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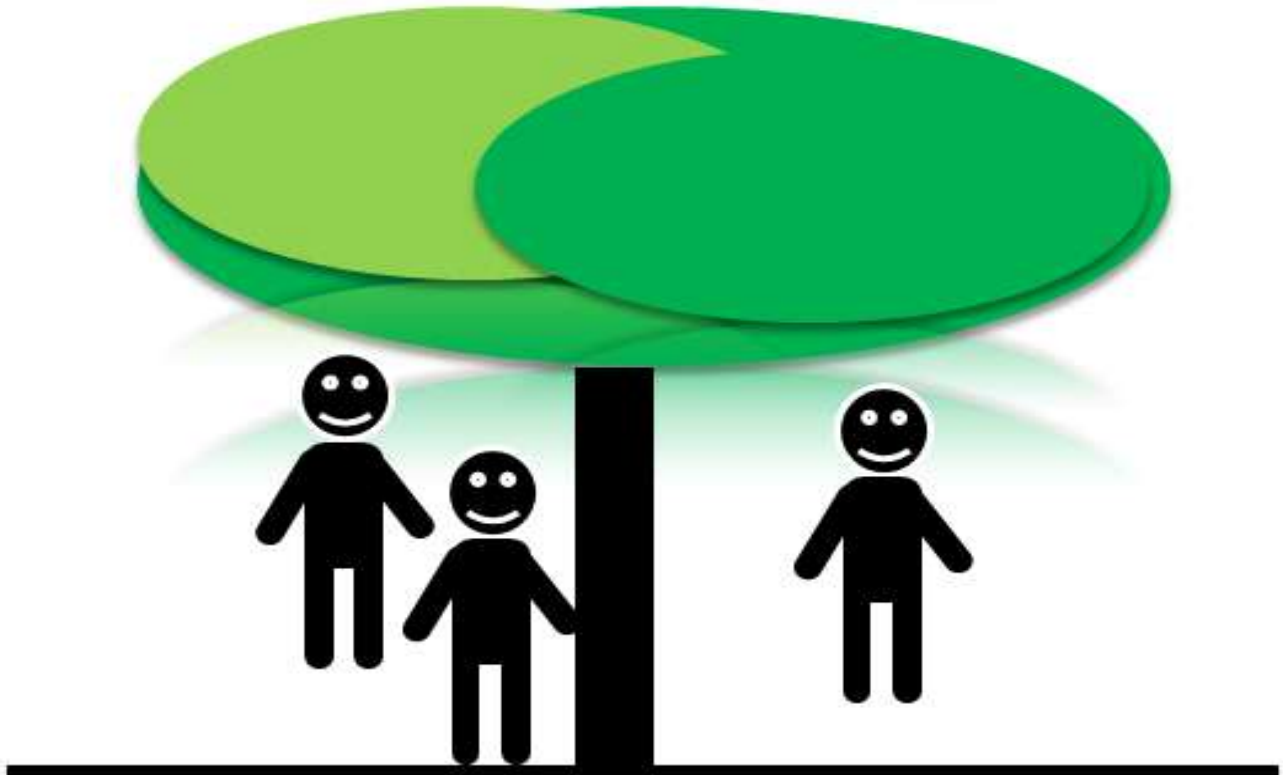

MATHEMATICS

O LEVEL

BOOK 7

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Algebraic Functions

Table of contents

FUNCTIONAL TABLES

To draw a curve you need to find the values of x and y. This is possible when using a functional table which is used to find the corresponding values of y given x-axis coordinate

We are going to use a standard question to demonstrate all the principles in this ebook. Also note that the first concept that is tested on almost all Functional graph question is to find the missing values in a given functional table

QUESTION

The following is an incomplete table of values for the function $y = x(4 – x)$.

x	-1	0	1	2	3	4	5
y	p	0	3	4	q	0	-5

(a) Calculate the value of p [1]
(b) Calculate the value of q. [1]

Solution

To find the values of p and q we use the equation and substitute the corresponding value of x into the equation. Also note that the values of p and q are the values of y when x = -1 and when x = 3.

(a) Corresponding x-value of p is -1, therefore we substitutes -1 for x into the equation

$y = x (4 – x)$

$p = -1 (4 – (-1)) = -1(5) = - 5$

(b) $y = x (4 – x)$

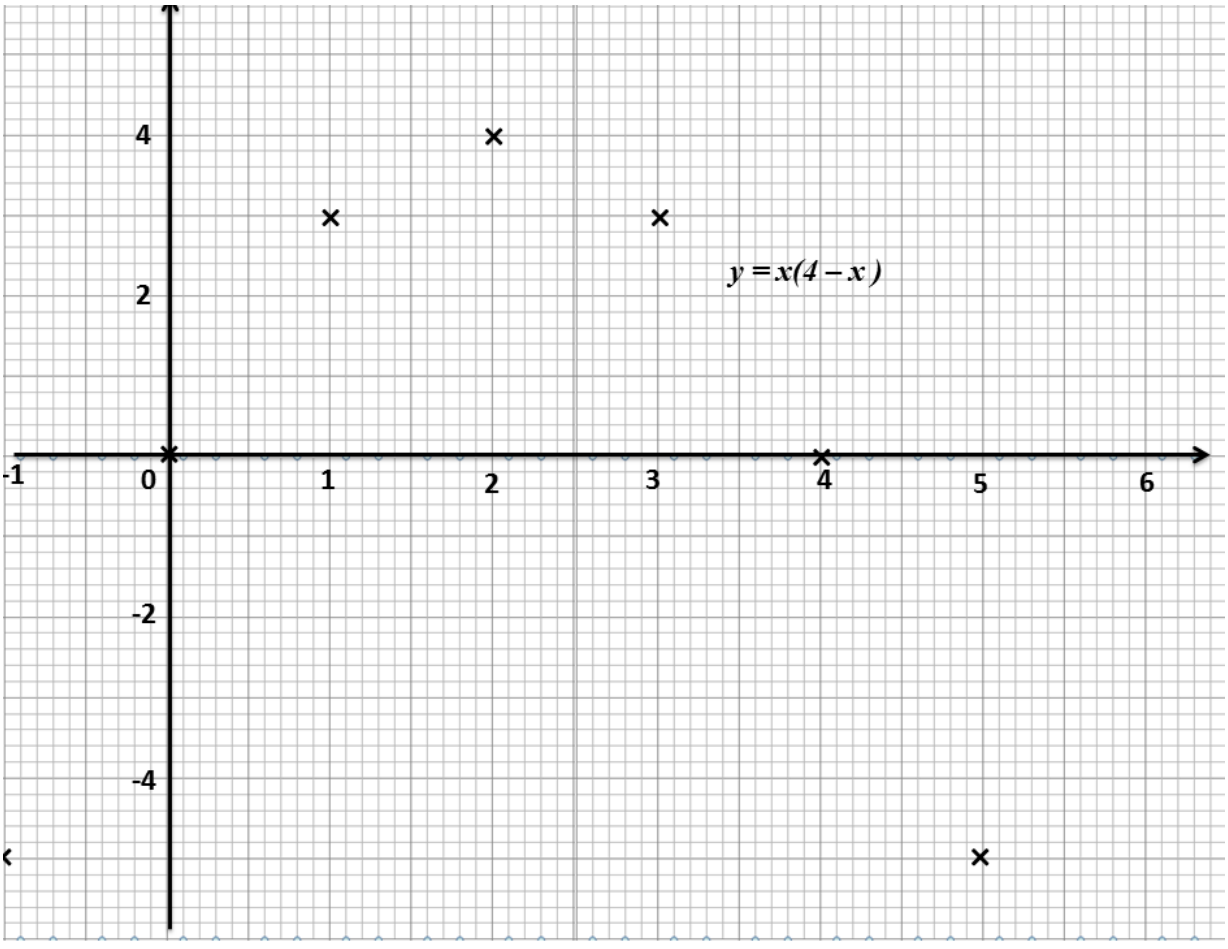
$$q = 3\,(4 - 3) = 3(1) = 3$$

2. PLOTTING AND DRAWING

We now have our complete functional table and can use it to plot and draw our points. This our complete Functional Table

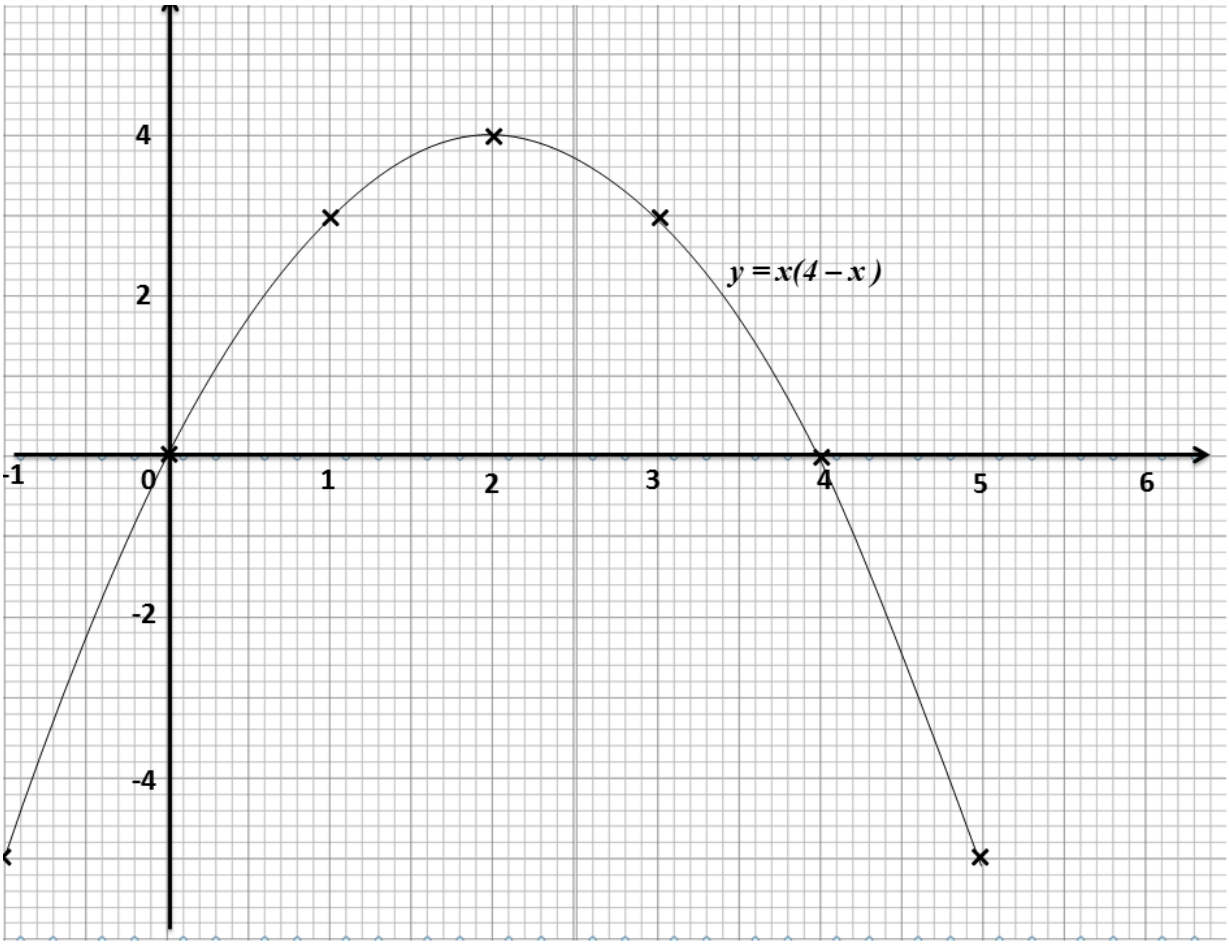
x	-1	0	1	2	3	4	5
y	-5	0	3	4	3	0	-5

Using a scale of 2cm to represent 1 unit x-axis and 2cm to represent 2 units on y – axis , lets now plot our graph…



You can see that I have plotted my points on the graph according to the functional table given above. It's now time to join the points by a smooth curve.

NB: Your curve must pass through all the plotted points as shown below



Done!!!

In the next chapter, we are going to look at how to locate the roots of the equation on the graph.

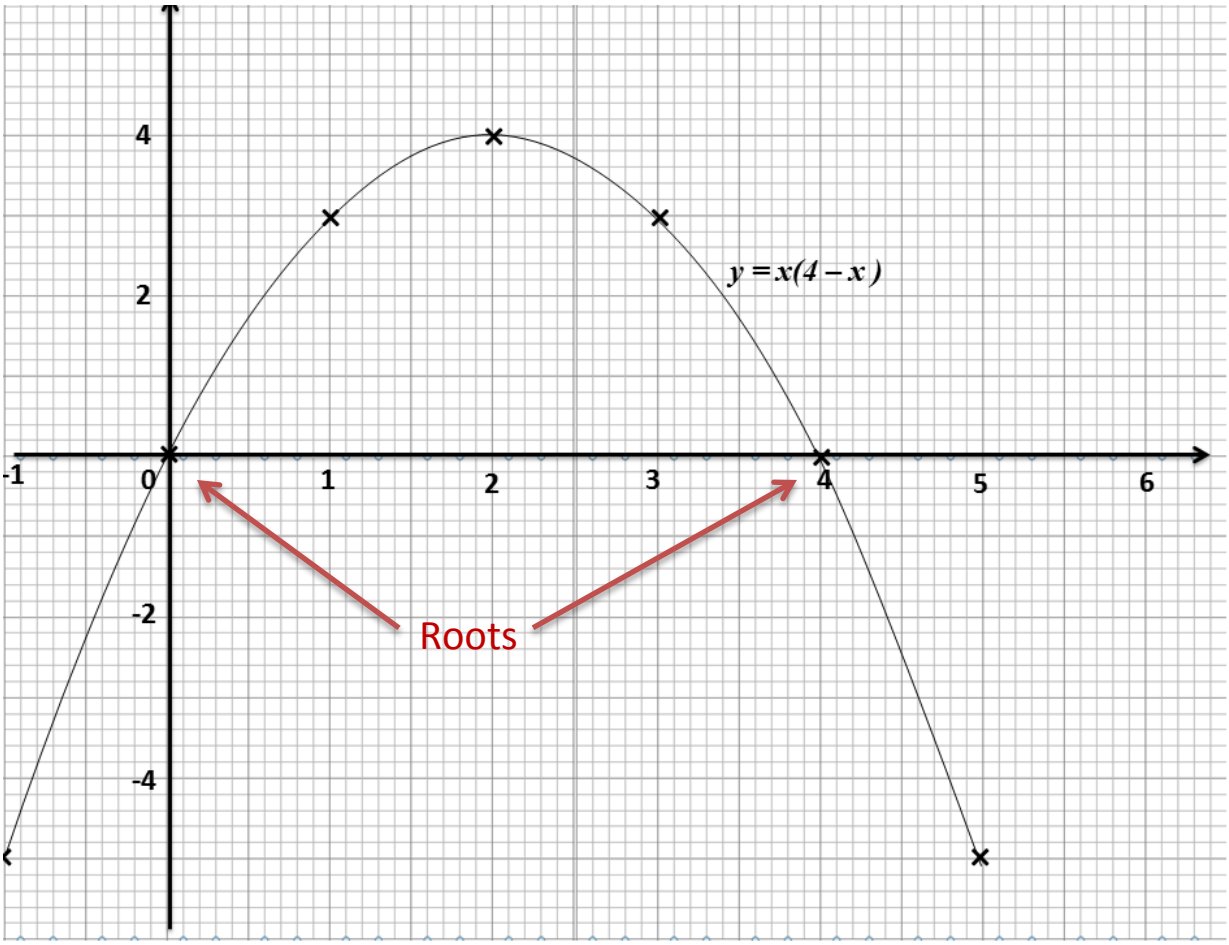
3. ROOTS OF THE EQUATION

You can be asked to write down the roots of the equation using the graph.

The roots of the equation are the points where the curve cuts across the x-axis. We only take the x-axis value and that will be the solution of the equation.

If it is a quadratic function, there will be two roots since the curve cuts the x-axis twice. If it's a cubic function, the curve cuts the x-axis thrice and this means there are three roots.

(a) Quadratic Function

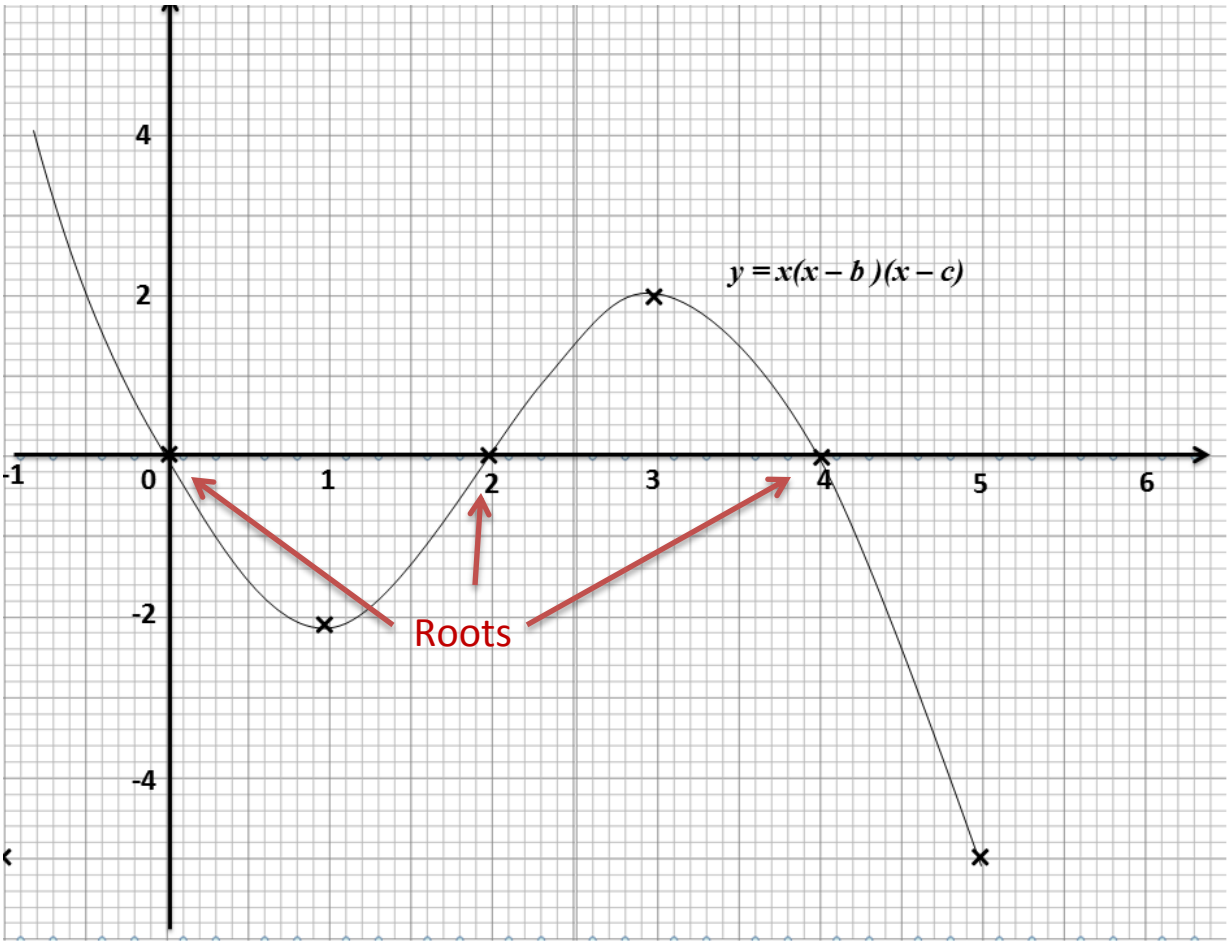


From the graph we drew in the last chapter, the solution of the equation is

$x = 0$ and $x = 4$

(b) Cubic Function

A cubic function will have 3 roots of the equation and the curve cuts across the x-axis thrice as shown.

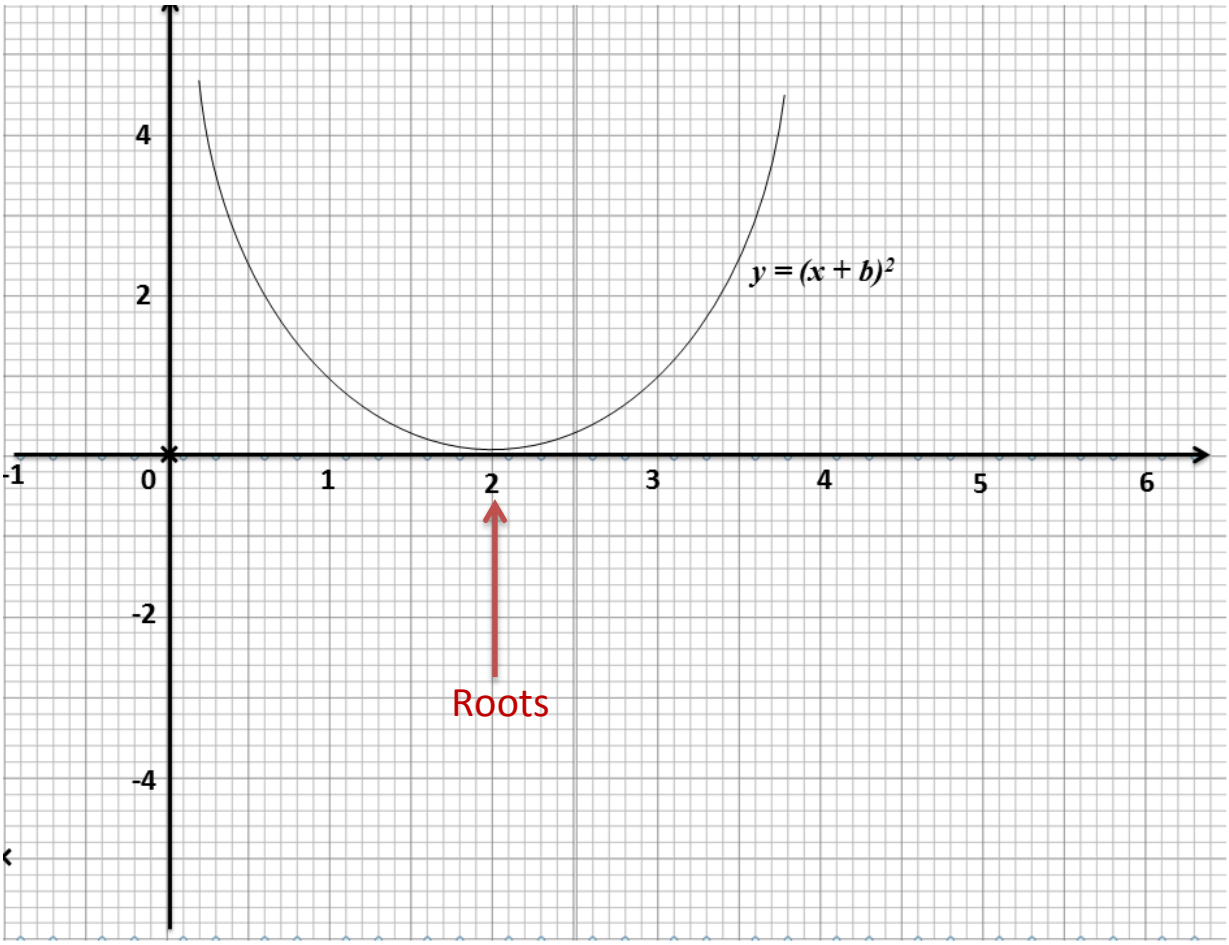


The solution for this graph will be

x = 0, x = 2 or x = 4

(c) Twice Roots

If the curve does not cut across but touches the x-axis, it has a twice root.



In this case, the solution of the equation will be

$x = 2$ *twice*

These are the three cases you are likely to meet in an exam. Its imperative that you master the concepts and know where to find roots in each of these cases.

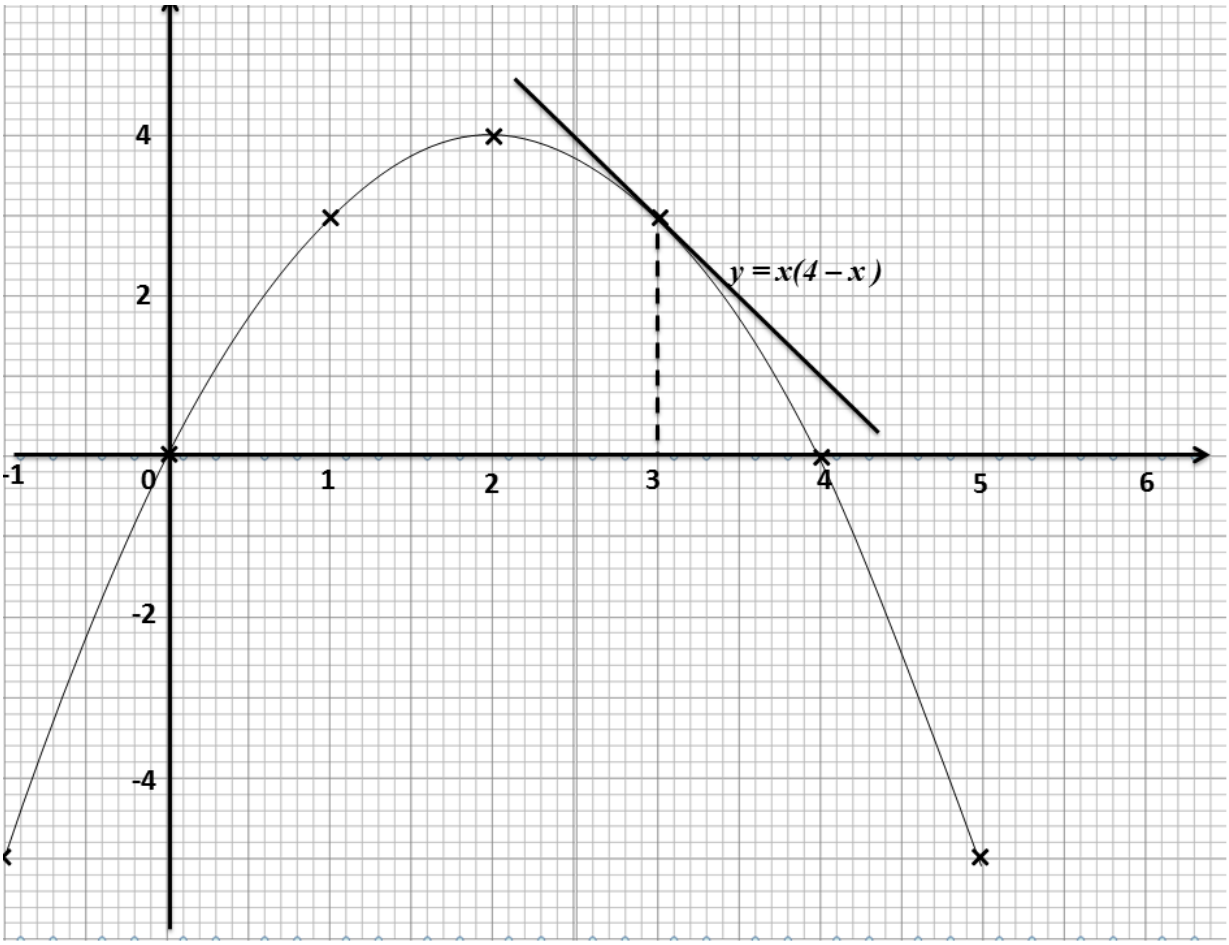
4. GRADIENT OF THE CURVE

In this section we want to find the gradient of the curve at a given point. For simplicity sake lets continue with the question we used in the first chapter. The function was $y = x(4 - x)$

This was part (c) of the question

> (c.) Use the graph in b(i) to find the gradient of the curve when $x = 3$. [2]

NB: Draw a tangent on the curve at the point where $x = 3$ and then calculate the gradient of that tangent. The gradient of the tangent is equal to the gradient of the curve at any given point



Now, we have to calculate the gradient of the tangent. We can pick any two points where the line passes through.

Let me use the point of tangency (3;3) and another point (4:1).

$$m = \frac{3-1}{3-4} = \frac{2}{-1} = -2$$

Therefore, the gradient of the curve at the point where x = 3 is -2

5. Line of symmetry

A line of symmetry is a line that divides a shape into two identical parts. For example a circle can be divided into two identical semi circle. This means a diameter is a line of symmetry.

The same applies to functional graphs. They can be U shaped of n-shaped and can have a line of symmetry.

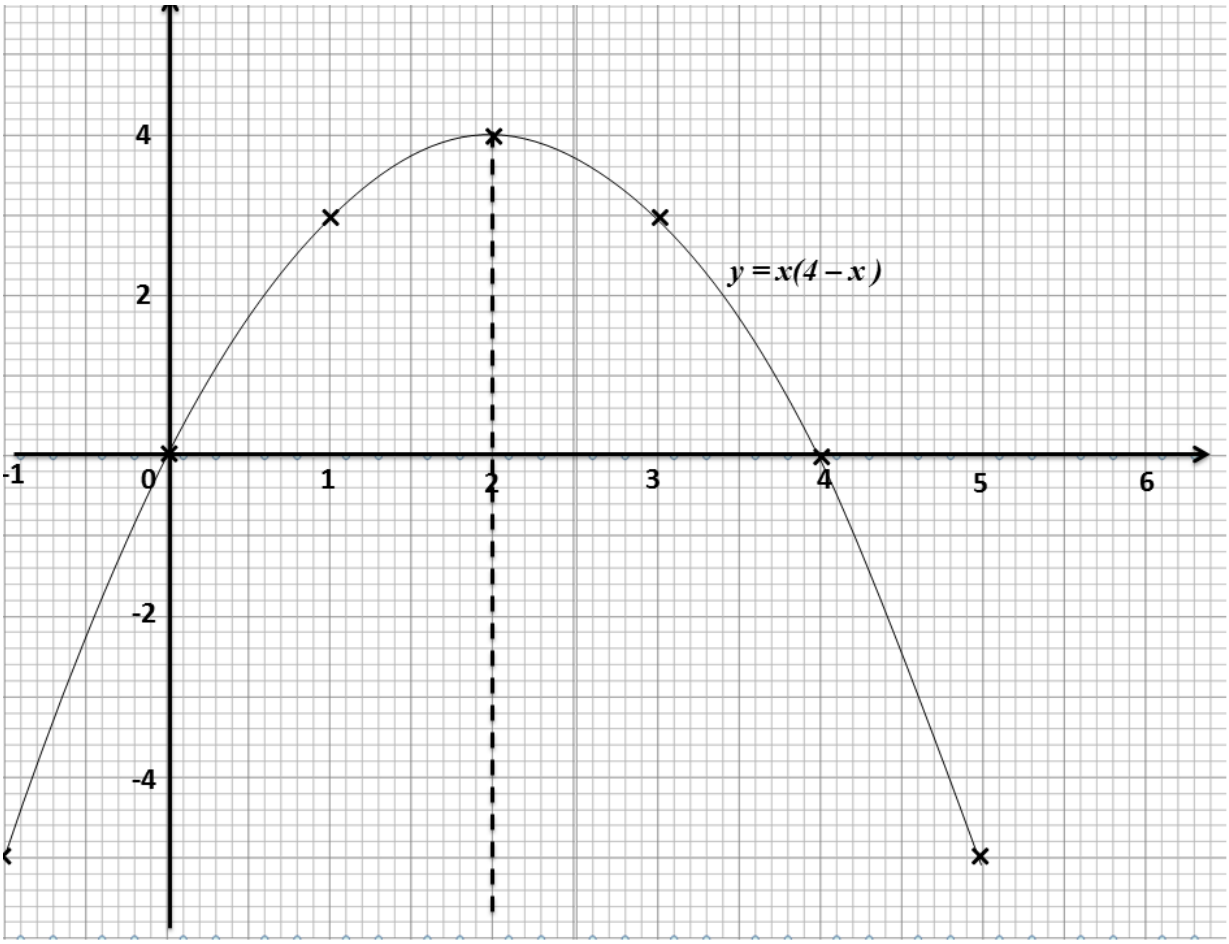
Let's continue with our question from the first chapter.

To find the line of symmetry, take two points where the curve cuts across x-axis (roots) and find a midpoint. In this case we will take

$x = 0$ and $x = 4$.

Therefore our midpoint is $x = \frac{0+4}{2} = 2$

This means the equation of the line of symmetry is $x = 2$. You must therefore draw the line $x = 2$ as shown in the following diagram



You can see that my line of symmetry divides the curve into two identical parts.

Now let’s move on to another concept under Algebraic Functions. For video explanations Download Our Android App, The Figtree App or WhatsApp +263 733 796 118 for assistance

Drawing a line

In almost all algebraic questions, you can be asked to draw a straight line or find a solution to a given equation (simultaneous) using the graph. You must be able to use the coordinate geometry principles to draw a straight line.

It's advisable to use the table of values given in the question.

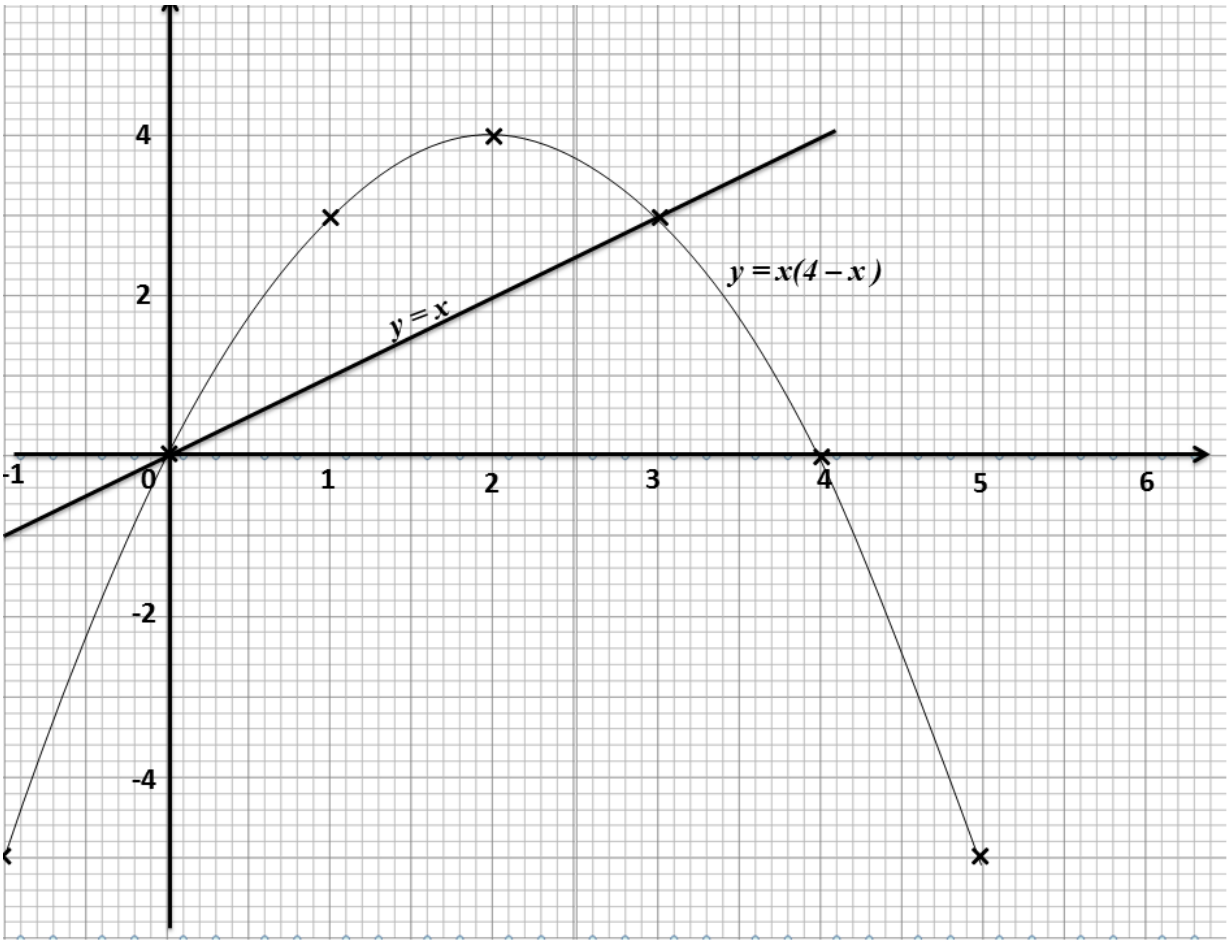
In this question, you were required to draw the line $y = x$. We are going to use the table of values to find the coordinates of the line, at least two.

$y = x$

x	-1	0	1	2	3	4	5
y	-1	0	1	2	3	4	5

You can notice that the values used for x (dependent variable) are the same with those used for the curve.

Let's now plot and draw our line on the same graph with the curve



Remember to label your line and curve with respective equations as shown above.

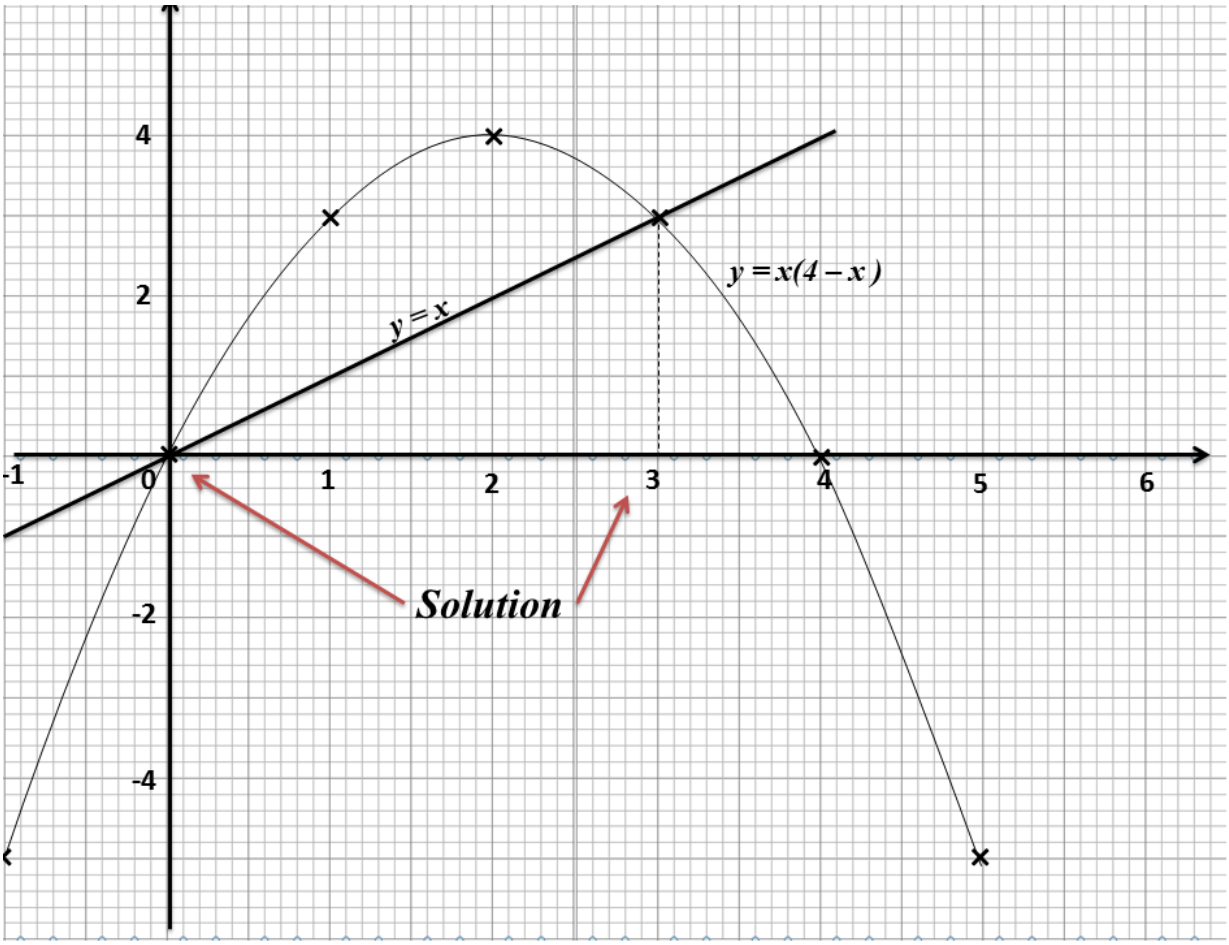
This curve and the line will be used to solve simultaneous equation in the next chapter.

6. Roots of simultaneous equations

The roots of the simultaneous equations are the coordinates of points where the two curves or a curve and a line intersect. You can be tested in different ways but they usually require candidates to find the solution of the equation (in this case)

$x(4-x) = x$.

This equation is solved using your graph but neither by calculations nor using quadratic formula. You must have two curves $y = x(4-x)$ and $y = x$



By using the graph, the solution for the equation is

$x = 0$ or $x = 3$ (Take x-values)

7. Translated curve

In other questions on functional graphs you can be required to find solution of the equation but equal to a given value.

In the previous chapters we learnt that the roots of the equation on a functional graph is found where the curve cuts the x-axis if it equals zero

In this case roots of $x(4 - x) = 0$ is found where the curve cuts the x-axis and the solutions are $x = 0$ or $x = 4$.

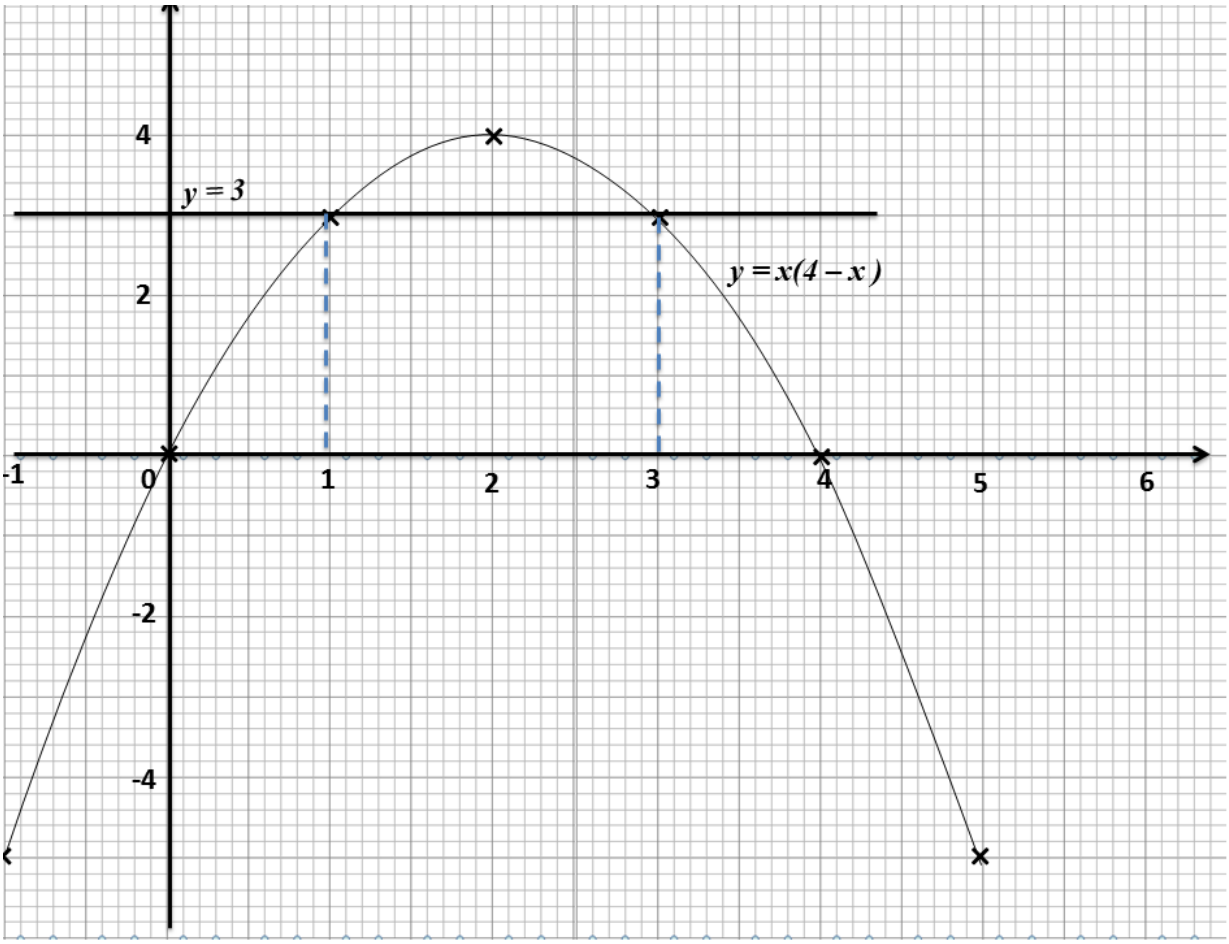
You can be required to use your graph to find the solution for

$x(4 - x) = 3$.

This one is found by drawing the line y = 3 and take the x-values of the points where the line cuts the curve (as shown below)

Therefore the solution of the equation, using my graph is

$x = 1$ or $x = 3$


4
2
-2
-4
-1
0
1
2
3
4
5
6
y = 3
y = x(4 – x)

8. Area under a curve

In this question, the last part was to find the area bounded by the curve

$y = x(4 - x)$ and the line $y = x$

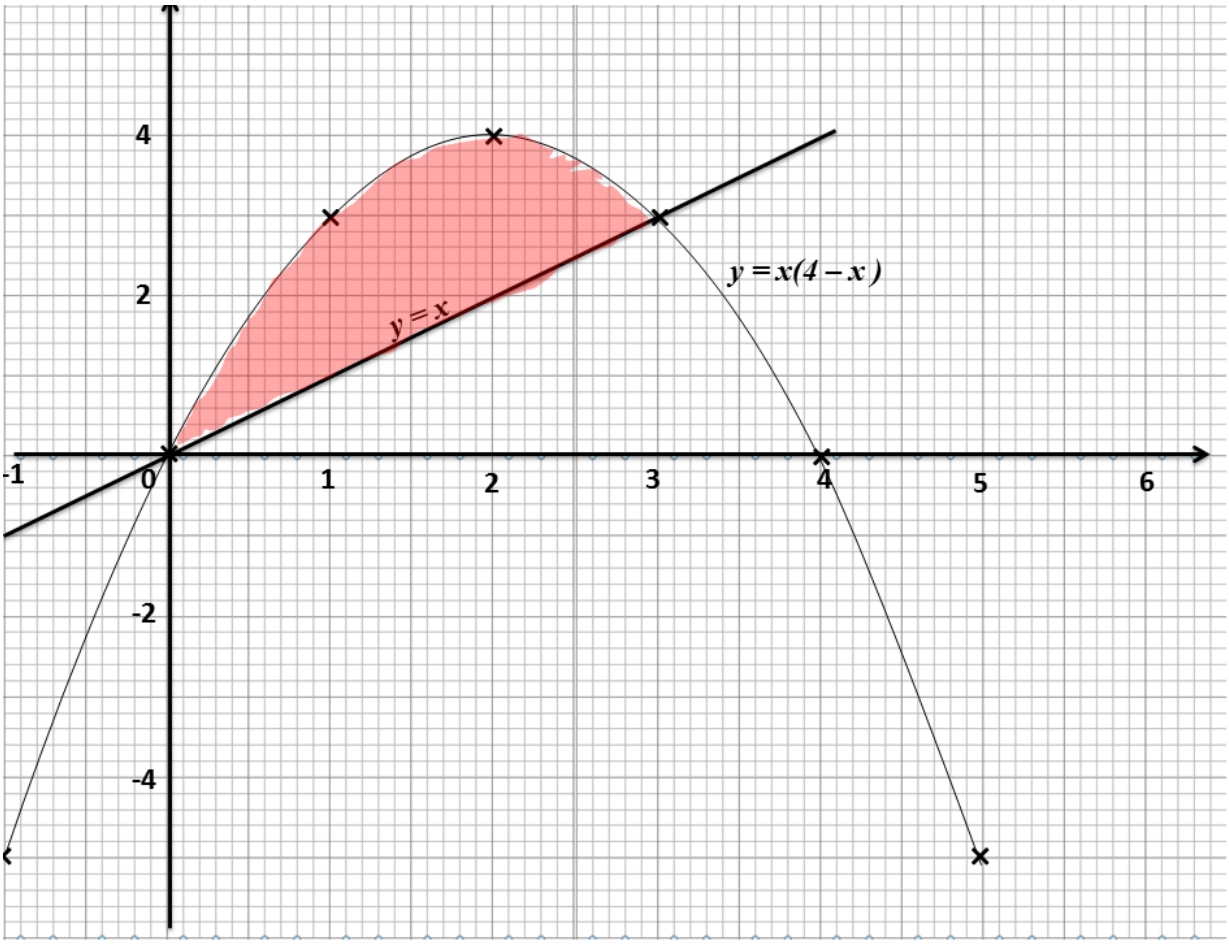
the area must be inside the region specified and you must be able to identify it. Shade the region and then calculate the area of the squares in the region bounded by the line and the curve.

Each large square area = (1×2) cm = 2 cm^2

NB: You take the value of each side on the x-axis and y-axis. Check the axis labels

The next step will be to estimate how many small squares ($2mm^2$) are in the region. In the 2cm square there are 100 small squares.

Try to estimate number of small squares in the shaded region below



My estimate was 200 small squares which is equal to 2 large (2cm) squares.

You then multiply by the value of each squares by the number of available squares.

2 large squares × $2cm^2$ = 4 cm^2 (area)

Also note that the value of each square differs with your scale. Use your scale to calculate the area of each large square.

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9. Velocity Functions

Velocity functions are part of functional graphs and almost the same with quadratic graphs except few differences in terminology used.

Also Note that its different from Speed Time graphs that are usually tested in paper 1.

These are some of the things to note

(a) y – axis : Velocity Axis
(b) x – axis: Time Axis
(c) gradient: is acceleration
(d) Area : distance covered

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10. Interpreting a curve

You can be given a already drawn curve and be required to calculate gradient, find roots, area bounded by the curve, translate or find maximum or minimum points of the curve.

This is what we have been learning throughout this eBook.

Use all the principles to interpret the curve. For revision sake, I will put an exercise on that and expected answers.

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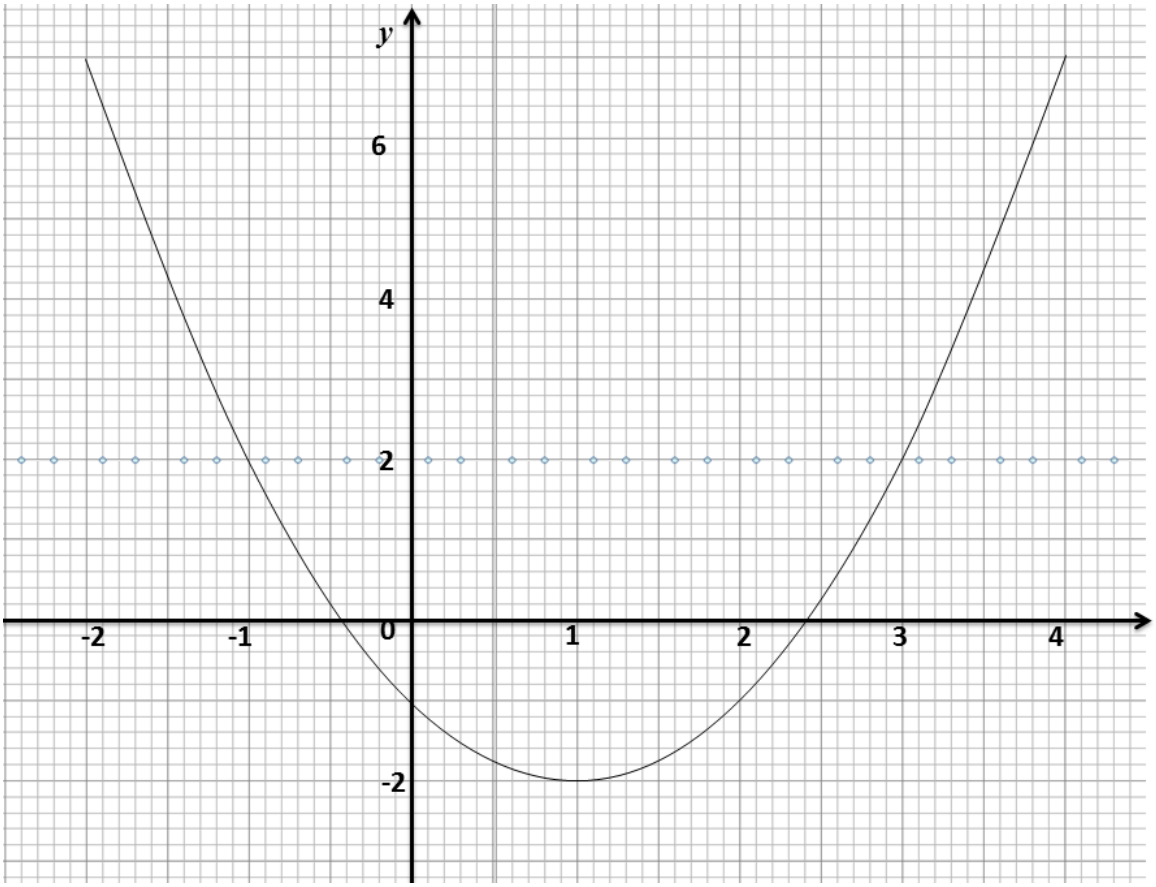
EXERCISE



The diagram shows the graph of the function $y = x^2 - 2x - 1$. Use the graph to find

a) The roots of the equation $x^2 - 2x - 1 = 0$.
b) The minimum value of $x^2 - 2x - 1$.
c) The equation of the line of symmetry
d) The area bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

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