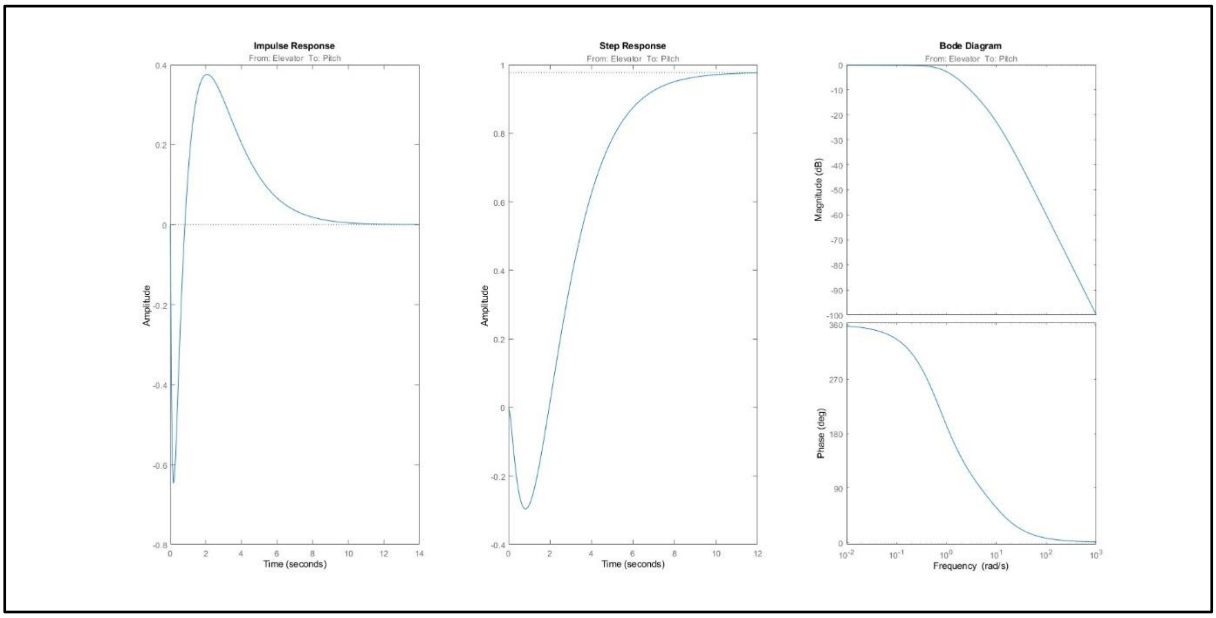

Figure 4.4: Impulse, step, and frequency response of the pitch with the linear quadratic regulator controller.

The pitch is now stable, no overshoot is present, and the system settles in 8.4598[s]. The gain and phase margins for the pitch are, 1.4966 and infinite respectively.

Figure 4.5 displays the impulse, step, and frequency response of the yaw with respect to the aileron and the rubber.

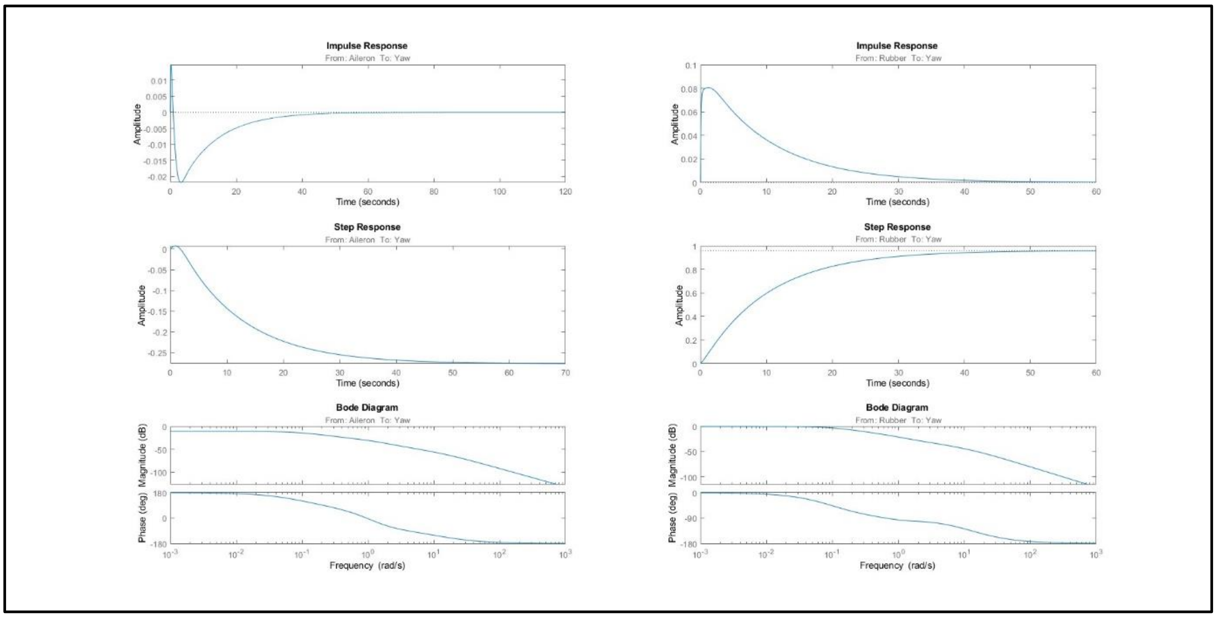

Figure 4.5: Impulse, step, and frequency response of the yaw with the linear quadratic regulator controller.

The yaw is stable, no overshoot is present and the response time is 39.5122. The gain and phase margin of the yaw are, 3.6090 and infinite with the respect to the aileron, and infinite and infinite with respect to the rubber.

Figure 4.6 displays the impulse, step, and frequency response of the roll with respect to the aileron and rubber.

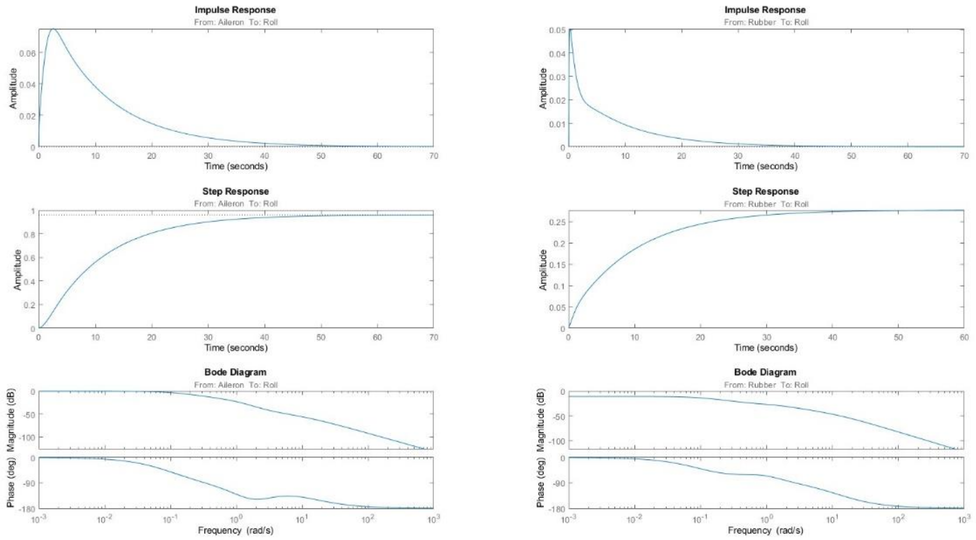

Figure 4.6: Impulse, step, and frequency for the roll with the linear quadratic regulator controller.

The roll is stable, no overshoot is present, and the settling time is 41.9780. The gain and phase margins for the roll are, infinite and infinite with respect to the aileron, and infinite and infinite with respect to the rubber.

5. Conclusion

After simulating the two systems, it was found that the linear quadratic regulator controller is the better design. The optimal control controller leads to the best response times and no overshoots.

6. References

[1] "Control System Toolbox," *Control System Toolbox Documentation*. [Online]. Available: https://www.mathworks.com/help/control/. [Accessed: 03-Dec-2022].

[2] R. L. Williams and D. A. Lawrence, *Linear State-Space Control Systems*. Hoboken, NJ: John Wiley & Sons, 2007.