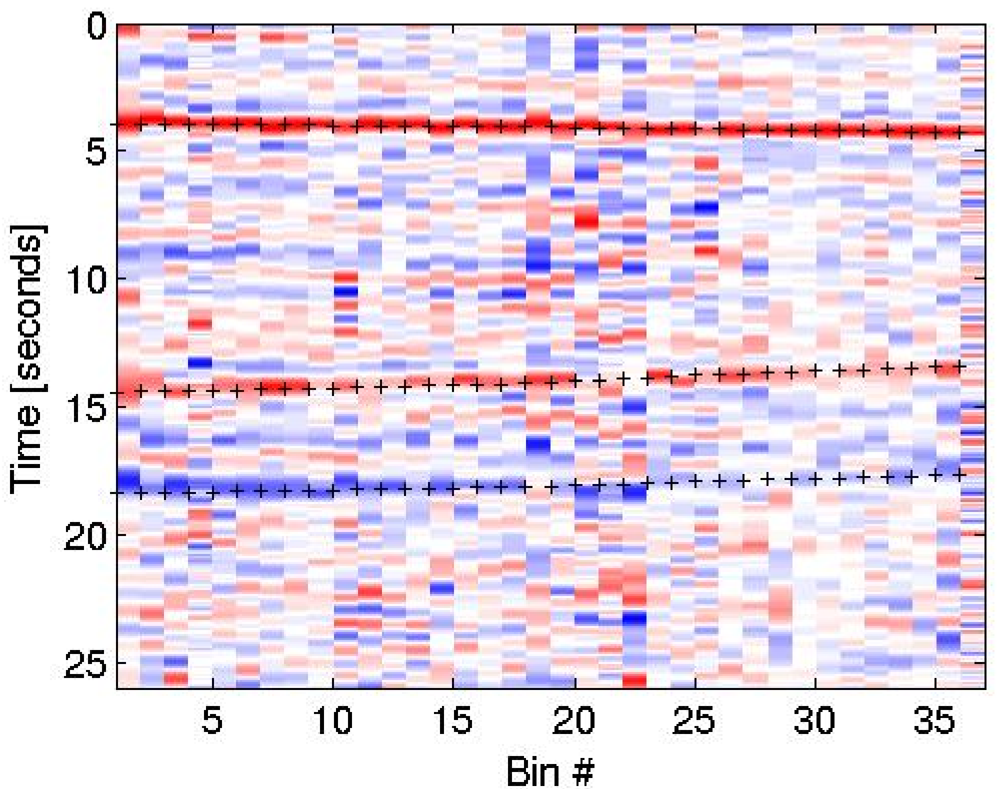
Sparsity Promoting L1 Deconvolution

application for a global seismology parameter search

Gian Matharu & Ben Postlethwaite



Project Summary

- Attempting deconvolution of a noisy approximation of a source from a seismogram.
- Wavelet transforms are employed to promote sparsity.
- Multiple sources and seismograms are solved for during each run.
- SPGL1 is used as the L1 solver.
- Comparisons are made between choices of wavelets.
- Comparison is also made against solutions from Simultaneous Deconvolution with Generalized Cross Validation regularization.

Motivation: The Problem

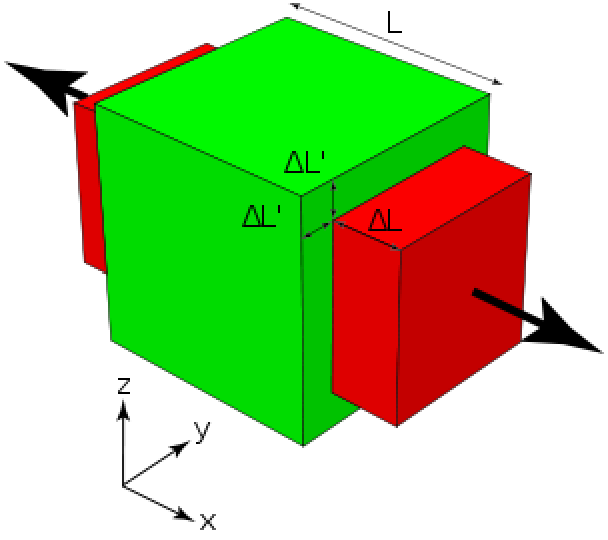
- Extracting material properties from seismic data has a long history.
- For bulk crustal composition under receivers the seismic velocity ratio

$$R=\frac{V_p}{V_s}$$

has been used.
- This provides Poisson's ratio:

$$\sigma=\frac{1}{2}\frac{\left(\frac{V_p}{V_s}\right)^2-2}{\left(\frac{V_p}{V_s}\right)^2-1}$$

which is useful for constraining crustal composition.
- Further use of seismic data to constrain crustal composition would require knowledge of explicit values of V_p or V_s.



Poisson's Ratio: It is the ratio of lengthening along one axis versus compression along the other two.

Motivation: The Problem

- Many previous studies have used a grid search for the seismic velocity ratio $R = V_p/V_s$ and crustal thickness H. Kanamori (2000)
- Some a priori information is required, namely an estimate P-wave velocity V_p.
- Here we attempt to recover V_p and seismic velocity ratio R, without any assumption on V_p.
- For accurate resolution of V_p in solution we require sharp deconvolution (close to spike train as possible).
- We try damped-L2 based deconvolution as well as sparsity promoting wavelet-based L1 minimization.

Motivation: The Problem

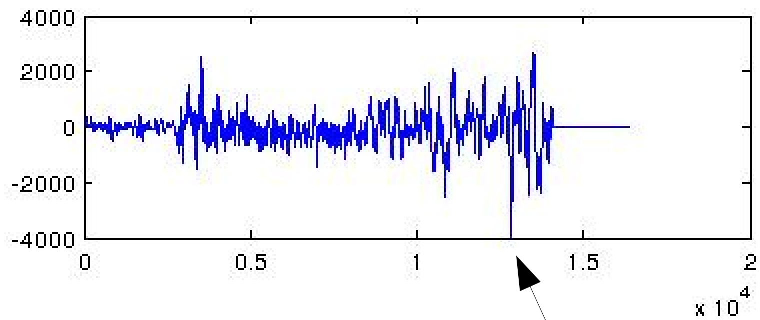
Pre-process Seismic Data

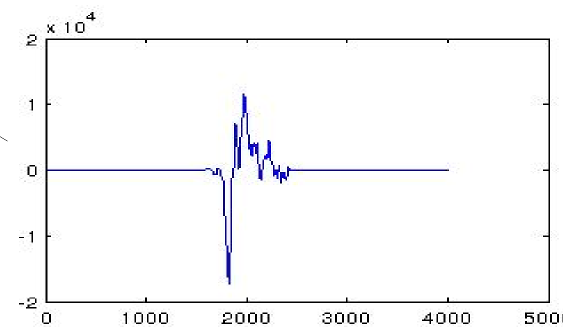
Select P-coda as source approx

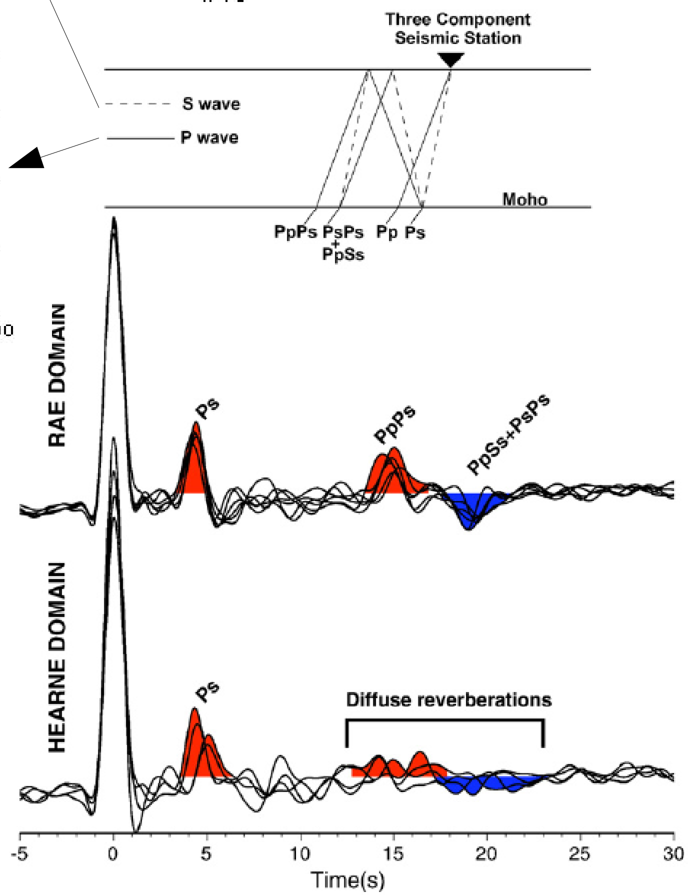
Deconvolution

Non-Linear Newton + L1 fit for first arrival time

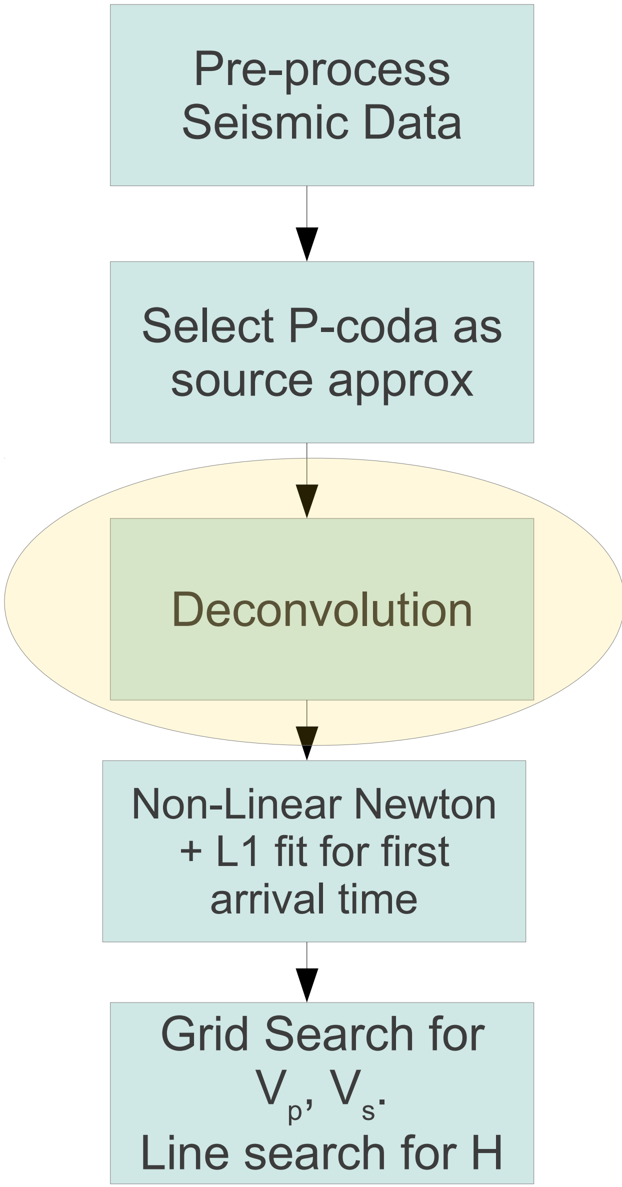
Grid Search for V_p, V_s.
Line search for H







Motivation - The Problem



- Currently using a **Simultaneous Deconvolution with Generalized Cross Validation** regularization.
- $$rec(t) = \mathrm{F}^{-1}[G(\omega)] = \mathrm{F}^{-1}\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right]$$

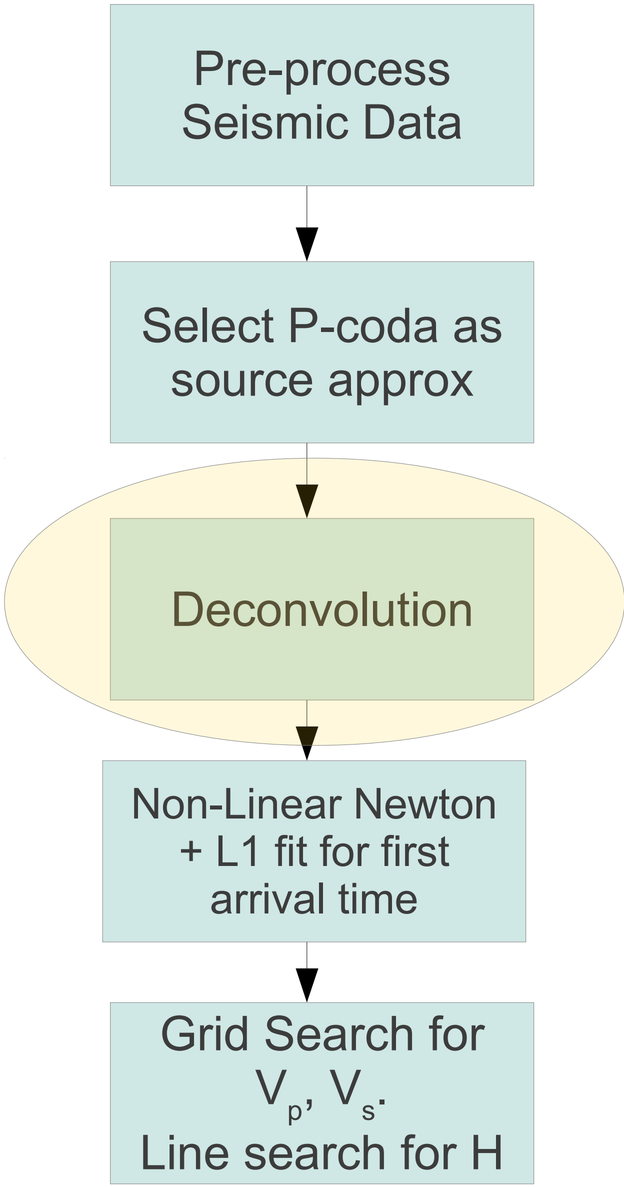
  where *F* is the Fourier transform matrix, *G* is the model, *S* is the S-wave seismogram , *P* is the P-wave source coda, * is the complex conjugate and δ is the regularization. δ is chosen by minimizing the Generalized Cross Validation function:

Misfit: more accurate with δ -> 0. Less stable.

$$min \left(\frac{\sum_n^N \sum_m^M \left(S_n(\omega_m) - P_n(\omega_m)\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right]\right)^2}{\left(NM - \sum_m^M \frac{\sum_m^M P_n(\omega) P_n^*(\omega)}{\sum_m^M P_n(\omega) P_n^*(\omega) + \delta}\right)^2} \right) w.r.t\ \delta$$

As δ increases denominator increases, promotes stability.

Motivation - The Problem



- Currently using a **Simultaneous Deconvolution with Generalized Cross Validation** regularization.
- $$rec(t) = \mathrm{F}^{-1}[G(\omega)] = \mathrm{F}^{-1}\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right]$$

  where F is the Fourier transform matrix, G is the model, S is the S-wave seismogram , P is the P-wave source coda, * is the complex conjugate and δ is the regularization. δ is chosen by minimizing the Generalized Cross Validation function:

Misfit: more accurate with δ -> 0. Less stable.

$$min\left(\frac{\sum_n^N \sum_m^M \left(S_n(\omega_m) - P_n(\omega_m)\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right]\right)^2}{\left(NM - \sum_m^M \frac{\sum_m^M P_n(\omega) P_n^*(\omega)}{\sum_m^M P_n(\omega) P_n^*(\omega) + \delta}\right)^2}\right) w.r.t\ \delta$$

As δ increases denominator increases, promotes stability.

Motivation: The Problem

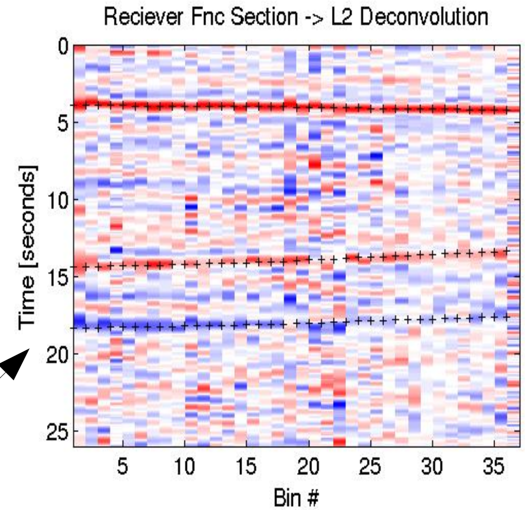
Pre-process Seismic Data

Select P-coda as source approx

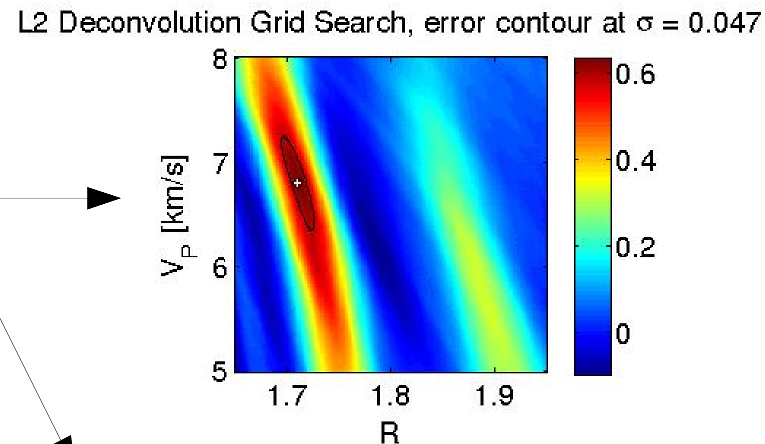
Deconvolution

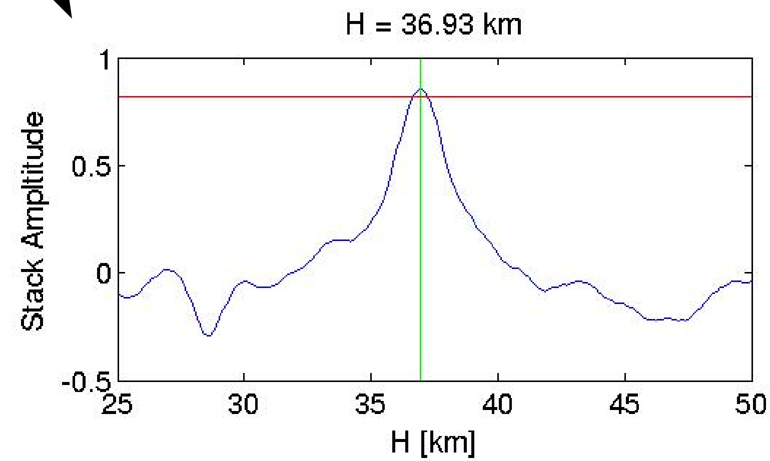
Non-Linear Newton + L1 fit for first arrival time

Grid Search for V_p, V_s.
Line search for H



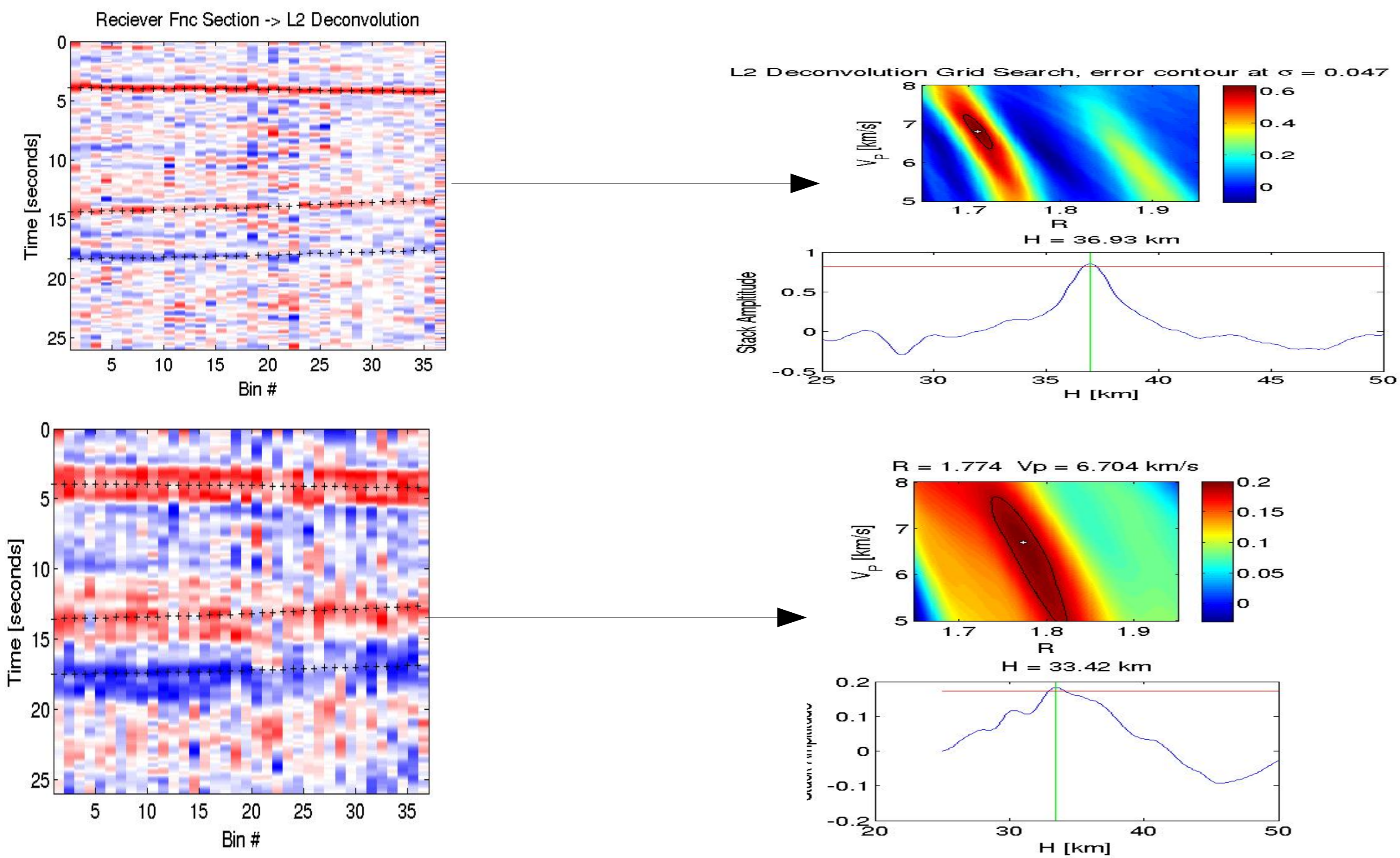
Using ratios of travel-time equations for phases tps, tpps & tpss





Motivation - The Problem

- Confidence of final results highly dependent on sharpness of deconvolution.



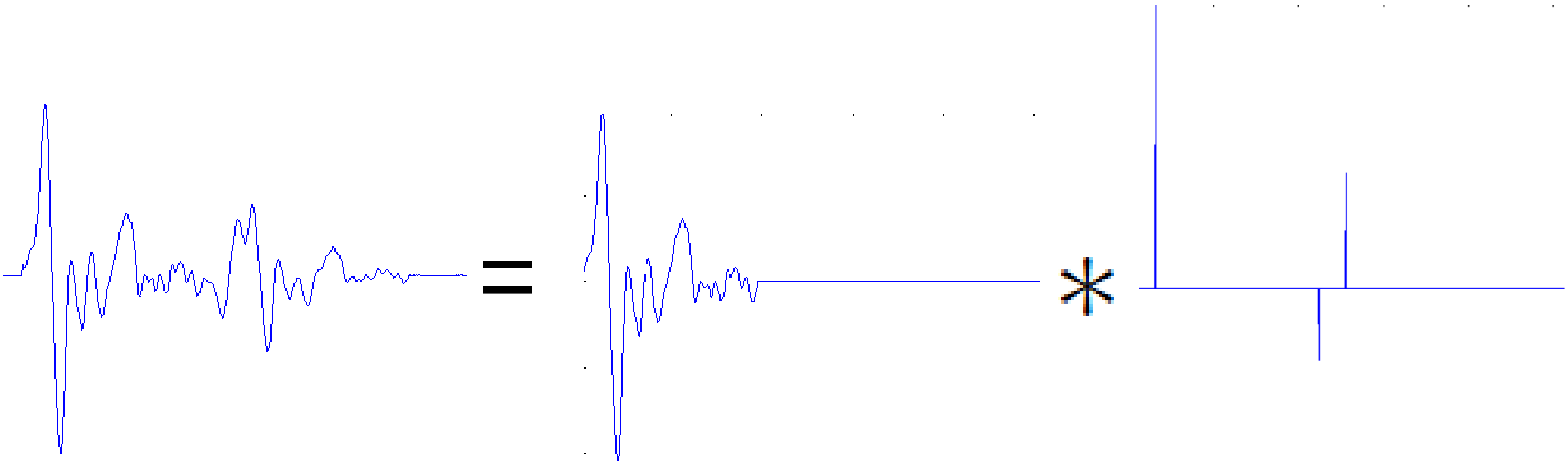
Sparsity promoting L1 Deconvolution.

- Take advantage of the fact that an optimal receiver function has sparse wavelet representation.
- Design a deconvolution algorithm that leverages sparsity.
- See if it can beat the tried and true L2 deconvolution.

Deconvolution

$$u(t) = s(t) * g(t)$$

u(t) – Recorded seismogram s(t) – source function g(t) – receiver function



Produce synthetic seismograms from a synthetic receiver function convolved with P-Codas obtained from windowed P waves of actual data.

Deconvolution

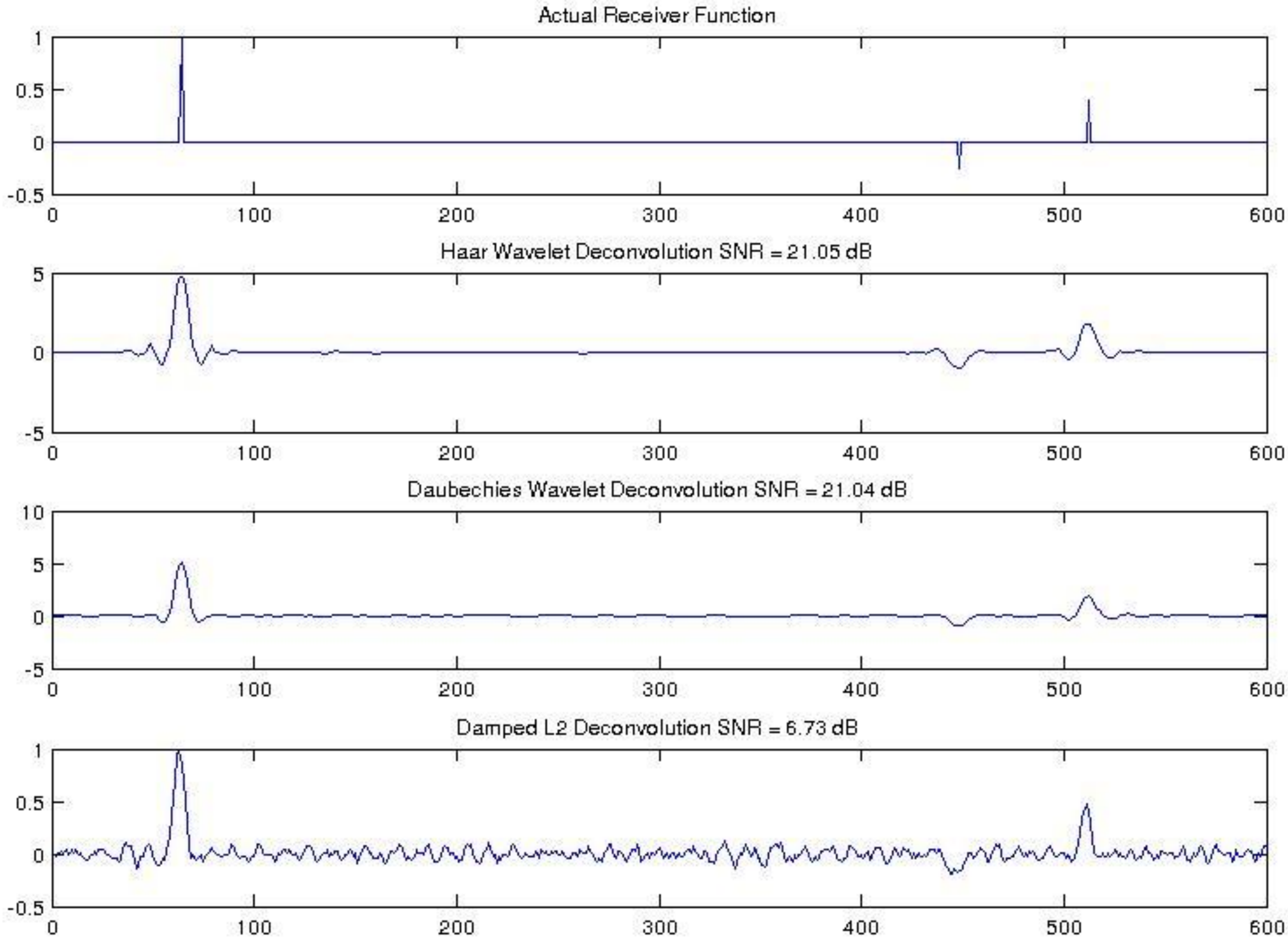
- **Aim:** Produce g(t) by deconvolving source functions from multiple seismograms.
- Construct the problem as an optimization problem.
- Seek a solution which is sparse in the wavelet domain.
- X is the wavelet transform of g(t)
- y is a vector of seismograms in the time domain.

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

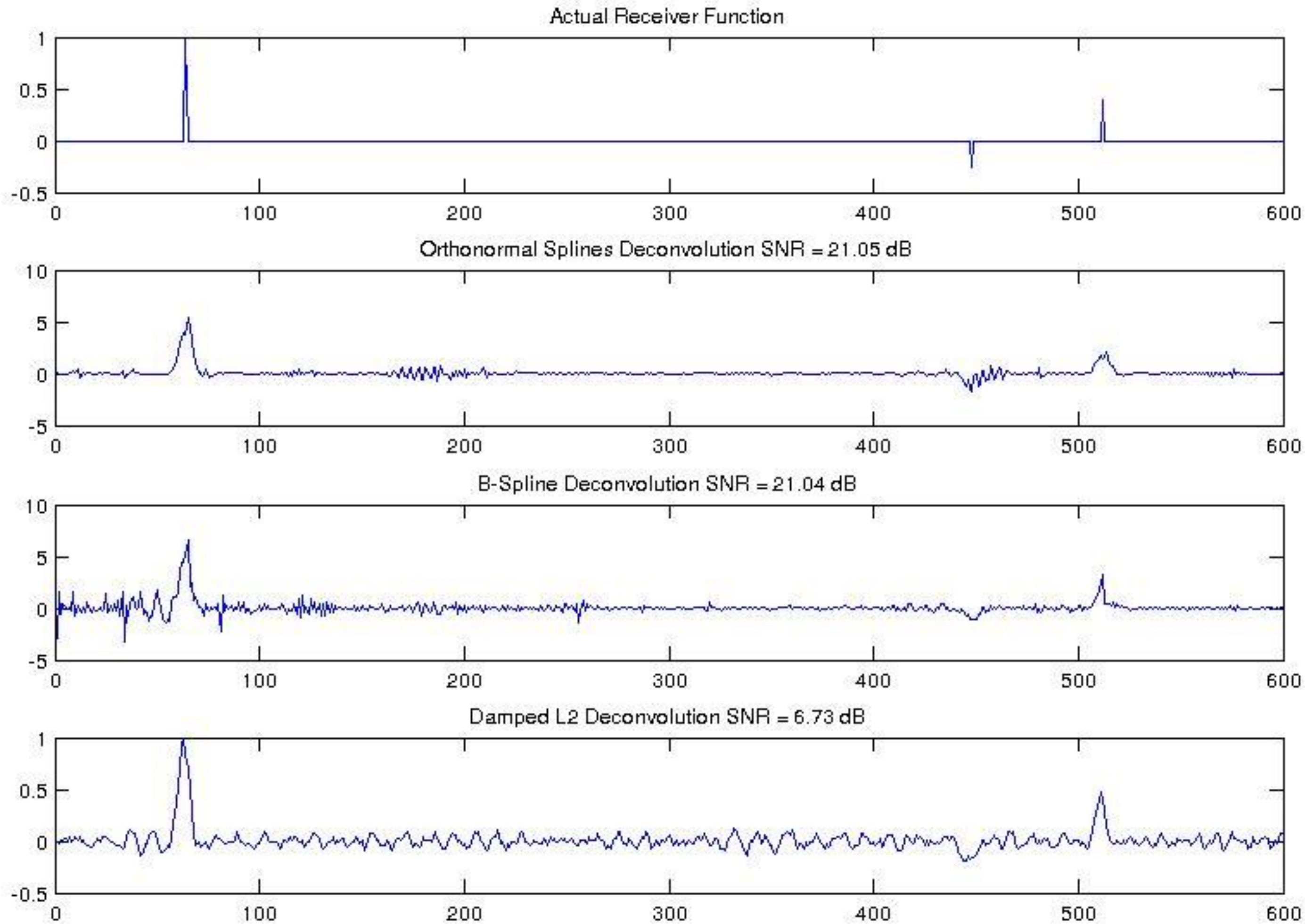
- Seek to minimize $||\mathbf{x}||_1$ s.t. $||\mathbf{A}\mathbf{x} - \mathbf{y}||_2 \leq \sigma$

$$\mathbf{y} = [\underbrace{\mathcal{F}^T \mathcal{F}(s) \mathcal{F}(g)}_{\text{Convolution operator}}] \mathcal{W}^T \mathbf{x}$$

Synthetic Tests – Standard wavelets

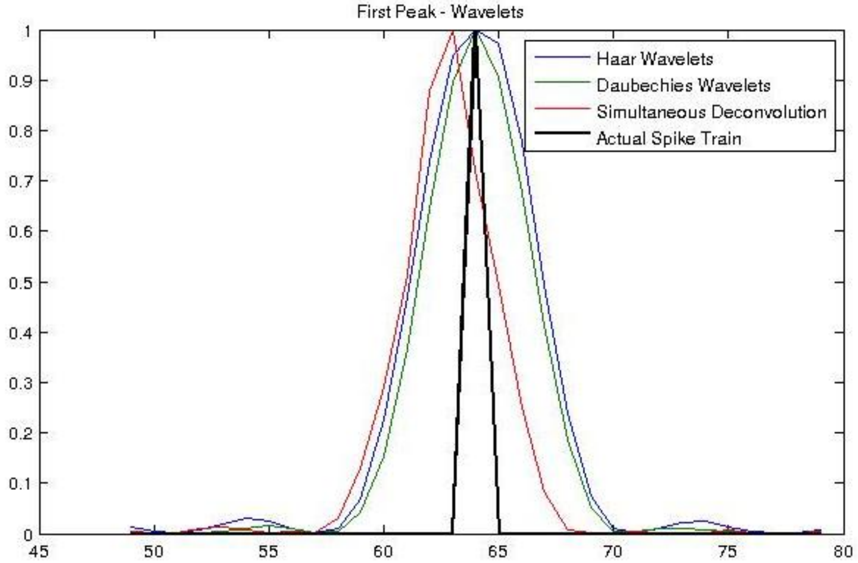


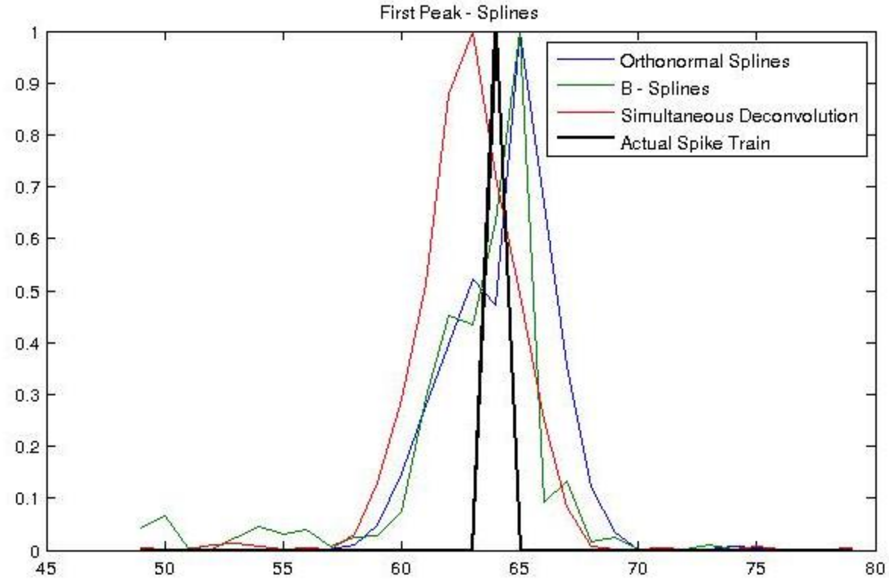
Synthetic Tests – Spline wavelets



Peak comparison

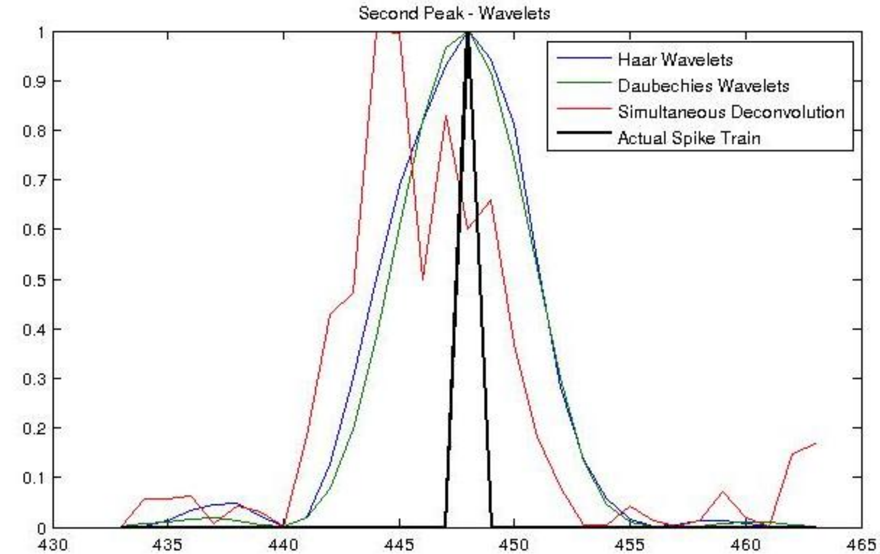
• SNR is an uninformative measure on the quality of fit in this case.
• Whilst simple wavelets provide a sparse solution we also aim for a sharper peak which allows us to better constrain parameters computed later.
• Compare the normalized peak recoveries for a range of wavelet types for each peak in the receiver function.

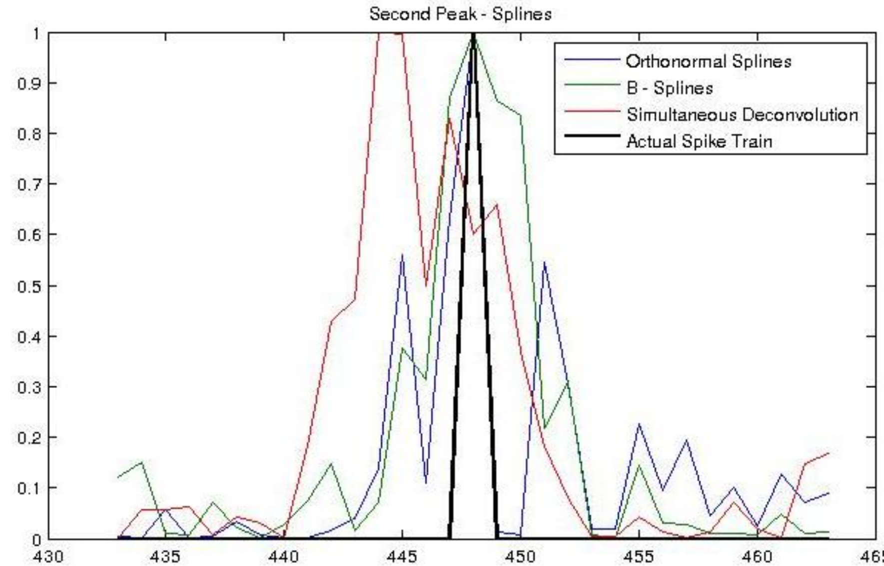




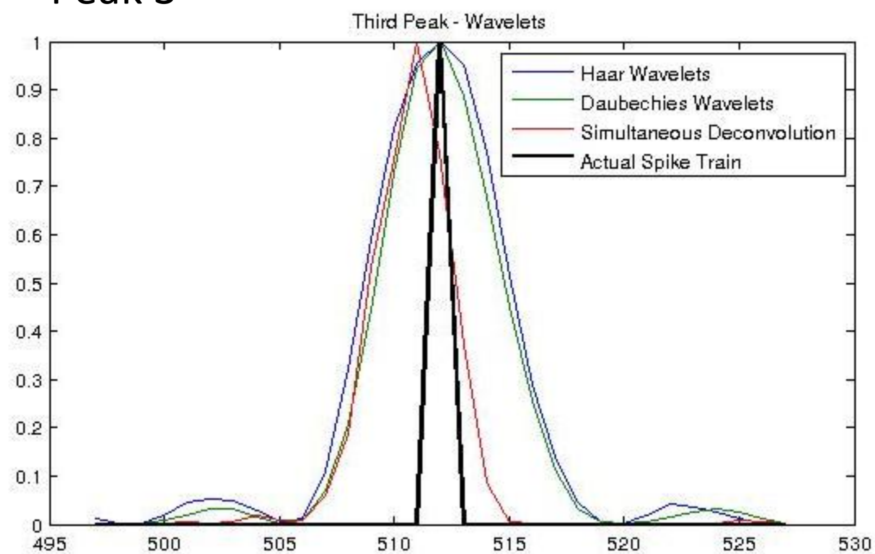
Peak comparison

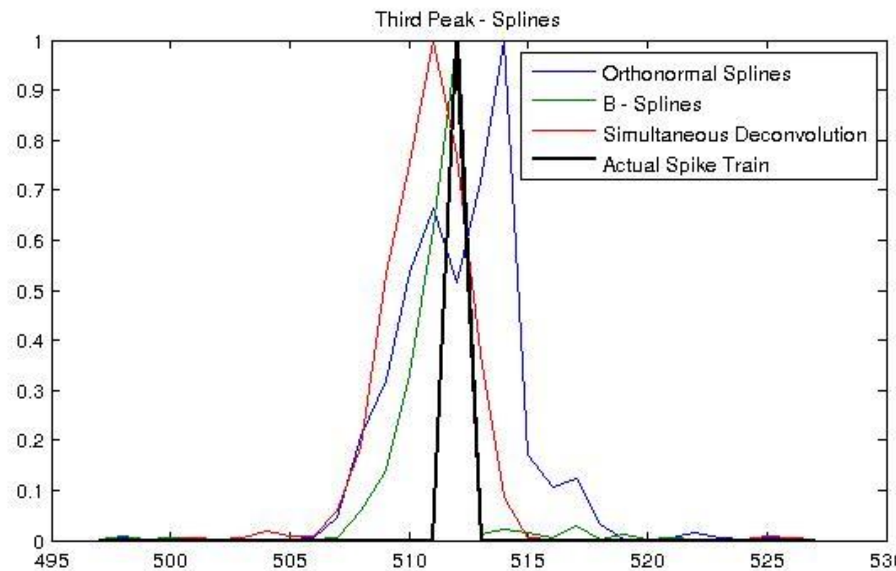
Peak 2





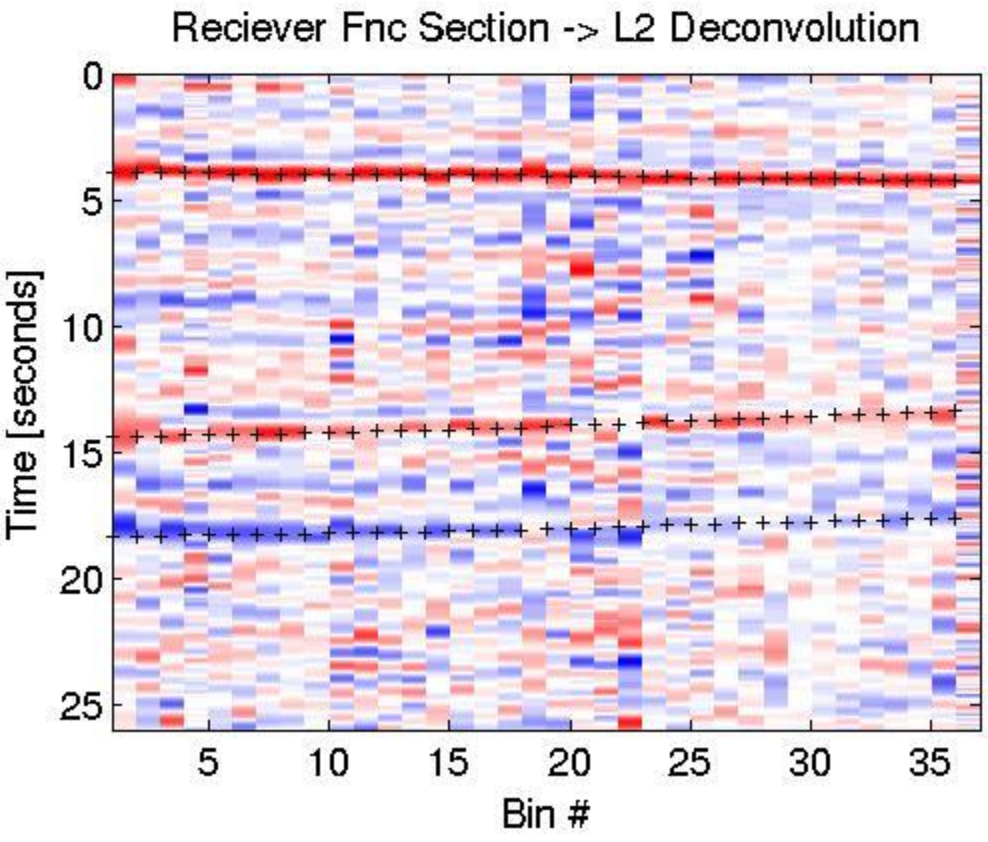
Peak 3

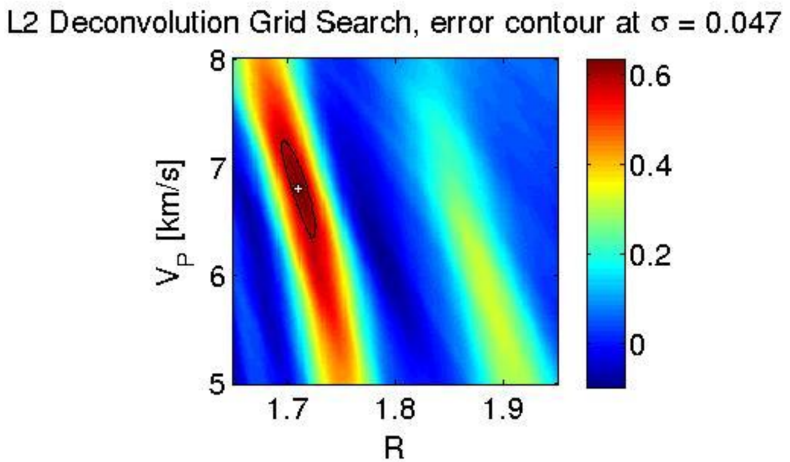


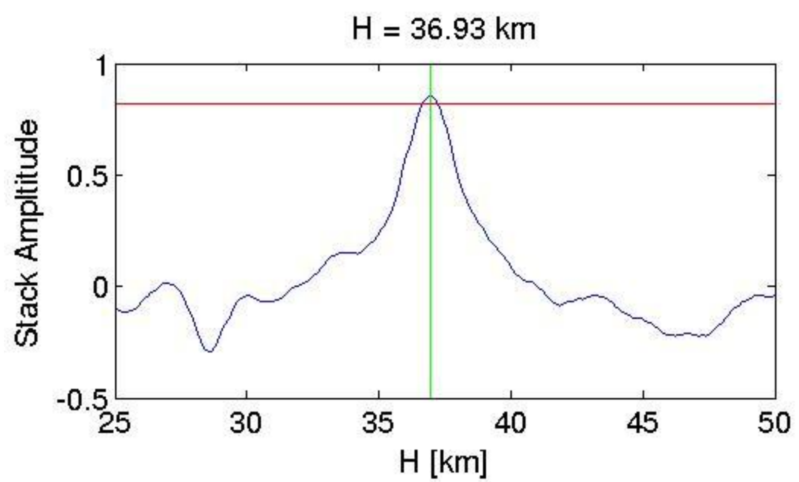


Results – Simultaneous Deconvolution

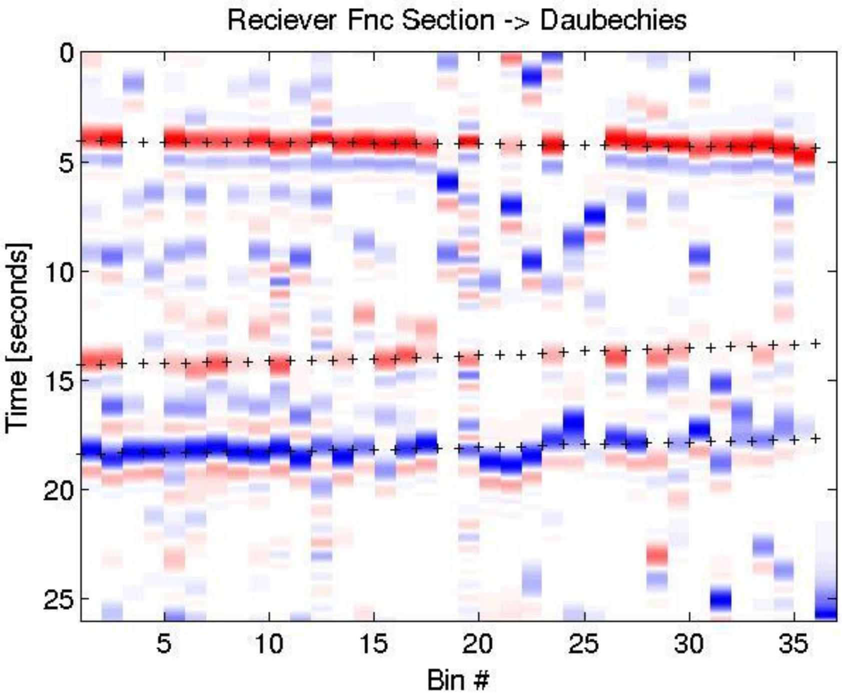
- Seismograms are binned depending on their slowness, p.
- For each bin an L1 optimization problem is solved producing a receiver function.
- Receiver function traces are collected and plotted by bin.

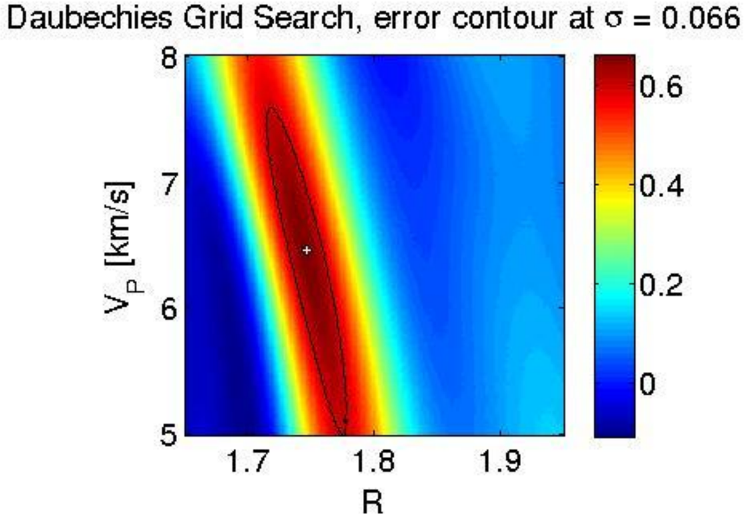


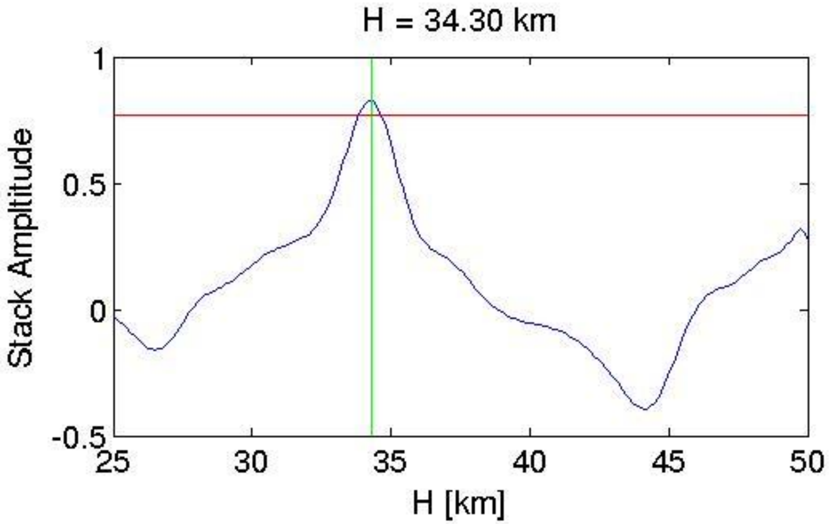




Results – Daubechies Wavelets

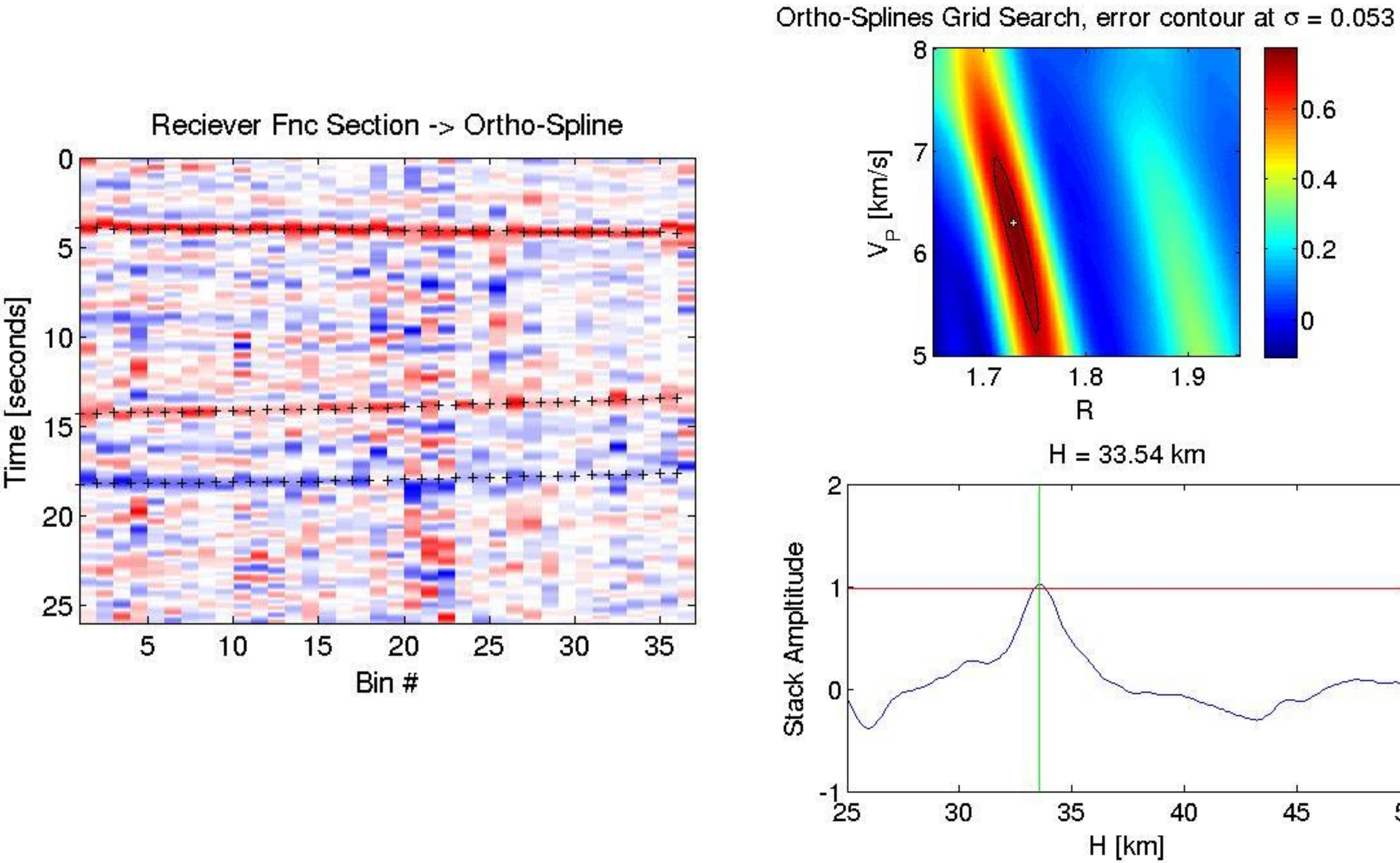




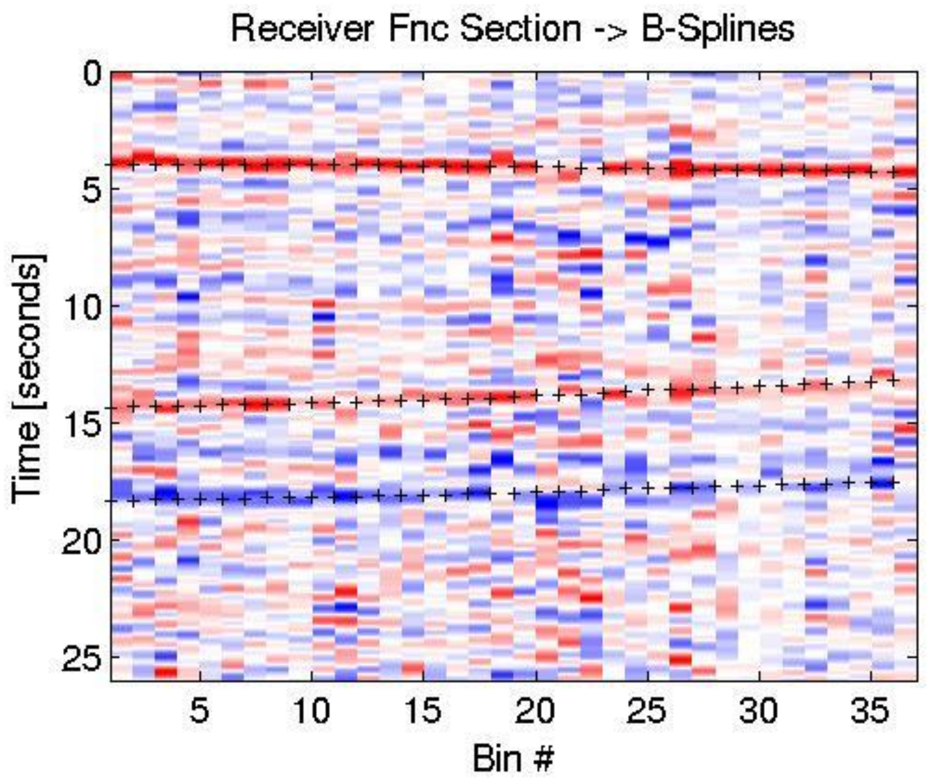


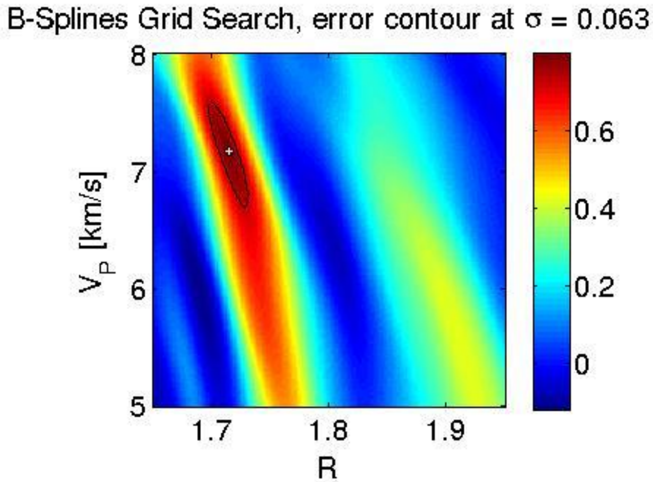
• As earlier observed, Daubechies wavelets provides a much sparser representation but peaks are less tightly constrained.

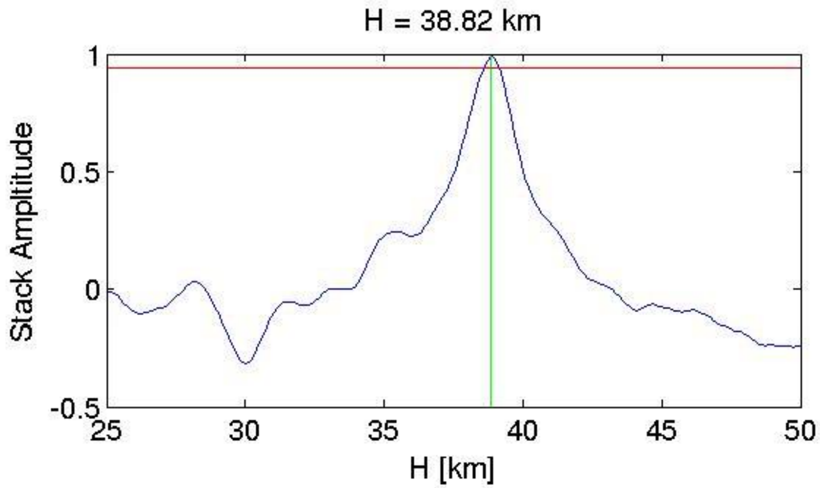
Results – Ortho Spline Wavelets



Results – B Spline Wavelets







Summary

- Performed a deconvolution to acquire receiver functions from recorded seismograms.
- Solved an L1 optimization problem and obtained a solution which promoted sparsity for the receiver function in the wavelet domain.
- Compared a range of wavelets with an L2 type algorithm.
- L1 algorithms failed to reproduce results which were as tightly constrained as the L2 method but show promising initial results which can be further developed.