1 Iterative Solvers

To minimize the error produced in the grid search for improperly chosen *tps* we find optimal α, β and crustal thickness h which define the minimal residual curve.

$$Tps = h\left(\sqrt{\frac{1}{\beta^2} - p^2} - \sqrt{\frac{1}{\alpha^2} - p^2}\right) \tag{1}$$

1.1 Newton's Method

To find the minimum we seek to minize the sum of the squares of the residual $r = (t_i - f(\mathbf{x}))$ where $f(\mathbf{x}) = Tps$ as in *1)* and $\mathbf{x} = [\alpha, \beta, h]^T$. Thus we seek

$$\min |r^T r| = \min |\phi| \tag{2}$$

which is true when $\frac{d\phi}{d\mathbf{x}} = g(\mathbf{x}^*) = 0$. The solution $\mathbf{x}^*$ is the initial guess $\mathbf{x}_0+$ the vector $\mathbf{m}$ which finds the minimum $g(\mathbf{x}^*) = 0$. Taking the Taylor approximation gives,

$$g(\mathbf{x} + \mathbf{m}) = g(\mathbf{x}) + \mathbf{Hm} + O(\|mathbfm\|) \tag{3}$$

The hessian H is equal to $\nabla^2\phi$ which is given by $\sum_{i,j=1}^{n} \frac{\partial^2\phi}{\partial x_i \partial x_j}$. As $g(\mathbf{x}+\mathbf{m}) = 0$ we can rewrite *3)* as

$$\mathbf{m} = -\mathbf{H}^{-}1g(mathbfx) \tag{4}$$

and iterate until convergence with

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{m} \tag{5}$$