Sparsity Promoting L1 Deconvolution: Application for a Global Seismology Parameter Search

Abstract. This report will discuss an attempt to utilize the inherent sparsity of an assumed layered earth model in order to provide better deconvolution results than the often utilized damped-L2 methods. The sparsity of the layered earth model is achieved through the wavelet transform, and this allows the use of sparse solvers such as SPGL1. Synthetic tests are performed using the damped L2 convolution as well as the sparse wavelet transform L1 minimization using several wavelet basis. Real data is employed and the deconvolved results from the competing methods are used for an actual parameter grid search. The final results of the grid search are then compared. Further work on the sparse-in-wavelets L1 algorthim will be required if it is to beat the damped-L2 algorthim currently being employed.

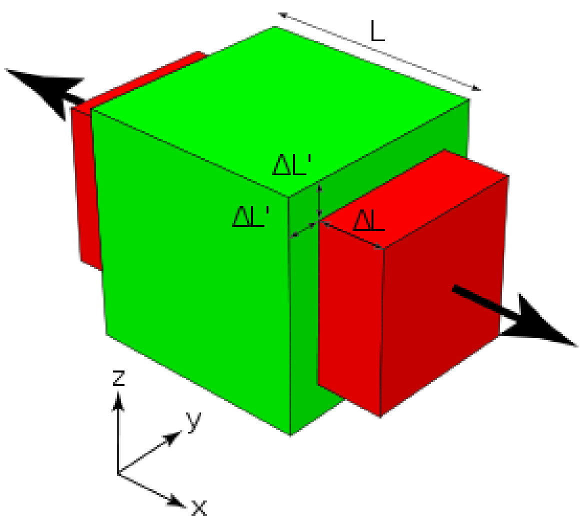

Figure 1. Poissons Ratio is the ratio of compression along one axis to extension along another

1. Introduction

Extracting material properties from seismic data spans industry and the research community. In the global seismic community material properties of the crust have been sought to validate existing geological models or provide clues for new ones. A good example of this is Poisson's ratio, which is the ratio between compression along one axis to extension on another when a material is stretched (see figure 1). Poisson's ratio is given by:

$$\sigma = \frac{1}{2}\frac{\left(\frac{V_p}{V_s}\right)^2 - 2}{\left(\frac{V_p}{V_s}\right)^2 - 1}$$

where the seismic velocities V_p and V_s determine the property entirely. Because of the direct link between seismic properties and Poisson's ratio, this material property has been sought in numerous seismic studies, such as Kanamori (2000) and Snyder (2010). All of these studies have performed a parameter search for $R = \frac{V_p}{V_s}$ and provided a background estimate for V_p. This allows for fast computation through a grid search and avoids the problem of the low-dependency of V_p in the grid search equations. Although Poisson's ratio can be estimated from R better geological constraints would be possible if V_p and V_s were resolved uniquely.

2. The Problem

In order to differentiate between V_p and V_s the travel times from seperate arrivals of energy must be compared. The first arrival a seismic station will receive is from the higher velocity P-wave. The impulsive character and high amplitude of the P-wave arrival allow it to be windowed and filtered and used as an approximation for the source. There are three more arrivals of interest. The Ps arrival, which is P-energy converted to S-wave energy at the Moho boundary between the crust and the mantle. Two further arrivals follow which are the reflections from the free surface which have reflected again off the Moho and recorded at the station, figure (2). These reflected arrivals are usually of lower amplitude and often less well defined in the seismogram. If the three S-wave arrivals, the Ps, PpPs and PsPs can be located intime. With this information a grid search over the travel time equations with respect to V_p and V_s can be performed. The paramater V_p is not very sensitive to the traveltime differences between the S-wave phases, thus a very impulsive deconvolution result is needed to steepen the gradient in V_p space.

3. Deconvolution - L2

Currently simultaneous deconvolution with generalized cross validation is being employed as it performs well and is very quick. Since division in the fourier basis is deconvolution in time we can write:

$$\mathbf{R} = \mathbf{F}^{-1}[\mathbf{G}(\omega)]$$

where

$$\mathbf{G}(\omega) = \left[\frac{\sum_n^N \mathbf{S}_n \mathbf{P}_n^*}{\sum_n^N \mathbf{P}_n \mathbf{P}_n^* + \delta}\right]$$

. Here $\mathbf{F}$ is the Fourier transform, $\mathbf{G}$ is the model, $\mathbf{S}$ is a matrix of S-wave seismograms, $\mathbf{P}$ is a matrix of P-wave source coda, $*$ is the complex conjugate and δ is the regularization parameter. δ is chosen by minimizing the Generalized Cross

0148-0227/12/$9.00

Validation function with respect to δ:

$$\min \left(\frac{\sum_n^N \sum_m^M \left(\overbrace{\mathbf{S}_n(\omega_m) - \mathbf{P}(\omega_m)\left[\frac{\sum_n^N \mathbf{S}_n\mathbf{P}_n^*}{\sum_n^N \mathbf{P}_n\mathbf{P}_n^* + \delta}\right]}^{\text{Misfit: more accurate with } \delta \to 0\text{. Less Stable}} \right)^2}{\left(\underbrace{NM - \sum_m^M \frac{\sum_n^N \mathbf{P}_n\mathbf{P}_n^*}{\sum_n^N \mathbf{P}_n\mathbf{P}_n^*+\delta}}_{\text{As } \delta \text{ increases denominator increases, promotes stability.}} \right)^2} \right)$$

Since the sharpness of the deconvolution helps determine the confidence level contours on the final parameter estimation the prospect of a better deconvolution methods is very appealing. As the earth model being used in this parameter search is a simple layered model and the data is piecewise smooth the wavelet transform could be effectively used to promote sparsity. If we could leverage this fact and use an L1 technique which promotes sparse solutions we may be able to increase the accuracy of the deconvolved receiver functions.

4. Future Work

Even with a fairly preliminary L1 minimization setup the results are encouraging. Further work could focus on the 2D reciever function plots and usage of curvelets to interpolate between poor data. This would significantly improve the estimates made by the grid search. While redundant wavelet dictionaries were employed in the non-spline cases, these authors are currently unsure if the B-spline methods were shift-invariant or not. This will require further investigation. Also, during a recent presentation of the work contained in this report Felix Herrmann suggested looking at Radon Transformations. Both authors are unsure of the utility of these transformations and this will also require further research. However, with a sophisticated tool-chain in place we believe that L1 minimization and the utilization of sparsity inherent in this problem will result in better deconvolution than the current L2 method being employed. Since there is a large body of global seismological work which has been built on results from L2 deconvolution there is a lot of potential to apply these methods to other problems. Along the same lines, once sparsity is being utilized, the larger compressive sensing framework should surely find a place in the global seismic research community. There is vast potential in this arena and likely many exciting avenues for phd research. Both the authors of this report are interested in exploring this new paradigm further in the context of global seismology.

Acknowledgments. (Text here)

References

Kanamori, H. (2000), Moho depth variation in southern California from teleseismic receiver functions, *Geophysical Research, 105*, 2969–2980.

Thompson, D. A., Bastow, I.D., Helffrich, G., Kendall, J-M., Wookey, J., Snyder, D.B., Eaton, D.W., (2010), Precambrian crustal evolution: Seismic constraints from the Canadian Shield, *Earth and Planetary Science Letters, 297*, 655–666.

Bostock, M. G., Kumar, M. R., (2010), Bias in seismic estimates of crustal properties, *Geophysical Journal International, 182*, 403–407.

Herrmann, F. J., (2012), Class Material from eos 513, *University of British Columbia*

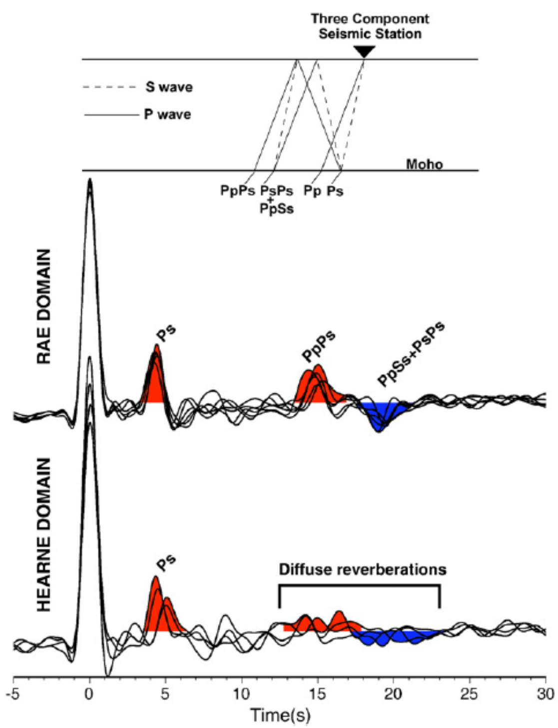

Figure 2. Top: Shows direct phases Pp and Ps from a seismic event hitting receiver. Also the reflected phases PpPs and PsPs are shown. The phases ending with 's' are the phases hitting the station as S-waves. These S-wave phases are used as the primary data, the direct Pp phase is used as the source, and the two are deconvolved to attain the receiver functions shown at Bottom: The receiver functions attained from deconvolution. Depending on the quality of the data and the sharpness of the deconvolution, the results may look like those given for the Rae Domain -better- or the Hearne Domain -worse.