Supplementary information on the computation of the bounds

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This document provides supplemental information to the source code available in `https://github.com/OguzKislal/ErrorDetection_InfoTheory` for the manuscript "Undetected Error Probability in the Short Blocklength Regime: Approaching Finite-Blocklength Bounds with Polar Codes". Specifically, we detail how the two bounds reported in the paper are simulated.

For the block-memoryless phase-noise channel (see Section V.B of the paper), we evaluate numerically the information-theoretic bounds provided in Theorem 2 (see `DeltaBitMethod_QPSK.m`) and Theorem 3 (see `ThrMethod_QPSK.m`) as follows:[1]

- For a given $\mathbf{x}$ and $\mathbf{y}$, we search for a parameter ζ^* satisfying $\left|\left(\gamma^{(n)}(\zeta^*)\right)' - \omega\right| \leq 10^{-3}$. Note that ω here depends on $\mathbf{x}$ and $\mathbf{y}$, and takes different values depending on which bound we want to evaluate.
- We then evaluate the CGF $\gamma^{(n)}(\zeta)$ and its first two derivatives, $\left(\gamma^{(n)}(\zeta)\right)'$ and $\left(\gamma^{(n)}(\zeta)\right)''$, for $\zeta = \zeta^*$ using the closed-form expressions given in (26)–(28).
- Then, we compute the pairwise error probability using the saddlepoint approximation provided in Theorem 4.
- Finally, we use this result to evaluate the bounds in (12) and (17). These bounds are estimated via a Monte Carlo simulation, which is used to approximate the outer expectations in the two bounds. This involves repeating the previous steps for each new value of $\mathbf{x}$ and $\mathbf{y}$.

For the BiAWGN case, we leverage the efficient saddlepoint method proposed in [R1]. This method, which is not applicable to the case of block-memoryless phase-noise channel, avoids the time-consuming steps of searching for the optimal ζ^* at each Monte-Carlo iteration (the first step in our procedure) and reduces the simulation time significantly. For the evaluation of the bound presented in Theorem 2, we further rely on a second saddlepoint approximation proposed in [R1] to further reduce the simulation time. For details on the implementation, we refer the interested reader to [R1]. We have also updated footnote 9 to clarify this point.

References

[R1] J. Font-Segura, G. Vazquez-Vilar, A. Martinez, A. Guillén i Fàbregas and A. Lancho, "Saddlepoint approximations of lower and upper bounds to the error probability in channel coding," *52nd Annual Conference on Information Sciences and Systems (CISS)*, Princeton, NJ, USA, 2018.

[1] We use the notation introduced in Section III of the paper.