Pollen Prediction

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Introduction

Goal: Model the distribution of allergenic pollen concentrations using historical data

Figure 1: Alder Tree (genus *Alnus*)

Alnus Pollen Data in Bechej, Serbia

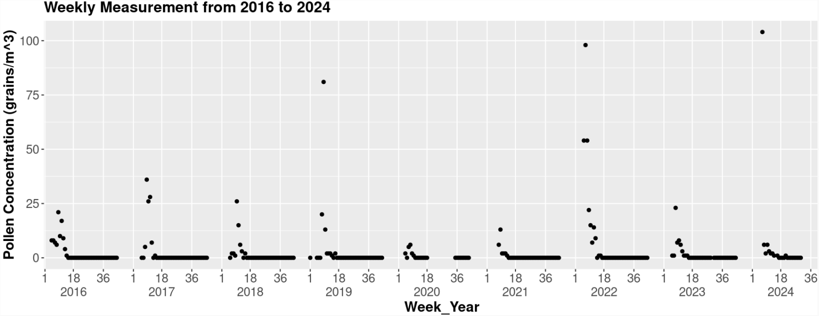


Figure 2: Weekly pollen concentration after data pre-processing

Models

$i \in \{1, \ldots, 53\}$ is the week, $j \in \{2016, \ldots, 2024\}$ is the year.

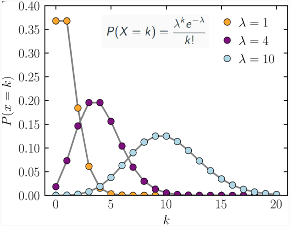
Both models group by year j

Model 1: $$y_{ij} \sim Poisson\left(\alpha_0 + \alpha_j\right)$$

Model 2: $$y_{ij} \sim Poisson(\alpha_0 + \alpha_j + \beta_j \cdot week_{ij})$$

$$\alpha_0 \sim Prior\left(\alpha_0\right) \qquad \tau_\alpha \sim Prior\left(\tau_\alpha\right) \qquad \tau_\beta \sim Prior\left(\tau_\beta\right)$$

$$\alpha_j \sim normal\left(0, \tau_\alpha\right) \qquad \beta_j \sim normal\left(0, \tau_\beta\right)$$



Model 1: $y_{ij} \sim Poisson\,(\alpha_0 + \alpha_j)$

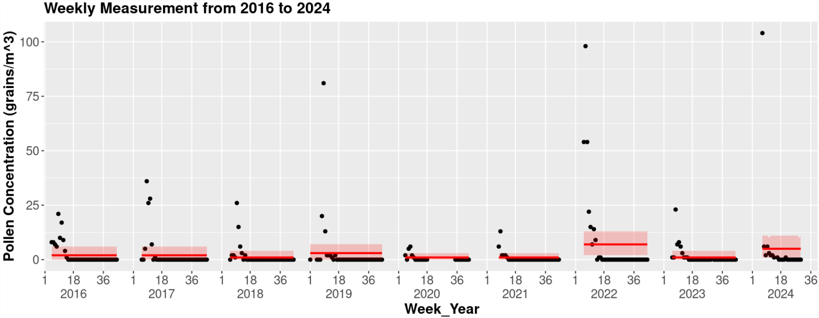


Figure 3: Best fit. Bayesian $R^2 = 0.03$. Informative prior on α_0, τ_α is normal$(0, 10)$

Model 2: $y_{ij} \sim Poisson(\alpha_0 + \alpha_j + \beta_j \cdot week_{ij})$

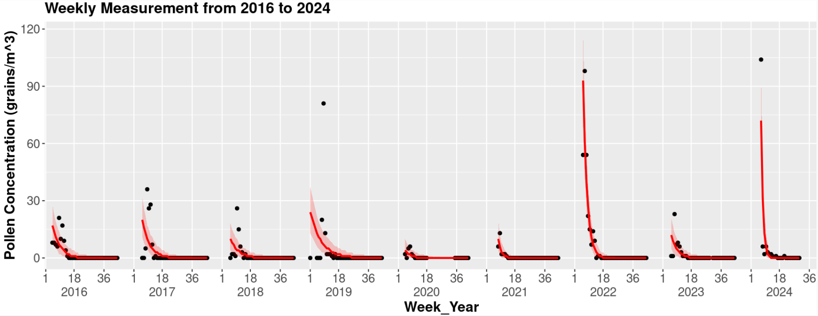


Figure 4: Best fit. Bayesian $R^2 = 0.6$. Informative prior on $\alpha_0, \tau_\alpha, \tau_\beta$ is normal$(0, 10)$

Convergence diagnostic

Predictive performance for model 2

$y_{ij} \sim Poisson(\alpha_0 + \alpha_j + \beta_j \cdot week_{ij})$. Adapt step is $\delta = 0.92$

Prior on $\alpha_0, \tau_\alpha, \tau_\beta$	$\hat{R}$	ESS_{bulk} (max 4000)	Divergences	Bayesian R^2
student's t(3,0,2.5)	1	1200	4	0.597
beta(1,10)	1.59	7	1230	/
gamma(10,1)	1.03	236	646	0.601
gamma(5,1)	1	784	43	0.599
gamma(1,1)	1	801	581	0.596
normal(0,4)	1	1123	3	0.597
normal(0,10)	1	889	3	0.598

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Power-scaling sensitivity for Intercept α_0 for model 2

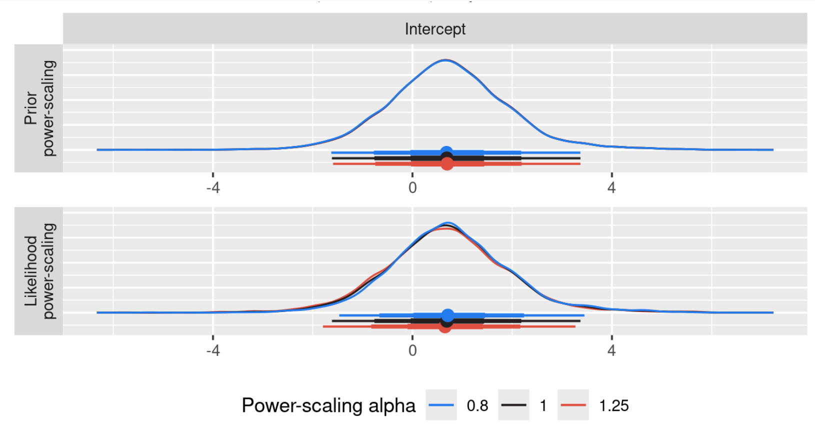
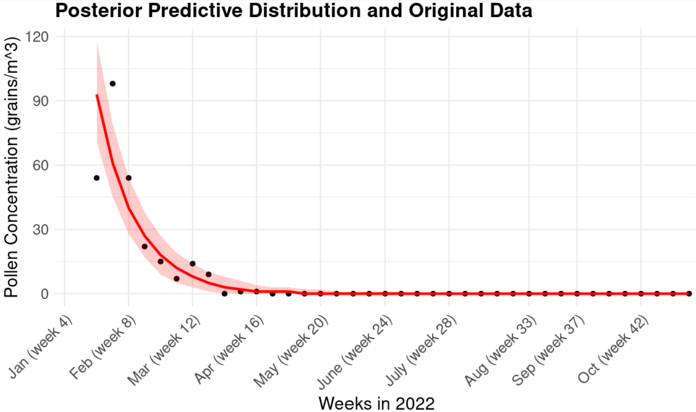


Figure 5: Prior scaling $p(\theta|y) \propto p(\theta)^{\alpha} p(y|\theta)$ and likel. scaling $p(\theta|y) \propto p(\theta) p(y|\theta)^{\alpha}$

Conclusion and Discussion



1. $y_{ij} \sim Poisson(\alpha_0 + \alpha_j + \beta_j \cdot week_{ij})$
2. Potential improvement: Zero-inflated models

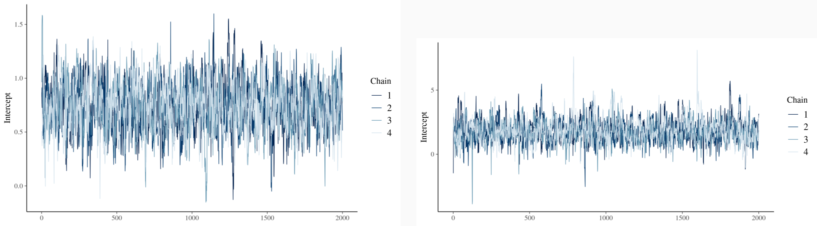
Appendix: Code and MCMC options

Listing 1: Main code

```
formula <- bf(y ~  1 + (1 + week | year), family = "poisson")

fit <- brm(formula,
           data = data,
           prior = priors,
           cores = 4, # default is 1
           chains = 4,
           iter = 4000,                        # default is 2000
           warmup = floor(iter/2),
           control = list(adapt_delta = 0.92)  # default is 0.8
)
```

Appendix: Convergence diagnostic



(a) Model 1 **(b)** Model 2

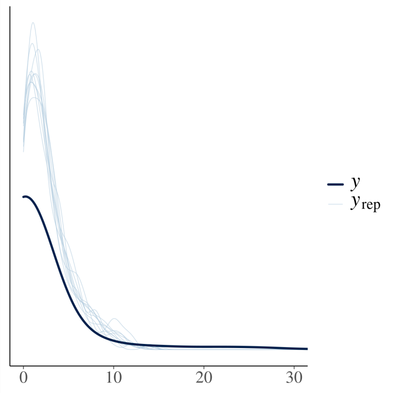
Figure 6: MCMC Trace for the intercept term α_0. The chains are mixing. Both models have priors of normal$(0, 10)$.

Appendix: Convergence diagnostic

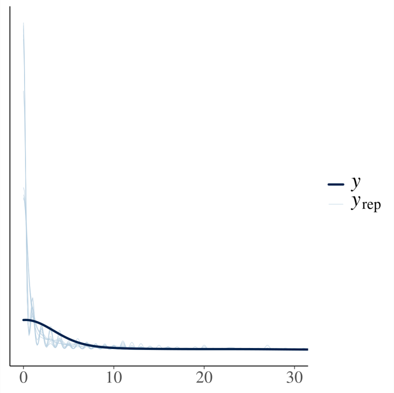
Table 1: Performance for model $y_{ij} \sim Poisson(\alpha_0 + \alpha_j)$.

Prior on α_0, τ_α	δ	$\hat{R}$	ESS_{bulk}	ESS_{tail}	Divergence	Bayesian R^2
gamma(5,1)	0.8	1	617	821	No	0.03
gamma(10,1)	0.92	1	737	1029	Yes(1)	0.03
normal(0,10)	0.8	1	1257	2164	No	0.03

Appendix: Posterior predictive checks



(a) pp_check of model 1,
$y \sim 1 + (1|year)$. Bayes $R^2 = 0.03$



(b) pp_check of model 2,
$y \sim 1 + (1 + week|year)$. Bayes $R^2 = 0.6$

Figure 7: Posterior predictive Checks