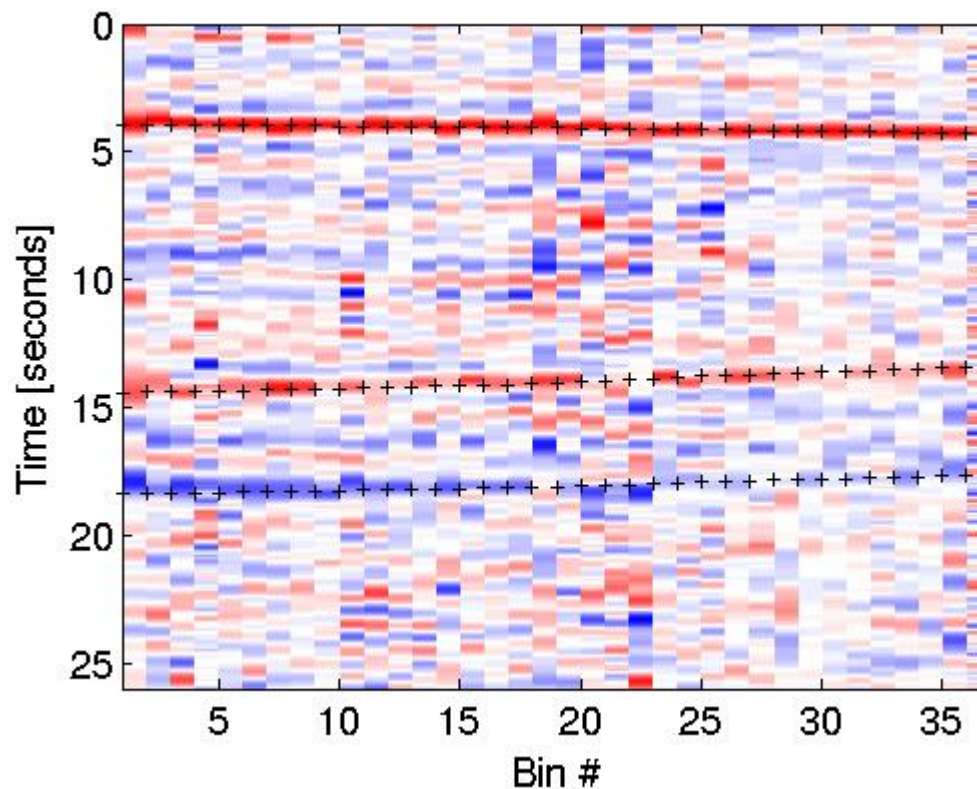# Sparsity Promoting L1 Deconvolution
application for a global seismology parameter search
Gian Matharu & Ben Postlethwaite

Project Summary

- Attempting deconvolution of a noisy approximation of a source from a seismogram.

- Wavelet transforms are employed to promote sparsity.

- Multiple sources and seismograms are solved for during each run.

- SPGL1 is used as the L1 solver.

- Comparisons are made between choices of wavelets.

- Comparison is also made against solutions from Simultaneous Deconvolution with Generalized Cross Validation regularization.

Motivation: The Problem

- Extracting material properties from seismic data has a long history.
- For bulk crustal composition under receivers the seismic velocity ratio
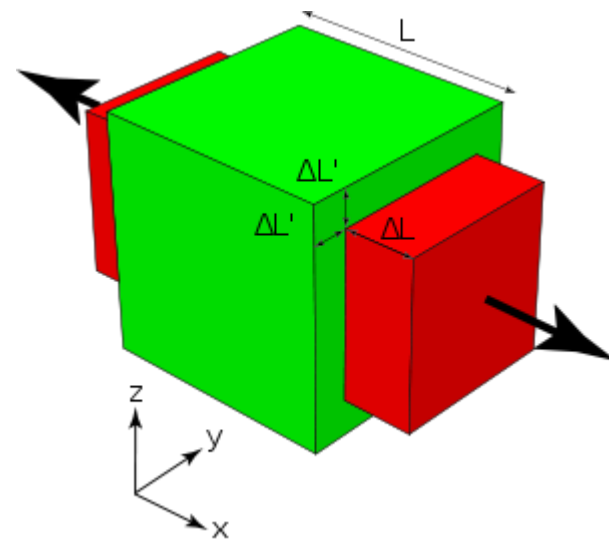
$$R = \frac{V_p}{V_s}$$

has been used.
- This provides Poisson's ratio:

$$\sigma = \frac{1}{2} \frac{\left(\frac{V_p}{V_s}\right)^2 - 2}{\left(\frac{V_p}{V_s}\right)^2 - 1}$$

which is useful for constraining crustal composition.
- Further use of seismic data to constrain crustal composition would require knowledge of explicit values of V_p or V_s.
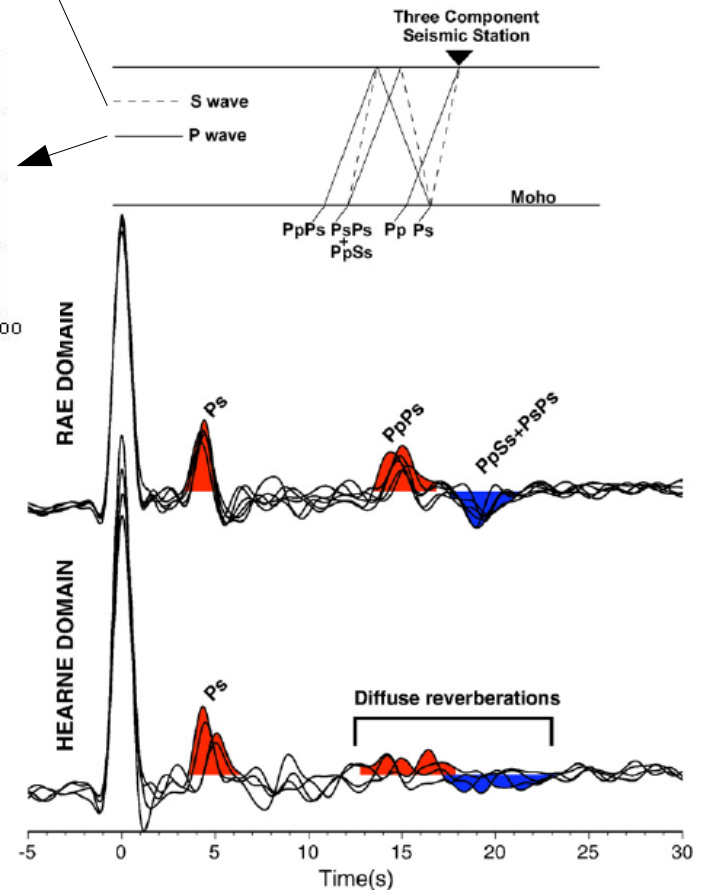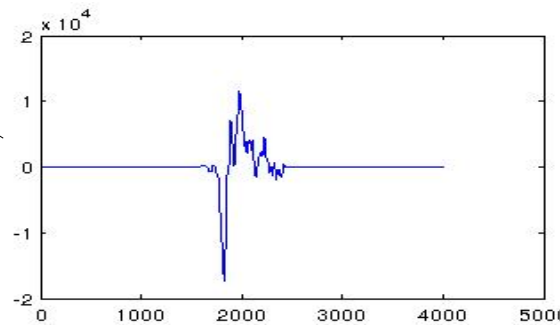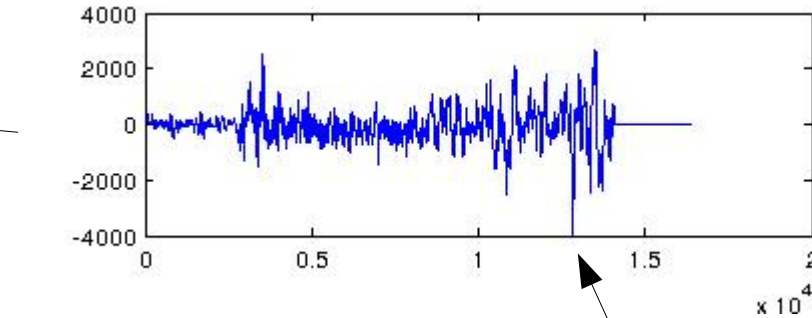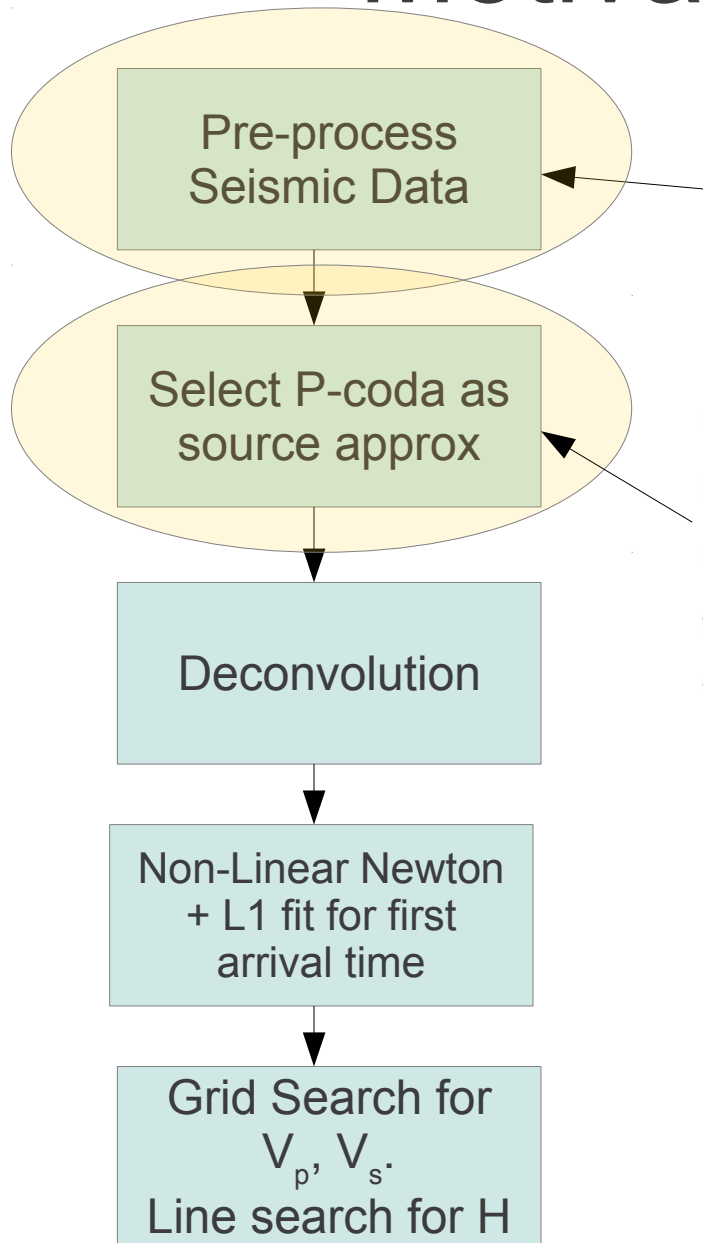

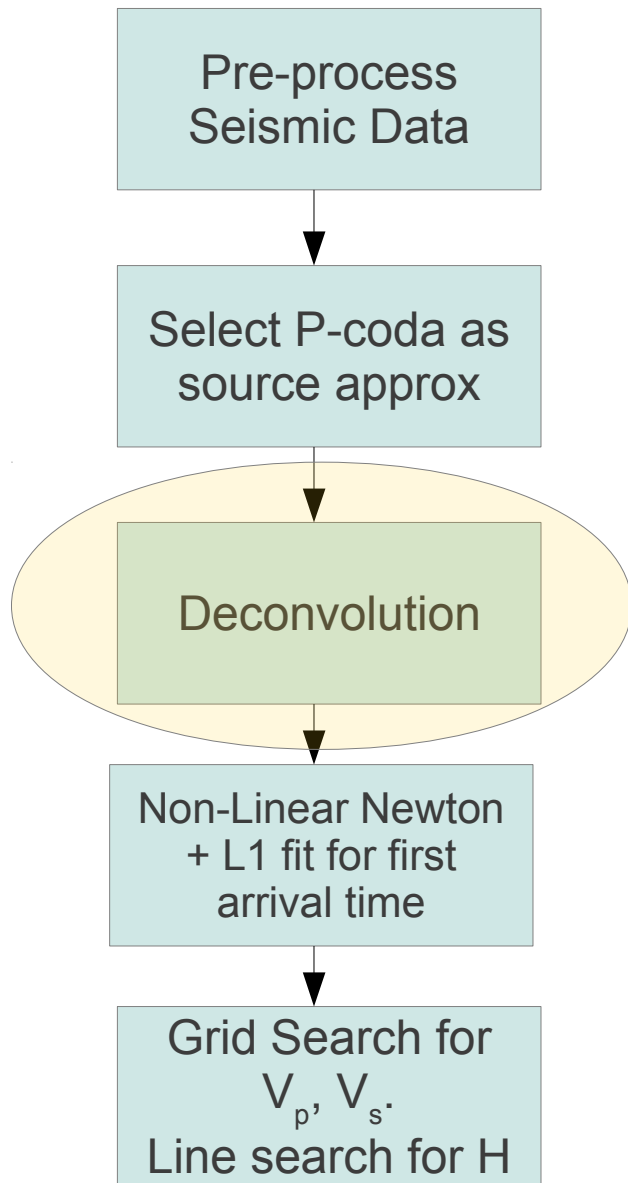Poisson's Ratio: It is the ratio of lengthening along one axis versus compression along the other two.

Motivation: The Problem

- Many previous studies have used a grid search for the seismic velocity ratio $R = V_p/V_s$ and crustal thickness H. Kanamori (2000)

- Some a priori information is required, namely an estimate P-wave velocity V_p.

- Here we attempt to recover V_p and seismic velocity ratio R, without any assumption on V_p.

- For accurate resolution of V_p in solution we require sharp deconvolution (close to spike train as possible).

- We try damped-L2 based deconvolution as well as sparsity promoting wavelet-based L1 minimization.

Motivation: The Problem

Motivation - The Problem

Pre-process
Seismic Data

Select P-coda as
source approx

Deconvolution

Non-Linear Newton
+ L1 fit for first
arrival time

Grid Search for
V_p, V_s.
Line search for H

- Currently using a **Simultaneous Deconvolution with Generalized Cross Validation** regularization.

- $$rec(t) = \mathrm{F}^{-1}[G(\omega)] = \mathrm{F}^{-1}\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right]$$
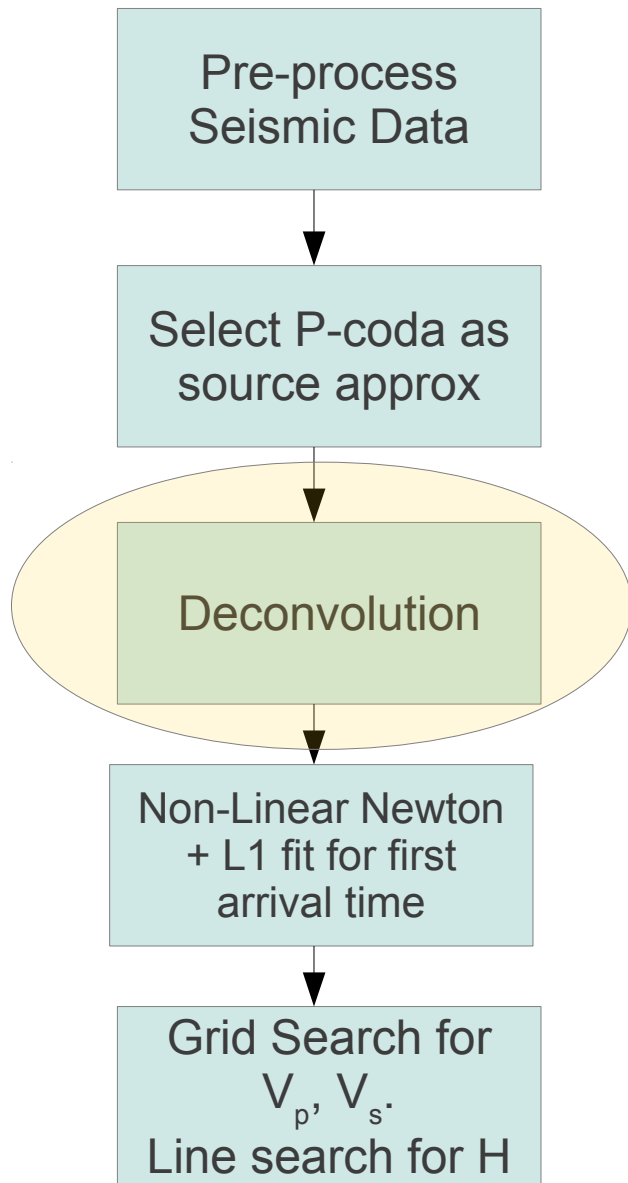
where F is the Fourier transform matrix, G is the model, S is the S-wave seismogram , P is the P-wave source coda, $*$ is the complex conjugate and δ is the regularization. δ is chosen by minimizing the Generalized Cross Validation function:

Misfit: more accurate with δ -> 0. Less stable.

$$min\left(\frac{\sum_n^N \sum_m^M (S_n(\omega_m) - P_n(\omega_m)\left[\frac{\sum_n^N S_n P_n^*}{\sum_n^N P_n P_n^* + \delta}\right])^2}{(NM - \sum_m^M \frac{\sum_m^M P_n(\omega) P_n^*(\omega)}{\sum_m^M P_n(\omega) P_n^*(\omega) + \delta})^2}\right) w.r.t\ \delta$$

As δ increases denominator increases, promotes stability.

Motivation - The Problem

Pre-process Seismic Data

↓

Select P-coda as source approx

↓

Deconvolution

↓

Non-Linear Newton + L1 fit for first arrival time

↓

Grid Search for V_p, V_s. Line search for H

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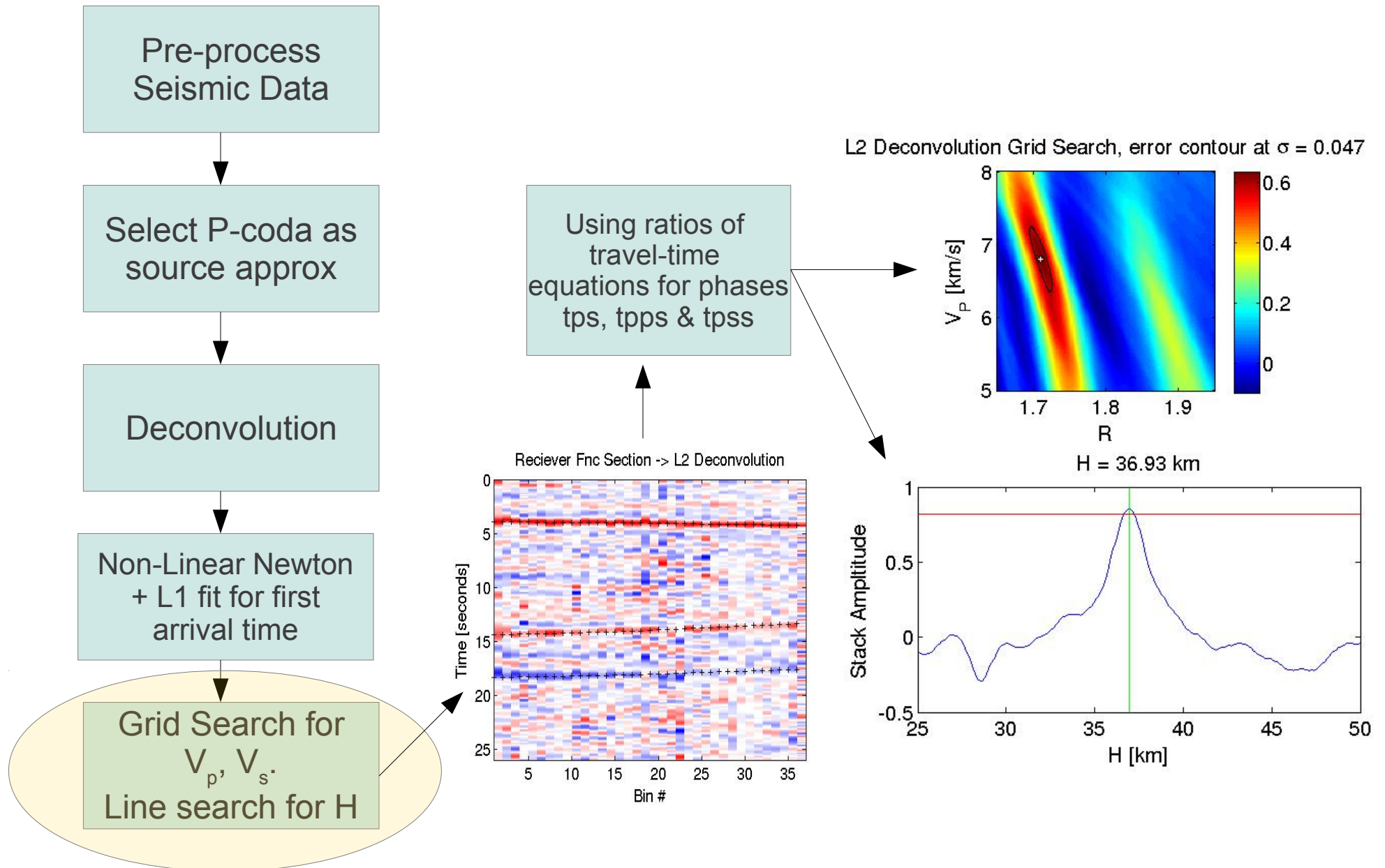
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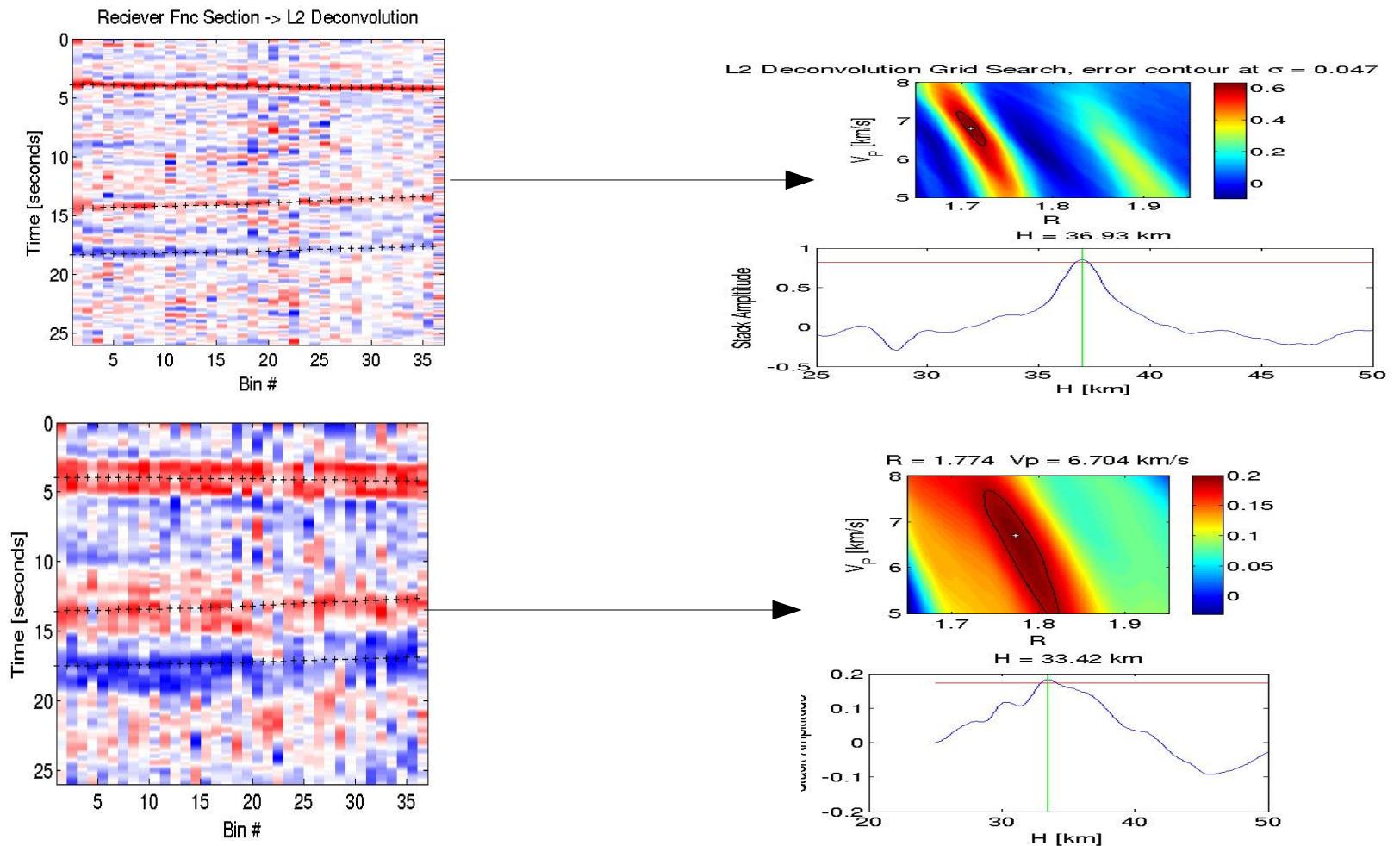
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Motivation: The Problem

Motivation - The Problem

- Confidence of final results highly dependent on sharpness of deconvolution.

Sparsity promoting L1 Deconvolution.

- Take advantage of the fact that an optimal receiver function has sparse wavelet representation.

- Design a deconvolution algorithm that leverages sparsity.

- See if it can beat the tried and true L2 deconvolution.