Structure preserving regularization for sparse deconvolution

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Summary

This paper presents a new scheme for preserving structural information in high-resolution seismic deconvolution. We use adaptive FX filtering in sparse deconvolution to enhance the coherence of seismic events across midpoints. The combination of adaptive FX filtering and sparse inversion provides multi-channel solutions that are sparse in the vertical direction and coherent in the lateral direction. The robustness of this technique is validated by tests on both synthetic and field data.

Introduction

Deconvolution is an important step for enhancing data resolution in seismic processing. It can be posed as an inverse problem in which we attempt to remove the signature of the wavelet. The basic assumption is that a seismic trace can be described by convolving the reflectivity series with a wavelet and then adding some noise. Deconvolution problems are classified into deterministic and indeterministic approaches, depending on if the wavelet is known or unknown. In this paper, we assume that the wavelet is known and band-limited, having been obtained via wavelet extraction based on well log information or perhaps alternatively via the relaxation method (Canadas, 2002). In the context of land production processing, the proposed technique would typically be applied poststack after industry-standard surface-consistent deconvolution and time-variant spectral whitening. Thus, our "wavelet" can be thought of as the embedded wavelet which exists after conventional wavelet processing. The goal of this paper is to reconstruct the sparse reflectivity series by collapsing this wavelet in the presence of noise.

It has been shown that the assumption of sparseness can be incorporated in seismic inverse problems to enhance resolution (Sacchi and Ulrych, 1995). For deconvolution, we can assume the reflectivity series is sparse in the vertical direction. Conventional sparse deconvolution uses only *a* priori information within a single trace. Mathematically, this is realized by introducing various norms (Oldenburg et al., 1983; Debeye and van Riel, 1990) with sparse flavor in the cost function. One challenge is that the quality of trace-by-trace sparse deconvolution may be compromised in the presence of noise and wavelet estimation error. One way to stabilize the solution is to use multi-channel information. The simplest method is to smooth seismic trace in various spatial directions; however when the geological structure is not flat, simple smoothing will smear the solution. On the other hand, FX filtering is a more sophisticated technique to enhance dipping events and suppress noise. With proper implementation, the method can handle multiple dips at the same time and no event picking is required. Furthermore, as we will show in this paper, adaptive FX filtering in conjunction with imposing a sparseness constraint can better preserve and enhance subtle structures compared to simple sparse inversion.

Methodology

FX filtering

FX filtering is a well-established method to reduce noise in seismic data. It can be applied to both prestack and poststack data with stable output. The basic idea is that in the FX domain we can predict one trace with the preceding several traces, and so on for the opposite direction. It is instructive to see the following 1-D formulas (Spitz, 1991):

$$g_k(f) = \sum_{j=1}^{L} P_j(f) g_{k-j}(f), k = L+1, ..., N, \tag{1}$$

$$g_k^*(f) = \sum_{j=1}^{L} P_j(f) g_{k+j}^*(f), k = 1, ..., N-L, \tag{2}$$

where g_k is the value of kth trace in FX domain, and $\mathbf{P}(f)$ is a frequency dependent prediction filter. Once this filter is inverted from the above equations, we can apply it to the input data to remove noise. Furthermore, it has been shown that the method is also applicable for multi-dimensional data with proper implementation (Chase, 1992; Wang, 1996).

Structure preserving sparse deconvolution

Conventional trace-by-trace sparse deconvolution seeks to minimize the following cost function:

$$J = ||\mathbf{Wr} - \mathbf{s}||^2 + \rho R(\mathbf{r}), \tag{3}$$

where $\mathbf{W}$ is the convolution matrix containing the known wavelet, $\mathbf{r}$ is the reflectivity trace, and R is a function to force the solution to be sparse. Here we use the Cauchy norm as the function R:

$$R(\mathbf{r}) = \sum_{i=1}^{n} ln(1 + r_i^2/\sigma_r^2), \tag{4}$$

where σ_r^2 is a scale parameter. The problem can be solved by the iterative reweighted least-squares (IRLS) (Scales and Smith, 1994) method with several iterations of the linear conjugate gradient algorithm. The explicit expression for the ith iteration solution is

$$\mathbf{r}_i = (\mathbf{W}'\mathbf{W} + \rho\mathbf{Q}_{i-1})^{-1}\mathbf{W}'\mathbf{s}, \tag{5}$$

where $\mathbf{Q}$ is a diagonal matrix, and its diagonal elements are calculated by

$$Q_{jj} = 1/(1 + r_j^2/\sigma_r^2). \tag{6}$$

According to our experience, the quality of the inversion could be affected by factors like wavelet estimation error and noise etc. These factors might introduce unwanted artifacts and deteriorate the interpretability of the result. One remedy is to force lateral coherence by borrowing information from neighboring traces. As we know, FX filtering is very efficient at detecting and enforcing local linear patterns. Therefore, we propose a cost function as bellow:

$$J = \sum_{k=1}^{NTR} ||\mathbf{W}\mathbf{r}_k - \mathbf{s}_k||^2 + \rho \sum_{k=1}^{NTR} R_k(\mathbf{L}_{FX}\mathbf{r}_k), \tag{7}$$

where $\mathbf{r}_k$ is the k-th reflectivity trace, $\mathbf{s}_k$ is the k-th seismic trace, R_k is the k-th regularization function, $\mathbf{L}_{FX}$ is the FX filtering operator, and NTR is the total trace number. The processing flow is generalized as the following:

- Initialize the diagonal weighting matrices (Q_{jj}) with identity matrices. Note that each seismic trace has one diagonal weighting matrix.
- Solve equation 5 for each seismic trace.
- Apply FX filtering to the deconvolved traces (i.e., the $\mathbf{r}_i$ in equation 5).
- Update the diagonal weighting matrices with the FX filtered traces

Repeating the last three steps for 3-4 iterations will lead to a satisfactory solution.

The described approach is not a direct multi-channel deconvolution problem since we still solve separate small linear systems. However by adaptive FX filtering, we gradually adjust the diagonal weighting and balance the energy among neighboring traces. The following tests show that this method significantly improves the quality of sparse inversion.

Wedge model

To validate the idea of combining sparse deconvolution and FX filtering, we prepared a wedge model data set. The data contain one horizontal event and one dipping event. We added a large amount of white noise (s/n = 2.0) to complicate the situation, as shown in Figure 1a.

We compare three kinds of processing: trace-by-trace sparse deconvolution, "two-pass" sparse deconvolution (FX filtering followed by trace-by-trace sparse deconvolution), and finally the proposed structure preserving sparse deconvolution. Figure 1b, 1c and 1d show that the quality of sparse deconvolution with adaptive FX filtering is the best. It can be seen that in all cases sparse regularization helps to suppress random noise, but the sparse regularization alone can sometimes suppress useful signal. On the other hand, adaptive FX filtering can help to avoid this trap and preserve structural information. The "two-pass" processing is better than the simple trace-by-trace method, but it is not as good as the proposed adaptive multi-channel method. Additional tests (not displayed) show that the process of trace-by-trace sparse deconvolution followed by FX filtering also fails to provide results of the same quality as that of the proposed method.

Field data

We also tested the method with a field data set. The input time migrated section is shown in Figure 2 (upper left). The data contain 81 migrated traces. Considering the more complicated structure, we applied two 1-D FX filters to two overlapping windows respectively. After four iterations of the IRLS algorithm, we obtained a sparse and coherent image (shown in the lower right panel of Figure 2). For comparison, results after trace-by-trace sparse deconvolvolution and after "two-pass" sparse deconvolution are also displayed. In all cases, the vertical resolution is enhanced, and overlapping events are nicely separated. In addition, it is evident that the regularization in the FX domain increases the coherence of seismic events across traces. Note that the method can be extended to 3-D case using FXY filtering. In that case, we can use *a* priori information from two directions.

Conclusions and discussion

We have proposed a robust iterative structural regularization in the FX domain for geophysical inverse problems. The tests of sparse deconvolution show that the method can preserve coherence information in various directions. The regularization can be implemented together with other regularizations like smoothness and sparseness to acquire specific model features under different circumstances. One potential application of adaptive FX filtering is the sparse least-squares migration (Wang and Sacchi, 2005), which aims at good structure and coherent amplitude at the same time.

Acknowledgments

We thank Geo-X Systems Ltd. for allowing us to publish this paper. We thank Mark Ng for providing the field data. We also our friends at Geo-X and the University of Alberta for their beneficial suggestions.

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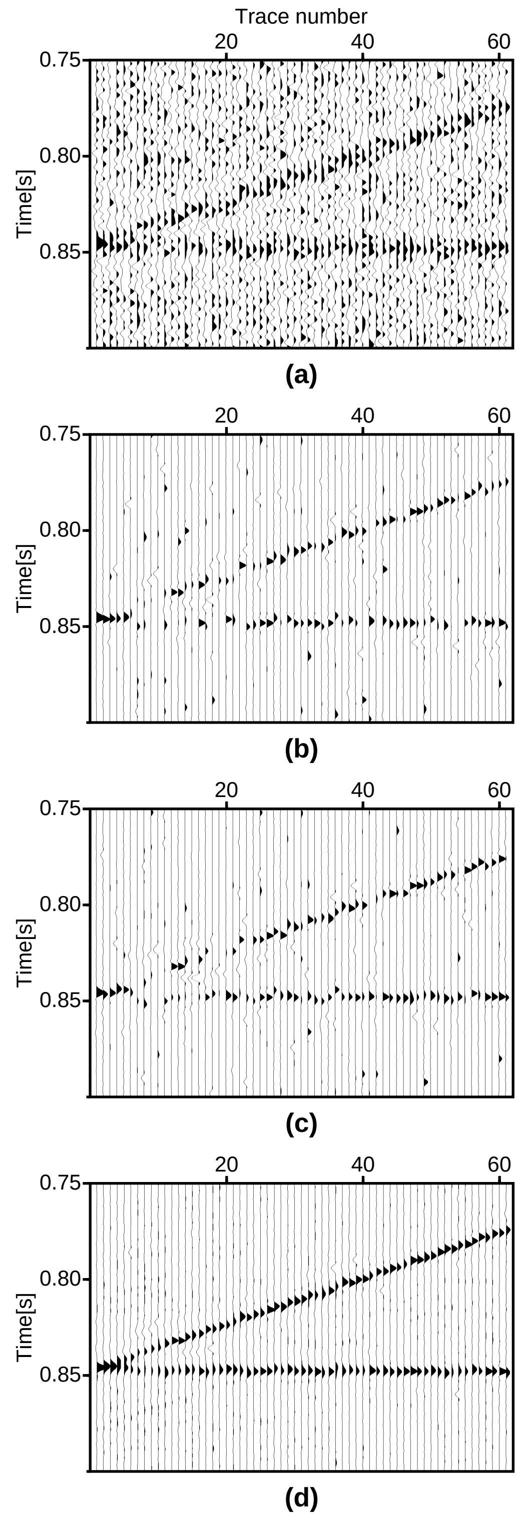

Fig. 1: Comparison of the results of the wedge model. (a) Input data. (b) Trace-by-trace sparse deconvolution. (c) "Two-pass" sparse deconvolution (FX filtering followed by trace-by-trace sparse deconvolution. (d) Multi-channel sparse deconvolution.

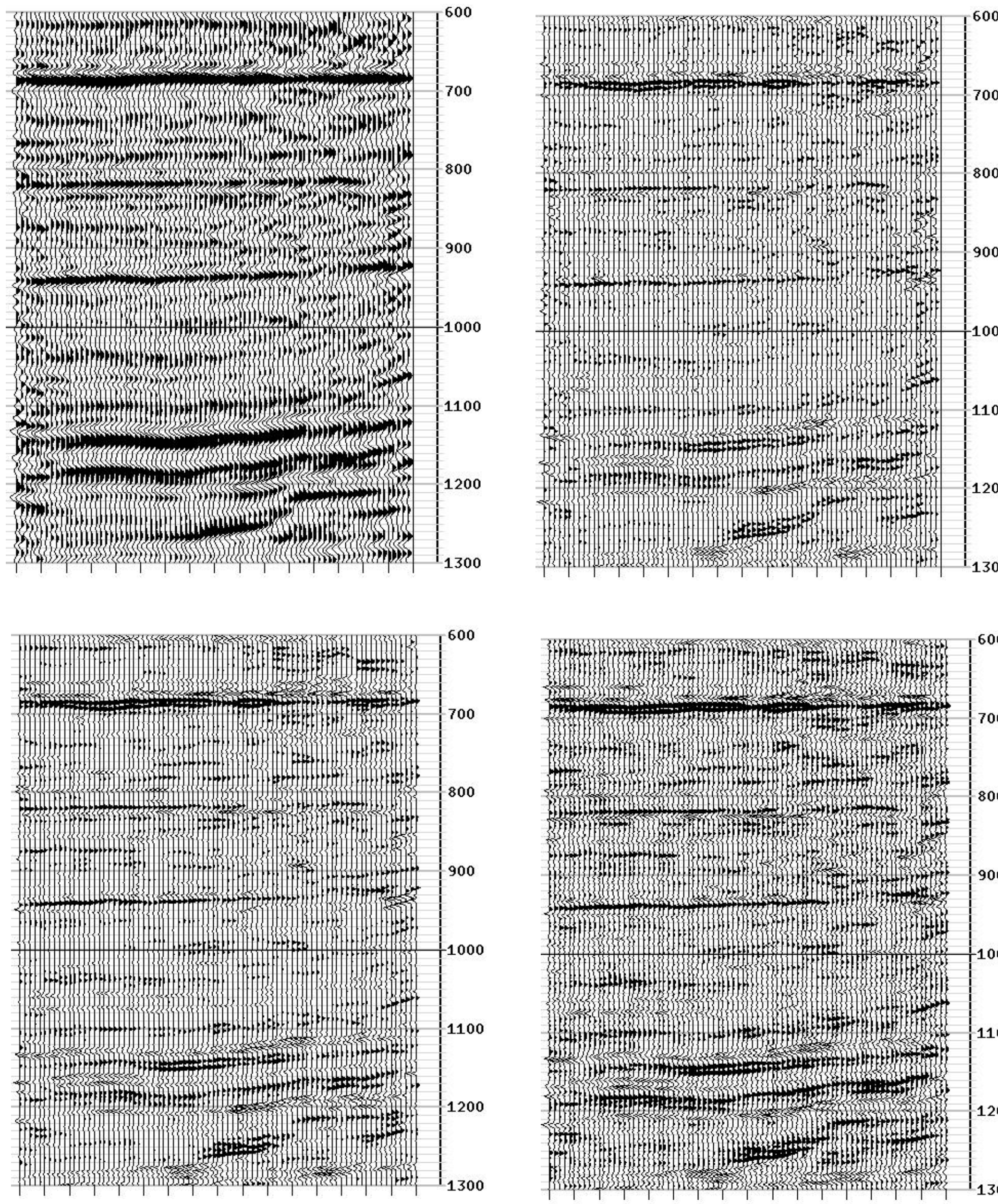

Fig. 2: Comparison of the results of the field data. Upper left: input data. Upper right: trace-by-trace sparse deconvolution. Lower left: "two-pass" sparse deconvolution (FX filtering followed by trace-by-trace sparse deconvolution). Lower right: multi-channel sparse deconvolution. Vertical unit: milisecond.

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