N3

$$\prod_i p(x_i) \to \max_{\mu,\sigma} \qquad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Для логарифма проще:

$$\ln \prod_i = \sum_i \ln p(x_i) = \sum_i \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right) =$$

$$= \sum_i \left(\ln \frac{1}{\sqrt{2\pi\sigma^2}} + \ln e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right) = \sum_i \left(\ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

$$\frac{\partial \ln \prod}{\partial \mu} = -2 \sum_i \frac{(x_i-\mu)}{2\sigma^2} \cdot (-1) = 2 \sum_i \frac{x_i-\mu}{2\sigma^2} = 0 \Rightarrow \sum_i (x_i-\mu) = 0$$

$$n\mu = \sum x_i \Rightarrow \boxed{\mu = \langle x \rangle}$$

$$\frac{\partial \ln \prod}{\partial \sigma} = \sum_i \left(\sqrt{2\pi\sigma^2} \cdot \frac{1}{\sqrt{2\pi}} \left(-\frac{1}{\sigma^2}\right) + 2\frac{(x_i-\mu)^2}{2\sigma^3}\right) = \sum_i \left(-\frac{1}{\sigma} + \frac{(x_i-\mu)^2}{\sigma^3}\right) = 0$$

$$\sum_i \frac{(x_i-\mu)^2 - \sigma^2}{\sigma^3} = 0; \quad \sum_i \left((x_i-\mu)^2 - \sigma^2\right) = 0$$

$$n\sigma^2 = \sum_i (x_i-\mu)^2; \quad \boxed{\sigma^2 = \frac{1}{n} \sum (x_i - \langle x \rangle)^2 = Dx}$$