Задание 3.

1) $ay^3+d=0$ $\qquad a=1\pm10^{-3}\qquad d=8\pm10^{-3}$

$$y=\sqrt[3]{-\frac{d}{a}}=-2$$

$$\varepsilon_y=\sqrt{\left(\varepsilon_d\tfrac13\right)^2+\left(\varepsilon_a\tfrac13\right)^2}=\sqrt{\left(\tfrac13\varepsilon_d\right)^2+\left(\tfrac13\varepsilon_a\right)^2}=\frac13\sqrt{\varepsilon_d^2+\varepsilon_a^2}=\frac13\sqrt{\left(\frac{10^{-3}}{8}\right)^2+\left(\frac{10^{-3}}{1}\right)^2}\approx0{,}34\cdot10^{-3}$$

$$\delta_y=\varepsilon_y\cdot|y|=0{,}7\cdot10^{-3}$$

$$y=-2\pm7\cdot10^{-4}$$

2) $$u'(x)\cong\frac{u(x-2h)-8u(x-h)+8u(x+h)-u(x+2h)}{12h}$$

$$u(x+\Delta x)=u(x)+u'(x)\Delta x+\frac12u''(x)(\Delta x)^2+\frac16u'''(x)(\Delta x)^3+\frac1{24}u^{IV}(x)(\Delta x)^4+\frac1{120}u^{V}(x)(\Delta x)^5+o((\Delta x)^5)$$

$$u(x-2h)=u(x)-2u'(x)h+2u''(x)h^2-\frac43u'''(x)h^3+\frac23u^{IV}(x)h^4-\frac4{15}u^{V}(x)h^5+o((2h)^5)$$

$-$

$$8u(x-h)=8\left[u(x)-u'(x)h+\frac12u''(x)h^2-\frac16u'''(x)h^3+\frac1{24}u^{IV}(x)h^4-\frac1{120}u^{V}(x)h^5+o(h^5)\right]$$

$+$

$$8u(x+h)=8\left[u(x)+u'(x)h+\frac12u''(x)h^2+\frac16u'''(x)h^3+\frac1{24}u^{IV}(x)h^4+\frac1{120}u^{V}(x)h^5+o(h^5)\right]$$

$-$

$$u(x+2h)=u(x)+2u'(x)h+2u''(x)h^2+\frac43u'''(x)h^3+\frac23u^{IV}(x)h^4+\frac4{15}u^{V}(x)h^5+o((2h)^5)$$

$\parallel$

$$\cancel{u}-2u'h+\cancel{2u''h^2}-\frac43\cancel{u'''h^3}+\frac23\cancel{u^{IV}h^4}-\frac4{15}u^{V}h^5-\cancel{8u}+8u'h-\cancel{4u''h^2}+\frac43\cancel{u'''h^3}-\frac13\cancel{u^{IV}h^4}+$$
$$+\frac1{15}u^{V}h^5+\cancel{8u}+8u'h+\cancel{4u''h^2}+\frac43\cancel{u'''h^3}+\frac13\cancel{u^{IV}h^4}+\frac1{15}u^{V}h^5-\cancel{u}-2u'h-\cancel{2u''h^2}-$$
$$-\frac43\cancel{u'''h^3}-\frac23\cancel{u^{IV}h^4}-\frac4{15}u^{V}h^5+o(h^5)=12u'h-\frac25u^{V}h^5+o(h^5)$$

$$\left|u'(x)-\frac{u(x-2h)-8u(x-h)+8u(x+h)-u(x+2h)}{12h}\right|=$$

$$=\left|u'(x)-u'(x)+\frac1{30}u^{V}h^4+o(h^4)\right|\le\frac1{30}M_5h^4$$

$$\varepsilon=\varepsilon_{\text{округл}}+\varepsilon_{\text{метод}}=\frac{\Delta u+8\Delta u+8\Delta u+\Delta u}{12h}+\frac{M_5h^4}{30}=\frac32\frac{\Delta u}{h}+\frac{M_5h^4}{30}\to\min$$

$$\varepsilon'_h=-\frac{3\Delta u}{2h^2}+\frac2{15}M_5h^3=0$$

4-й порядок аппроксимации ($o(h^4)$)

$$\frac2{15}M_5h^5=\frac{3\Delta u}{2}\Rightarrow h_{opt}=\sqrt[5]{\frac{45\Delta u}{4M_5}}$$

Задание 4

$x^2 - 2x + 0{,}99999375 1 = 0$

$\Delta x \sim 10^{-4}$ – 4 верных знака

$c = 2x - x^2$

$\Delta c = |2\Delta x - 2x\Delta x| = |2\Delta x(1-x)|$

$x_1 \simeq 1{,}0025$

$x_2 \simeq 0{,}9975$

$\Rightarrow \quad \Delta c \approx |2 \cdot 10^{-4} \cdot 0{,}0025| \approx 5 \cdot 10^{-7} \Rightarrow$

7 верных знаков

Задание 5

$x_n, \quad n = 0, 1, \ldots$

$5x_{n+1} - x_n = 4 \qquad \Delta x_0 = 10^{-6}$

$x_n = C\left(\frac{1}{5}\right)^n + 1$

$x_0 = C + 1$

$\Delta x_0 = \Delta C$

$\Delta x_n = \Delta C \cdot \left(\frac{1}{5}\right)^n = \Delta x_0 \left(\frac{1}{5}\right)^n$

$\varepsilon_{x_n} = \frac{\Delta x_n}{x_n} = \Delta x_0 \frac{\left(\frac{1}{5}\right)^n}{C\left(\frac{1}{5}\right)^n + 1} = \Delta x_0 \frac{1}{C + 5^n}$

Если C достаточно отрицательный, то несколько первых членов будут иметь возрастающую отн. погрешность. В общем же случае, начиная с некоторого $n = N$ отн. погрешность будет уменьшаться