APPLIED GEOPHYSICS, Vol.2, No.2 (June 2005)

The Second-generation Wavelet Transform and its Application in Denoising of Seismic Data

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Abstract: This paper discusses the principle and procedures of the second-generation wavelet transform and its application to the denoising of seismic data. Based on lifting steps, it is a flexible wavelet construction method using linear and nonlinear spatial prediction and operators to implement the wavelet transform and to make it reversible. The lifting scheme transform includes three steps: split, predict, and update. Deslauriers-Dubuc (4, 2) wavelet transforms are used to process both synthetic and real data in our second-generation wavelet transform. The processing results show that random noise is effectively suppressed and the signal to noise ratio improves remarkably. The lifting wavelet transform is an efficient algorithm.
Keywords: wavelet transform, second-generation, and denoise.

The traditional wavelet transform methods have been applied in many fields, such as signal and image processing. Sweldens (1994) proposed another wavelet transform based on lifting steps. It is a flexible wavelet construction method by using linear and nonlinear spatial prediction and operators to implement the wavelet transform and to make it reversible. The lifting scheme is independent of the Fourier transform and is called the second-generation wavelet transform. It has the following features: (1) keeps the same character as first-generation wavelet transforms; (2) is independent of the Fourier transform; (3) is a computationally fast algorithm; (4) is convenient for reverse transform; and (5) makes non-linear wavelet transforms possible, e.g. an integer to integer wavelet transforms (Sweldens, 1997; Donoho, 1995; Sweldens, 1998).

De-noising by second-generation wavelet transform is divided into three steps: wavelet decomposition (split), wavelet coefficient computation (predict), and data reconstruction (update). The common wavelet de-noising methods include soft-threshold and hard-threshold methods. The soft-threshold method is used in this paper. The Deslauriers-Dubuc wavelet (4, 2) is used in the wavelet transform.

Based on the transform methods above, we conduct three levels of lifting-step reverse transforms for examples of synthetic data with noises and real seismic data. At each level, we use the soft threshold method to compute wavelet coefficients, and then reconstruct the data. As a consequence, the noise can be reduced significantly.

The results show that the wavelet transform algorithm based on the lifting scheme is flexible in application, fast in computation, and efficient in denoising.

Basic principles of the second-generation wavelet transform

Assume that the original signals c_j are decomposed into low-frequency signals c_{j-1} and high-frequency detail signals d_{j-1}. The transformation by the lifting scheme includes three steps: split, predict, and update (Zhang Xueying, et al, 2004; Feng Lin, et al, 2004).

The split step

The original signals c_j are split into two non-intersecting subsets c_{j-1} and d_{j-1}. The greater the correlation between them, the better the split effect is. Usually, a

Manuscript received by the Editor March 13, 2005; revised manuscript received April 13, 2005.
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signal sequence is split into odd and even sequences as

$$c_{j-1} = c[2n], \quad d_{j-1} = c[2n-1]. \tag{1}$$

In terms of the lifting algorithm, the split method looks simple but smart. It provides the basis for the following two steps of predict and update with the local similarity of signals c_j.

The predict step

Using the similarity of data, we can predict d_{j-1} from c_{j-1} by using a predict operator P which is independent of the dataset. The resulting difference is called the wavelet coefficients $d[n]$ and represents the closeness of the two data sequences. If the prediction is good, the difference dataset contains much less information than in the original subset. The predict process can be described by

$$d_{j-1} = c[2n+1] - p(c_{j-1}) = d_{j-1} - p(c_{j-1}). \tag{2}$$

The update step

Some characteristics of the coefficient subset c_{j-1} derived from the above two steps are inconsistent with that of the original dataset. Therefore, the update step is necessary. We use an update operator U to generate a better subset $c[n]$ to keep some characteristics of the original dataset, that is,

$$c_{j-1} = c[2n] + u(d_{j-1}) = c_{j-1} + p(d_{j-1}) \ . \tag{3}$$

The above three steps constitute a lifting scheme. With iteratively lifting, we can obtain the approximate signals c_{j-n} and the high-frequency detail signals d_{j-n}. After n times of decomposition, the wavelet transform of the original data can be described by $\{c_{j-n}, d_{j-n}, d_{j-n+1}, \cdots, d_{j-1}\}$. In fact, each wavelet can be divided into a series of lifting schemes. Generally, the predict step is aimed at reducing the low-order polynomial components and at preserving the high-order detail signals. The update step is designed to preserve the low-order polynomial components at coarse scales.

By changing operation orders and signs, we can easily get the reconstruction formulae as follows,

$$\begin{aligned} c_{j-1} &= c_{j-1} - p(d_{j-1}), \\ d_{j-1} &= d_{j-1} + p(s_{j-1}). \end{aligned} \tag{4}$$

Based on the above algorithm, the complete lifting wavelet decomposition and reconstruction can be performed and shown in the sketch map below (figure 1):

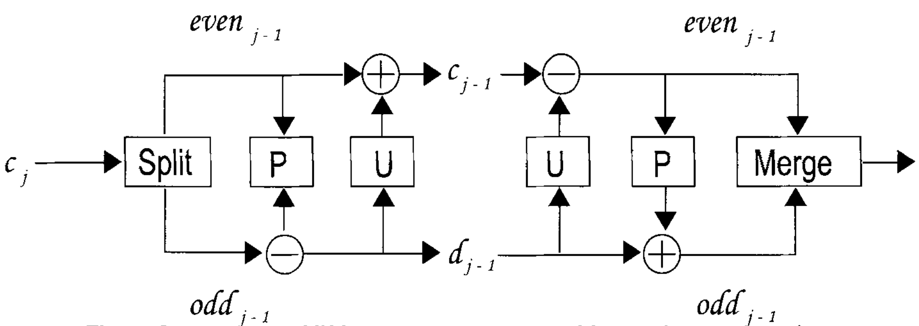

Fig.1 Sketch map of lifting wavelet decomposition and reconstruction.

The lifting scheme and Deslauriers-Dubuc (4, 2) wavelet transforms

Sweldens (1994) proved that the lifting scheme is applicable to any wavelet transform. The simplest transform is the S-transform:

$$\begin{aligned} d_{j-1} &= d_{j-1} - p(c_{j-1}), \\ c_{j-1} &= c_{j-1} + p(d_{j-1}). \end{aligned} \tag{5}$$

After S-transform, the linear wavelet transform prediction can be applied to new high-pass coefficients based on the low-pass coefficients s_{j-1}. In this paper, we use Deslauriers-Dubuc (4, 2) wavelet transforms:

Split: $\quad c_{j-1} = c[2j], d_{j-1} = c[2j+1]. \qquad (6)$

Forward transform:

$$d_{j-1,1} = c_{j,2l+1} - [9/16(c_{j,2l} + c_{j,2l+2}) - 1/16(c_{j,2l-2} + c_{j,2l+4}) + 0.5], \tag{7}$$

$$c_{j-1,1} = c_{j,2l} + 0.25(d_{j-1,l-1} + d_{j-1,1} + 0.5). \tag{8}$$

Reverse transform:

$$c_{j,21} = c_{j,2l} - 0.25(d_{j-1,l-1} + d_{j-1,1} + 0.5), \tag{9}$$

$$d_{j-1,1} = c_{j,2l+1} + [9/16(c_{j,2l} + c_{j,2l+2}) - 1/16(c_{j,2l-2} + c_{j,2l+4}) + 0.5]. \tag{10}$$

Based on these formulae, the three step reversible transform has been applied to the noise-containing signals in this paper. The seismic data signals can be de-noised using the methods described in the following sections.

The application of the second-generation wavelet transform in denoising of seismic data

Seismic data contains both signal and noise parts. There is not an absolute definition for noise. Noise represents the unneeded part of a seismic section. Land seismic data usually contains several different types of noise. In this paper we are focusing on the elimination of random noise which we presume to be Gaussian white noise.

The denoising procedure using the wavelet transform of discrete signals can be divided into three steps: wavelet decomposition, wavelet coefficient reduction (truncating the noise portion), and reducing the composition of wavelet coefficients. Currently the most popular denoising methods include the soft-threshold method and the hard-threshold method. In this paper we use the soft-threshold method:

$$d_\tau(n) = sign(d(n))\left|d(n)-\tau\right| = \begin{cases} 0, |x| \le \tau \\ d(n)-\tau, x > \tau. \\ d(n)+\tau, x < \tau \end{cases} \tag{11}$$

$$\tau = \sigma\sqrt{2\log_e^{(N)}}, \sigma = 1/0.6745Med(|d|)$$

where τ is the computation threshold, σ is the standard deviation of the noise estimation, and Med(·) is the median function.

In the denoising procedure, we first perform the lifting wavelet transform for the seismic data and then process the detail signals at each level using the soft-threshold method. As a result, the noise part in the wavelet coefficient is reduced.

Denoising of seismic records

Denoising of single-channel synthetic seismic data

The de-noising of synthetic seismic data using the lifting-based Deslauriers-Dubuc (4, 2) wavelet transform is described in this section. Figure 2 shows a noisy signal of Ricker wavelets with Gaussian white noise added. Figure 3 shows the de-noised signal using the lifting algorithm. In Figure 2, the energy of the random noise is distributed in both high and low frequency bands, while the energy of the signal is basically in the range of 0 to 100 Hz. The signal in Figure 3 has been processed so that most of the random noise has been removed. However, there are still some residual noise components left which shows the poor waveforms continuity of the seismograms.

In seismic exploration, Signal-to-noise ratio (SNR) is commonly used to measure noise intensity in seismic records. Li Qingzhong (1994) defined the visual signal-to-noise ratio (VINSR) as

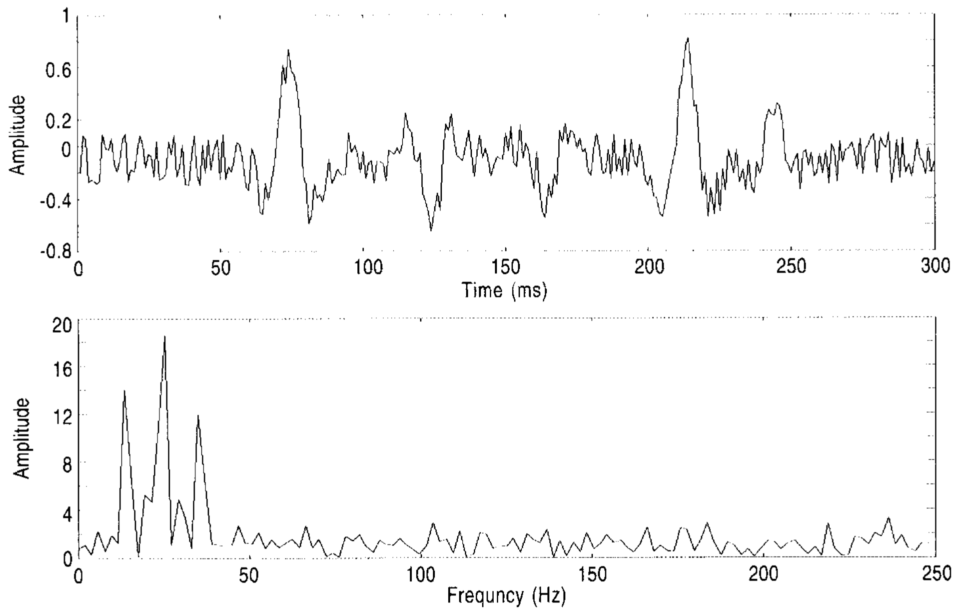

Fig. 2 One-channel seismic signal (upper) and its spectrum (lower).

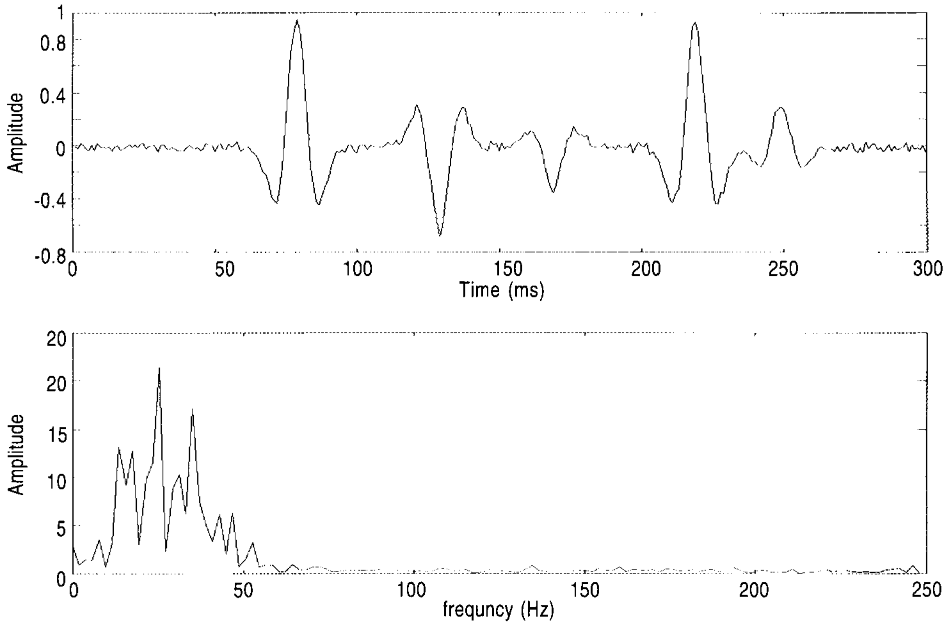

Fig. 3 The denoised seismic signal (upper) and its spectrum (lower).

$$VISNR = \frac{\sum A_X(f) R_S(f)}{\sum A_X(f) R_N(f)}, \qquad (12)$$

$$R_S(f) = \frac{A_S(f)}{A_S(f) + A_N(f)}, \qquad (13)$$

where $A_X(f)$ is the seismic data with signal and noise, i. e. the amplitude spectrum of the seismic channel, $R_S(f)$ and $R_N(f)$ are the signal ratio and the noise ratio in the frequency domain, and $A_S(f)$ and $A_N(f)$ are the signal amplitude spectrum and noise amplitude spectrum.

From the above formulae, the computed SNR of the seismic signal in Figure 2 is 2.7651and is 6.6182 after denoising in Figure 3. This suggests that the method can improve the SNR and efficiently reduce the noise.

Denoising of real seismic records

Figure 4 shows a section of seismic traces with intense random noise, poor lateral continuity, and low SNR. Figure 5 shows the section after processing with the lifting wavelet transform algorithm. It is evident that the SNR of the section significantly increases and the lateral continuity of the seismic traces improves as well. This result indicates the effectiveness of the method on real data.

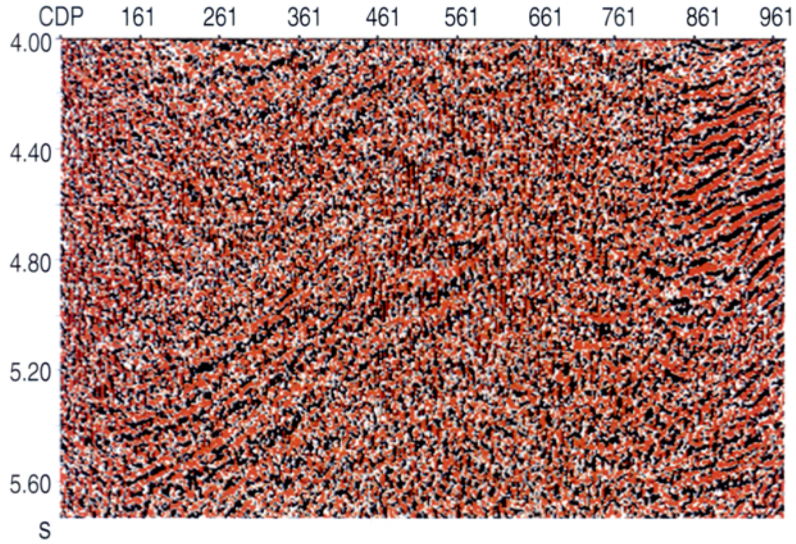

Fig.4. Seismic traces with noise.

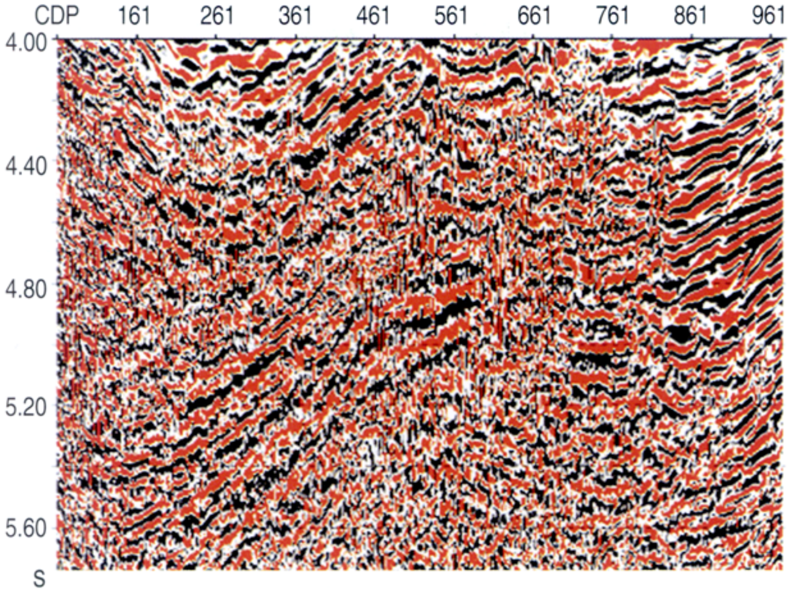

Fig.5. Seismic traces after denoising.

Conclusions

The second-generation wavelet transform is a development of the traditional waveform theory. The study on this field is tentative and it is deserving of in-depth theoretical study. In thise paper, we discuss the principles and the transformation procedures of the second-generation wavelet transform and use Deslauriers-Dubuc (4, 2) wavelet transforms to process both synthetic and real data. From the processed seismic section, it is evident that the random noise is suppressed effectively, and the signal to noise ratio improves remarkably. The results indicate that the lifting wavelet transform is an efficient algorithm. It is believed that more applications are anticipated as the study of the second-generation wavelet transform goes deeper.

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Cao Siyuan received his Ph.D. degree from the University of Petroleum in 1994. His interests are in the areas of seismic data processing, neural networks, wavelet analysis, and fractals.