EE 361 ELECTROMECHANICAL CONVERSION I
MAGNETIC CIRCUITS, SOLVED PROBLEMS

Q **1** The single magnetic loop shown in Fig.1a is excited by a permanent magnet whose characteristic (MPN) is shown in Fig.1b. Find the length of the magnet (ℓ_m) to produce maximum energy in the air-gap, (neglect fringing). $A_g = A_m = A_s = 10\text{cm}^2$ $\ell_s = 100$ cm, $\ell_g = 1$ cm, $\mu_r = 500$ (steel).

SOLUTION

$$H_m\ell_m + H\ell_s + H_g\ell_g = 0,\ A_m = A_s = A_g \rightarrow B_m = B_s = B_g$$

$$\phi = B_m A_m = B_s A_s = B_g A_g,\ B_m = \mu_r H_m,\ B_s\mu_s,\ B_g = \mu_0 H_g$$

Equation of the operating line $\overline{\text{OP}}$:

$$H_m = -\frac{A_m}{\ell_m}\left[\frac{\ell_g}{\mu_0 A_g} + \frac{\ell_s}{\mu_s A_s}\right]B_m$$

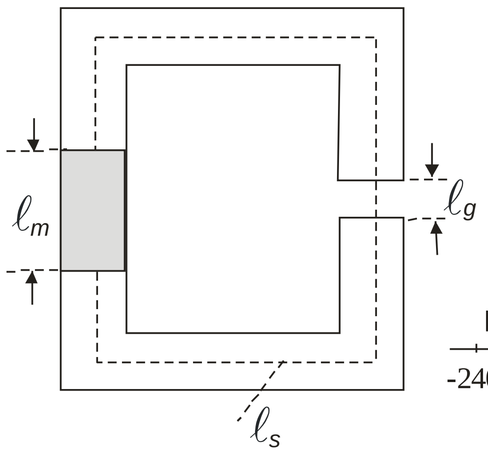


Fig.1a

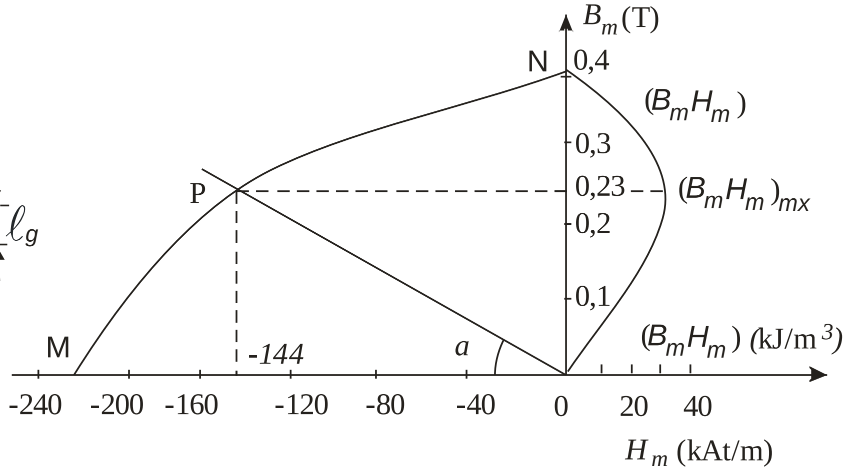


Fig.1b

$$H_m = -\frac{1}{\ell_m}\times\left[\frac{0.01}{4\pi.\times10^{-7}} + \frac{1}{500\times4\pi\times10^{-7}}\right]\times B_m,\ H_m = -\frac{9549.3}{\ell_m}\times B_m,\ B_m = -\frac{\ell_m}{9549.3}\times H_m$$

$$At(B.H)_{\max}:\ P\ (B_m = 0.23\ \text{Wb/m}^2,\ H_m = -144\times10^3\ \text{At/m})$$

$$\tan\alpha = \frac{0.23}{144\times10^3} = \frac{\ell_m}{9549.3},\ \ell_m \cong 1.53\text{cm}$$

Q2 Consider the single-phase 50-Hz transformer and its approximate hysteresis characteristic shown in Figs.2a-b. A=25 cm^2, ℓ=40 cm, N_1=259 turns, N_2=18 turns.
a) Show that, for a sinusoidal flux density distribution in the core (B$\leq$1.6 T), the rms value of the induced emf in the primary winding is:

$$E_1 = 4.44 N_1 f \Phi_{\max}$$

where f and $\Phi_{\max}$ are the frequency and maximum value of the mutual sinusoidal flux, respectively.
b) By neglecting the leakage flux and the primary winding resistance, determine the maximum value of E_1. Compute also the rms value of the magnetizing primary current I_m. Draw, e_1 (t) and i_1(t) waveforms on the common axes. What is the hysteresis loss?
c) Find the rms value of the open-circuited secondary voltage E_2.

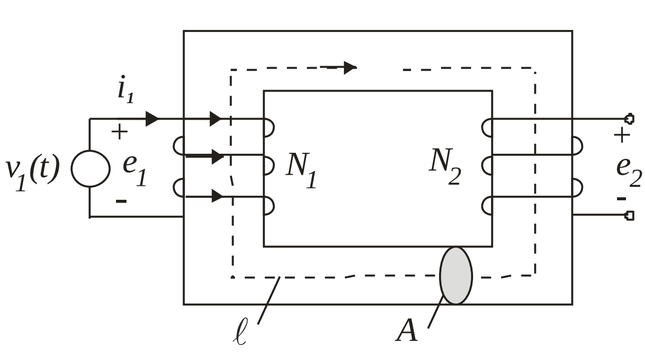


Fig.2a

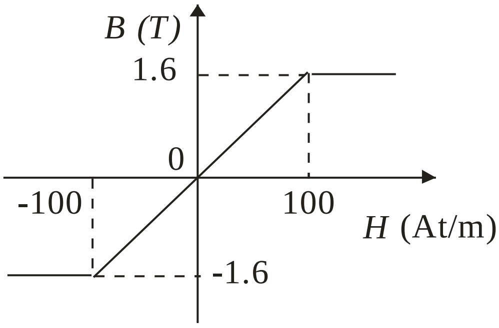


Fig.2b

SOLUTION

$a)\ \phi(t)=\phi_m \sin\omega t,\ \ e_1 = N_1\dfrac{d\phi}{dt} = \phi_m N_1 \omega\cos\omega t = \sqrt{2}E_1\cos\omega t,\ \ \omega=2\pi f,\ E_1 = \dfrac{2\pi}{\sqrt2} f N_1\phi_m = 4.44\, f\, N_1 \phi_m$

b) For $B_m \le 1.6\ T$ (linear region):

$\phi_m = B_m.A = 1.6\times 25\times 10^{-4} = 0.004$ Wb, $E_1 = 4.44\times 50\times 259\times 4\times 10^{-3} \cong 230$ V

$v_1(t) = e_1(t) = \sqrt{2}\times 230\ \cos\omega t = 325.3\times\cos\omega t$ V, $\ N_1 I_{1m} = H_m.\ell,\ H_m = 100$ At/m

$I_{1m} = \dfrac{H_m \ell}{N_1} = \dfrac{100\times 0.4}{259} = 155$ mA, $I_{1rms} = 109.2$ mA, $\ i_1(t) \cong 0.155\times\sin\omega t$ A

The hysteresis loss is zero as the hysteresis loop area is zero.

$b)\ L = 10^{-3}\ H,\ N = 40$ turns, $\Re = \Re_g, \Re_c = 0,\ L = N^2/\Re_g,\ \Re_g = 1.6\times 10^6\ H^{-1}$

$\Re_g = \dfrac{\ell_g}{\mu_0 A_g},\ \ell_g = \Re_g\, \mu_0 A_g = 1.6\times 10^6\times 4\pi\times 10^{-7}\times 10^{-3} = 0.002$ m

c) I = 10 A, N = 40 turns, $\Re = 1.6\times 10^6,\ \ \phi = \dfrac{\mathrm{NI}}{\Re} = \dfrac{40\times 10}{1.6\times 10^6} = 2.5\times 10^{-4}$ Wb

$B = \dfrac{\phi}{A_c} = \dfrac{2.5\times 10^{-4}}{10^{-3}} = 0.25\ T \le 0.5$ T, $\mathrm{W_f} = \dfrac{1}{2}\mathrm{LI}^2 = \dfrac{1}{2}\times 10^{-3}\times 10^2 = 0.05$

c) $e_2 = N_2\dfrac{d\phi}{dt} = 18\times\dfrac{d(4\times 10^{-3}\sin\omega t)}{dt}$

$e_2 = 4\times 10^{-3}\times 18\times\omega\times\cos\omega t$

$E_{2rms} = \dfrac{4\times 18\times 10^{-3}\times 100\pi}{\sqrt2} = 16$ V

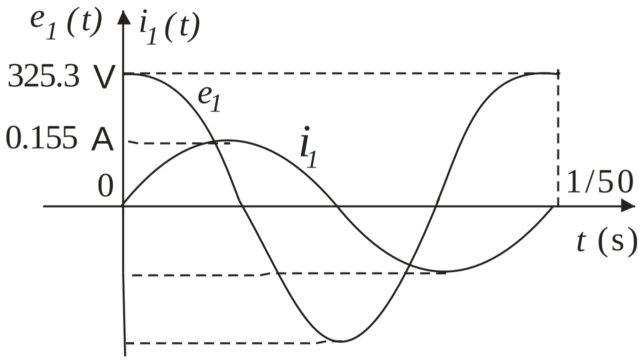


Fig.2c

3 Consider the magnetic circuit of an electromechanical device shown in Fig.3a, where the traced part corresponds to a permanent magnet material. During the horizontal movement of the moving part in both directions, the length of the air gap is changed periodically between ℓ_{gmn} and ℓ_{gmx}.

a) Explain clearly how one can magnetize the permanent-magnet portion of the circuit in the figure, by assuming that it is initially de-magnetized. Sketch the magnetic circuit and show the modifications needed for magnetization. Sketch a hysteresis loop to illustrate the trajectory of magnetic operating point during magnetization of the permanent magnet material. Sketch the waveform of the electrical quantity which magnetizes the permanent magnet.

b) Assume now that Samarium-cobalt has been used in the magnetic circuit,(Fig.3b). Compute the operating points which correspond to ℓ_{gmn} and ℓ_{gmx} and mark them on the B_m-H_m characteristic. Mark the locus of the operating point during the periodic movement of the moving part also on the same graph. Determine the optimum operating point. Discuss the success of the design. Neglect fringing. ($\mu_0 = 4\pi 10^{-7}$ H/m).

c) If Alnico 5 were used in place of Samarium-cobalt in the above circuit, what would the locus of the operating point be, (Fig.3f)? Roughly sketch it. Compare the usefulness of Samarium-cobalt and Alnico-5 materials in this particular application considering that magnet length can also be changed.

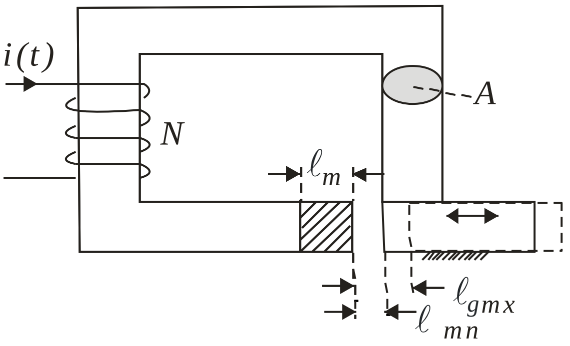


Fig.3a

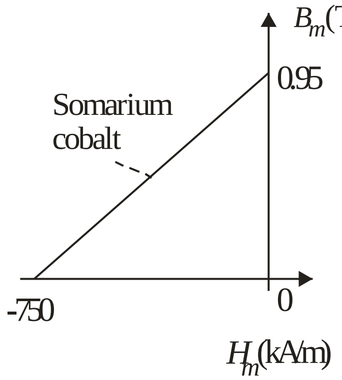


Fig.3b

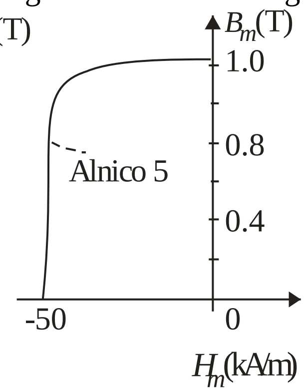


Fig.3c

SOLUTION

a) In order to magnetize the permanent-magnet portion, a positive current pulse with sufficiently high magnitude is passed through the coil with N turn, short-circuiting the air-gap with a high permeability ($\mu \to \infty$) material. The shape of the current pulse which is not significant and the corresponding hysteresis loop are shown in Figs.3d-e.

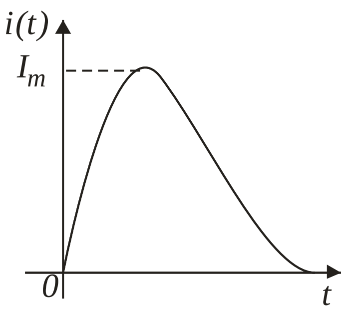


Fig.3d

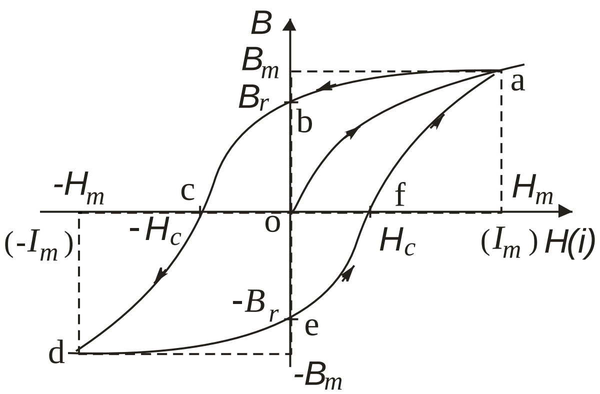


Fig3e

b) Somarium cobalt:

The B_m-H_m characteristic can be approximated by a straight line, (Fig.3f).

$B_m = aH_m + b$, $H_m = 0$ A/t, $B_m = b = 0.95$ T, $B_m 0$ T, $0 = a(-730) + 0.95$

$$a = \frac{0.95}{730 \times 10^3} = 1.3 \times 10^{-6},\ B_m = 1.3 \times 10^{-6} H_m + 0.95$$

$$\sum Ni = \sum H\ell,\ 0 = H_g \ell_g + H_m \ell_m,\ \phi = B_g A_g = B_m A_m,\ A_g = A_m$$

$$B_g = \mu_0 H_g = \mu_0 H_m = B_m,\ B_m = -\frac{\ell_m}{\ell_g} H_m, \text{ (load-line)}$$

$\ell_m = 5$ mm, $\ell_{gmn} = 4 \times 10^{-3}$ m

$$B_m = -\frac{5}{4} \times 4\pi \times 10^{-7} H_m = -15.7 \times 10^{-7} H_m,\ 1.3 \times 10^{-6} H_m + 0.95 = -15.7 \times 10^{-7} H_m$$

$$H_m = -\frac{0.95}{28.7\times10^{-7}} \cong -331\ \text{kA/m},\ B_m \cong 0.52\ \text{T}$$

$$\ell_m = 5\ \text{mm},\ \ell_{g_{mx}} = 8\ \text{mm},\ B_m = 7.85\times10^{-7} H_m$$

$$H_m = -\frac{0.95}{20.85\times10^{-7}} \cong -455\ \text{kA/m},\ B_m \cong 0.358\ \text{T}$$

The recoil line ($\overline{PQ}$) coincides with the magnet characteristic (Fig.3f).

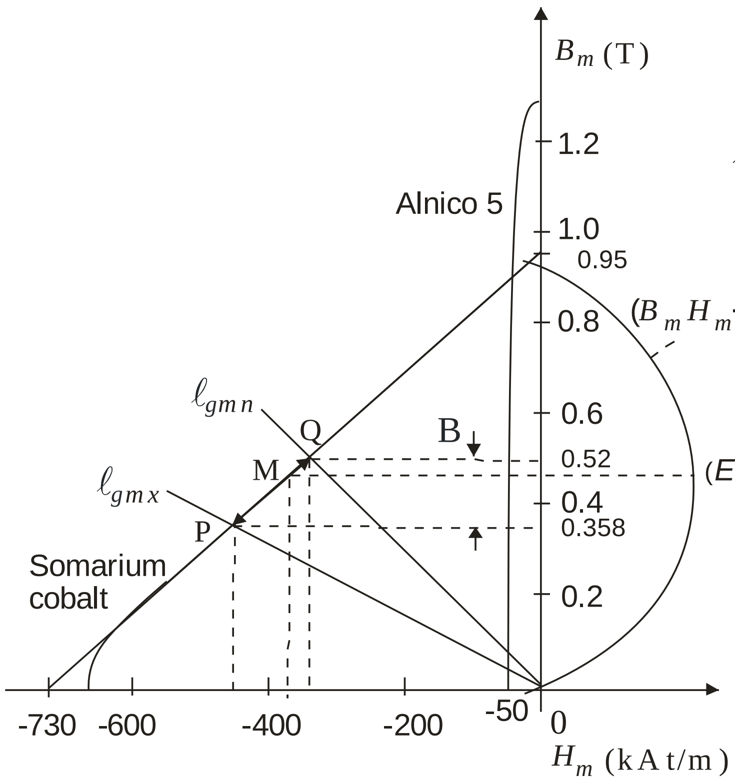


Fig.3f

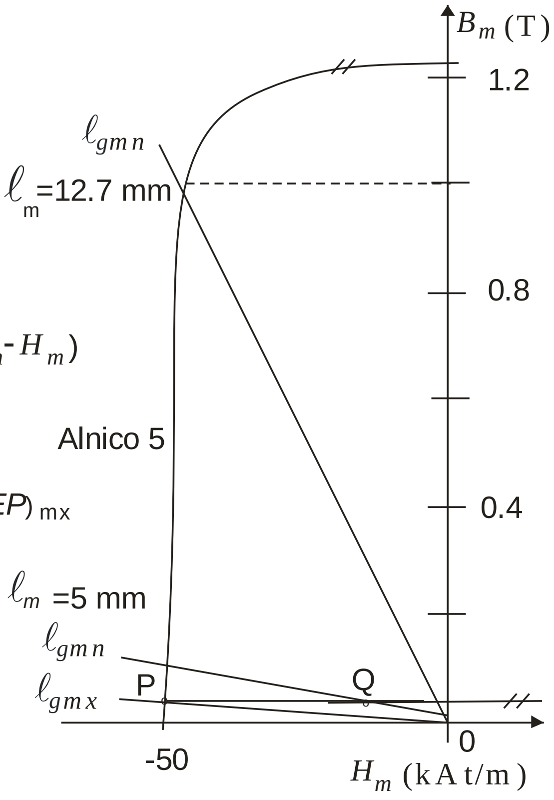


Fig.3g

c) Operation takes place around the optimum operating point M which corresponds to maximum magnet energy product $(EP)_{mx}$. In view of this, the design is successful. Smaller ΔB is more desirable. $\Delta B = 0.52 - 0.358 \approx 0.162\,\text{T}$, (Fig.5f).

$$(B_m - H_m)\,\text{curve: } H_m B_m = 1.3\times10^{-6}\times H_m^{\ 2} + 0.95\times H_m,\ \frac{d(H_m B_m)}{dH_m} = 2.6\times10^{-6} H_m + 0.95 = 0$$

$$H_m = -\frac{0.95}{2.6\times10^{-6}} = -365.4\ \text{kA/m},\ B_m \cong 0.475\ \text{T},\ (H_m B_m)_{opt} \cong 173.6\ \text{kJ/m}^3,\ \text{M}(0.475; -365.4)$$

d) For Alnico 5: $4 \le \ell_g \le 8$ mm and $\ell_m = 5$ mm, operating point P is around (0.05 T, -50 kAt/m), (Fig.3g). The recoil line ($\overline{\text{PQ}}$) does not coincide with the magnet characteristic. Inspection of the $(BH)_m - H_m$ curve indicates that the optimum operating point is around $B_m \cong 1\ T$. For optimum operation, the slop of the load-line ($-\mu_0\,\ell_m/\ell_g$) should be increased considerably, (Fig.3f).

Take $\ell_g = 8$ mm, $H_m \cong -50$ kA/m, $B = 1$ T.

The length of the magnet for optimum operation is

$$\ell_m = -\frac{B_m}{H_m}\cdot\frac{\ell_g}{\mu_0} = \frac{1}{50\times10^{-3}}\cdot\frac{8\times10^{-3}}{4\pi10^{-7}} \cong 12.7\ \text{cm}.$$ More Alnico 5 should be used.

Q4 A choke (inductor) composed of a single loop magnetic core and a winding with N turns is connected in series with a dc network to smooth out the current waveform. The average core length is ℓ and the core cross sectional area is A. The coil is excited by a dc current I. The flux density in the magnetic core should be less than or equal to 0.5 T over the entire operating range in order to guaranty the linearity of the magnetic circuit. The B-H characteristic is given in Fig.4.

a) Show that the self-inductance of the coil operating on the linear part of the B-H characteristic ($\overline{\mathrm{OP}}$) is given by $\mathrm{L} = \mathrm{N}^2/\Re$, where $\Re$ is the core reluctance. Neglect the leakage flux.

b) An air gap is introduced into the core in order to obtain a self inductance L=1mH. What should the air-gap length be?

c) Compute the flux density created in the magnetic circuit at the rated dc current of I=10A, and test whether it exceeds the maximum safe value of 0.5 T or not. Find the stored magnetic energy.

SOLUTION

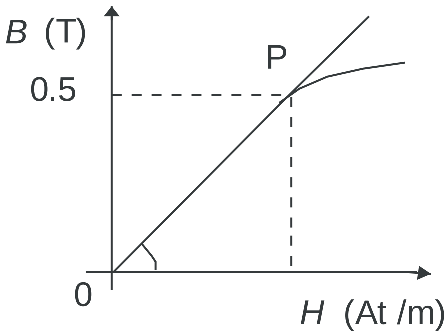


Fig.4

a) For ($\overline{\mathrm{OP}}$) region, $B \le 0.5$ T, $B = \mu H$

$\mu = \tan\theta = \text{constant},\ \phi_\ell = 0$, (Fig.1.14)

$$N^2 i = N\phi\Re = \lambda\Re,\ L = \frac{d\lambda}{dI} = \frac{N^2}{\Re}$$

$$\Im = Ni = \phi\Re,\ \lambda = N\phi = N^2 i/\Re,\ L = \frac{d\lambda}{di}$$

$b)\ L = 10^{-3}\ H,\ N = 40$ turns, $\Re = \Re_g, \Re_c = 0,\ L = N^2/\Re_g,\ \Re_g = 1.6\times10^6\ H^{-1}$

$$\Re_g = \frac{\ell_g}{\mu_0 A_g},\ \ell_g = \Re_g\,\mu_0 A_g = 1.6\times10^6\times4\pi\times10^{-7}\times10^{-3} = 0.002\ \text{m}$$

c) $\mathrm{I} = 10$ A, N = 40 turns, $\Re = 1.6\times10^6$, $\phi = \dfrac{\mathrm{NI}}{\Re} = \dfrac{40\times10}{1.6\times10^6} = 2.5\times10^{-4}$ Wb

$$B = \frac{\phi}{A_c} = \frac{2.5\times10^{-4}}{10^{-3}} = 0.25\ T \le 0.5\ \text{T},\ W_f = \frac{1}{2}LI^2 = \frac{1}{2}\times10^{-3}\times10^2 = 0.05\ \text{J}$$

Q5 The magnetic circuit given in Fig.5a has two windings and two air gaps. The core can be assumed to be of infinite permeability. The equivalent magnetic circuit is shown in Fig.5b, (neglect fringing).

a) Assuming that the coil 1 to be carrying a current I_1 and the current in coil 2 to be zero. Calculate;

(i) the magnetic flux density in each of the air gaps, (ii) the flux linkage of winding 1 and, (iii) the flux linkage of winding 2.

b) Repeat part (a), assuming zero current in winding 1 and a current I_2 in winding 2.

c) Repeat part (a), assuming the current I_1 in winding 1 and the current I_2 in winding 2.

d) Find the self-inductance of windings 1 and 2 and the mutual inductance between the windings.

SOLUTION

$$a)\ \Re_1 = \frac{g_1}{\mu_0 A_1},\ \Re_2 = \frac{g_2}{\mu_0 A_2},\ \Im_1 = N_1 I_1,\ \Im_2 = 0$$

$$i)\ \phi_1 = \frac{\Im_1}{\Re_1},\ \phi_2' = \frac{\Im_1}{\Re_2},\ \phi' = \phi_1 + \phi_2',\ \ ii)\ \lambda_1' = N_1\phi' = N_1(\phi_1' + \phi_2'),\ \ iii)\ \lambda_2' = N_2\phi_2 = N_2\frac{\Im_1}{\Re_2}$$

$$b)\ \Im_1 = 0,\ \Im_2 = N_2 I_2$$

$$i)\ \phi_1'' = 0,\ \ \phi'' = \phi_2'' = \frac{\Im_2}{\Re_2},\ \ ii)\ \lambda_1'' = N_1\phi'' = N_1\frac{\Im_2}{\Re_2},\ \ iii)\ \lambda_2'' = N_2\phi_2'' = N_2\frac{\Im_2}{\Re_2}$$

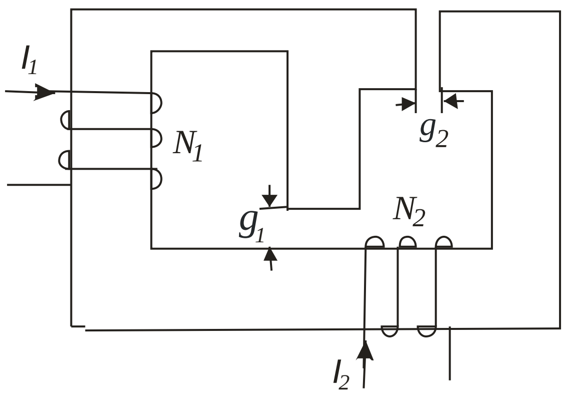


Fig.5a

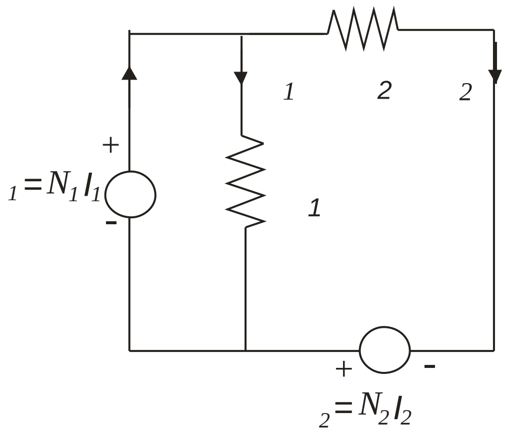


Fig.5b

$$c)\ \Im_1 + \Im_2 = \Re_2 \phi_2\,,\ \ \phi_2 = \frac{\Im_1}{\Re_2} + \frac{\Im_2}{\Re_2} = \phi'_2 + \phi''_2$$

$$\Im_1 = \Re_1 \phi_1,\ \ \phi_1 = \frac{\Im_1}{\Re_2} = \phi'_1 + \phi''_1,\ \ \phi = \phi_1 + \phi_2 = (\phi'_1 + \phi''_1) + (\phi'_2 + \phi''_2)$$

As the circuit is linear, superposition of cases (a) and (b):

$$\lambda_1 = N_1 \phi_1 = N_1(\phi'_1 + \phi''_1) = \lambda'_1 + \lambda''_1,\ \ \lambda_2 = N_2 \phi_2 = N_2(\phi'_2 + \phi''_2) = \lambda'_2 + \lambda''_2$$

$$d)\ \frac{1}{\Re} = \frac{1}{\Re_1} + \frac{1}{\Re_2},\ \ \Re = \frac{\Re_1 \Re_2}{\Re_1 + \Re_2}$$

$$L_{11} = \frac{\lambda'_1}{I_1} = \frac{N_1 \phi'}{I_1} = \frac{N_1(\phi'_1 + \phi'_2)}{I_1} = \frac{N_1}{I_1}\left(\frac{\Im_1}{\Re_1} + \frac{\Im_1}{\Re_2}\right) = \frac{N_1^2}{\Re}$$

$$L_{22} = \frac{\lambda''_2}{I_2} = \frac{N_2 \Im_2}{I_2 \Re_2} = \frac{N_2^2}{\Re_2},\ L_{12} = \frac{\lambda''_2}{I_1} = \frac{N_2 \Im_1}{I_1 \Re_2} = \frac{N_1 N_2}{\Re_2},\ L_{21} = \frac{\lambda'_1}{I_2} = \frac{N_1 \Im_2}{I_2 \Re_2} = \frac{N_1 N_2}{\Re_2},\ L_{12} = L_{21}$$

Q6 The magnetic circuit shown in Fig.6a is composed of a permanent magnet with length ℓ_m and cross sectional area A_m , an air gap with length ℓ_g and a high permeability ferromagnetic part. The movable part in the magnetic circuit can only move in vertical direction. The B-H characteristics of two alternative permanent magnet materials, I and II, are as given in Fig.3b. $A_m=A_g=10\ cm^2$ and $\mu_0=4\pi\times 10^{-7}$ H/m.

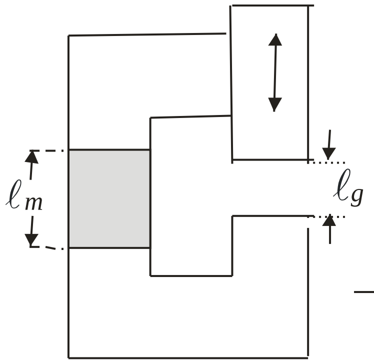
a) Calculate the length and volume of the permanent magnet section for both materials by assuming that 0.5 T is produced in the air gap for $\ell_g = 2$ mm

b) Which material is to be chosen in view of the cost if the permanent magnet material I is five times more expensive than II on volumetric basis?

c) For both materials and for the magnet lengths calculated in part (a), find the variation of the air-gap flux density as the gap length ℓ_g is periodically varied between 2 and 5 mm? Which material is more suitable if the air-gap flux density has to be 0.5 T when the air gap length is reduced from 5 mm to 2 mm, i.e. if a permanent demagnetization is not desired? (Neglect fringing).

<u>SOLUTION</u>

$$a)\ \ \ell_m H_m + \ell_c H_c + \ell_g H_g = 0\,,\quad \phi = A_m B_m = A_g B_g\,,\quad A_m = A_g\,,\quad B_m = B_g$$



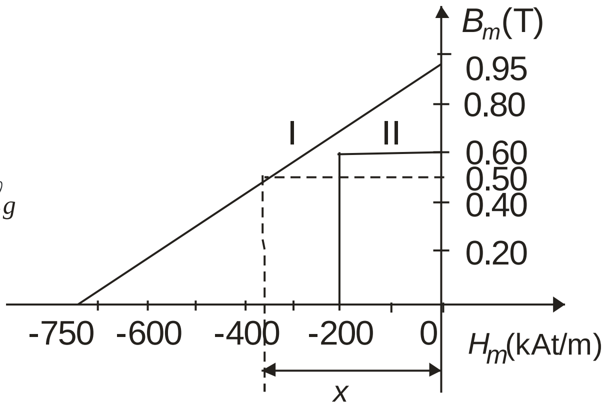


Fig.6a Fig.6b

$$B_m = \mu_m H_m = \mu_0 H_g \rightarrow H_g = \frac{B_m}{\mu_0},$$

$$\ell_m H_m + \ell_g \frac{B_m}{\mu_0} = 0,\ H_m = -\left(\frac{\ell_g}{\ell_m}.\frac{B_m}{\mu_0}\right) \text{ and } \ell_m = -\frac{\ell_g}{\mu_0}\cdot\frac{B_m}{H_m}$$

$$\ell_g = 2\times 10^{-3}\,\text{m},\quad \ell_m = ?,\quad B_g = B_m = 0.5\ \text{T},\quad \mu_0 = 4\pi\times 10^{-7}\,\text{H/m}.$$

Magnet I: $B_m = 0.5\ T,\quad \dfrac{x}{750} = \dfrac{0.95-0.5}{0.95},\ x = H_m = -\dfrac{0.45}{0.95}\times 750 = -355.26\ \text{kA.t/m}$

$$\ell_{mI} = \frac{2\times 10^{-3}\times 0.5}{4\pi\times 10^{-7}\times 355.26\times 10^{3}} = 0.00224\ \text{mm} = 2.24\times 10^{-3}\ \text{m}$$

$$(vol)_I = A_m \ell_{mI} = 10^{-3}\times 2.24\times 10^{-3} = 2.24\times 10^{-6}\ \text{m}^3$$

Magnet II: $B_m = 0.5\,\text{T},\ H_m = -200\ \text{kA.t/m}$

$$\ell_{mII} = \frac{2\times 10^{-3}\times 0.5}{4\pi\times 10^{-7}\times 200\times 10^{3}} = 3.98\times 10^{-3}\ \text{m},\quad (vol)_{II} = 3.98\times 10^{-3}\times 10^{-3} = 3.98\times 10^{-6}\ \text{m}^3$$

b) $vol_I < vol_{II},\ cost_I \approx 5\times vol_I,\ cost_{II} \approx vol_{II},\quad 5\times 2.24\times 10^{-6} > 3.98\times 10^{-6}$

$Cost_I > Cost_{II}$; the permanent magnet II should be selected.

c) Magnet I: $\ell_{mI} = 2.24\times 10^{-3}\,m,\ A_m = 10^{-3}\ \text{m}^2,\ 2\ \text{mm} < \ell_g < 5\,\text{mm}$

$$B_m = -\frac{\ell_m}{\ell_g}\mu_0 H_m,\ \ell_g = 2\ \text{mm},\ B_m = -\frac{2.24}{2}\times 4\pi\times 10^{-7} H_m = -1.407\times 10^{-6} H_m$$

$$\ell_g = 5\ \text{mm},\ B_m = -\frac{2.24}{5}\times 4\pi\times 10^{-7} H_m = -5.63\times 10^{-7} H_m$$

Equation of the permanent magnet (I) characteristic:

$$B_m = 0.95 + \frac{0.95}{750\times 10^3} H_m = 0.95 + 1.267\times 10^{-6} H_m$$

Operating points for magnet I: $\ell_{mI} = 2.24\times 10^{-3}\,\text{m},\ \ell_g = 2\ \text{mm}$:

$$1.407\times 10^{-6} H_m = 0.95 + \frac{0.95}{750\times 10^3} H_m,\ H_m = -355.3\times 10^3\ \text{kAt/m},\ B_m \cong 0.5\ \text{T}$$

$\ell_g = 5\ \text{mm}$:

$$-5.63\times 10^{-7} H_m = 0.95 + \frac{0.95}{750\times 10^2} H_m,\ H_m = -519.22\times 10^{3}\ \text{kA.t/m},\ B_m = 0.292\ \text{T}$$

When the air-gap length is varied between 2 and 5 mm, the operating point traces $\overline{MN}$ line which to the re-coil line, (Fig.6a).

When ℓ_{gI} is reduced to 2 mm, $B_m \cong 0.5$ T is preserved

Operating points for magnet II:

$H_m = -200$ kAt/m, $\ell_{mI} = 3.98$ mm

$\ell_g = 2$ mm: $B_m = \dfrac{3.98}{2} \times 4\pi \times 10^{-7} \times 200 \times 10^3 = 0.5$ T

$\ell_g = 5$ mm: $B_m = \dfrac{3.98}{5} \times 4\pi \times 10^{-7} \times 200 \times 10^3 = 0.2$ T

At point Q; $H_m = \dfrac{0.2}{0.5} \times 200 = -80$ kAt/m

For permanent magnet II, B=0.5 T is achieved initially at $\ell_g = 2$ mm (point R). For $\ell_g = 5$ mm, P is the operating point. If ℓ_g is reduced to 2 mm, the operating point becomes Q and B is reduced to 0.2 T. $\overline{PQ}$ is the re-coil line, (Fig.6d)

Conclusion: Magnet I is suitable to produce 0.5 T for $2\,\text{mm} < \ell_g < 5$ mm without demagnetization

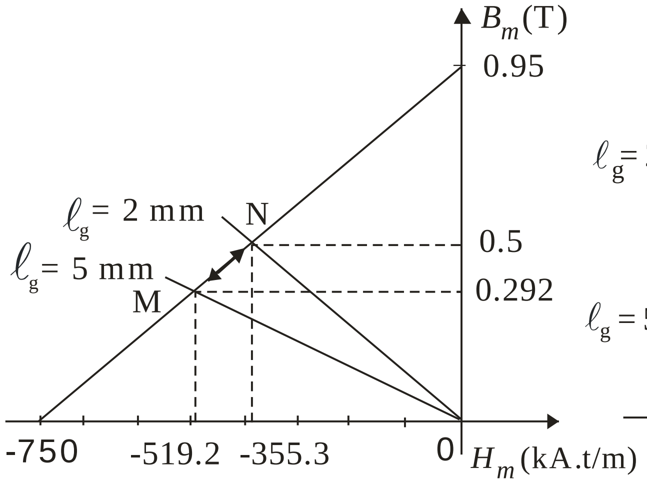


Fig.6c

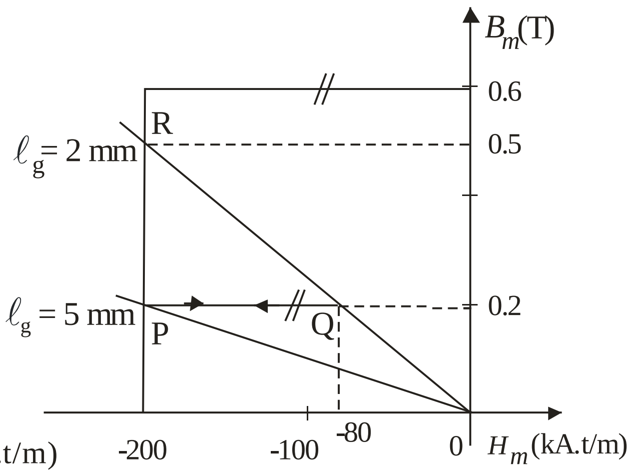


Fig.6d