Research Proposal

Quan Shi
shiquan@std.uestc.edu.cn

September 2023

My research interests lie in foundations of cryptography and computational complexity. The first section is ***Fine-Grained Cryptography against Bounded-Circuit-Depth*** and the second section is ***Cryptography from Time-Bounded Kolmogorov Complexity***. This document interprets my preliminary understanding of theoretical cryptography. In future, I will try to apply this lightweight scheme to IoT transaction system, remote medical monitoring system, and smart homes.

Future Research Proposal

1 Overview & Significance

We are interested in how to construct cryptography based on much mild assumptions or which form of security cryptography can be achieved if all classical assumptions do not hold. (Classical systems based on them woud become insecure once quantum computers are developed.) For example, can we have a meaningful notion of cryptography even if we live in Pessiland or Heuristica? [1]

Generally, fine-grained cryptography is the study of cryptographic primitives that are:
◇ 1. Secure against adversaries with bounded resources;
◇ 2. Computable with fewer resources than these adversaries.

Fine-grained is not a strict concept, depending on the different assumptions and primitives. Based on established hardness conjectures about different problems, we hope to construct more primitives, such as average-case hard problems, one-way functions, NIZK, and public-key encryption.

[1] Impagliazzo defined five worlds, which capture the state of cryptography. The three worlds worst for cryptography are Algorithmica (NP in BPP), Heuristica (NP is not in BPP but NP problems are easy on average) and Pessiland (there are NP problems that are hard on average but solved hard instances are hard to sample, and OWFs do not exist).

A prevalent area is related to fine-grained cryptography – characterizing the existence of One-way funtion from the hardness of time-bounded Kolmogorov Complexity. What makes the string 10101010 less random than 57198325? The notion of Kolmogorov complexity measures the amount of "randomness". We could provide a win-win paradigm, where either OWF exists, or we solve all NP problems in practice.

We believe these two tasks could be the way forward towards relations between worst-case & average-case complexities and OWF,PKE, eventually removing the computational assumptions necessary for doing cryptography.

2 Proposed Research

2.1 Aim #1: Fine-Grained Cryptography against Bounded Circuit Depth – Enrich & Recast

In this section, I'll make conclusion and prospect about ACC^0 & AC^0 -fine-grained cryptography. My goal is **enriching its available tools, recasting normal cryptography** under such circuit restriction.

2.1.1 Motivation

[DVV16] proposed fine-grained cryptographic primitives against adversaries captured by two (non-uniform) classes of adversaries, which are AC^0 and NC^1 (logarithmic-depth polynomial-sized) circuits consisting of AND, OR, and NOT gates of fan-in 2. They first constructed an unconditionally secure pseudorandom generator with arbitrary polynomial stretch, a weak pseudorandom function, and a secret-key encryption scheme, all of which are computable in AC^0 and secure against adversaries that are AC^0 circuits. Then, under the widely believed separation assumption $NC^1 \neq \oplus L/poly$, they constructed a OWF, a pseudorandom generator, a collision-resistant hash function, and a semantically secure PKE scheme that are computable in NC^1 and secure against NC^1 circuits. In the end, it left open problems:

Open Problem 1: *Unconditionally lower-bounds are known for slightly larger classes like $AC^0[p]$ when p is a prime power. Can we get cryptographic primitives from those lower-bounds?*

Open Problem 2: *Construct a public key encryption scheme secure against AC^0.*

These questions are interesting. The concept of $AC^0[p]$ is also known as $ACC0$. Compared to AC^0, it extends to a more general family of circuits, and the simplest extension is to allow other logic gates besides the $\vee$ and $\wedge$ gates,

while still guaranteeing that the depth of the circuit is $O(1)$.

2.1.2 Introduction & Related Work

Fine-Grained Cryptography [DVV16] and [BRSV17] were the first to propose the notion of *"Fine-Grained Cryptography"*. But similar motivations can be found in Merkle public-key exchange [Mer78].

Fine-grained cryptography [DVV16] designs cryptographic schemes in a setting where adversaries have only bounded resources and honest users have no more resources than adversaries, so that we may construct more efficient schemes and base their security on weaker, or extremely mild assumptions.

There is a scarcity of research accomplishments in the field of fine-grained cryptography, primarily focusing on primitives now. [DVV16] develops cryptographic protocols secure against adversaries that are at most as powerful as low circuit classes within P such as NC^1 — this is more fine-grained but does not address runtime. More recently, [BRSV17][BRSV18] provide several problems that are provably hard on average, under $SETH$ or the $3-SUM$ or $APSP$ hypotheses. Then they use these problems to construct a Proof of Work scheme.

Circuit-Depth-Bounded Cryptography

In NC^1-fine-grained, [2]as we do not know any lower bounds against NC^1, we are forced to rely on an unproven assumption $NC^1 \neq \oplus L/poly$. Here $\oplus L/poly$ is the class of languages with polynomial-size branching programs, and all languages in NC^1 have polynomial-size branching programs of constant width [Bar86]. In AC^0-fine-grained,[3] circuit lower bounds could transform to meaningful cryptography, the first was one-way permutations against AC^0 adversaries.

Recast Normal Cryptography under circuit classes In the recent years, we are interested in which kind of cryptosystems can be constructed in this setting. We highlight the constructions of OWFs , symmetric-key and (leveled fully homomorphic) public-key encryption [CG18][DVV16], verifiable computation [CG18], hash proof systems (HPS) [EWT21], non-interactive zero-knowledge (NIZK) proof systems [WP22a][WP22b], attribute-based encryption and digital signature [WPC23]. However, due to the restriction on running resources, many important primitives remain unknown.

2.1.3 Proposed Plan

A. What hindrance is troubling us?

[2] NC^i : $\log^i$-depth circuits with bounded fan-in AND/OR/NOT gates.

[3] AC^i : $\log^i$-depth circuits with unbounded fan-in AND/OR/NOT gates.(become ACC if add MOD_p gates)

Setting	Paper	Assumption	Crypto Primitives	Honest	Adversary
Time-Bounded	[Mer78]	Random Oracles	Key Exchange	$O(N)$	$O(N^2)$
	[BGI08]	Exponentially -Strong OWFs	Key Exchange	$O(N)$	$O(N^2)$
	[BRSV18]	WC 3-Sum, OV, APSP, or SETH	Proof of Work	$O(N^2)$	/
	[LLVW19]	Zero-k-Clique or k-Sum	OWFs, Key Exchange, PKE	$O(N)$ $O(N)$	$O(N^{1+t})$ $O(N^{1.5-t})$
Storage-Bounded	[CM97]	/	Key Exchange	$O(S)$	$O(S^2)$
Circuit-Bounded (Parallel -time)	[DVV16]	$NC^1 \neq \oplus L/poly$	OWFs, PRGs, CRHFs, PKE	NC^1	NC^1
	[CG18]		SHE, VC		
	[EWT21]		OWP, HPS, CCA PKE, TDF		
	[BDSK20]		full domain TDF		
	[WPC23]		ABE, Quasi-Adaptive NIZK		
	[WP22a]		NIZK		
	[Has87]	/	OWP	AC^0	AC^0
	[DVV16]		weakPRGs, SKE, CRHFs		
	[WP22b]		NIZK		

Table 1: A table of previous works' result in this area. There have been several results characterizing different aspects of Fine-Grained Cryptography. The primitives and assumptions were previously studied in bounded amount of resources.

We have the following inclusions that are either proven strict ($\subsetneq$) or believed to be strict ($\subseteq$) : $\mathrm{NC}^0 \subsetneq \mathrm{AC}^0 \subsetneq \mathrm{ACC}^0[p] \subsetneq \mathrm{NC}^1 \subseteq \mathrm{L} \subseteq \oplus\mathrm{L}$. We note that the inclusion $\mathrm{ACC}^0[p] \subseteq \mathrm{NC}^1$ is only known to be strict when p is prime.

Some NC^1 primitives can not be directly derived from existing ones by adopting previous generic conversions in the polynomial-time world since the resulting primitive may not be in NC^1 any more. For example, in standard settings the pseudorandom functions (PRF) are well known that can be constructed from OWF/OWPs. But in NC^1, ones in NC^1 are neither implied by NC^1-OWP nor the OWF. This necessities us to construct fine-grained primitives in *another way*.

Furthermore, all systems for unconditional AC^0 are harder to be constructed than $NC^1 \neq \oplus L/poly$. Many cryptographic primitives rely on the algebraic structures of pairing groups, which are not available in fine-grained settings. In NC^1 world, at least we could rely on [AIK06][IK00]. Now let's look at the case of AC^0-cryptography (unconditionally secure). The AC^0-fine-grained-system only involves simple operations in GF(2).

Fact 1 ([FSS81],[Ajt83]) $PARITY(\oplus) \notin \mathrm{AC}^0$

So a main technical hurdle is that in the AC^0-fine-grained setting, many standard operations, such as computing the sum of a polynomial number of random elements and multiplication of two random matrices, are not allowed.

Fact 2([[Raz87],[Smo87]]) *For different primes p and q, the function MOD_p does not belong to $AC^0[q]$.*

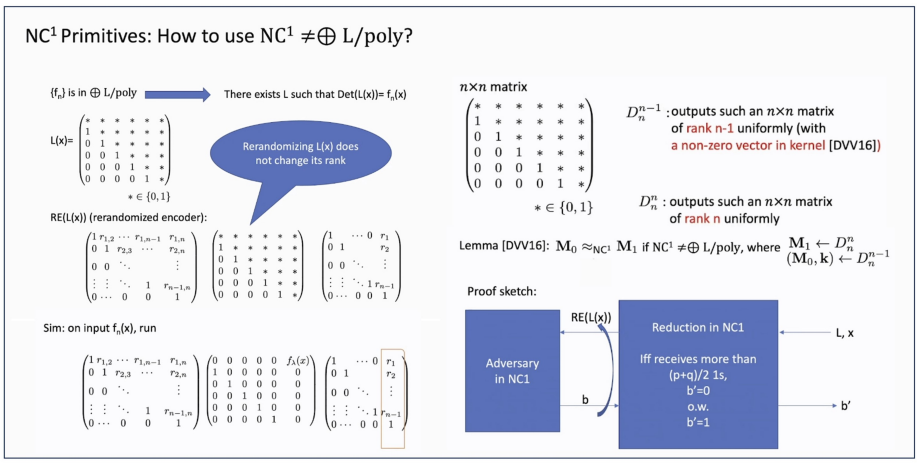
This shows that ACC^0 is different from ${\mathrm{AC}^0}_{CM}[2]$ in [CG18] . Although there are many works about $\mathrm{AC}^0[2]$-grained settings, it cannot transform to *Open Problem 1.*

B. Which techniques can potentially help us solve the problem?

♦ NC^1**-fine-grained** what can we get from the assumption? As shown in the figure, if assumption is $NC^1 \neq \oplus L/poly$, it uses randomized encodings of [IK00][AIK06] to construct the sampling distributions. By using LSamp, RSamp, it outputs ZeroSamp and OneSamp. These two matrices have ranks of n-1 and full, and exhibit indistinguishability in NC^1. [DVV16] uses these techniques to construct OWF, PKE, PRG in NC^1

Based on the same assumption, [CG18] constructs Somewhat homomorphic encryption (SHE) , however it can only compute $\mathrm{AC}^0_Q[2]$ against adversaries in NC^1.

♦ AC^0**-fine-grained** In simple terms, we can look for primitives – are known to construct crypto-systems, which can also be constructed by techniques and primitives already available in AC^0-Minicrypt world.(e.g.OWF, SKE, wPRG,



OR-Proof)

For example, it seems that PKE were necessary for NIZK in the standard model. (NIZK seemed to be in Cryptomania world) Even in the NC^1-fine-grained setting, NIZK systems [WP22a] require the unproven assumption which also implies PKE schemes [DVV16]. [WP22b] found that OR-Proof techniques [GOS12][Ràf15] can also construct a NIZK system for circuit SAT. They can transform NIZK for AC^0-linear-language into OR-proof for the SAT of *1-out-of-2* statements, which could be extended to a fully-fledged one for *1-out-of-poly*. So the solution is to construct a AC^0-linear-language NIZK.

C. Subsequent Work

♦ In NC^1-fine-grained setting obtain a strong NC^1-FHE scheme and FHE-based application

We notice that after [CG18] constructs the somewhat homomorphic encryption (SHE) in the NC^1-fine-grained setting, there are no works which concentrate on the power of homomorphic computation and FHE-based applications. Recently, based on OR-Proof model [WP22a], we've already constructed a strong FHE in $AC^0[2]$ setting, which improves the previous SHE. Now I'm thinking about how to lower the public key size, and make such FHE a NC^1 scheme used XOR and AND gates.

♦ In AC^0-fine-grained setting, obtain unconditionally secure Searchable Encryption, Proxy Re-Encryption (PRE) and Identity-Based Encryption (IBE).

We've already make breakthrough, jumping out of AC^0-Minicrypt with the help of some newest techniques in computational complexity. OR-proof is a

great mediation.

♦ Try to find fixed polylog-*wise* independent distribution that fools AC^0 circuits of arbitrary depth.

This is different from [Bra08] shows that any $n^\epsilon - wise$ independent distribution fools all AC^0 circuits. It may be the most difficult problem, which will imply PKE.

2.2 Aim #2: Cryptography against Bounded Running Time – Kolmogorov Complexity

2.2.1 Motivation

Recently, a series of elegant works by Liu and Pass [LP20] [LP21a] [LP21b] [LP] [LP23]show the connection between OWFs and Kolmogorov Complexity. We call it MK^tP-Cryptography. The reason I'm interested in this topic is that fine-grained Cryptography is related to MK^tP-Cryptography (program is taken to be a time-bounded Turing Machine). Fine-grained works have in common that if we start off with a fine-grained lower bounds, the resulting cryptographic primitive (such as fine-grained OWFs[4]) we get will also only be secure against weak (a-priori bounded polynomial-time) attackers. MK^tP-Cryptography shows how to get different versions of OWFs parametrized by a threshold s. Sometimes, the gap between the time needed to evaluate and invert is super-polynomial (fine-grained cryptography needs fixed polynomial), from very weak fine-grained lower bounds.

We believe that more cryptographic primitives could be constructed based on techniques in MK^tP-Cryptography. As said in [LP23], an unpublished paper [BLMP23] demonstrates how to use Liu's techniques (and in particular how restricting attention to an appropriate analog of computational depth) can be used to get a characterization of *key-exchange agreement* using the worst-case hardness of a Kolmogorov complexity-style problem. I can't read it yet, anyway. But It encourages me to find the relationship between fine-grained world and Kolmogorov world, consider the similar equicalances hold in the fine-grained world, use the different versions of OWFs to construct other primitives.

[4]Fine-Grained One Way Function [BRSV17][LLVW19]: We say a function $f : \{0,1\}^* \to \{0,1\}^*$ is (t, ϵ)-one-way if it can be computed in $O\left(t(n)^{1-\delta}\right)$ time for some $\delta > 0$, but for any $\delta' > 0$, any $O\left(t(n)^{1-\delta'}\right)$-time algorithm A, and all sufficiently large n,

$$\Pr_{x \leftarrow \{0,1\}^n}\left[A(f(x)) \in f^{-1}(f(x))\right] \leq \epsilon\left(n, \delta'\right)$$

2.2.2 Introduction & Related Work

Time-Bounded Fine-Grained Cryptography Fine-grained complexity is built upon "fine-grained" hypotheses on the (worst-case) hardness of a small number of key problems. Fine-grained uses fine-grained reductions between problems in a very tight way[5], and obtains strong lower bounds for many problems. A few works developing cryptographic schemes assuming fine-grained hardness of some computational problems. (i.e.assuming hardness $n^{1+\alpha} - time$ attackers for som *fixed* constant $\alpha > 0$). [BRSV17][GR18][BABB21][DLW20] shows worst-case to average-case reductions for certain natural classes of problems in the fine-grained regimes. (OV,3SUM,APSP). [BRSV18] shows the existence of "proof of work" assuming fine-grained worst-case hardness of these problems. [LLVW19][DLW20] constructs a fine-grained analog of OWF, assuming fine-grained average-case harness of many languages.

Different Assumptions Some efficient cryptosystems like DSA, Diffie-Hellman have weaknesses – for instance, they are completely broken in a postquantum world as Shor's algorithm breaks their assumptions in essentially quadratic time [Sho94]. Thus, it makes sense to look at the cryptosystems based on other assumptions. Some of the main hypotheses in fine-grained complexity (see [Wil15]) set K to be CNF-SAT (with $T(n) = 2^n$, where n is the number of variables), or the k-Sum problem (with $T(n) = n^{\lceil k/2 \rceil}$), or the All-Pairs Shortest Paths problem (with $T(n) = n^3$ where n is the number of vertices), or one of several versions of the k-Clique problem in weighted graphs. [BRSV17] used their fine-grained-complexity problems to build cryptographic primitives, but gave a barrier for their approach: extending their approach would falsify the NSETH, which encourages us to find different hardness assumption.

Win-win Paradigm and OWFs from Average-case Harness It is commonly known that if a problem is average-case hard and it is possible to efficiently sample an input from the hard distribution along with the corresponding solution, then this implies a one-way function[6] (OWF). OWF is usually the simplest cryptographic primitives. Ideally, we would want an assumption that leads to a win-win scenario:
$\diamond$ If these problems are hard then we have secure OWFs (It can securely implement many other primitives in Minicrypt);
$\diamond$ If not, get some breakthroughs - such as we rule out "Pessiland", get efficient algorithms for optimal file compression, inductive reasoning, optimal Machine Learning.

On OWFs and Kolmogorov Complexity The notion of Kolmogorov

[5] [VW15] Fine-grained reduction: if problem A has requires running time $a(n)^{1-o(1)}$, and one obtains an $(a(n), b(n))$-fine-grained reduction from A to B, then problem B needs runtime $b(n)^{1-o(1)}$.

[6] A one-way function (OWF) is a function f that can be efficiently computed in polynomial time, yet no probabilistic polynomial time (PPT) algorithm can invert f with inverse polynominal probability for infinitely many input lengths n.

complexity (K-complexity), introduced by [Sol64][Kol65][Cha69], provides an elegant method for measuring the amount of "randomness" in individual strings[7]. The notion of $t(\cdot)$-time-bounded Kolmogorov Complexity (K^t-complexity). [Kol65] [Ko86][Sip83][Har83][All01][ABK+06] considers a time-bounded version of this problem (MKTP). Given a string x describing a truthtable, let $K^t(x)$ denote the t-bounded Kolmogorov complexity of x-that is, the length of the shortest string Π such that for every $i \in [n], U(\Pi, i) = x_i$ within time $t(|\Pi|)$, where U is a fixed Universal Turing machine. Given a threshold, $s(\cdot)$, and a polynomial time-bound, $t(\cdot)$, let $\text{MK}^t\text{P}[s]$ denote the set of strings x such that $K^t(x) \leq s(|x|)$.

2.2.3 Proposed Plan

A. Which techniques can potentially help us solve the problem?

There are many ways to define time-bounded Kolmogorov complexity. We have introduced the "local compression" version in 6.2.2. All the versions are showed in the table.

MKTP K-complexity	K(x)=min|M|s.t.U(M)=x
MKtP Kt-complexity	K(x)=min|M|s.t. s.t.U(M)=x within time
MK^tP Kt-complexity	K(x)=min|M|+logt s.t.U(M)=x within time t
MKTP KT-complexity	K(x)=min|M|+T s.t.U(M)=x within time t
conditional Fixed threshld	cK,McKP MKtP[n/2]

[LP20] recently showed that when the threshold s(·) is *"large"*, when s(n)=n-clogn for some constant c, then mild average-case hardness of this language w.r.t. the uniform distribution of instances, is equivalent to the existence of OWF. [LP21a] considered a notion of mild average-case hardness, and characterized quasi-polynomial or subexponential OWF. [LP23] considered the worst-case hardness of $\text{MK}^t\text{P}[s]$ and construct OWF-complete.

Existence of OWF	[LP20]	K^t w.r.t uniform distribution MK^tP[n-O(logn)] w.r.t uniform distribution
	[LP22]	cK^t w.r.t uniform distribution cK^t is NP-complete

Continued on next page

[7]The K-complexity of a string is the length of the shortest program (to be run on some fixed universal Turing machine U) that outputs the string x

(Continued)

	[LP21b],[RS21]	Kt w.r.t uniform distribution Kt is EXP-complete
	[IRS22],[LP]	MpKpolyP is (mildly) HOA w.r.t any distribution D MpKpolyP is (mildly) HOA w.r.t uniform distribution
	[LP23]	worst-case hardness w.r.t BPP
OWF in NC^0	[RS21]	MKTP w.r.t uniform
	[ACM+]	McKTP w.r.t uniform McKTP is NP-complete
	[LP21b]	using space-bounded variant
SubExp OWF	[LP21a]	MK^tP polylogn w.r.t uniform Threshold s determines OWF hardness

B. Subsequent Work

♦ Can we get exponentially-hard OWFs from $MK^tP[s]$ problems?

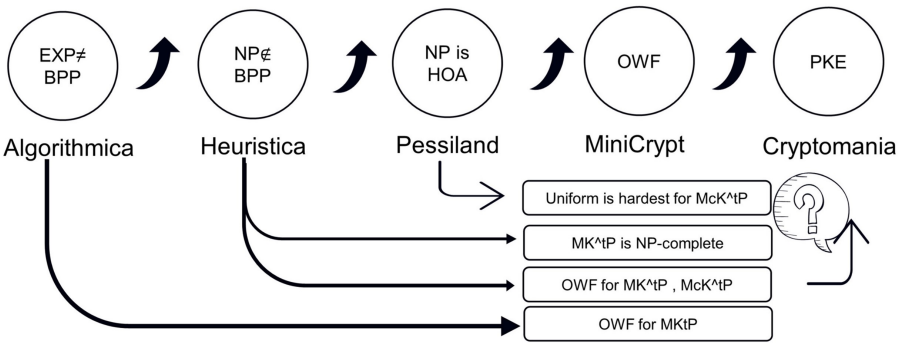
[LP21a] [LP23] all relies on a "nice" function classes, where the class of functions $\mathcal{F}$ is said to be "nice" if: 1. for every function $T \in \mathcal{F}, T$ is time-constructible and strictly increasing in the sense that there exists a constant $\nu > 0$ such that for all $n > 1, T(n+1) \geq T(n) + \nu$; 2. $\mathcal{F}$ is closed under (sublinear) polynomial compositions: for any $T \in \mathcal{F}$, for all $0 < \varepsilon_1, \varepsilon_2 < 1$, $\left(T\left(n^{\varepsilon_1}\right)\right)^{\varepsilon_2} \in \mathcal{F}$.

$\mathcal{F}_{\text{subexp}} = \left\{2^{cn^{\varepsilon}}\right\}_{c>0,0<\varepsilon<1}$ and $\mathcal{F}_{\text{qpoly}} = \left\{n^{c\log n}\right\}_{c>0}$ are considered as "nice" classes. They rely on this notion so as to capture classes of "polynomially-related" functions.[8]

However, characterizing the exponentially secure OWF is out of "nice" classes, and all theorems in their works would lose efficacy. We need to find another notion to show the relationship between $MK^tP[s]$.

♦ Can we construct PKE and NIZK from worst-case hardness of $MK^tP[s]$ problems?

[8] Almost all the reductions (considered in their works) are of form "if A is $T(n)$-hard, B is $\left(T\left(n^{\Omega(1)}\right)^{\Omega(1)}/n^{O(1)} - n^{O(1)}\right)$-hard".



We believe that there are various primitives in Cryptomania based on the hardness of Kolmogorov complexity problems. The key is to restrict attention to an appropriate analog of computational depth. Our goal is to find its application in Cryptomania, such us PKE and NIZK, which is similar to our discussion in fine-grained setting.

♦ Can we get a characterization w.r.t a single (non-meta) problem?

Briefly recall the high-level approach fistly used in [LP20]: An object called an entropy-preserving pseudorandom generator (EP-PRG) was introduced. Roughly speaking, an EP-PRG is a pseudorandom generator that expands n-bits to $n + O(\log n)$ bits, having the property that the output of the PRG is not only pseudorandom, but also preserves the entropy of the input (i.e., the seed): The Shannon-entropy of the output is $n - O(\log n)$. [LP20] [LP21a] constructed a relaxed form of an EP-PRG, called a conditionally secure entropy-preserving PRG (cond-EP PRG), and showed how such a cond EP-PRG can be constructed from OWFs, and next showed that the existence of cond EP-PRGs implies that $\mathrm{MK}^t\mathrm{P}[n - O(\log n)]$ is mildly HoA.

The key obstacle is that their works relies on the security of a primitive *cond EP-PRF* for which it is hard to check if an attacker manages to break its security. [LP23] deals with this obstacle without non-uniform advice, but still relies on cond EP-PRF.(not only $\mathrm{MK}^t\mathrm{P}[s]$)

Meta-complexing problem means that we need a family of problems to characterize a primitive. Our goal is to get a characterization w.r.t a single (non-meta) problem, and design a totally different proof system.

3 Broader Impact

Crypto vs Complexity Achievements in cryptography theory often hinge upon captivating contributions in the realm of complexity, such as the groundbreaking work by the work [AIK06] that led to the development of circuit-bound-setting cryptography. Attaining success in this field requires a combination of luck and diligent effort. To further enhance my understanding, I aim to delve

deeper into the vast array of works related to complexity theory (FOCS, STOC, SODA, CCC) as well as Cryptography (EuroCrypt, Crypto, AsiaCrypt).

Conjunction Use The fine-grained and Kolmogorov primitives can be used in conjunction with other primitives that are secure against many versions of adversaries under different assumptions; this would result in hybrids that are secure against polynomial-time adversaries under these stronger assumptions while also being secure against bounded adversaries under weaker assumptions. Our hope is that we can trade a small loss of efficiency for a big improvement in security.

Ephemeral security Primitives with limited security guarantees is best suited for applications that do not need additional security. One application scenario is cellular mobile communication, where the system is considered secure as long as an adversary cannot crack the encryption within a limited time and communication overhead. Another scenario is the stock market, where various pieces of information exist only for a few seconds. Any adversary who spends too much time disrupting these primitives will only obtain information that has already lost its value. In such situation, security is transient and only needs to be maintained for a short period of time.

Potential Risks

While Kolmogorov complexity and fine-grained complexity provide a method to measure the complexity of information, they are not complete. In future research, it is necessary to thoroughly evaluate their applicability and limitations. Currently, the primitives obtained in this field are highly secure but have complex constructions and low practicality. We need to further optimize performance while ensuring security.

References

[ABK+06] Eric Allender, Harry Buhrman, Michal Kouckỳ, Dieter Van Melkebeek, and Detlef Ronneburger. Power from random strings. *SIAM Journal on Computing*, 35(6):1467–1493, 2006.

[ACM+] Eric Allender, Mahdi Cheraghchi, Dimitrios Myrisiotis, Harsha Tirumala, and Ilya Volkovoch. One-way functions and a conditional variant of mktp. In *41st IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2021)*.

[AIK06] Benny Applebaum, Yuval Ishai, and Eyal Kushilevitz. Cryptography in nc^0. *SIAM Journal on Computing*, 36(4):845–888, 2006.

[Ajt83] Miklós Ajtai. $\sum_1^1$-formulae on finite structures. *Annals of pure and applied logic*, 24(1):1–48, 1983.

[All01] Eric Allender. When worlds collide: Derandomization, lower bounds, and kolmogorov complexity. In *FST TCS 2001: Foundations of Software Technology and Theoretical Computer Science: 21st Conference Bangalore, India, December 13–15, 2001 Proceedings 21*, pages 1–15. Springer, 2001.

[BABB21] Enric Boix-Adserà, Matthew Brennan, and Guy Bresler. The average-case complexity of counting cliques in erdos–renyi hypergraphs. *SIAM Journal on Computing*, (0):FOCS19–39, 2021.

[Bar86] David A Barrington. Bounded-width polynomial-size branching programs recognize exactly those languages in nc. In *Proceedings of the eighteenth annual ACM symposium on Theory of computing*, pages 1–5, 1986.

[BDSK20] Marshall Ball, Dana Dachman-Soled, and Mukul Kulkarni. New techniques for zero-knowledge: leveraging inefficient provers to reduce assumptions, interaction, and trust. In *Annual International Cryptology Conference*, pages 674–703. Springer, 2020.

[BGI08] Eli Biham, Yaron J Goren, and Yuval Ishai. Basing weak public-key cryptography on strong one-way functions. In *Theory of Cryptography: Fifth Theory of Cryptography Conference, TCC 2008, New York, USA, March 19-21, 2008. Proceedings 5*, pages 55–72. Springer, 2008.

[Bra08] Mark Braverman. Polylogarithmic independence fools ac 0 circuits. *Journal of the ACM (JACM)*, 57(5):1–10, 2008.

[BRSV17] Marshall Ball, Alon Rosen, Manuel Sabin, and Prashant Nalini Vasudevan. Average-case fine-grained hardness. In *Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing*, pages 483–496, 2017.

[BRSV18] Marshall Ball, Alon Rosen, Manuel Sabin, and Prashant Nalini Vasudevan. Proofs of work from worst-case assumptions. In *Advances in Cryptology–CRYPTO 2018: 38th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 19–23, 2018, Proceedings, Part I 38*, pages 789–819. Springer, 2018.

[CG18] Matteo Campanelli and Rosario Gennaro. Fine-grained secure computation. In *Theory of Cryptography Conference*, pages 66–97. Springer, 2018.

[Cha69] Gregory J Chaitin. On the simplicity and speed of programs for computing infinite sets of natural numbers. *Journal of the ACM (JACM)*, 16(3):407–422, 1969.

[CM97] Christian Cachin and Ueli Maurer. Unconditional security against memory-bounded adversaries. In *Annual International Cryptology Conference*, pages 292–306. Springer, 1997.

[DLW20] Mina Dalirrooyfard, Andrea Lincoln, and Virginia Vassilevska Williams. New techniques for proving fine-grained average-case hardness. In *2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, pages 774–785. IEEE, 2020.

[DVV16] Akshay Degwekar, Vinod Vaikuntanathan, and Prashant Nalini Vasudevan. Fine-grained cryptography. In *Annual International Cryptology Conference*, pages 533–562. Springer, 2016.

[EWT21] Shohei Egashira, Yuyu Wang, and Keisuke Tanaka. Fine-grained cryptography revisited. *Journal of Cryptology*, 34(3):23, 2021.

[FSS81] Merrick Furst, James B Saxe, and Michael Sipser. Parity, circuits, and the polynomial-time hierarchy. In *Proceedings of the 22nd Annual Symposium on Foundations of Computer Science*, pages 260–270, 1981.

[GOS12] Jens Groth, Rafail Ostrovsky, and Amit Sahai. New techniques for noninteractive zero-knowledge. *Journal of the ACM (JACM)*, 59(3):1–35, 2012.

[GR18] Oded Goldreich and Guy Rothblum. Counting t-cliques: Worst-case to average-case reductions and direct interactive proof systems. In *2018 IEEE 59th Annual Symposium on Foundations of Computer Science (FOCS)*, pages 77–88. IEEE, 2018.

[Har83] Juris Hartmanis. Generalized kolmogorov complexity and the structure of feasible computations. In *24th Annual Symposium on Foundations of Computer Science (sfcs 1983)*, pages 439–445. IEEE, 1983.

[Has87] Johan Hastad. One-way permutations in nc0. *Information Processing Letters*, 26(3):153–155, 1987.

[IK00] Yuval Ishai and Eyal Kushilevitz. Randomizing polynomials: A new representation with applications to round-efficient secure computation. In *Proceedings 41st Annual Symposium on Foundations of Computer Science*, pages 294–304. IEEE, 2000.

[IRS22] Rahul Ilango, Hanlin Ren, and Rahul Santhanam. Robustness of average-case meta-complexity via pseudorandomness. In *Proceedings of the 54th Annual ACM SIGACT Symposium on Theory of Computing*, pages 1575–1583, 2022.

[Ko86] Ker-I Ko. On the notion of infinite pseudorandom sequences. *Theoretical Computer Science*, 48:9–33, 1986.

[Kol65] Andrei N Kolmogorov. Three approaches to the quantitative definition ofinformation'. *Problems of information transmission*, 1(1):1–7, 1965.

[LLVW19] Rio LaVigne, Andrea Lincoln, and Virginia Vassilevska Williams. Public-key cryptography in the fine-grained setting. In *Annual International Cryptology Conference*, pages 605–635. Springer, 2019.

[LP] Yanyi Liu and Rafael Pass. One-way functions and the hardness of (probabilistic) time-bounded kolmogorov complexity w.r.t. samplable distributions. In *Advances in Cryptology – CRYPTO 2023*, pages 645–673.

[LP20] Yanyi Liu and Rafael Pass. On one-way functions and kolmogorov complexity. In *2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, pages 1243–1254. IEEE, 2020.

[LP21a] Yanyi Liu and Rafael Pass. Cryptography from sublinear-time average-case hardness of time-bounded kolmogorov complexity. In *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*, pages 722–735, 2021.

[LP21b] Yanyi Liu and Rafael Pass. On the possibility of basing cryptography on exp $\neq$ bpp. In *Advances in Cryptology–CRYPTO 2021: 41st Annual International Cryptology Conference, Virtual Event, August 16–20, 2021, Proceedings, Part I 41*, pages 11–40. Springer, 2021.

[LP22] Yanyi Liu and Rafael Pass. On one-way functions from np-complete problems. *Cryptology ePrint Archive*, 2022.

[LP23] Yanyi Liu and Rafael Pass. On one-way functions and the worst-case hardness of time-bounded kolmogorov complexity. *Cryptology ePrint Archive*, 2023.

[Mer78] Ralph C Merkle. Secure communications over insecure channels. *Communications of the ACM*, 21(4):294–299, 1978.

[Ràf15] Carla Ràfols. Stretching groth-sahai: Nizk proofs of partial satisfiability. In *Theory of Cryptography: 12th Theory of Cryptography Conference, TCC 2015, Warsaw, Poland, March 23-25, 2015, Proceedings, Part II 12*, pages 247–276. Springer, 2015.

[Raz87] Alexander A Razborov. Lower bounds on the size of bounded depth circuits over a complete basis with logical addition. *Mathematical Notes of the Academy of Sciences of the USSR*, 41(4):333–338, 1987.

[RS21] Hanlin Ren and Rahul Santhanam. Hardness of kt characterizes parallel cryptography. *Cryptology ePrint Archive*, 2021.

[Sho94] Peter W Shor. Algorithms for quantum computation: discrete logarithms and factoring. In *Proceedings 35th annual symposium on foundations of computer science*, pages 124–134. Ieee, 1994.

[Sip83] Michael Sipser. A complexity theoretic approach to randomness. In *Proceedings of the fifteenth annual ACM symposium on Theory of computing*, pages 330–335, 1983.

[Smo87] Roman Smolensky. Algebraic methods in the theory of lower bounds for boolean circuit complexity. In *Proceedings of the nineteenth annual ACM symposium on Theory of computing*, pages 77–82, 1987.

[Sol64] RJ Solmonoff. A formal theory of inductive inference. i. *II Information and Control*, 7:224–254, 1964.

[VW15] Virginia Vassilevska Williams. Hardness of easy problems: Basing hardness on popular conjectures such as the strong exponential time hypothesis (invited talk). In *10th International Symposium on Parameterized and Exact Computation (IPEC 2015)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2015.

[WP22a] Yuyu Wang and Jiaxin Pan. Non-interactive zero-knowledge proofs with fine-grained security. In *Annual International Conference on the Theory and Applications of Cryptographic Techniques*, pages 305–335. Springer, 2022.

[WP22b] Yuyu Wang and Jiaxin Pan. Unconditionally secure nizk in the fine-grained setting. In *International Conference on the Theory and Application of Cryptology and Information Security*, pages 437–465. Springer, 2022.

[WPC23] Yuyu Wang, Jiaxin Pan, and Yu Chen. Fine-grained secure attribute-based encryption. *Journal of Cryptology*, 36(4):33, 2023.