PS

1) Trace the simplex method on following linear programming problem

maximize $3x + y$

subject to $-x + y \leq 1$

$2x + y \leq 4$

$x \geq 0, y \geq 0$

Start with transforming the problem into standard form:

maximize $3x + y$

subject to $-x + y + u = 1$

$2x + y + v = 4$

$x, y, u, v \geq 0$

Create simplex tableau.

$$\begin{array}{c|cccc|c} & x & y & u & v & \\ \hline u & -1 & 1 & 1 & 0 & 1 \\ v & 2 & 1 & 0 & 1 & 4 \\ \hline & -3 & -1 & 0 & 0 & 0 \end{array}$$

~~pivot~~ select smallest negative

Since we want to maximize, we will select the minimum negative value on the last row.

Right Hand Side values.

	x	y	u	v	
u	-1	1	1	0	1
← v	2 (pivot)	1	0	1	4
	-3	-1	0	0	0

Now calculate the ratios of Right Hand Side values / selected column values. Find the smallest nonnegative ratio among them. That is the pivot.

$1/-1 = -1$

$4/2 = 2$ ⇒ This value is the pivot

	x	y	u	v	
u	-1	1	1	0	1
← v	2 (pivot)	1	0	1	4
	-3	-1	0	0	0

→ Update this row by dividing all values by pivot value.

You will also need to update other rows of the table. Use following equation for updating the other rows.

$$\text{new row} = \underbrace{\text{old row}}_{a} - \underbrace{\text{pivot column value} \times \text{new pivot row}}_{b}$$

old tableau ⇒

	x	y	u	v	
u	-1	1	1	0	1
v	2	1	0	1	4
	-3	-1	0	0	0

new tableau ⇒

	x	y	u	v	
u					
x	1	1/2	0	1/2	2

→ new pivot row

old row ⇒ $\boxed{-3 \quad -1 \quad 0 \quad 0 \quad 0}$ a

pivot column value ⇒ -3

pivot column value × new pivot row ⇒ $-3\times 1 \quad -3\times\frac{1}{2} \quad -3\times 0 \quad -3\times\frac{1}{2} \quad -3\times 2$

$\boxed{-3 \quad -3/2 \quad 0 \quad -3/2 \quad -6}$ b

new row ⇒ $-3-(-3) \quad -1-(-3/2) \quad 0-0 \quad 0-(-3/2) \quad 0-(-6)$

$\boxed{0 \quad 1/2 \quad 0 \quad 3/2 \quad 6}$

new tableau ⇒

	x	y	u	v	
u					
x	1	1/2	0	1/2	2
	0	1/2	0	3/2	6

Apply the same logic to find the new values of the first row too.

$$\text{new row} = \underbrace{\text{old row}}_{a} - \underbrace{\text{pivot column value} \times \text{new pivot row}}_{b}$$

old row $\Rightarrow$ $\underbrace{\boxed{-1 \quad 1 \quad 1 \quad 0 \quad 1}}_{a}$

pivot column value $\Rightarrow -1$

pivot column value $\times$ new pivot row $\Big\}$ $-1\times 1 \quad -1\times\frac{1}{2} \quad -1\times 0 \quad -1\times\frac{1}{2} \quad -1\times 2$

$$\underbrace{\boxed{-1 \quad -\tfrac{1}{2} \quad 0 \quad -\tfrac{1}{2} \quad -2}}_{b}$$

new row $\Rightarrow$ $-1-(-1) \quad 1-(-\frac{1}{2}) \quad 1-0 \quad 0-(-\frac{1}{2}) \quad 1-(-2)$

$$\boxed{0 \quad \tfrac{3}{2} \quad 1 \quad \tfrac{1}{2} \quad 3}$$

new tableau $\Rightarrow$

	x	y	u	v	
u	0	$\frac{3}{2}$	1	$\frac{1}{2}$	3
x	1	$\frac{1}{2}$	0	$\frac{1}{2}$	2
	0	$\frac{1}{2}$	0	$\frac{3}{2}$	6

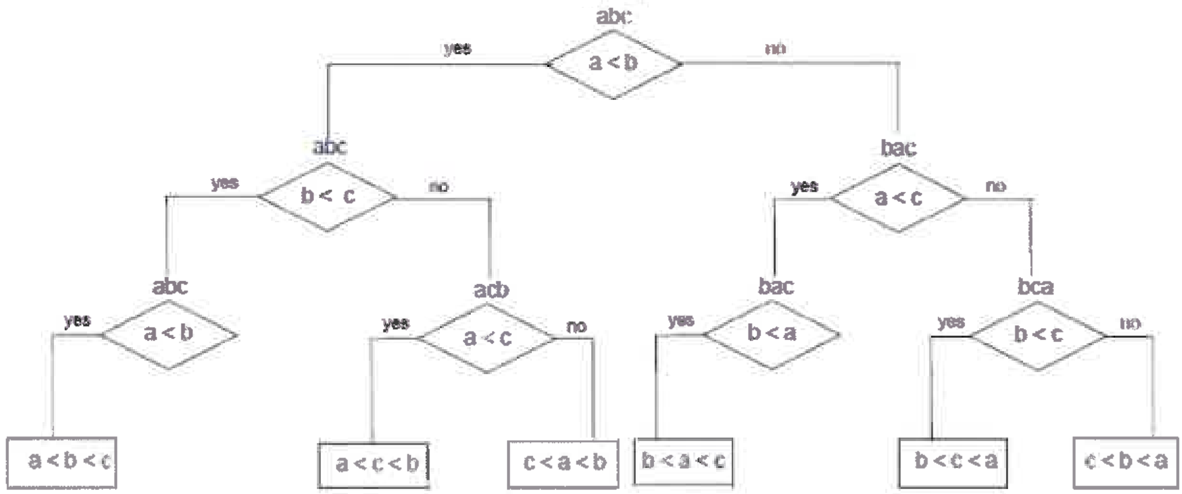
If the last row of your tableau has no negative value, then you found the optimal solution. If you still have negative values, you need to apply the whole process to the new tableau until you reach optimal solution. Since we reached to optimal solution ~~at~~ the first iteration, we stop here.

The values for optimal solution is $x=2$ and $y=0$ with the maximal value 6.

comes from right hand side part of the tableau.

2) Draw a decision tree and find the number of key comparisons in the worst and average cases for the three-element basic bubble sort.

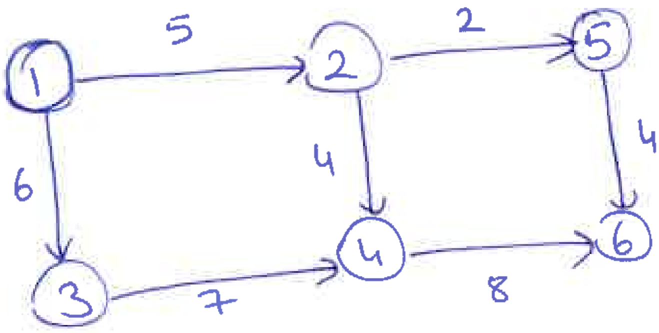
a. Here is a decision tree for sorting an array of three distinct elements a, b, and c by basic bubble sort:

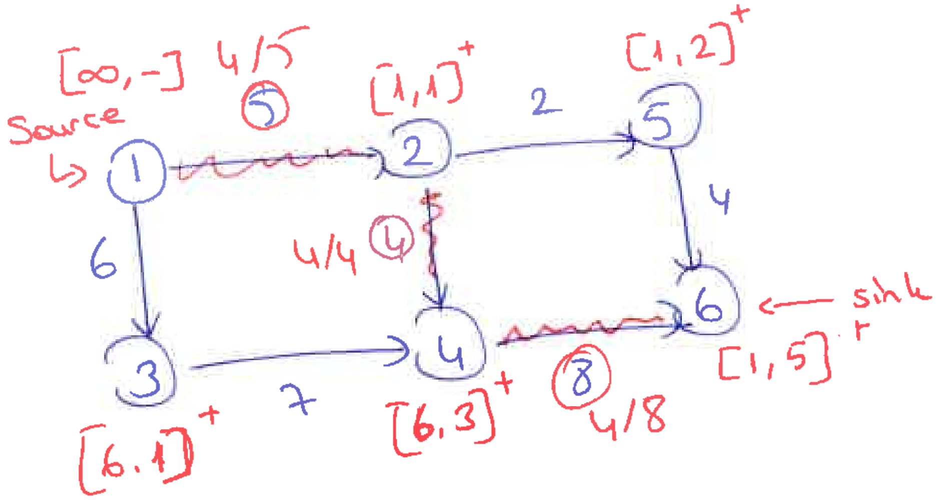


The algorithm makes exactly three comparisons on any of its inputs.

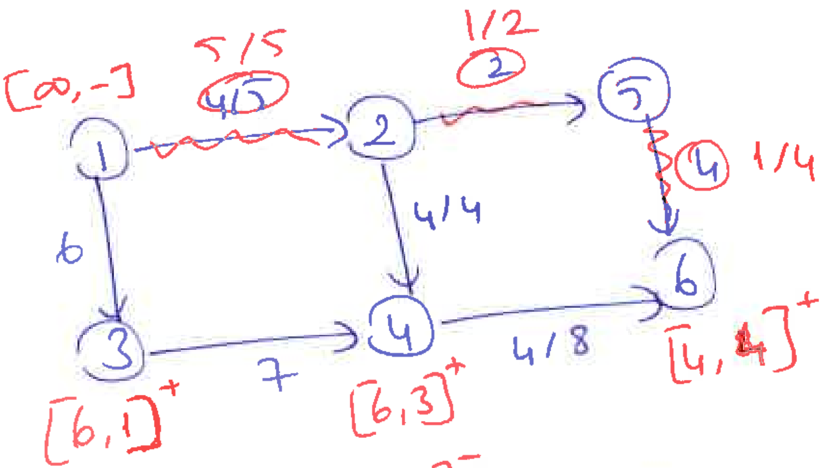
PS

① Apply shortest-augmenting path algorithm to find a maximum flow network.

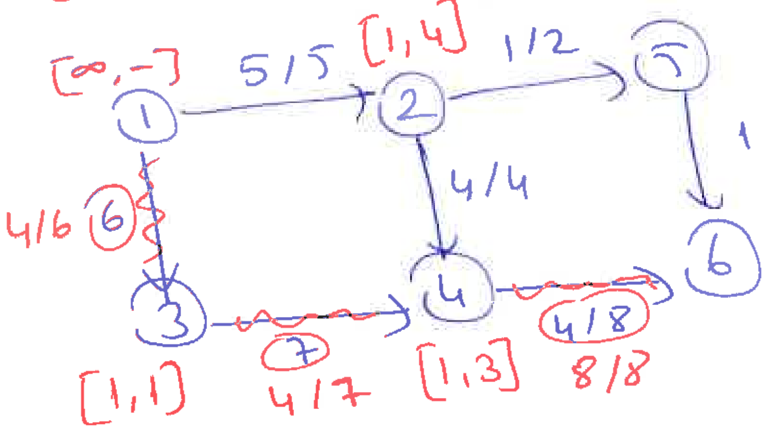




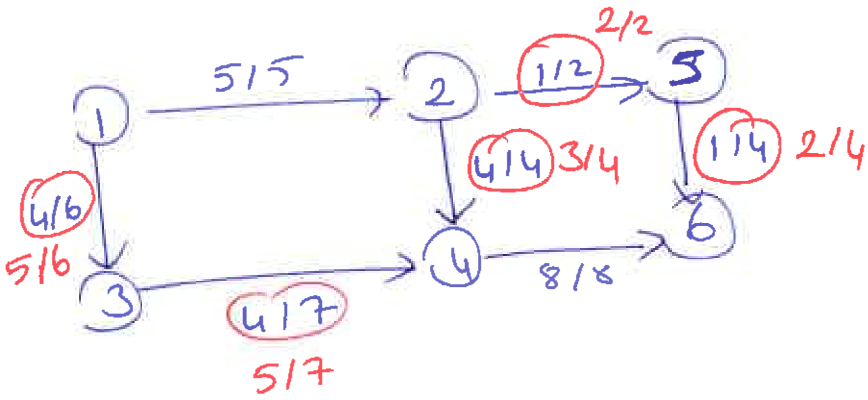
minimum capacity = 4

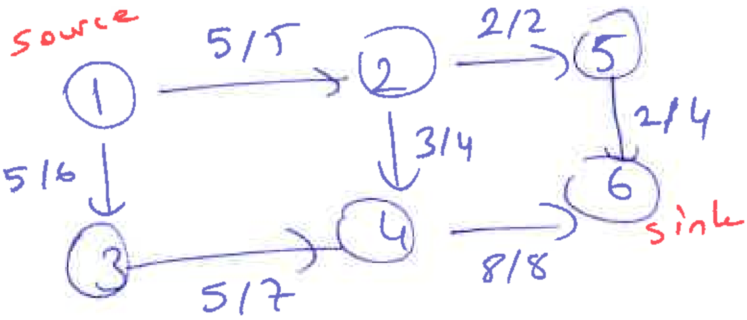


minimum capacity = 1



minimum capacity = 4





Maximum flow $2+8=10$