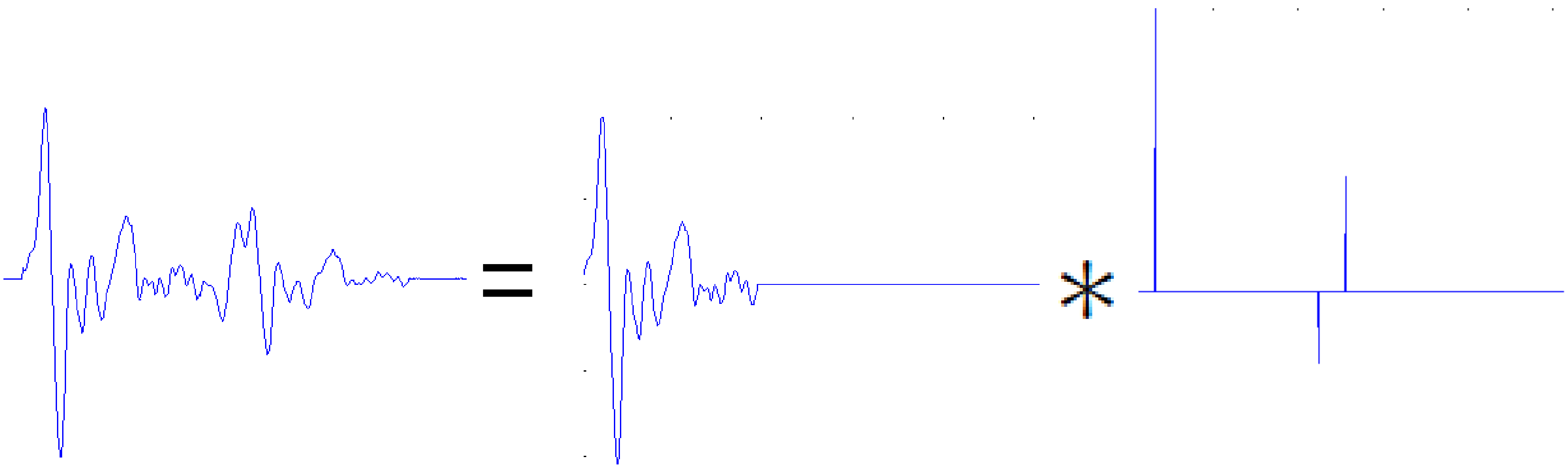
Deconvolution

$$u(t) = s(t) * g(t)$$

u(t) – Recorded seismogram s(t) – source function g(t) – receiver function



Produce synthetic seismograms from a synthetic receiver function convolved with P-Codas obtained from windowed P waves of actual data.

Deconvolution

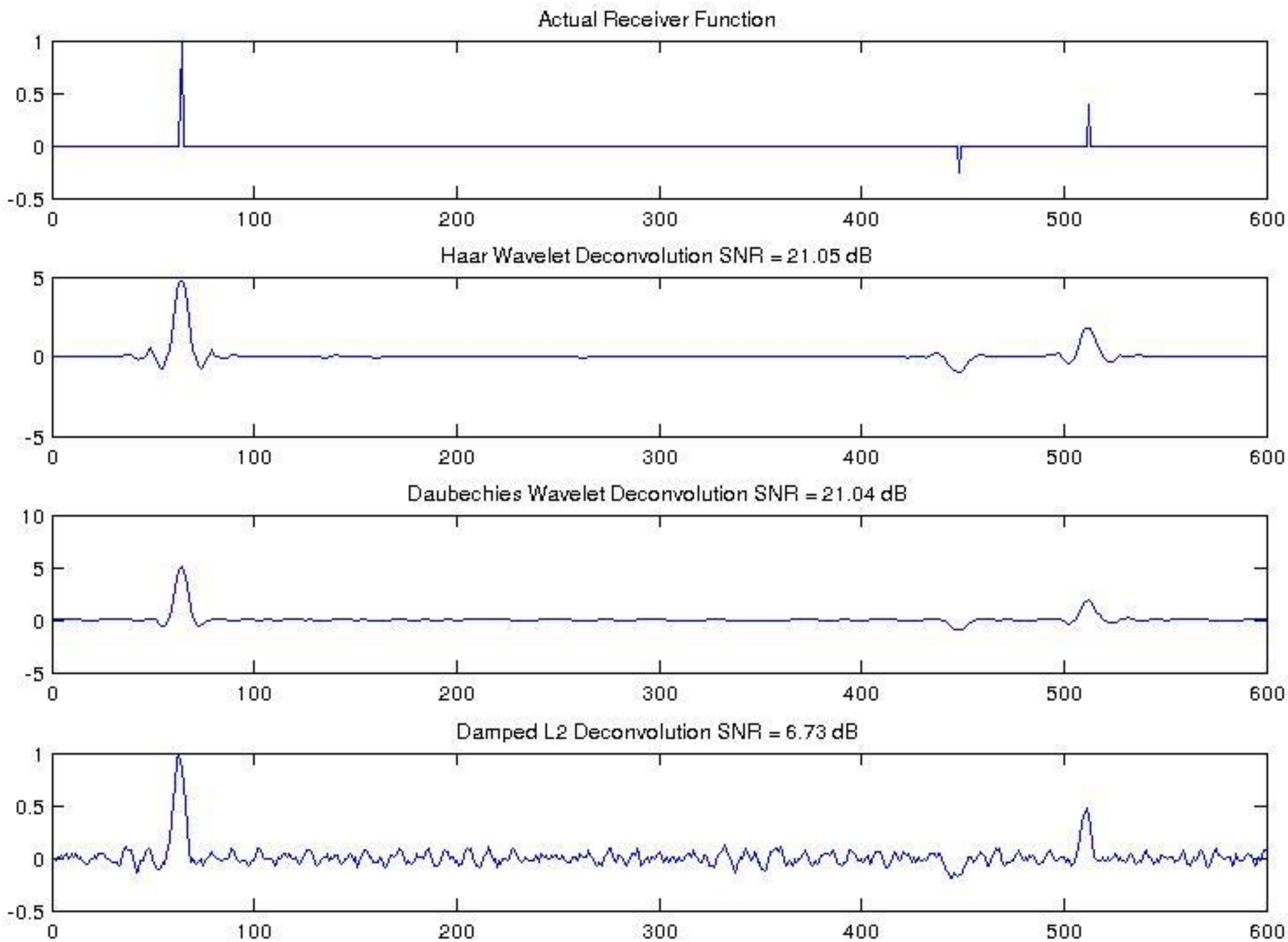
- **Aim:** Produce g(t) by deconvolving source functions from multiple seismograms.
- Construct the problem as an optimization problem.
- Seek a solution which is sparse in the wavelet domain.
- X is the wavelet transform of g(t)
- y is a vector of seismograms in the time domain.

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

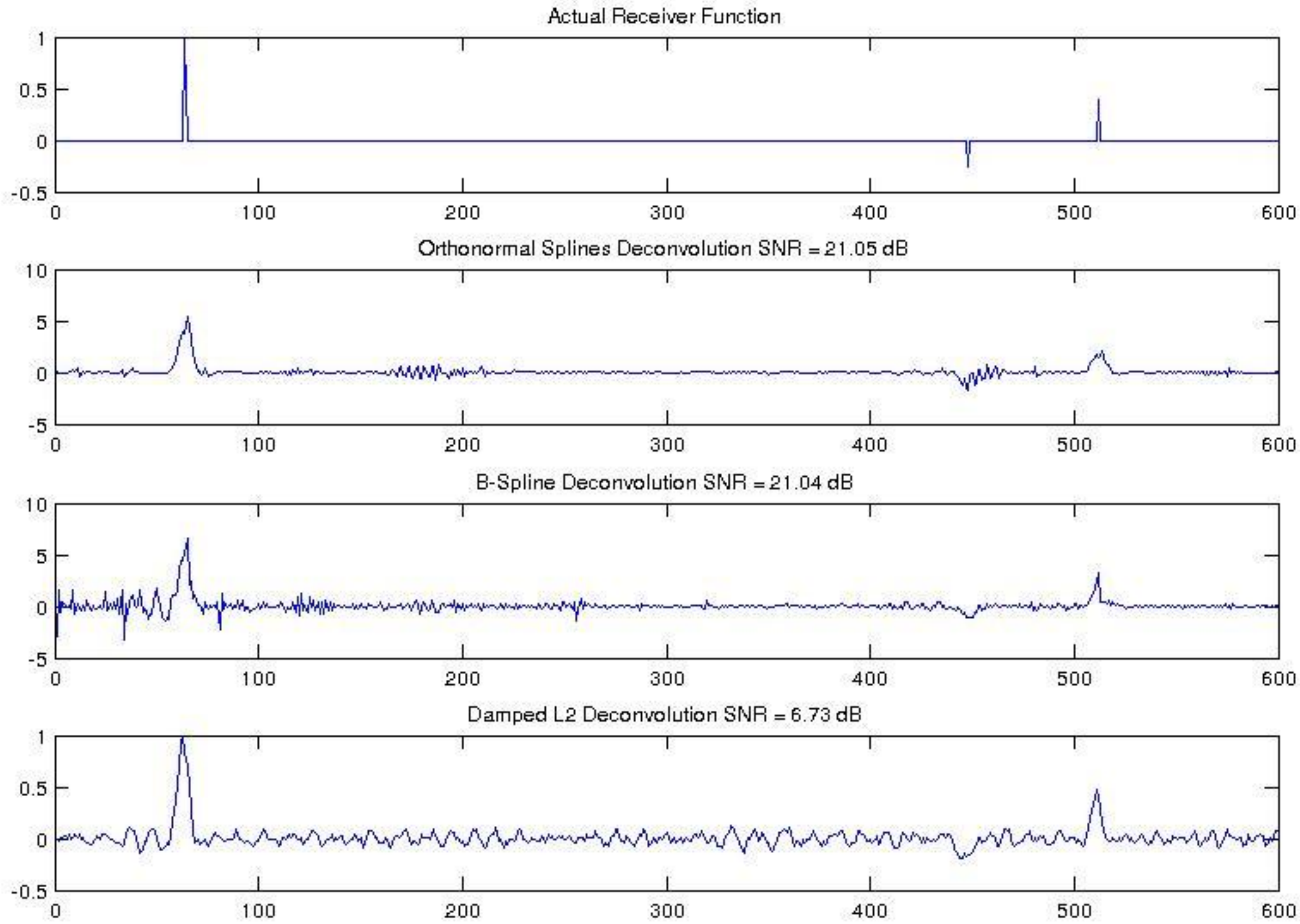
- Seek to minimize $||\mathbf{x}||_1$ s.t. $||\mathbf{A}\mathbf{x} - \mathbf{y}||_2 \leq \sigma$

$$\mathbf{y} = [\underbrace{\mathcal{F}^T \mathcal{F}(s) \mathcal{F}(g)}_{\text{Convolution operator}}] \mathcal{W}^T \mathbf{x}$$

Synthetic Tests – Standard wavelets

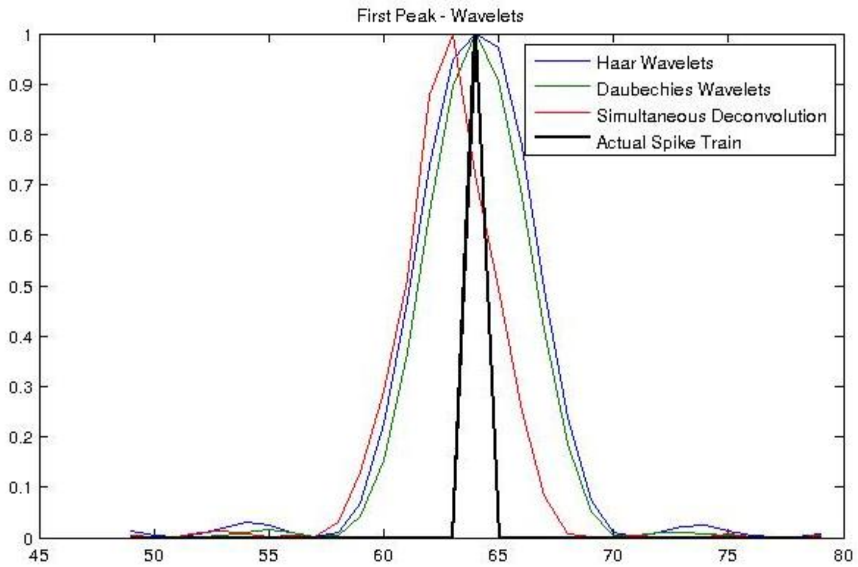


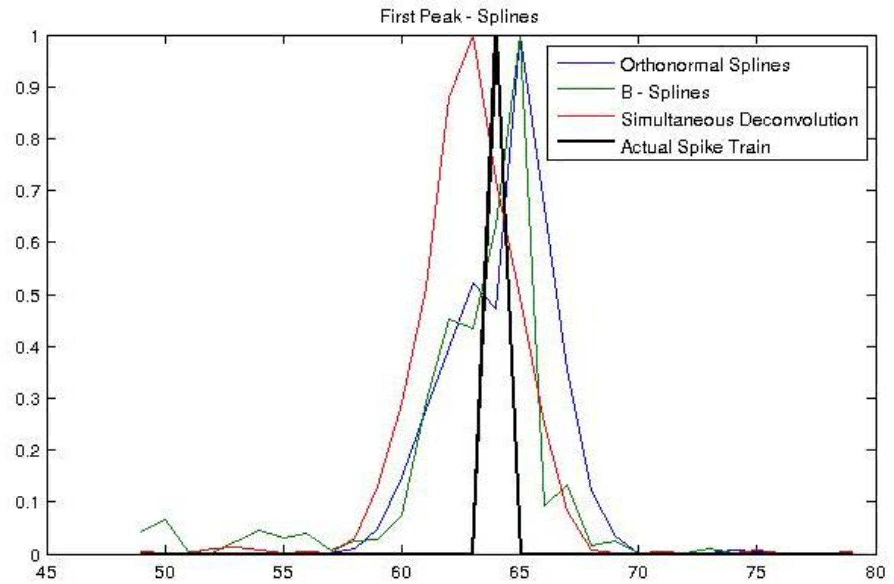
Synthetic Tests – Spline wavelets



Peak comparison

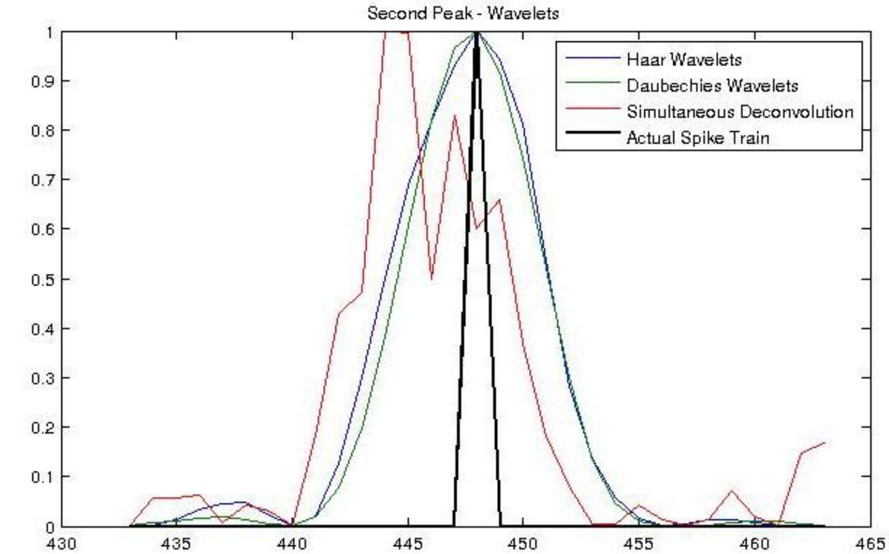
• SNR is an uninformative measure on the quality of fit in this case.
• Whilst simple wavelets provide a sparse solution we also aim for a sharper peak which allows us to better constrain parameters computed later.
• Compare the normalized peak recoveries for a range of wavelet types for each peak in the receiver function.

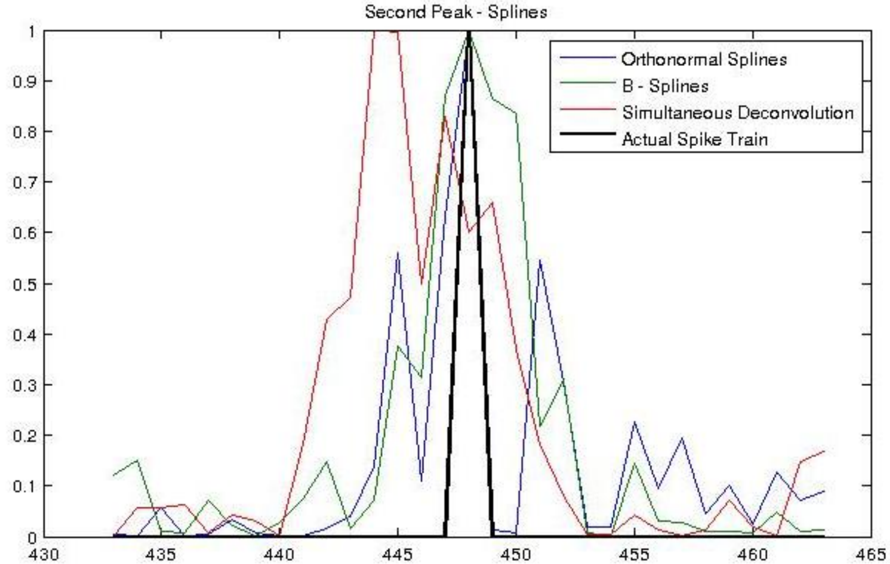




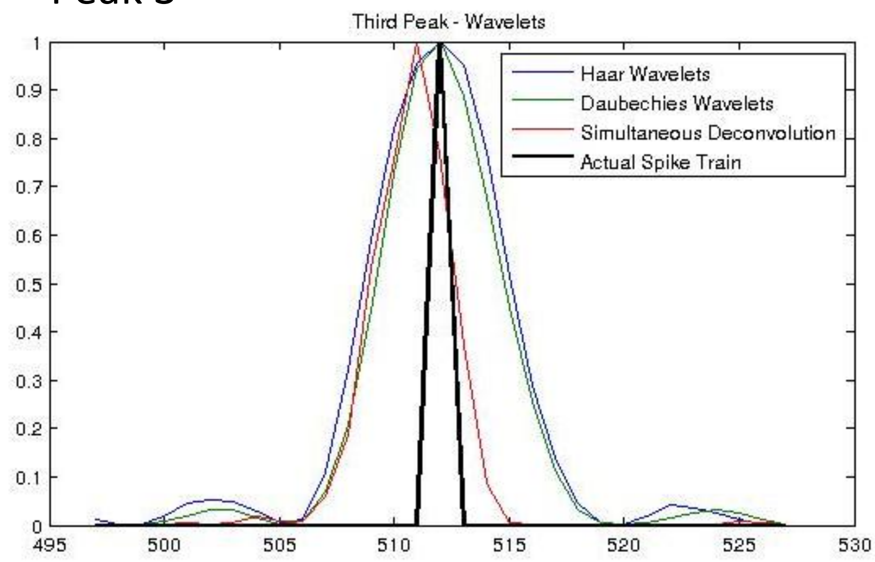
Peak comparison

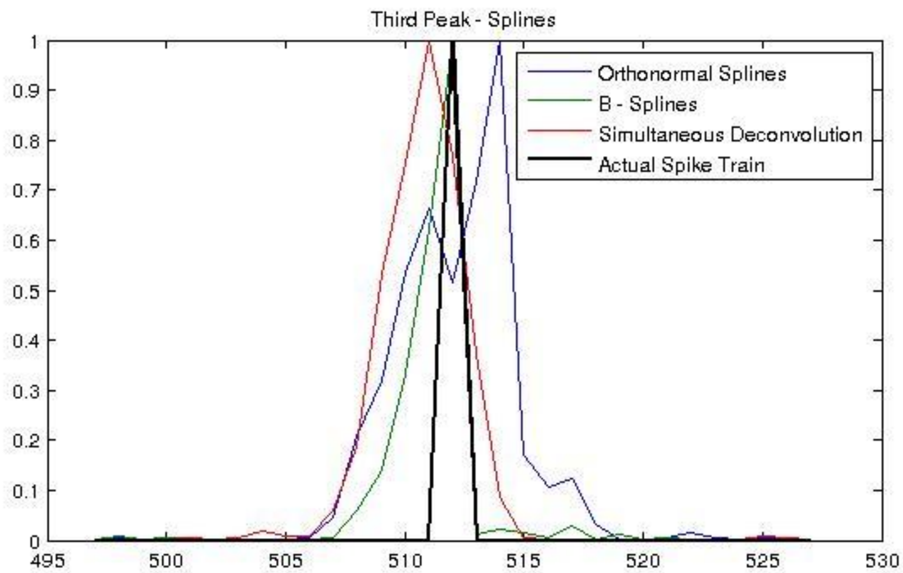
Peak 2





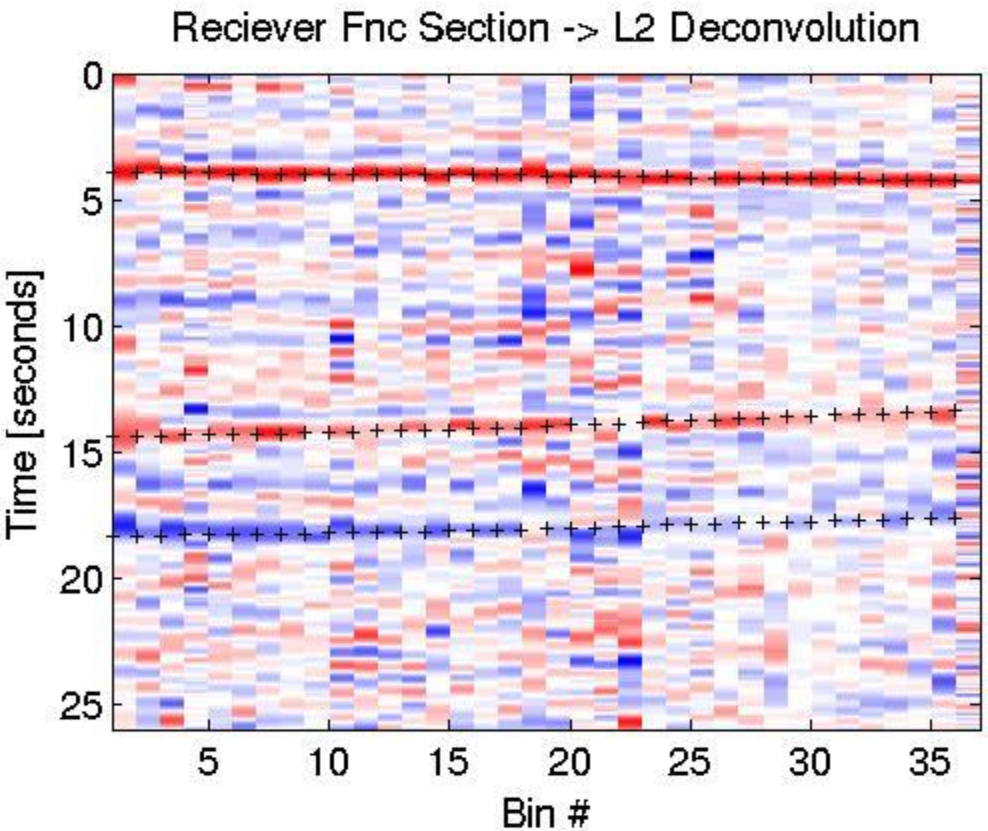
Peak 3

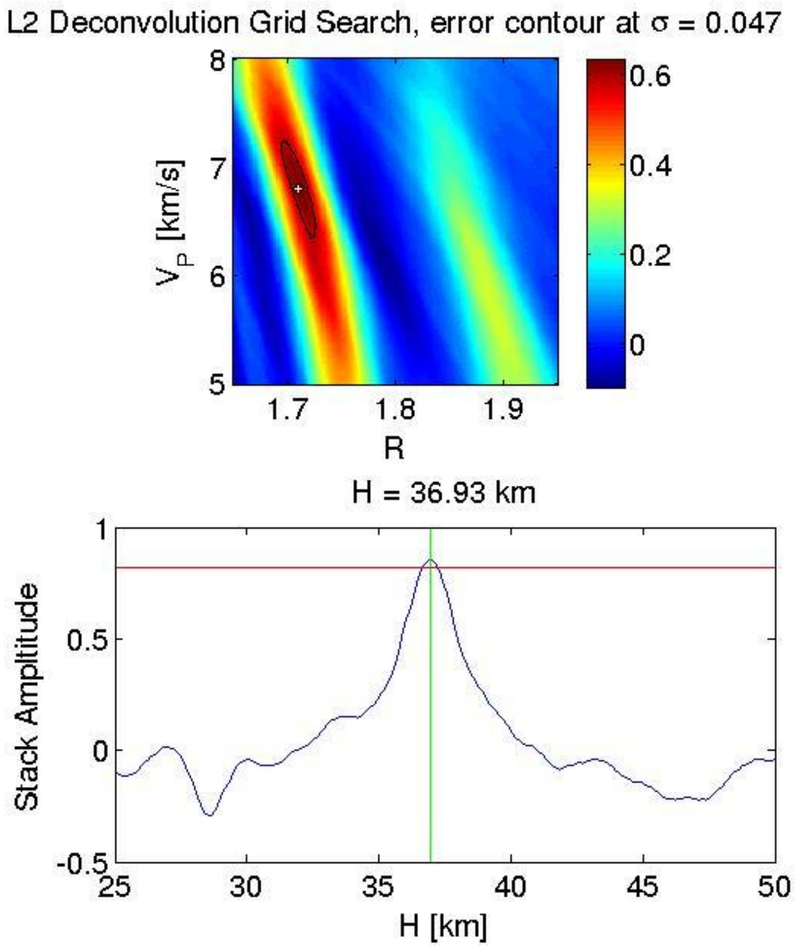




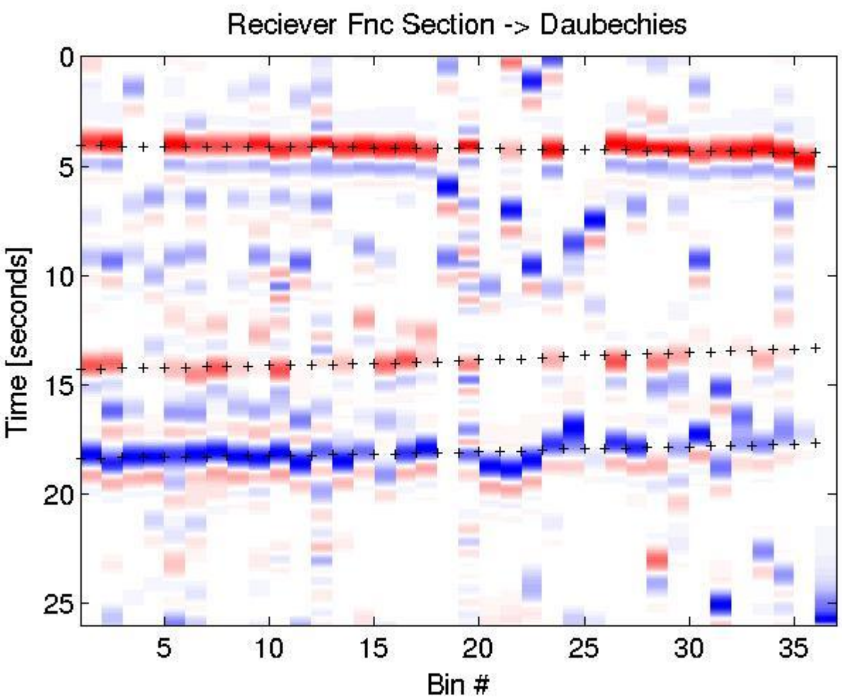
Results – Simultaneous Deconvolution

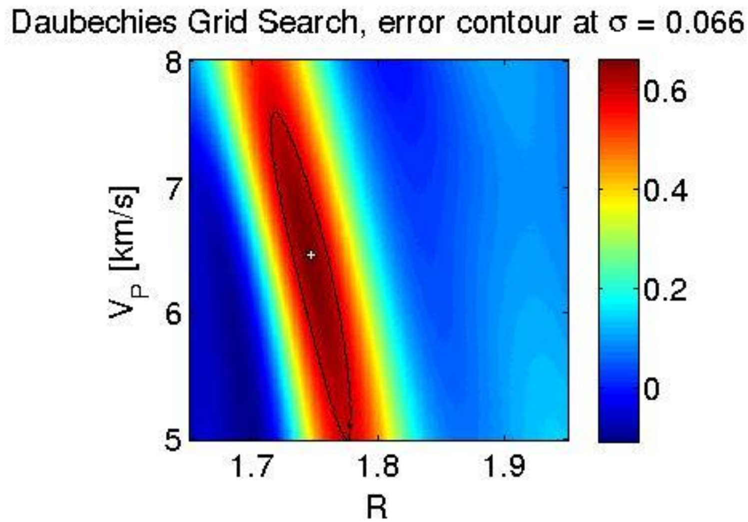
- Seismograms are binned depending on their slowness, p.
- For each bin an L1 optimization problem is solved producing a receiver function.
- Receiver function traces are collected and plotted by bin.

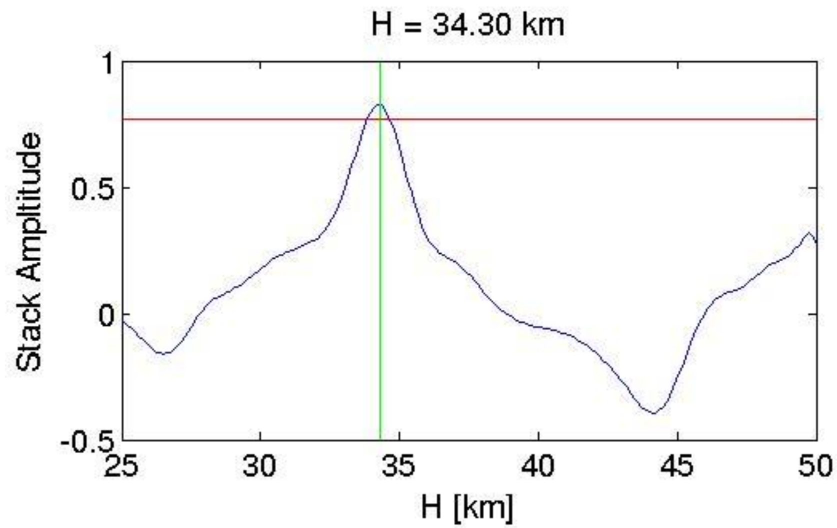




Results – Daubechies Wavelets

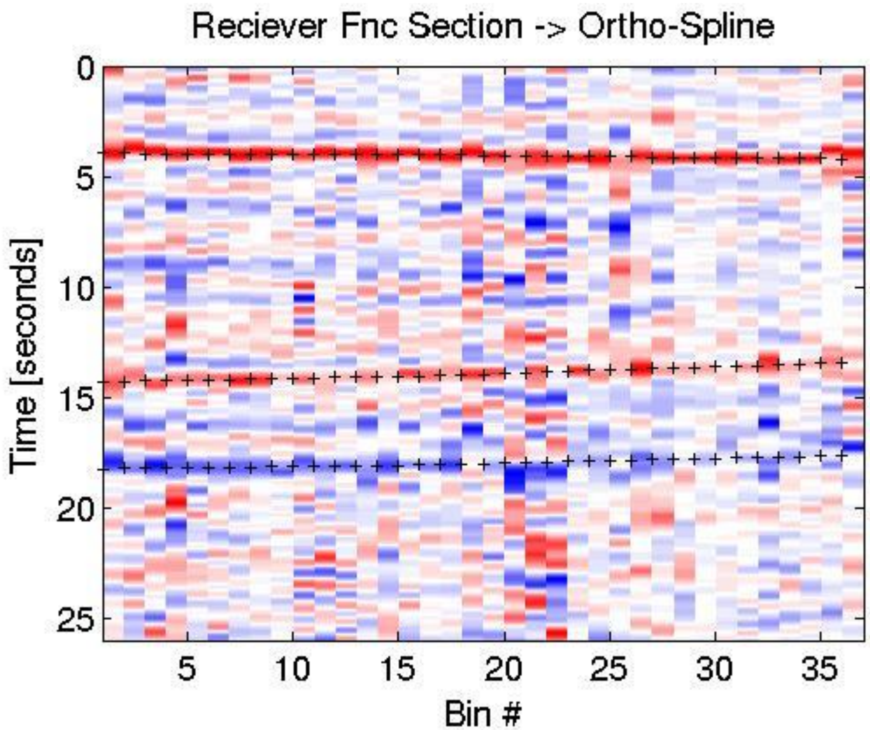


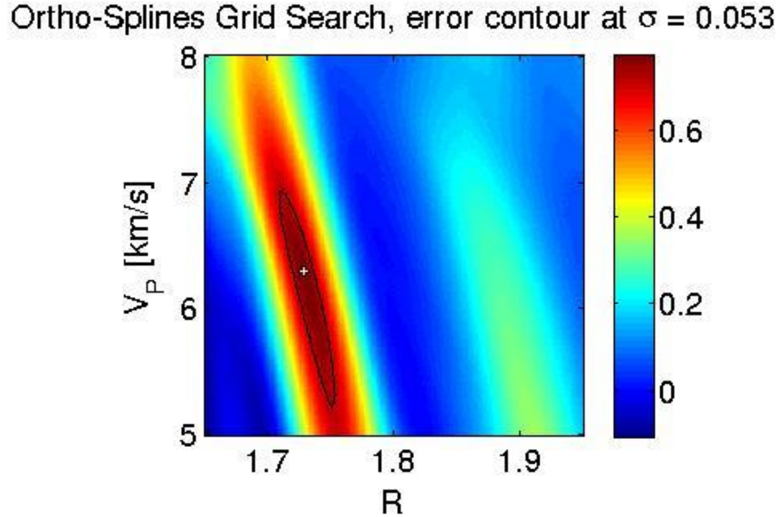


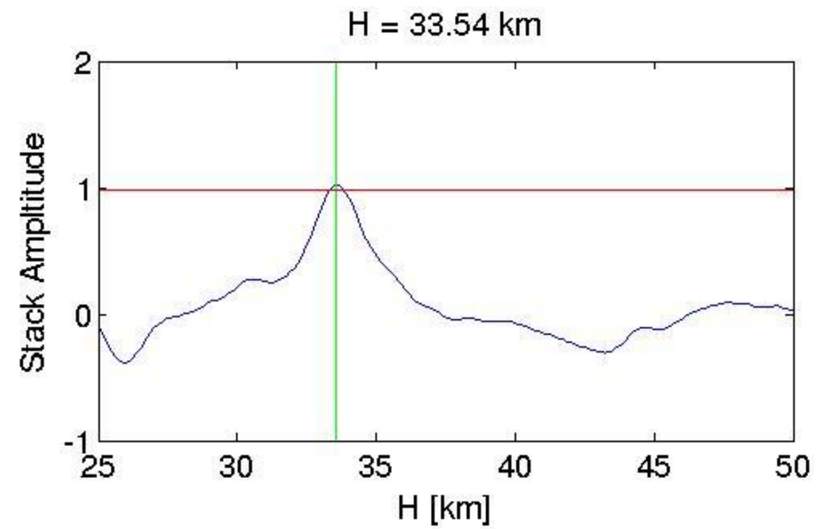


• As earlier observed, Daubechies wavelets provides a much sparser representation but peaks are less tightly constrained.

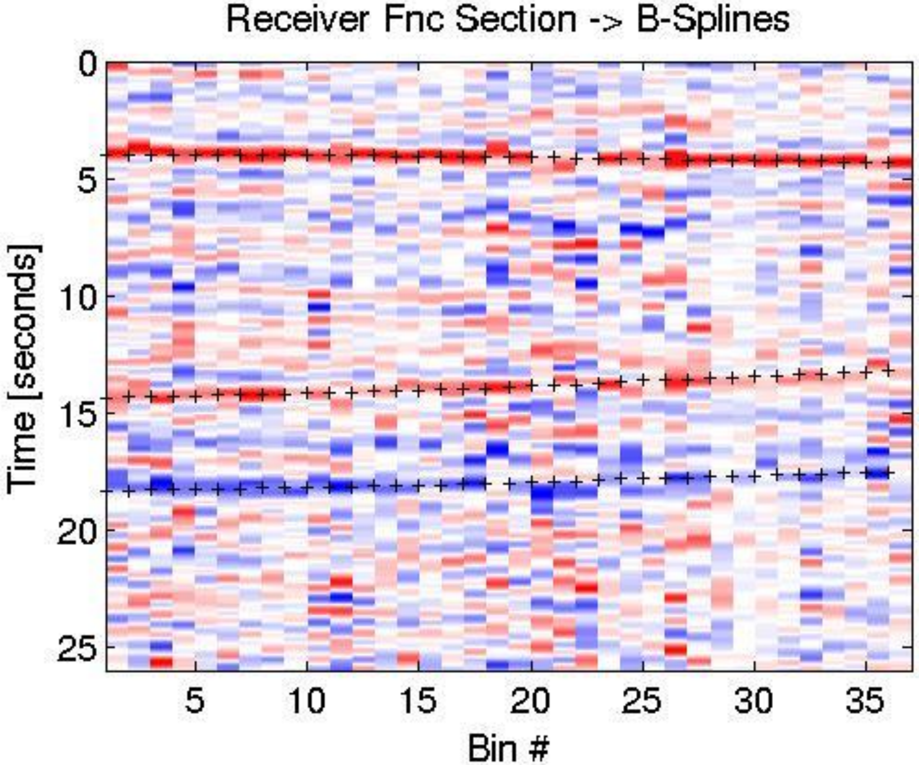
Results – Ortho Spline Wavelets

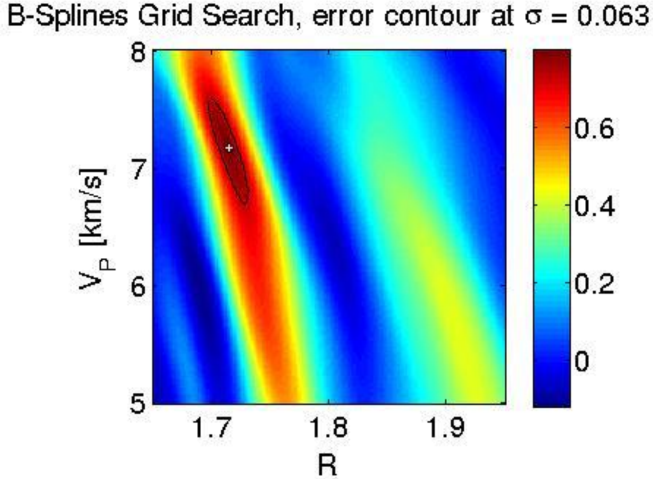


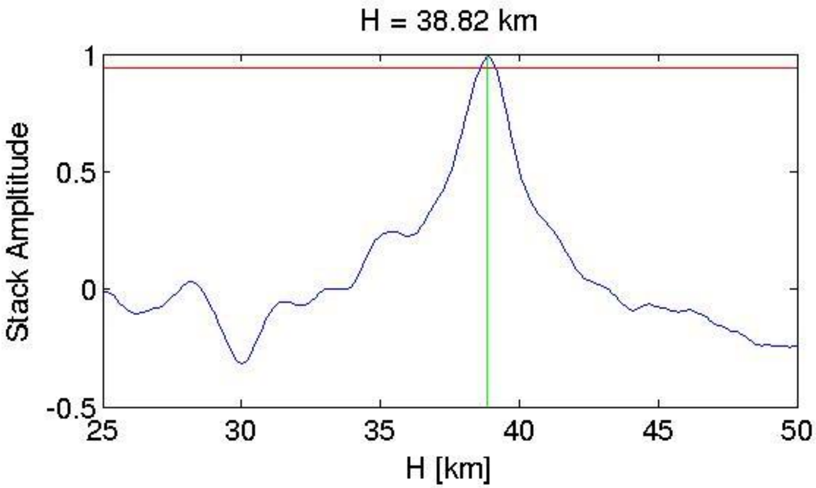




Results – B Spline Wavelets







Summary

- Performed a deconvolution to acquire receiver functions from recorded seismograms.
- Solved an L1 optimization problem and obtained a solution which promoted sparsity for the receiver function in the wavelet domain.
- Compared a range of wavelets with an L2 type algorithm.
- L1 algorithms failed to reproduce results which were as tightly constrained as the L2 method but show promising initial results which can be further developed.