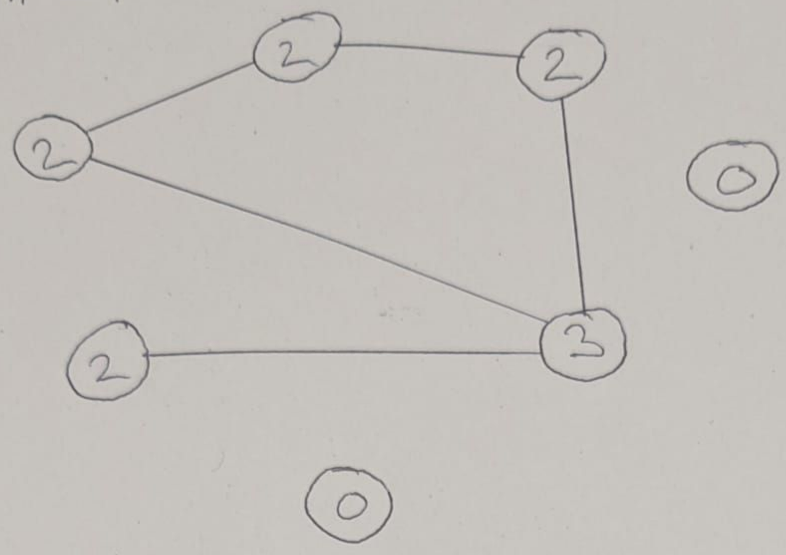
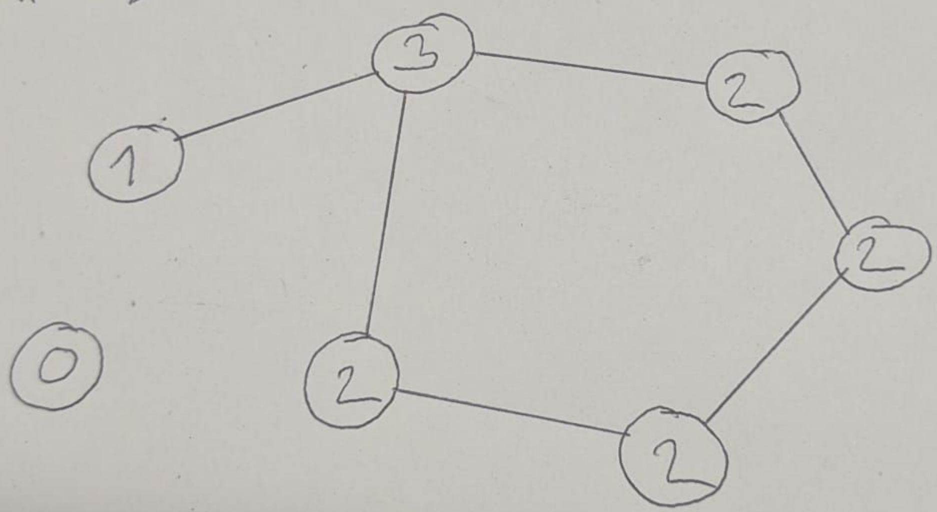
Problem 3

a) Graph G_1:



Graph G_2:



b) Iteration 1: Relabling each node with their respective multiset (Here in order of the nodes in the adjacency matrix):

Graph G_1: 2,23 ; 2,22 ; 2,23 ; 0 ; 3,122 ; 0 ; 1,3

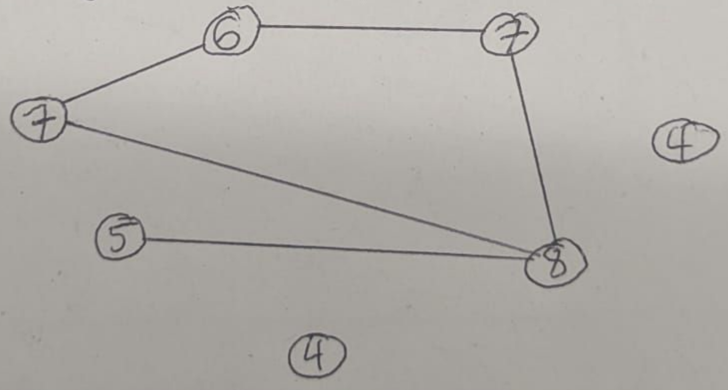
G_2: 1,3 ; 3,122 ; 2,23 ; 2,22 ; 2,22 ; 2,23 ; 0

Mapping each label to a new label:

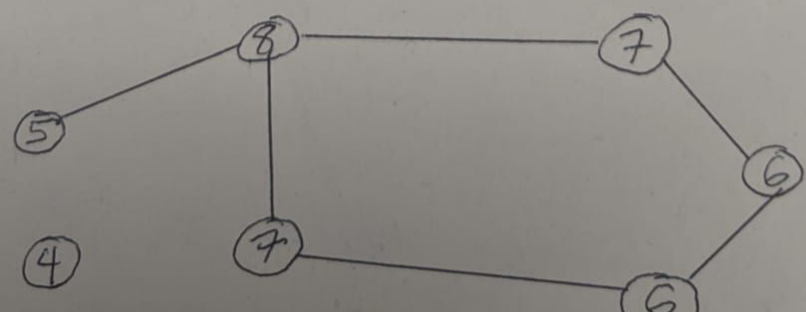
0 → 4
1,3 → 5
2,22 → 6
2,23 → 7
3,122 → 8

Relabeling the graphs:

G_1:



G_2:



Iteration 2: Relabeling each node with their multiset:

G_1: 7,68 ; 6,77 ; 7,68 ; 4 ; 8,577 ;
4 ; 5,8

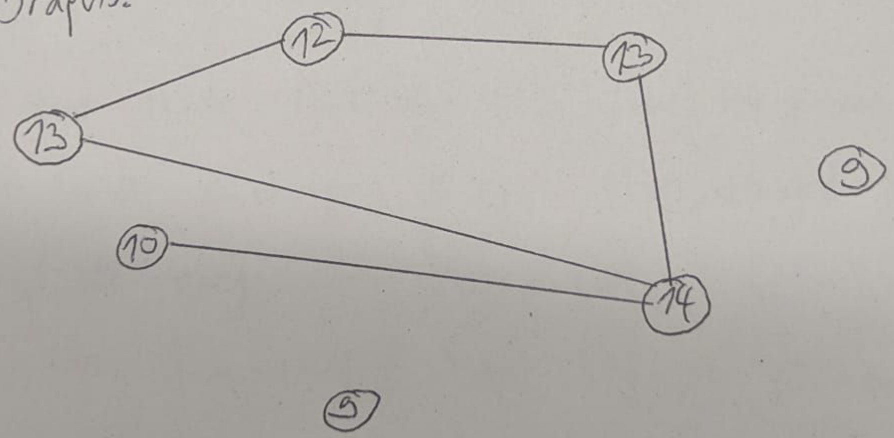
G_2: 5,8 ; 8,577 ; 7,68 ; 6,67 ; 6,67 ;
7,68 ; 4

Mapping multiset labels to new labels:

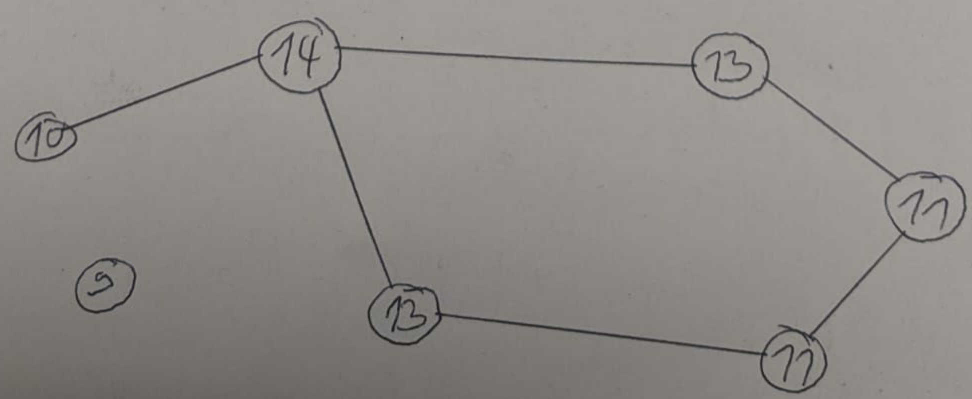
4 → 9	6,77 → 12
5,8 → 10	7,68 → 13
6,67 → 11	8,577 → 14

Relabeled Graphs:

G_1:

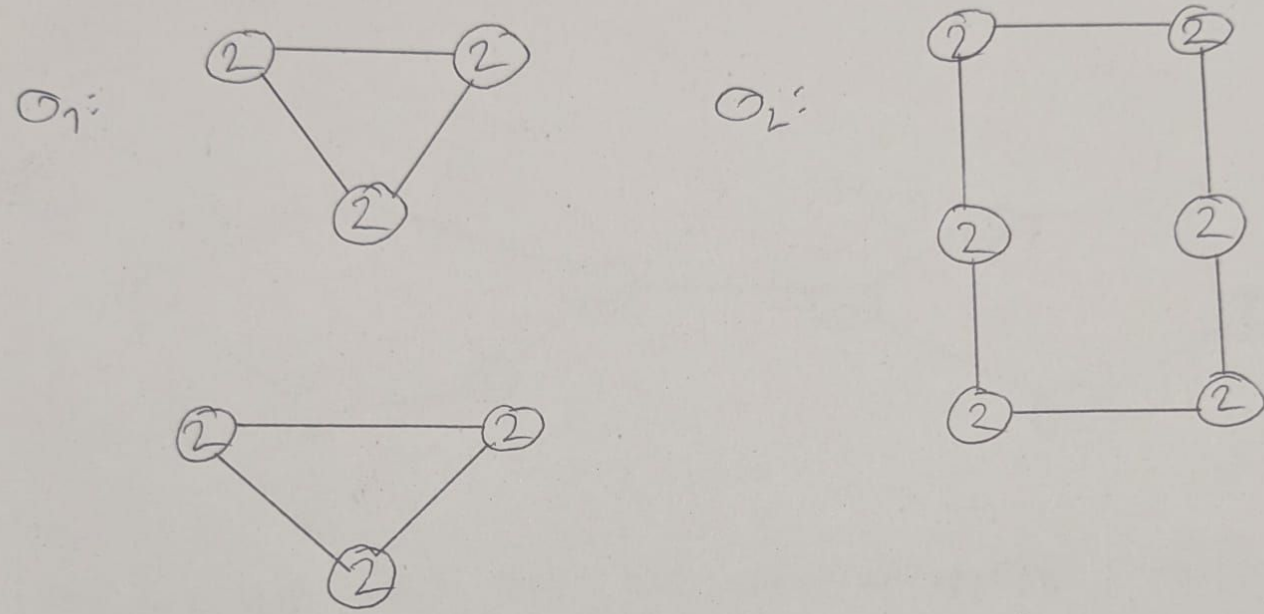


G_2:



The algorithm terminates after iteration 2, since the graphs have a different set of labels $\Rightarrow G_1$ and G_2 are not an isomorphism.

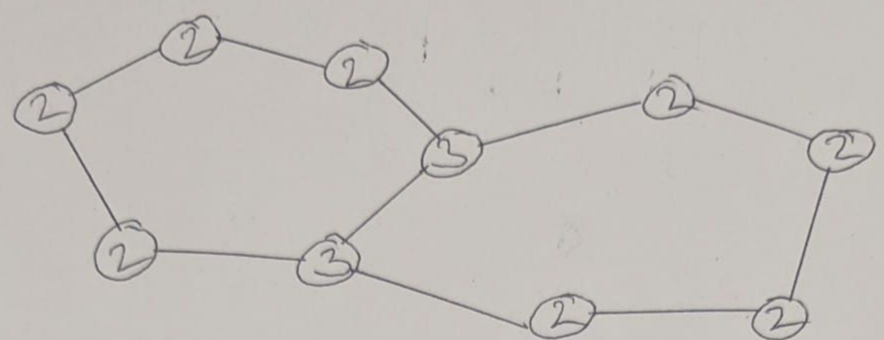
c) Counterexample 1:



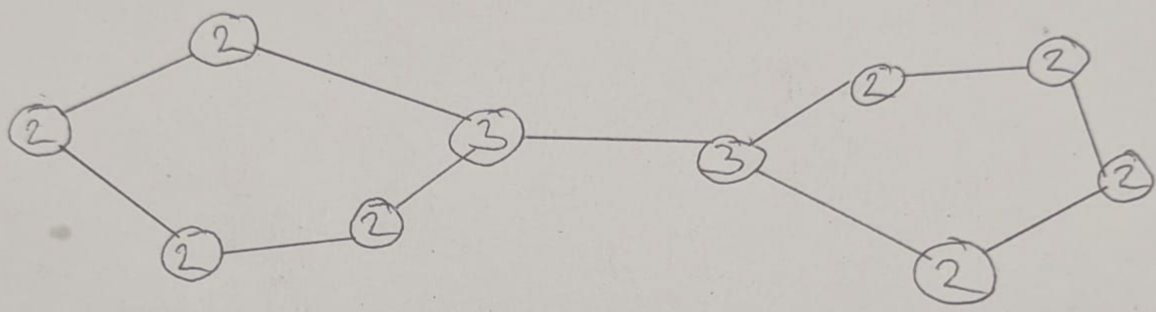
Since every node has exactly two neighbours, the algorithm will always assign the exact same multisets / relabeling and thus run indefinitely. However, since G_1 is disconnected and G_2 is connected, they are not an isomorphism.

Counter example 2

G_1:



G_2:



Here G_1 and G_2 are not an isomorphism, diffrent to counter example 1, both Graphs are connected graphs. Here the algorithm will also run infinitely, Since it will never assign diffrent multisets / relabeling to the diffrent graphs.