Middle East Technical University
Electrical and Electronics Eng. Dept.

EE 361 Solutions of Homework 1 due 22 Oct. 2010

Question1

The magnetic system may be represented by the equivalent magnetic circuit of Fig. 1.33, where $\mathscr{R}_{\mathrm{i}}$ is the reluctance of the cast-iron part and $\mathscr{R}_{\mathrm{s}}$ that of the cast-steel part.

It will be assumed that the flux density is uniform in each part of the system, so that H will be uniform in each part. Then by the circuital law,

$$\oint \vec{\mathbf{H}} \cdot \vec{\mathbf{dl}} = H_i l_i + H_s l_s = Ni \quad \text{A}$$

The value of H needed to produce 0.25×10^{-3} Wb in each part must be determined.

$$A_i = 25 \times 25 \times 10^{-6} \quad \text{m}^2$$
$$A_s = 12.5 \times 25 \times 10^{-6} \quad \text{m}^2$$

a)

$$B_i = \frac{\phi}{A_i} = \frac{0.25 \times 10^{-3}}{625 \times 10^{-6}} = 0.4 \quad \text{T}$$

From Fig. 1.7,

$$H_i = 710 \text{ A/m}$$
$$B_s = \frac{\phi}{A_s} = \frac{0.25 \times 10^{-3}}{312.5 \times 10^{-6}} = 0.8 \quad \text{T}$$

From Fig. 1.7,

$$H_s = 480 \text{ A/m}$$

$$l_i \simeq (80 - 25) + 2\left(100 - \frac{25 + 12.5}{2}\right) + \frac{2 \times 25}{2} = 242.5 \text{ mm} = 0.2425 \quad \text{m}$$

$$l_s = 30 \times 10^{-3} \quad \text{m}$$

$$i = \frac{H_i l_i + H_s l_s}{N} = \frac{710 \times 0.2425 + 480 \times 3 \times 10^{-2}}{500} = 0.373 \quad \text{A}$$

b) $$\mathscr{R} = \frac{\mathscr{F}}{\phi} = \frac{Ni}{\phi} = \frac{500 \times 0.373}{0.25 \times 10^{-3}} = 746 \times 10^3 \quad \text{A/Wb}$$

c)

$$\mu_r \mu_0 = \frac{B}{H}$$

$$\mu_{ri} = \frac{0.4}{4\pi \times 10^{-7} \times 710} = 448$$

Thus, cast iron is 448 times as effective as air in producing a magnetic field of this flux density.

$$\mu_{rs} = \frac{0.8}{4\pi \times 10^{-7} \times 480} = 1330$$

Cast steel is more effective than cast iron.

d) $$\mathcal{R}_i = \frac{l_i}{\mu_{ri}\mu_0 A_i} = \frac{0.2425}{448 \times 4\pi \times 10^{-7} \times 25^2 \times 10^{-6}} = 690 \times 10^3 \quad \text{A/Wb}$$

$$\mathcal{R}_s = \frac{l_s}{\mu_{rs}\mu_0 A_s} = \frac{3 \times 10^{-2}}{1330 \times 4\pi \times 10^{-7} \times 12.5 \times 25 \times 10^{-6}}$$
$$= 57.7 \times 10^3 \quad \text{A/Wb}$$

Check: $$\mathcal{R}_i + \mathcal{R}_s = (690 + 58)10^3 = 748 \times 10^3 = \mathcal{R}$$

Question 2

$$\mathcal{R}_1 = \mathcal{R}_3 = \frac{300 \times 10^{-3}}{2250 \times 4\pi \times 10^{-7} \times 200 \times 10^{-6}} = 0.531 \times 10^6 \quad \text{A/Wb}$$

$$\mathcal{R}_2 = \frac{100 \times 10^{-3}}{1350 \times 4\pi \times 10^{-7} \times 400 \times 10^{-6}} = 0.148 \times 10^6 \quad \text{A/Wb}$$

The equivalent magnetic circuit for this system is shown in Fig. 1.35, and the problem may be solved by writing mmf equations for the two loops employing branch fluxes. Thus

$$\mathcal{F} = \mathcal{R}_1\phi_1 + \mathcal{R}_2\phi_2$$
$$0 = \mathcal{R}_3\phi_3 - \mathcal{R}_2\phi_2$$

These are analogous to equations of potential difference for a dc circuit. Also,

$$\phi_1 = \phi_2 + \phi_3$$

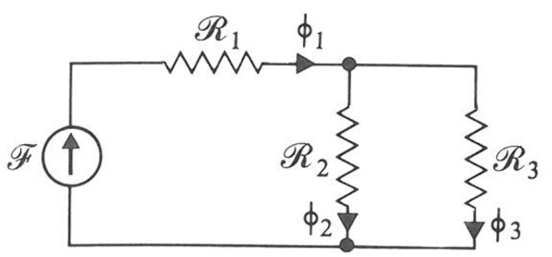

Fig. 1.35 Equivalent magnetic circuit for the system of Fig. 1.34.

This is analogous to a current equation at a node. Substitution of the values of mmf and reluctances in these equations gives

$$12.5 \times 10^{-6} = 0.531\ \phi_1 + 0.148\ \phi_2$$
$$0 = -0.148\phi_2 + 0.531\ \phi_3$$
$$0 = -\phi_1 + \phi_2 + \phi_3$$

Solution of these equations yields

$$\phi_1 = 19.3 \times 10^{-6} \quad \text{Wb}$$
$$\phi_2 = 15.1 \times 10^{-6} \quad \text{Wb}$$
$$\phi_3 = 4.21 \times 10^{-6} \quad \text{Wb}$$

from which the following values are obtained:

$$B_1 = \frac{\phi_1}{A_1} = \frac{19.3 \times 10^{-6}}{200 \times 10^{-6}} = 0.0965 \quad \text{T}$$

$$B_2 = \frac{\phi_2}{A_2} = \frac{15.1 \times 10^{-6}}{400 \times 10^{-6}} = 0.0377 \quad \text{T}$$

$$B_3 = \frac{\phi_3}{A_3} = \frac{4.21 \times 10^{-6}}{200 \times 10^{-6}} = 0.0210 \quad \text{T}$$

Question 3

It will not be necessary for us to convert the **B–H** curves for cast steel and sheet steel, as the two sections have the same length and area. The MMF equation is

$$\mathscr{F} = \mathrm{H}_1 l_1 + \mathrm{H}_2 l_2$$

Thus since $l_1 = l_2 = l$,

$$\frac{\mathscr{F}}{l} = \mathrm{H}_1 + \mathrm{H}_2$$

As a result, for part (a):

$$\frac{800}{0.4} = \mathrm{H}_1 + \mathrm{H}_2$$
$$2000 = \mathrm{H}_1 + \mathrm{H}_2$$

We thus have the construction of Figure 2.27. From the intersection of the curves, we obtain

$$\mathrm{H}_1 = 400 \text{ At/M} \qquad \mathscr{F}_1 = 160 \text{ At}$$
$$\mathrm{H}_2 = 1600 \text{ At/m} \qquad \mathscr{F}_2 = 640 \text{ At}$$
$$\mathrm{B}_1 = \mathrm{B}_2 = 1.35 \text{ T}$$

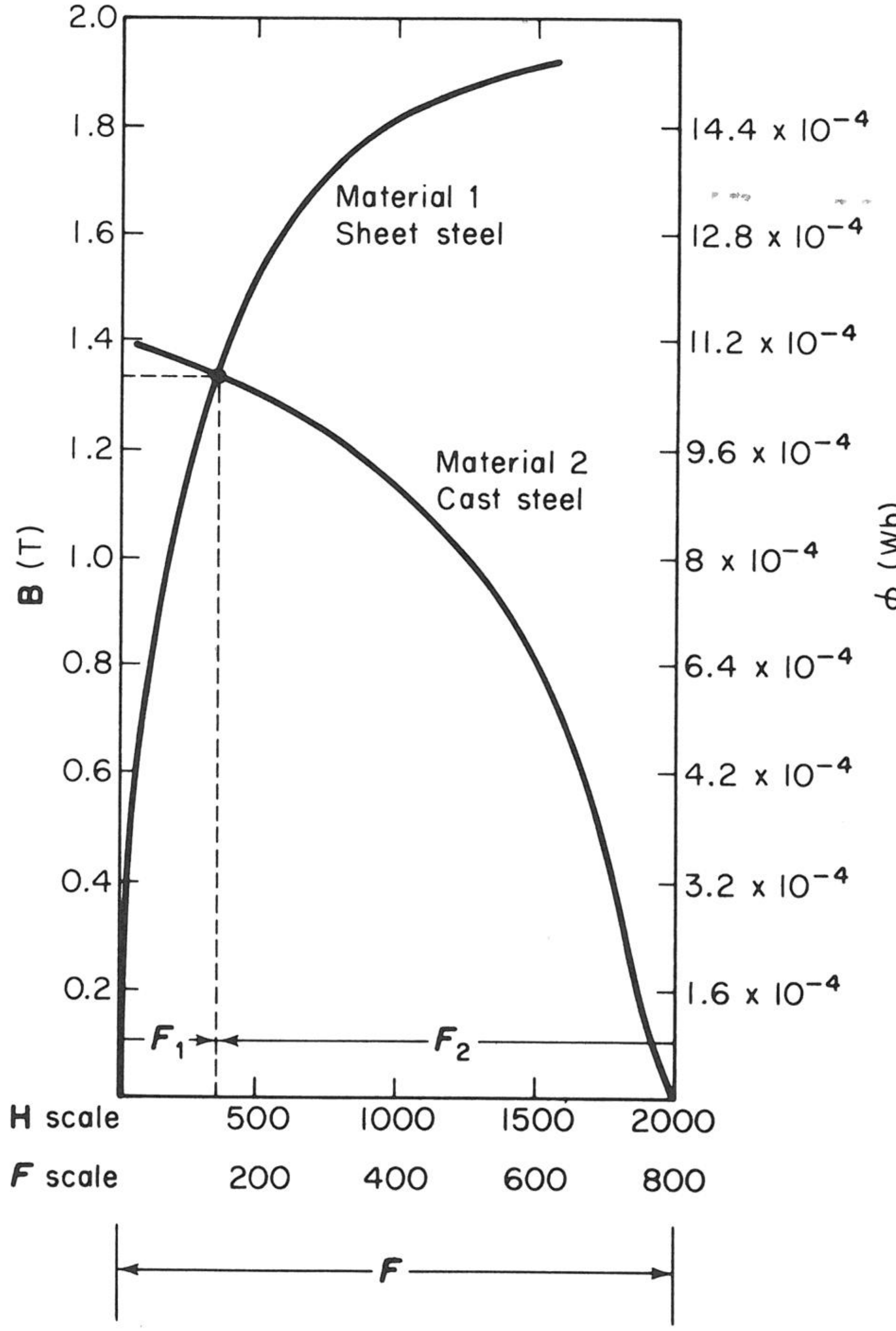

FIGURE 2.27 Solving Example 2.7 for $\mathcal{F} = 800$ At.

The flux is therefore given by

$$\Phi = 1.35 \times 8 \times 10^{-4} = 10.80 \times 10^{-4} \text{ Wb}$$

The construction for part (b) is shown in Figure 2.28. From the intersection of the two curves, we get

$$H_1 = 450 \text{ At/m} \qquad \mathcal{F}_1 = 180 \text{ At}$$

$$H_2 = 1550 \text{ At/m} \qquad \mathcal{F}_2 = 1020 \text{ At}$$

$$B_1 = B_2 \simeq 1.48 \text{ T}$$

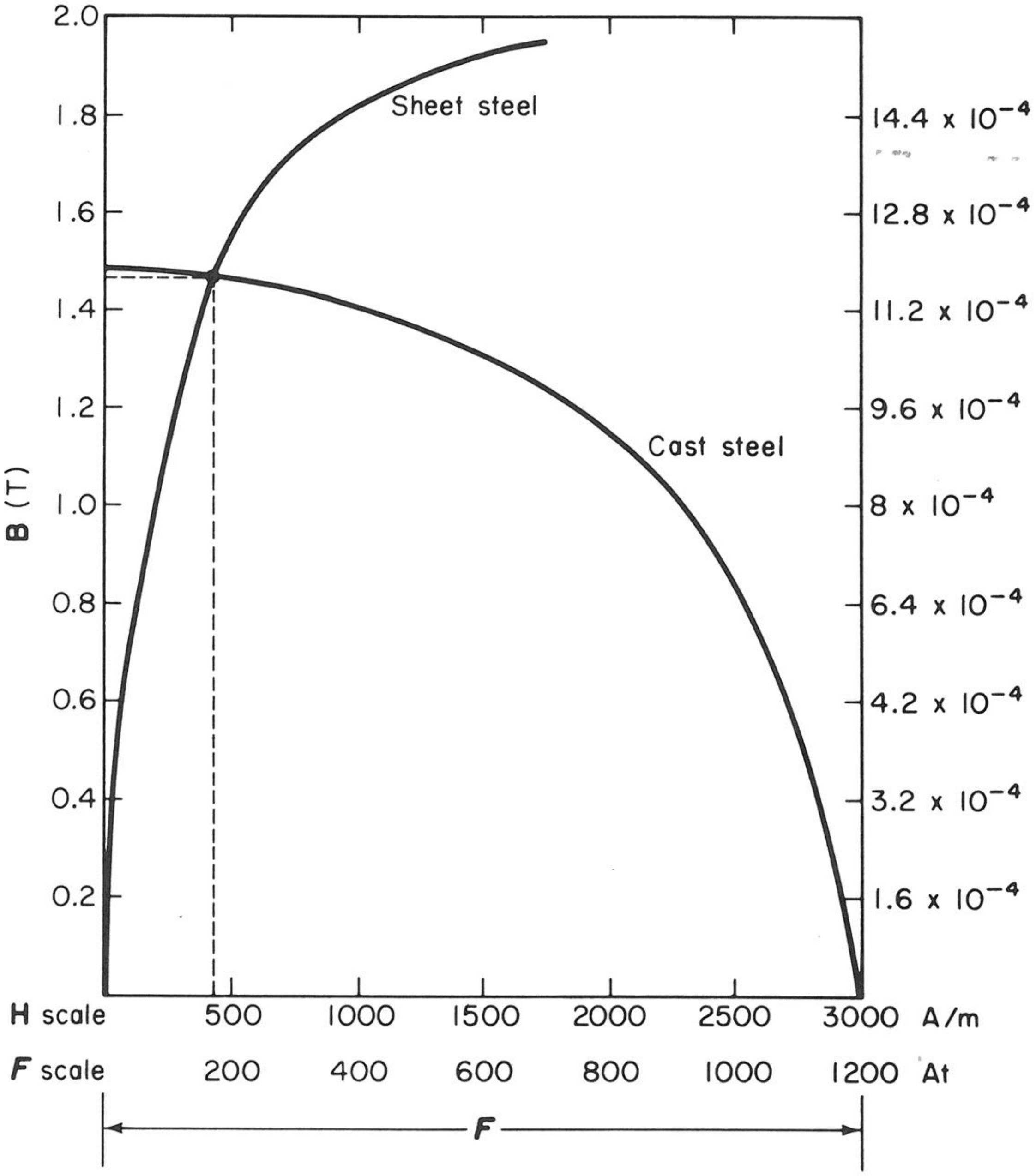

FIGURE 2.28 Solving Example 2.7 for $\mathcal{F} = 1200$ At.

The flux is therefore given by

$$\Phi = 1.48 \times 8 \times 10^{-4} = 11.84 \times 10^{-4} \text{ Wb}$$

For part (c), we have Figure 2.29, giving

$$H_1 = 550 \text{ At/m} \qquad \mathcal{F}_1 = 220 \text{ At}$$

$$H_2 = 1950 \text{ At/m} \qquad \mathcal{F}_2 = 1180 \text{ At}$$

$$B_1 = B_2 \simeq 1.49 \text{ T}$$

The flux is therefore given by

$$\Phi = 1.49 \times 8 \times 10^{-4} = 11.92 \times 10^{-4} \text{ Wb}$$

The reader should note that the results obtained here are approximate.

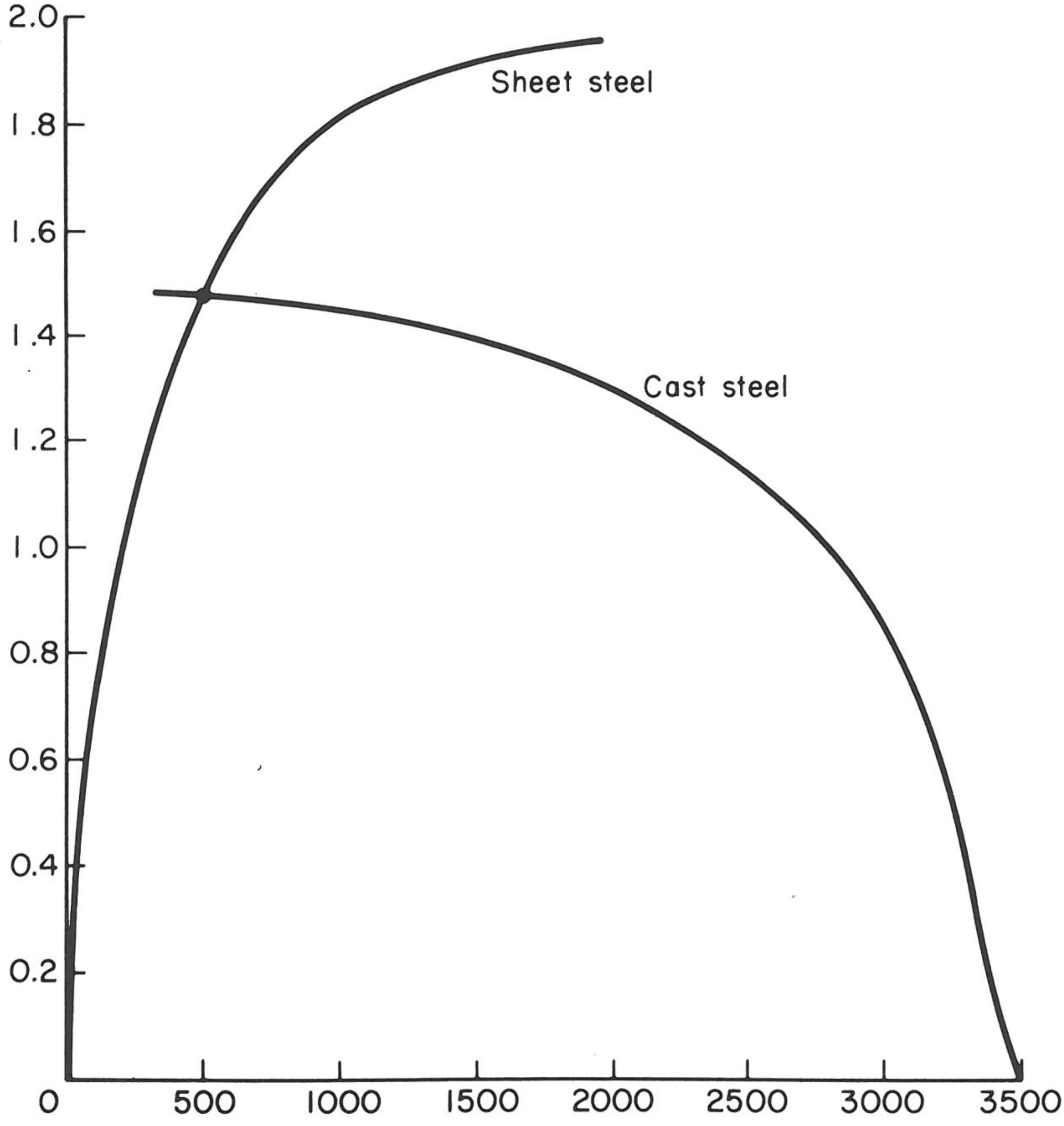

FIGURE 2.29 Solving Example 2.7 for $\mathcal{F} = 1400$ At.

Question 4

We will need the air-gap characteristic. Since the area of the structure is uniform ($A_i = A_g$), we need only work with flux densities.

$$\mathcal{F}_g = H_g l_g = \frac{B_g l_g}{\mu_0}$$

$$= \frac{B_g(0.4\pi \times 10^{-3})}{4\pi \times 10^{-7}}$$

Thus

$$\mathcal{F}_g = 1000 B_g$$

As a result, the air-gap line has a slope of 1000 At/T, or alternatively for $B_g = 1$ T, we have $\mathcal{F}_g = 1000$ At.

The H scale of the **B–H** characteristic of cast steel is converted to $\mathcal{F}$ scale by multiplying by the iron path length, which is 0.8 m. Thus a 3000 At/m is converted to 2400 At on the $\mathcal{F}$ scale as shown in Figure 2.31. From the construction, we find the flux density as

$$B \cong 1.27 \text{ T}$$

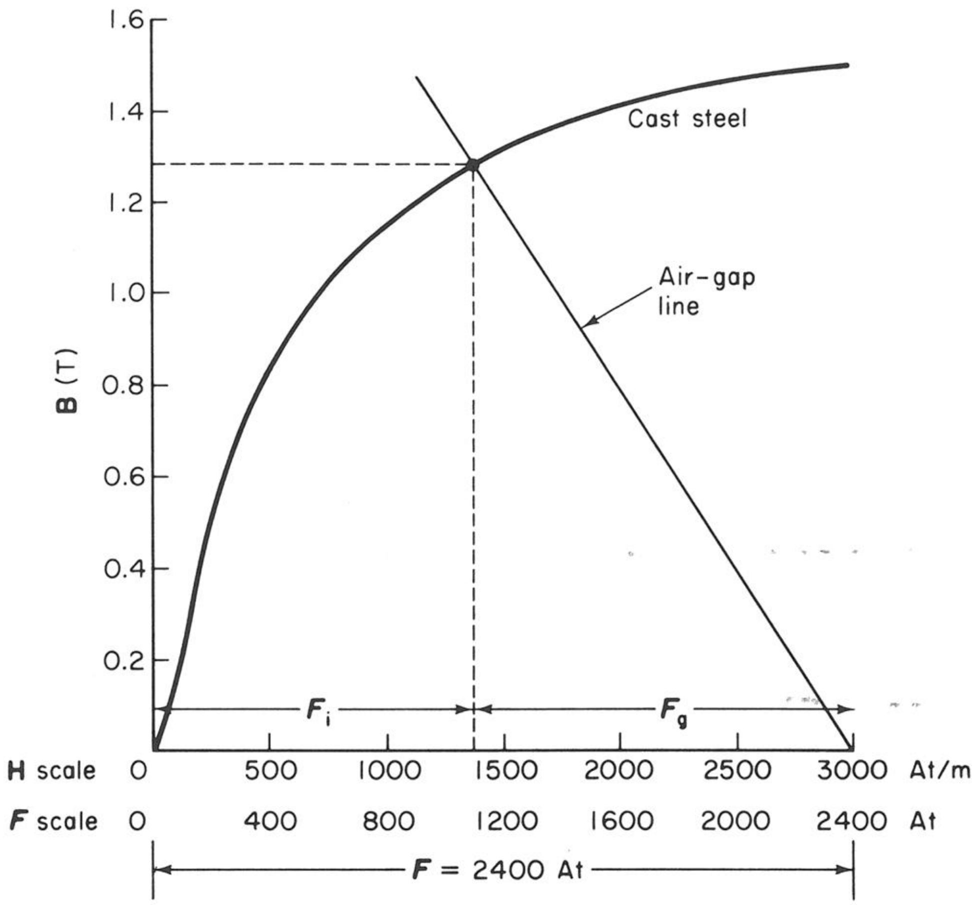

FIGURE 2.31 Construction for solving Example 2.8.