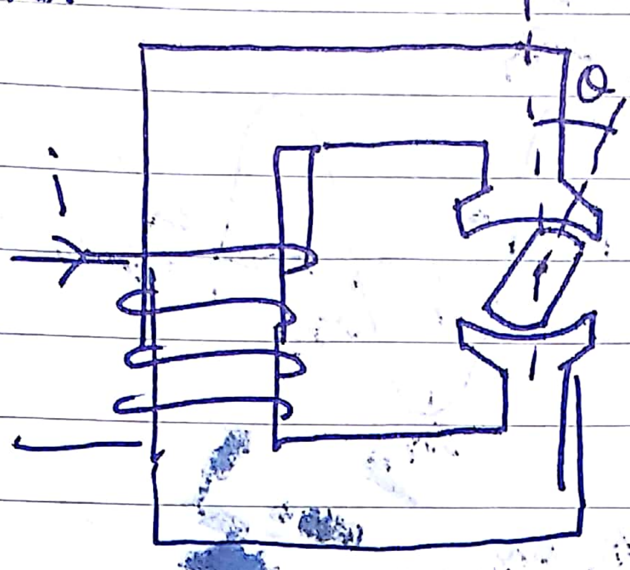
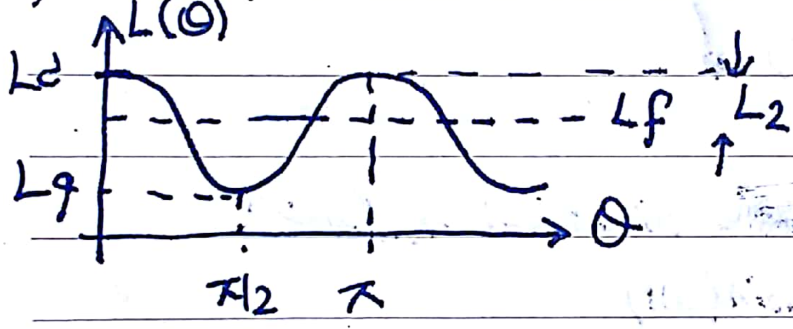
Ex.



Singly-excited reluctance motor.
$\curvearrowright T$

i) Estimate the inductance of the system as a function of θ



$L(\theta) = L_f + L_2 \cos(2\theta)$

$L_f + L_2 = L_d$ (max induct)
$L_f - L_2 = L_q$ (min induct

ii) Estimate the torque with constant DC current I_m

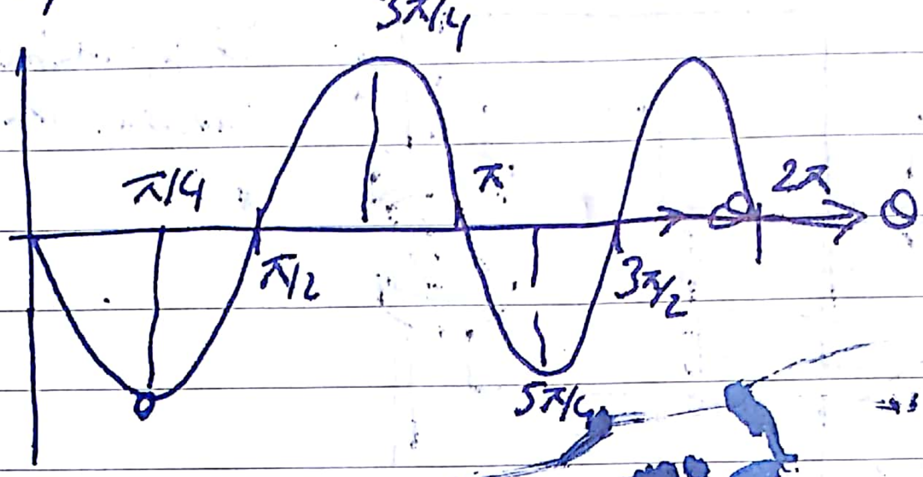
$$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$$

$$T = \frac{1}{2} I_m^2 \cdot \frac{d}{d\theta}\left(L_f + L_2 \cos(2\theta)\right)$$

$$T = -\frac{1}{2} I_m^2 \cdot L_2 \sin(2\theta) \cdot 2$$

$$= -I_m^2 L_2 \sin(2\theta)$$

Torque with constant current



$T \rightarrow max$ @ $\theta = \pi/4, 3\pi/4 \ldots$
$-\pi/4$

What about Tav?

↳ $T_{av} = 0$

Case II) Assume sinusoidal excitation:

$i(t) = I_m . \sin(\omega t)$

$$T = \frac{1}{2} . i^2 \frac{d L(\theta)}{d\theta}$$

$$= \frac{1}{2} I_m^2 . \sin^2(\omega t) . \frac{d}{d\theta}(L_f + L_2 \cos(2\theta))$$

$$= -I_m^2 L_2 . \sin(2\theta) . \sin^2(\omega t).$$

Assume the rotor is initially rotating with ω_m
mechanical speed.

$\theta = W_{mech} . t$ $\theta = \omega_m . t.$

$$T = -I_m^2 . L_2 . \sin^2(\omega t) . \sin(2\omega_m t)$$

(↳ google docs).

if $\omega_m = \omega \Rightarrow T_{av} = 0$

Case iii)

Rotor is rotating with ω_m, but there is a phase difference between electrical excitation and mechanical rotation.

$$\theta = \omega_m . t - \delta$$

$$I = I_m . \sin(\omega t)$$

$$T = \frac{1}{2} (I_m . \sin(\omega t))^2 . \frac{dL(\theta)}{d\theta}$$

$$= - I_m^2 . L_2 \sin(2\theta) . \sin^2(\omega t)$$

$$\theta = \omega_m t - \delta$$

$$T = -I_m^2 . L_2 . \sin(2(\omega_m t - \delta)) . \sin^2(\omega t)$$

mech. freq.   electrical freq.

$$\sin^2(A) = \frac{1}{2}(1 - \cos(2A))$$

$$T = -I_m^2 L_2 \sin(2(\omega_m t - \delta)) . \frac{1}{2} . (1 - \cos(2\omega t))$$

$$= \frac{-I_m^2}{2} L_2 \left[\sin(2(\omega t - \delta)) - \sin(2(\omega t - \delta)) . \cos(2\omega t) \right]$$

$$\sin A . \cos B = \frac{1}{2} \left[\sin(A+B) + \sin(A-B) \right]$$

$$T = \frac{-1}{2} I_m^2 L_2 \left[\sin(2(\omega t - \delta)) - \frac{1}{2} \left\{ \begin{array}{l} \sin 2(\omega_m t + \omega t - \delta) \\ + \sin 2(\omega_m t - \omega t - \delta) \end{array} \right\} \right]$$

if $\omega_m \neq \omega$ then $T_{av} = 0$

but if $\underline{\omega_m = \omega}$

$$T = -\frac{1}{2} I_m^2 L_2 \left[\sin(2(\omega_m t - \delta) - \frac{1}{2} \left\{ \begin{array}{l} \sin 2(2\omega t - \delta) \\ + \sin 2(-\delta) \end{array} \right\} \right]$$

$$\text{Torque} = -\frac{1}{2} \phi^2 \frac{dR(\theta)}{d\theta} \Big|_{\phi = \text{const.}}$$

$$= \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \Big|_{i = \text{const.}}$$
