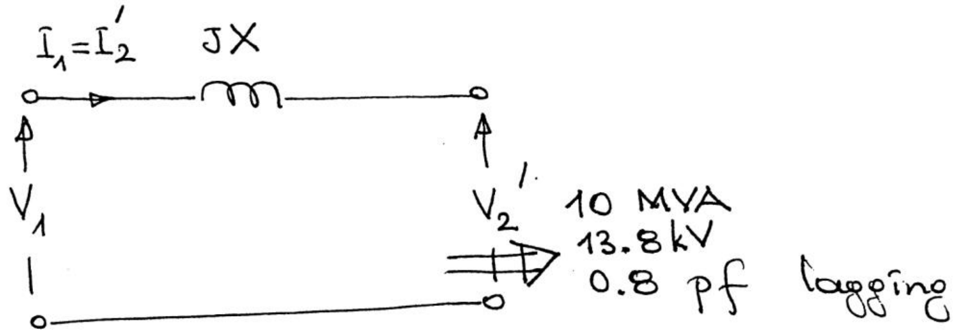
Ex  A 10 MVA 13.8/79.7 kV step-up transformer has a total leakage reactance of $X = 1.9\ \Omega$ referred to primary. Neglecting all transformer losses and magnetizing current, find the primary voltage when transformer is supplying the rated MVA at rated voltage and 0.8 pf lagging.



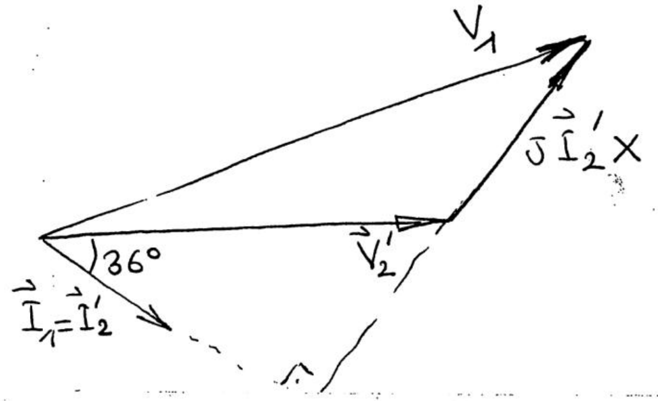

$P_{out} = V_2' I_2' \cos\theta$

$S_2 = V_2 I_2 = V_2' I_2' = 10$ MVA (Rated power of the transformer given on the nameplate is an output quantity)

$P_{out} = 10 \times 0.8 = 8$ MW

$I_2' = 10\times10^6 / 13.8\times10^3 = 725$ A

$I_2 = 725 \times (13.8/79.7) = 125$ A ($I_2 < I_2'$ because it is a step-up transformer)




phasor diagram

$$\vec{V}_1 = \vec{V}_2' + j\vec{I}_2' X$$

$$\vec{V}_1 = \underbrace{(13.8 + j0.0)}_{\text{reference phasor}} \times 10^3 + j\,1.9 \times \underbrace{725}_{|I_2'|} \underbrace{(0.8 - j0.6)}_{\text{0.8 pf lagging }(\cos 36^\circ - j\sin 36^\circ)}$$

$$\vec{V}_1 = 13800 \quad + j\,1102 + 826.5$$

$$\vec{V}_1 = 14626.5 + j\,1102$$

$$\vec{V}_1 = 14668 \angle 4.3^\circ$$

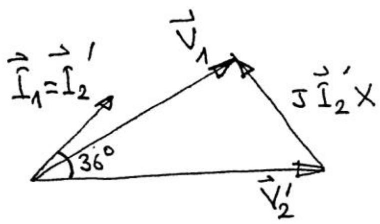
$$|\vec{V}_1| = 14.7 \text{ kV}$$

$$Reg = \frac{V_1 - V_2'}{V_2'} \times 100$$

$$= \frac{14.7 - 13.8}{13.8} \times 100$$

$$= +6.5 \%$$

Now solve the same problem for the leading load




$$\vec{V}_1 = 13.8 \times 10^3 + j1.9 \times 725 \underbrace{(0.8 + j0.6)}_{\text{leading pf}}$$

$$|\vec{V}_1| = \underbrace{13}_{\text{less than 13.8 kV because of the capacitive load}} \text{ kV}$$

$$Reg = \frac{13 - 13.8}{13.8} \times 100 = \underbrace{-}_{\text{because of the capacitive load}} 5.7 \%$$

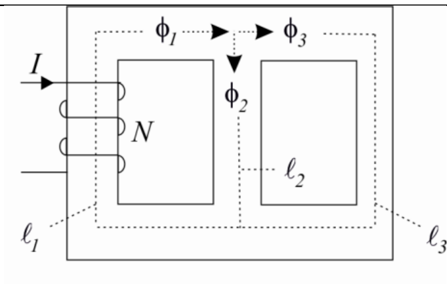
# EE 361 HOMEWORK 1

**DUE: 4 Nov. 2014**

**1.** For the magnetic circuit indicated in Fig. (1), find the excitation current $I$ to produce a flux density of 0.5 Wb/m$^2$ in the third leg. Find also the fluxes in other legs. A part of the $B$-$H$ curve is given below.

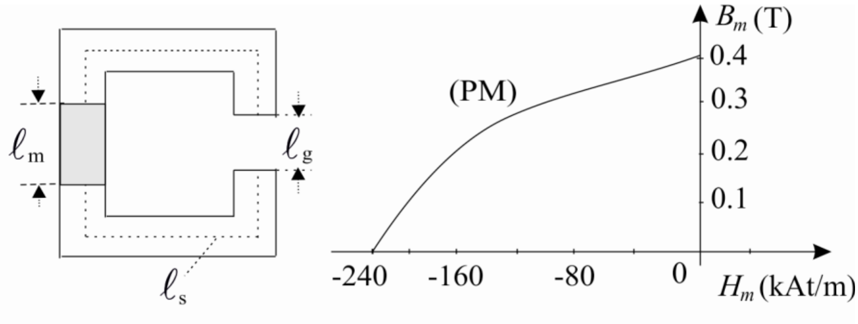
| $B$ (T) | 0.5 | 0.9 | 1 | 1.4 |
|---|---|---|---|---|
| $H$ (At/m) | 50 | 150 | 200 | 750 |

$A = 9.10^{-4}$ m$^2$, $\ell_1 = \ell_3 = 0.3$ m, $\ell_2 = 0.1$ m



(1)

**2.** The magnetic circuit shown in Fig. (2a) is excited by a permanent magnet whose characteristic (PM) is shown in Fig. (2b). The operating point on the magnet characteristic providing maximum energy product operation is estimated as P ($B_m \approx 0.23$T, $H_m \approx -144\times10^3$). Find the length of the magnet ($\ell_m$) to produce maximum magnet energy product density. Fringing is neglected. $A_g = A_m = A_s = 10\,\text{cm}^2$, $\ell_s = 100$ cm, $\ell_g = 1$ cm, $\mu_r = 500$ (steel).




**3.** Consider the magnetic circuit and its approximated $B$-$H$ characteristic shown in Figs. (3a,b; $\overline{OM}$ : linear part, $\overline{MN}$ : saturated part). The winding is excited by a square wave voltage source $v_s(t)$ as indicated in Fig. (3c). It is also given that 10% of the flux linking the coil is the leakage flux ($\phi_\ell$). Neglect the winding resistance.

**a)** Find the maximum value of the source voltage ($V_m$) such that the core does not saturate for this operation and draw the current waveform $i(t)$ under this condition. The initial value of flux in the core is zero.

**b)** Draw the $i(t)$ waveform when the supply voltage is as given in Fig. (3d) with the $V_m$ value found in part **(a)**, noting that the new voltage waveform is obtained by shifting the previous one by 1 ms in time, (Fig.(3d)).

EE 361 ELECTROMECHANICAL CONVERSION I
MAGNETIC CIRCUITS, SOLVED PROBLEMS

Q **1** The single magnetic loop shown in Fig.1a is excited by a permanent magnet whose characteristic (MPN) is shown in Fig.1b. Find the length of the magnet $(\ell_m)$ to produce maximum energy in the air-gap, (neglect fringing). $A_g = A_m = A_s = 10\text{cm}^2$ $\ell_s = 100$ cm, $\ell_g = 1$ cm, $\mu_r = 500$ ( steel).

SOLUTION

$$H_m\ell_m + H\ell_s + H_g\ell_g = 0,\ A_m = A_s = A_g \rightarrow B_m = B_s = B_g$$

$$\phi = B_m A_m = B_s A_s = B_g A_g,\ B_m = \mu_r H_m,\ B_s\mu_s,\ B_g = \mu_0 H_g$$

Equation of the operating line $\overline{\text{OP}}$:

$$H_m = -\frac{A_m}{\ell_m}\left[\frac{\ell_g}{\mu_0 A_g} + \frac{\ell_s}{\mu_s A_s}\right]B_m$$

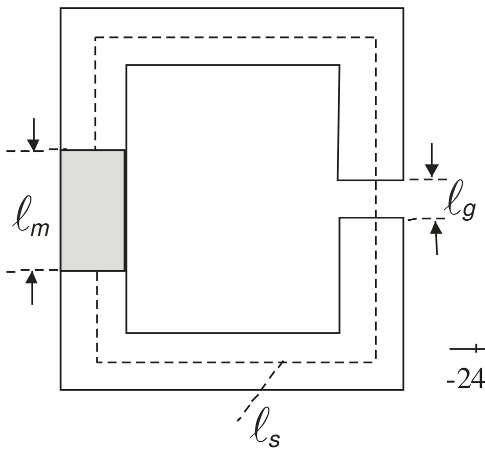



Fig.1a

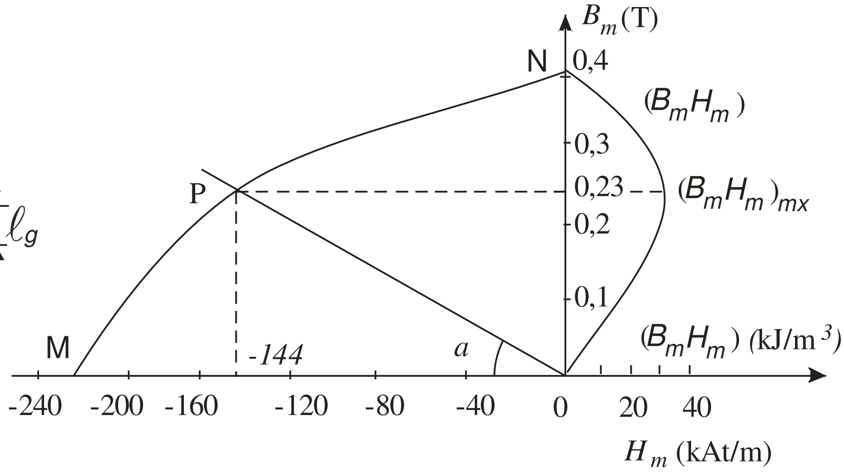



Fig.1b

$$H_m = -\frac{1}{\ell_m}\times\left[\frac{0.01}{4\pi.\times 10^{-7}} + \frac{1}{500\times 4\pi\times 10^{-7}}\right]\times B_m,\ H_m = -\frac{9549.3}{\ell_m}\times B_m,\ B_m = -\frac{\ell_m}{9549.3}\times H_m$$

$$At(B.H)_{\max}:\ P\ (B_m = 0.23\ \text{Wb/m}^2,\ H_m = -144\times 10^3\ \text{At/m})$$

$$\tan\alpha = \frac{0.23}{144\times 10^3} = \frac{\ell_m}{9549.3},\ \ell_m \cong 1.53\text{cm}$$

**Q2** Consider the single-phase 50-Hz transformer and its approximate hysteresis characteristic shown in Figs.2a-b. A=25 cm$^2$, ℓ=40 cm, $N_1$=259 turns, $N_2$=18 turns.

a) Show that, for a sinusoidal flux density distribution in the core ($B \leq 1.6$ T), the rms value of the induced emf in the primary winding is:

$$E_1 = 4.44 N_1 f \Phi_{\max}$$

where f and $\Phi_{\max}$ are the frequency and maximum value of the mutual sinusoidal flux, respectively.

b) By neglecting the leakage flux and the primary winding resistance, determine the maximum value of $E_1$. Compute also the rms value of the magnetizing primary current $I_m$. Draw, $e_1$ (t) and $i_1$(t) waveforms on the common axes. What is the hysteresis loss?

c) Find the rms value of the open-circuited secondary voltage $E_2$.

# Middle East Technical University
# Electrical and Electronics Engineering Department

## EE 361 Homework No.3 Due: 23 Nov. 2011

**Q1.** A 16.67-kVA, 7200/120 V, 60-Hz, two-winding transformer has the following actual equivalent circuit impedances.

$R_c = 311\ k\Omega$ ; $X_m = 54.8\ k\Omega$

$r_1 = 18.7\ \Omega$ ; $x_1 = 77.8\ \Omega$ ; $r_2 = 0.00519\ \Omega$ ; $x_2 = 0.0216\ \Omega$ ;

The voltage applied to the primary terminals is adjusted so that the secondary terminal voltage is 120 V, when the load on the secondary is 16.67 kVA at unity power factor.

Find the primary current and the required primary terminal voltage by the use of

a. The complete equivalent circuit,
b. The simplified equivalent circuit in which the exciting branch has been moved to the primary terminals, and
c. The simplified equivalent circuit containing only the primary and secondary leakage reactances.
d. Comment on the results.
e. Find the core and copper loss, and the transformer efficiency under these conditions by the use of the complete equivalent circuit.

**Q.2.** A certain power transformer is connected between a transmission line and a load. The secondary terminal voltage is $V_2$ = 707sin(377t) Volts and the load current is $i_2$ = 141.4 sin(377t-30$^o$) Amperes. The primary winding has 300 turns and a resistance of 2.0Ω. The secondary winding has 30 turns and a resistance of 0.02Ω. The leakage inductance of the primary is 0.0300H while that of secondary is $3.00\text{x}10^{-4}$H. The exciting current of this transformer is 0.707sin(377t-80$^o$).

a. Find turns ratio a
b. The primary and secondary induced rms voltages E1 and E2 and the terminal voltage V1 (rms).
c. Compare the actual terminal voltage ratio with the turns ratio.

EE 361 Homework No.3 Due: <u>23 Nov. 2011</u>
**SOLUTIONS**

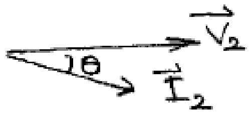
# Question 1.

Turns-ratio or Voltage Transformation ratio, $n = 7200/120 = 60$

$x_2' = n^2 \cdot x_2 = 60^2 \times 0.0216 = 77.76\ \Omega$ referred to primary

$r_2' = 60^2 \times 0.00519 = 18.684\ \Omega$ referred to primary

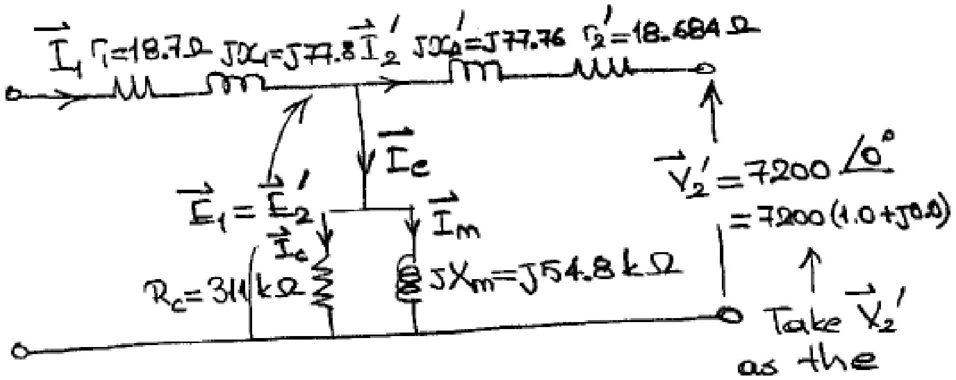
$V_2' = n \cdot V_2 = 60 \times 120 = 7200$ volts referred to primary




Power Factor, p.f. $= \cos\theta = 1.0$

$\Rightarrow \theta = 0°$


a.




Take $\vec{V}_2'$ as the reference phasor

The load on the secondary, $S_2 = V_2' I_2'$

$16670\text{ VA} = 7200\text{ Volts} \times I_2'$

$\Rightarrow I_2' = 2.315$ amps

Since p.f. is unity then $\vec{I}_2' = 2.315\angle 0°$ amps

$$\vec{E}_1 = \vec{E}_2' = \vec{V}_2' + (r_2' + jx_2')\vec{I}_2'$$

$$= 7200(1.0 + j0.0) + (18.684 + j77.76)\,2.315(1.0 + j0.0)$$

$$= 7243.3 + j180.0 \text{ volts}$$

$$= 7245.5 \angle{+1.42^\circ} \text{ volts}$$

$$\vec{I}_e = \vec{I}_c + \vec{I}_m$$

$$\vec{I}_c = \vec{E}_1 / R_c = 7245.5 \angle{+1.42^\circ} \,/\, 311 \times 10^3 \angle{0^\circ} = 0.0233 \angle{+1.42^\circ}$$

$$\vec{I}_m = \vec{E}_1 / jX_m = 7245.5 \angle{+1.42^\circ} \,/\, 54.8 \times 10^3 \angle{+90^\circ} = 0.1322 \angle{-88.58^\circ}$$

$$\therefore \vec{I}_e = 0.0233(\cos 1.42 + j\sin 1.42) + 0.1322[\cos(-88.58) + j\sin(-88.58)]$$

$$= 0.0233 + j0.0006 + 0.0248 - j0.1322$$

$$= 0.0481 - j0.1316 \text{ amps}$$

$$= 0.14 \angle{-69.9^\circ} \text{ amps}$$

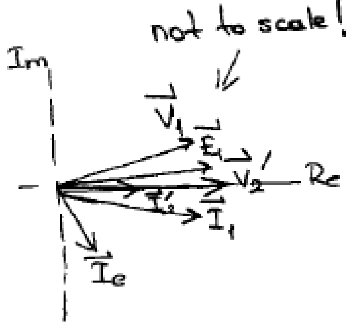
It is better to calculate $\vec{I}_e$ in the following way

$$\vec{I}_e = \vec{E}_1 (G_c - jB_m) = \vec{E}_1 [(1/R_c) - j(1/X_m)]$$

$$\vec{I}_1 = \vec{I}_e + \vec{I}_2' = 0.0481 - j0.1316 + 2.3150 + j0.0000$$

$$= 2.3631 - j0.1316 \text{ amps} //$$

$$= 2.367 \angle{-3.19^\circ} \text{ amps} //$$



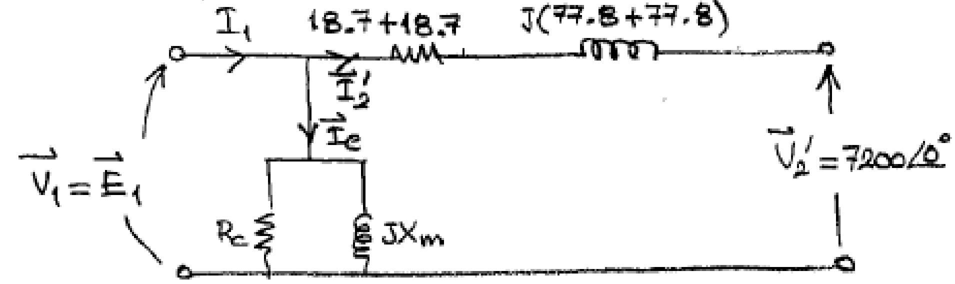

$$\vec{V}_1 = \vec{E}_1 + (r_1 + jx_1)\vec{I}_1$$

$$= 7243.3 + j180.0 + (18.7 + j77.8)(2.3631 - j0.1316)$$

$$= 7297.7 + 361.4 \text{ volts} //$$

$$= 7306.6 \angle{+2.8^\circ} \text{ volts} //$$

b.




$$\vec{V}_1 = \vec{V}_2' + (r_1 + r_2' + Jx_1 + Jx_2')\vec{I}_2'$$

$$= 7200 + (37.4 + J\,155.6) \times 2.3$$

$$= 7286 + J\,357.9 \text{ volts } //$$

$$= 7294.8 \angle +2.8^\circ \text{ volts } //$$

$$\vec{I}_e = \vec{V}_1 \left(\frac{1}{R_c} - J\frac{1}{X_m}\right)$$

$$= (7286 + J\,357.9)\left(\frac{1}{311 \times 10^3} - J\frac{1}{54.8 \times 10^3}\right)$$

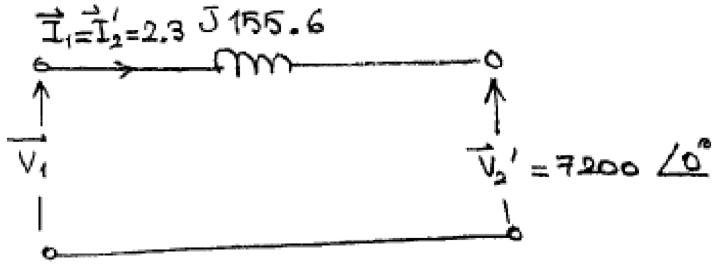
$$= 0.03 - J0.13 \text{ amps}$$

$$\therefore \vec{I}_1 = \vec{I}_2' + \vec{I}_e$$

$$= 2.34 - J0.13 \text{ amps } //$$

$$= 2.34 \angle -3.2^\circ \text{ amps } //$$

C.




$$\vec{I}_1=\vec{I}_2'=2.3\angle 0^\circ \text{ amps } //$$

$$\vec{V}_1=\vec{V}_2'+J(x_1+x_2')\vec{I}_2'$$
$$=7200+J155.6\times 2.3$$
$$=7200+J358 \text{ volts} //$$
$$=7209\angle +2.8^\circ \text{ volts}$$

Comment on the results

| | part (a) | part (b) | part (c) |
|---|---|---|---|
| $V_1\angle\delta$ | $7307\angle +2.8^\circ$ | $7295\angle +2.8^\circ$ | $7209\angle +2.8^\circ$ |
| $I_1\angle\varphi$ | $2.37\angle -3.2$ | $2.34\angle -3.2$ | $2.3\angle 0^\circ$ |

The equivalent crct in part (b) gives quite good results while reducing the computational labour.
Altough the equivalent crct in part (c) reduces the computations significantly, the results are not as accurate as those of (b) for small and medium size transformers.