Quantum conductivity in Graphene

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We observe the results of a measurements of QHE (Quantum Hall effect) in Graphene sample with the dimensions of $6 \times 20\ \mu m^2$, that as made in Lab C course in TAU. the electron density, quantities Resistance, filling factor, degeneracy and mobility as found from the measurement and compared to theory and known values. the results show the proprieties of Graphene and Quantum Hall effect.

I. INTRODUCTION

A. why electrons flow?

The flow of electrons is a phenomenon involving an ensemble of electrons, therefor it is appropriate to look at that phenomenon from a statistical mechanics point of view.

1. Dispersion relation and bend structure in lattice

Graphene, like many other materials is a lattice. which means that its structure is made of some unit cell that repeats itself all over the lattice. That repetitive structure is important for solid-state physics, it allows us to analyze those materials and use different approximations like nearly-free electrons or Tight binding. Those approximations can give us things like the dispersion relation of the electron's wave function. When writing the Dispersion relation $E(\vec{k})$ one would get the "bend structure" of the lattice, a typical bend structure is made of layers of bends which are the "allowed areas" where the electrons have some states to occupy and between each band there is some "forbidden areas" with no available states between each band - as shown in figure 1:

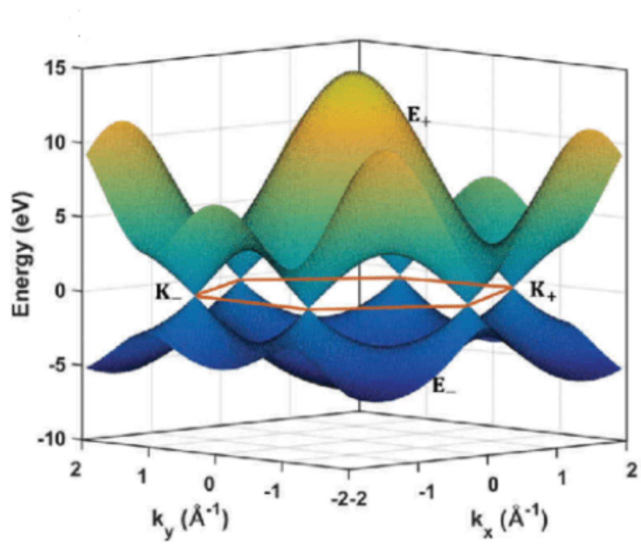

Figure 1. bend structure of grephene

2. Fermi - Dirac statistics

From Pauli exclusion principle, the electrons, which are fermions, cannot populate the same quantum state, so each state can be populated by only one electron. this causes electrons to obey to Fermi - Dirac distribution. Fermi - Dirac distribution says that in a system where one can control the chemical potential μ, the temperature T and the volume V the probability of a state with defined momentum number $\vec{k}$ to be populated is:

$$P(\vec{k}, T, V, \mu) = \frac{1}{1 + e^{\frac{\epsilon(\vec{k}) - \mu}{T * k_B}}} \tag{1}$$

When $\mu >> Tk_B$ we notice that Fermi - Dirac distribution becomes a step function $\theta(\mu - \epsilon(\vec{k}))$. So all state with energies lower then μ are occupied and all the states with energies higher then μ are not occupied. The highest occupied energy is called the fermi energy:

$$\epsilon_{fermi} = \mu \tag{2}$$

Another result of Fermi - Dirac distribution is when the number of identical fermions is fixed to be N. Because the equilibrium state is singular, if $\epsilon_{max} >> Tk_B$ one can find the first N lowest energy states of the system and know that they are the only occupied states. we can conclude:

$$\epsilon_{max} = \epsilon_{fermi} = \mu \tag{3}$$

3. adding a power source

From a statistical - mechanics point of view, adding an electric potential V to a system is like changing the chemical potential from μ to $\mu + 0.5eV$ from one side (we call it the left side) and from μ to $\mu - 0.5eV$ from the other side (the right side), we call that the electrochemical potential. Therefor an electric circuit power source can be described as connecting two systems that are not in equilibrium with one and other but because they are large enough each of them cannot be driven out-of-equilibrium. We would expect that by connecting those systems the electrons would flow from high electrochemical potential to lower electrochemical potential in any material, but as we all know not every material is a conductor. For a material to transport electrons, it needs to have some available states in the energy range:

$$\mu + 0.5eV > \epsilon > \mu - 0.5eV \tag{4}$$

Because if $\mu + 0.5eV < \epsilon$ both sides would want to empty the states, and if $\mu - 0.5eV > \epsilon$ both sides would want to fill the states, in both cases there will not be a any flow if there are no states in the energy range (4). If there are states in the energy range (4) then one sides always fills the conductor with electrons and one side always empty them, causing a net flow. In some materials the range (4) is between two bands of the band structure, therefor there are no electrons in the energy range (4). because of that not every material is a conductor.

B. Drude model and classical hall effect

Drude model is a model of electrons' transport in a conductor, which is based on calculating the motion of the average place of electrons. the model assumes that the electrons in the conductor are free, or at least that all the effects of the lattice can be reduced to some effective mass m_e, that contains all the interactions of the electron with the lattice. according to that model the average place of electrons moves in response to an external force as described in newton's second low. but once every characteristic time τ they are being scattered (typically *tau* is very small compared to one second), the equation of motion that Drude concluded from those assumptions is:

$$m_{eff}\frac{d\vec{v}}{dt} = \vec{F} - \frac{m_{eff}\vec{v}}{\tau} \tag{5}$$

$\vec{F}$ is the external force. If we assume that there is a constant electric field along the X axis, e is the electron's charge and at $t = 0$ there is no flow then:

$$m_{eff}\frac{dv_x}{dt} = -eE_x - \frac{m_{eff}v_x}{\tau}, v(0) = 0 \tag{6}$$

$$v(t) = -\frac{eE_x\tau}{m_{eff}}(1 - e^{\frac{t}{\tau}}) \tag{7}$$

So if $t >> \tau$ then v(t) is constant, this is the steady-state scenario. If we define the current $\vec{J}$ as:

$$\vec{J} = -en\vec{v} \tag{8}$$

where n is the density of the electrons, we get that at steady - state:

$$\vec{J} = \frac{e^2n\tau}{m_{eff}}E_x \tag{9}$$

we define σ_D as the electric conductivity and μ as the electric mobility:

$$\frac{e^2n\tau}{m_{eff}} = \sigma_D = ne\mu \tag{10}$$

1. *the classical hall effect*

to get the classical hall effect one could add a constant magnetic field along the Z axis to $\vec{F}$, assume some unknown electric field E_y of Drude's model and get:

$$m_{eff}\frac{d\vec{v}}{dt} = -e(\vec{E} + v \times \vec{B}) - \frac{m_{eff}\vec{v}}{\tau} \tag{11}$$

$$m_{eff}\frac{d\vec{v}}{dt} = -e(E_x\hat{x} + E_y\hat{y} + B_0(v_y\hat{x} - v_x\hat{y})) - \frac{m_{eff}\vec{v}}{\tau}$$

since in steady-state $v_y = 0$ (we assume an almost one dimensional conductor) the solution of v_x and $\vec{J}$ is the same as (7),(9):

$$E_y = B_0v_x = -\frac{B_0}{ne}J_x \tag{12}$$

so if w is the width of the conductor in the y axis then the hall voltage is:

$$V_y = -wE_y = \frac{wB_0}{ne}J_x \tag{13}$$

and if I is the total current then $I = wJ_x$ and:

$$V_y = \frac{IB_0}{ne} \tag{14}$$

We want to calculate the resistance in the x axis and in the y axis, we define those resistances as:

$$R_{xx} = \frac{V_x}{I_x} \tag{15}$$

$$R_{xy} = \frac{V_y}{I_x} = \frac{B_0}{ne}$$

C. Ballistic conductors

1. *What is ballistic conductor?*

As we know from Drude's model the electrons in a conductor are being scattered once every characteristic time τ, so if we know the average velocity of the electrons, we can have the characteristic length L_c. So, if we have a tiny conductor with length $L << L_c$ then the electrons would act like free electrons (without scattering)- this is a ballistic conductor. to understand how the current behaves in a ballistic conductor we recall the model described in chapter A3. we have two systems with defined electrochemical potential each, $\mu + 0.5eV$ from the left, $\mu - 0.5eV$ from the right and the Ballistic conductor connecting them.

2. quantum description of the free electrons

the Schrodinger equation of an electron in a ballistic conductor is that of a free electron in the x axis, and in the y axis a harmonic potential with frequency ω. let k be the momentum-number of the x axis and the Schrodinger equation of the problem is:

$$(\frac{\hbar^2 k^2}{2m} + \frac{P_y^2}{2m} + \frac{1}{2}\omega y^2)\psi = E\psi \tag{16}$$

and the energy levels are:

$$E = \frac{\hbar^2 k^2}{2m} + \hbar\omega(n + \frac{1}{2}) \tag{17}$$

k's quantization is determent by the boundary conditions and has an important physical interpretation, if $k > 0$ then the electron moves from left to right and if $k < 0$ then the electron moves from right to left, we call those states +k and -k and both have the same kinetic energy.

3. Ballistic conductors conductivity

To understand how the current behaves in a Ballistic conductor we recall the model described in chapter A3. We have two systems with defined electrochemical potential each, $\mu + 0.5eV$ from the left and $\mu - 0.5eV$ from the right and the Ballistic conductor connecting them. the main argument of this conductivity model is that electrons that move from the left side to the right side "feel" only the electrochemical potential of the left side and vice versa. We back it up with the understanding that if the electrochemical potential in both sides will be the same as the left side for example, them Fermi level of the electrons that go from left to right (k+) will be $\mu + 0.5V$, and if we suddenly change the electrochemical potential of the right side, because there is no scattering nor back scattering, there are no additional electrons that can occupy the +k states- just more electrons that go from right to left, so those states are not affected by the change and there electrochemical potential will not change for +k states.

4. calculating the Quantization of current

The net flow of the electrons is the flow caused by the electrons that move from right to left minus the flow caused by electrons that move from left to right. the group velocity of electrons with defined momentum k in the x axis is:

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k} \tag{18}$$

so the current carried by the +k states is:

$$I_+ = -nev = \sum_{k=1}^{k_{max}} -\frac{e}{L} f(E(k)) v_g(k) \tag{19}$$

1/L is the density of electrons (L is the length of the conductor) and f counts the number of electrons with momentum k. Now we change the sum to an integral on k and multiplying by a constant $2 * L/(2\pi)$- for spin and the place each state occupies:

$$I_+ = -\frac{2e}{h}\int_0^{k_{max}} f(E)\frac{\partial E}{\partial k}, dk \tag{20}$$

$$I_+ = -\frac{2e}{h}\int_0^{\epsilon_{max}} f(E) dE$$

the total current is $I_{tot} = I_+ - I_-$ and f(E) is the same for k+ and k- from (15) so the total current is:

$$I_{tot} = -\frac{2e}{h}\int_{\epsilon_{-max}}^{\epsilon_{+max}} f(E) dE \tag{21}$$

Because $\epsilon_{+max} - \epsilon_{-max} = eV << \mu$ the function f(E) is about constant, we can define M as that constant and get:

$$I_{tot} = M\frac{2e^2 V}{h} \tag{22}$$

5. the contact resistance

By deafening the resistance of a conductor as:

$$R = \frac{V}{I} = \frac{h}{2Me^2}$$

One can calculate the number of available modes M to get R. A mode can propagate only if $k_{-max} < k < k_{+max}$. because we assume that k_{-max} is about the same as k_{+max}:

$$M = k_{+max} - k_{-max} = \frac{\sqrt{2m}}{\hbar}(\sqrt{\epsilon_{+max}} - \sqrt{\epsilon_{+max}}) \tag{23}$$

from (23) as V gets bigger M gets bigger and R gets smaller.

D. Resistance measurement

Since inside the ballistic conductor there is no scattering and therefor there is no energy loss, the Resistance between two points in there is zero. The electrochemical potential in the conductor is an average of the two systems that are connected to each side of it which gives μ. between each contact point of the conductor and a system there is an electrochemical potential difference of 0.5V.

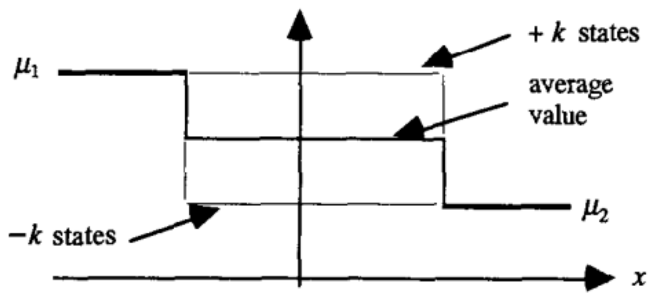

Figure 2. μ(x),$\mu_1 = \mu + 0.5V$, $\mu_2 = \mu - 0.5V$

E. Quantum hall effect

The full description of the Quantum Hall effect (QHE) is divided into two steps, solving the Schrodinger equation and a statistical - mechanics analyses based on that solution. The QHE is the quantization of the hall resistance - R_{xy} and oscillations in R_{xx}.

1. *Schrodinger equation and Landau levels*

The Schrodinger equation of a free electron with effective mass and a constant magnetic field $B_0\hat{z}$ is:

$$(\frac{(\vec{p}+e\vec{A})^2}{2m})\psi = E\psi \tag{24}$$

$\vec{A}$ is the magnetic vector-potential:

$$\vec{A} = B_0 y\hat{x} \tag{25}$$

so, the equation is:

$$(\frac{(p_x+eyB_0)^2}{2m} + \frac{p_y}{2m})\psi = E\psi \tag{26}$$

$$(\frac{(\hbar k+eyB_0)^2}{2m} + \frac{p_y}{2m})\psi = E\psi$$

for each k we define $y'eB_0 = \hbar k + eyB_0$:

$$(\frac{(eB_0y')^2}{2m} + \frac{p'_y}{2m})\psi = E\psi$$

the solutions are:

$$E_{n,k} = \hbar\omega_c(n+\frac{1}{2}), \omega_c = \frac{eB_0}{m_{eff}} \tag{27}$$

$$\psi_{n,k} = u_n(y+y_k), y_k = \frac{\hbar k}{eB_0}$$

u_n are the solutions of a quantum harmonic oscillator. and the energy levels are the Landau levels.
In our model there is another component V(y) which is an expression of boundary effects. so V(y) is about 0 in the middle of the sample and blows fast about the boundaries. To consider V(y) one can use perturbation

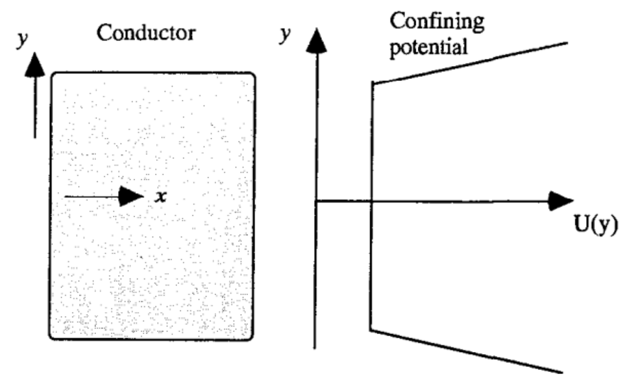

Figure 3. V(y) example

theory to estimate the effect of V(y). Since $\psi_{n,k}$ is an harmonic oscillator centered around y_k so states with the same k would be about the same place of the y axis, we assume that V(y) is about constant in that place:

$$E_{n,k} = \hbar\omega_c(n+\frac{1}{2}) + V(y_k) \tag{28}$$

$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{1}{\hbar}\frac{\partial V}{\partial k} = \frac{1}{eB_0}\frac{\partial V}{\partial y}$$

so if k is about 0 than the effect of V is insignificant and $v = 0$, the electrons with higher y flow from left to right and the electrons with lower y flow from right to left.

2. *Statistical-mechanics point of view*

Our material has some finite number of electrons, and all the quantum hall effect is caused by the different possible populations of the perturbated Landau levels. If the fermi energy is between landau levels as shown in figure.4 the material acts like a ballistic conductor, this is because electrons in the edges cannot back-scatter, since the only available states with a momentum in the opposite direction are at the other side of the sample, that makes the

correlation between the two wave functions very low and so as the possibility of back scattering. Because there is no back-scattering and similarly to a ballistic conductor, the electrons that move from right to left "fill" the electrochemical potential of the right side and vice versa. In this case where each side hold electrons that move together from left to right or from right to left with no interactions between both sides - we get a difference in the electrochemical potential (or the measured voltage) in the y axis and measure 0 voltage between to point on the x axis (for the same y value).
If the fermi energy is at a landau level there are available states in the middle, so the sample acts like a regular conductor and we can measure a voltage in the x axis. There are still electrons in the side so we can measure a voltage in the y axis the as in the previous case.

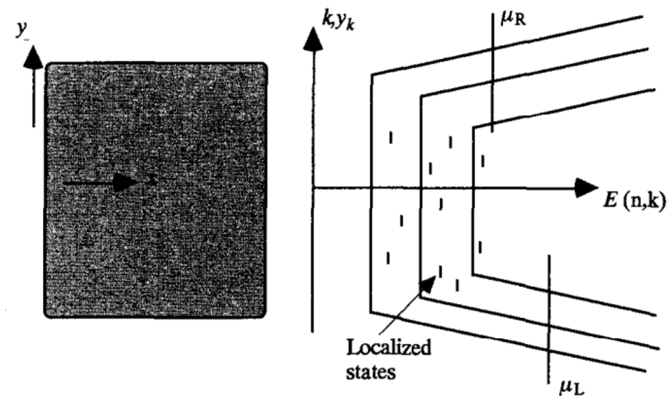

Figure 4. E(n,k) example

3. *Calculating the resistance*

Suppose there are M landau levels under the fermi level and the fermi level is in between two landau levels. The velocity of the electrons on the side of the sample is about the same at each landau level. So we can calculate the current I_x for one landau level and multiply it by M.

$$I_x = \frac{2Me}{2\pi}\int_{k_{min}}^{k_{max}} v(k)dk \tag{29}$$

$$I_x = \frac{2Me}{2\pi\hbar}\int_{k_{min}}^{k_{max}} \frac{\partial E}{\partial k}dk$$

$$I_x = \frac{2Me}{\hbar}\int_{\mu-0.5eV}^{\mu+0.5eV} dE$$

$$I_x = \frac{2Me^2}{\hbar}V$$

and because V is the also V_y

$$R_{xy} = \frac{\hbar}{2e^2M} \tag{30}$$

M is the number of occupied landau levels, as seen in figure.4 v_g is about the same in each perturbated level, therefor one can calculate the contribution of one landau level, multiply by M and get I_x as we did in (29)-(30).

4. *the filling factor*

At first this result can look a little bit odd since R_{xy} is independent of B_0. But as B_0 gets bigger the number of available states at each landau level increases, because of that, there are less landau levels occupied. that results R_{xy} to become bigger. from (27) and spin degeneration, the number of states at each landau level (per area unit) is:

$$N = \frac{2}{y_{k+1} - y_k} = 2\frac{eB_0}{h} \tag{31}$$

we define the filling factor ν as the number of occupied landau levels times the degeneracy of every level. n_e is the total electron density, from chapter A.2:

$$\nu = \frac{2n_e}{N} = \frac{hn_e}{eB_0} \tag{32}$$

generally, ν is not an integer because the last landau level is not always full, in those cases there is a hall resistance and a longitudinal resistance. When it is between landau levels ν is a from (30) integer and we get the relation:

$$R_{xy} = \frac{\hbar}{e^2\nu} \tag{33}$$

$$\frac{h}{e^2} = 25.812807[k\Omega]$$

When the highest occupied landau level is not full, ν just takes a non-integer value. We know in general that in a lattice the highest resistance occurs when the highest bend is half full (and ν takes half integer values), so if we take two following picks in the longitudinal resistance, we can get n_e:

$$2 = \nu_{i+1} - \nu_i = \frac{hn_e}{e}\left(\frac{1}{B_{i+1}} - \frac{1}{B_i}\right) \tag{34}$$

$$n_e = \frac{2e}{h(\frac{1}{B_{i+1}} - \frac{1}{B_i})}$$

5. *Quantum hall effect in graphene*

There are some differences between the general model of quantum hall effect and the specific model for graphene. the energy spectrum of the electrons in graphene that of a relativistic particle when $B_0 = 0$:

$$E_{\pm}(k) = \pm\sqrt{(mc^2)^2 + (\hbar ck)^2} \tag{35}$$

and when B_0 is nonzero the landau levels are:

$$E_{\pm}(n) = \pm\sqrt{(mc^2)^2 + 2\hbar n|eB_0|c^2} \tag{36}$$

and in graphene $m = 0$ and V_f is approximately c:

$$E_{\pm}(n) = \pm\sqrt{2\hbar n|eB_0|c^2} \tag{37}$$

The degeneracy of each landau level is different in graphene and equals 4 because there are two valleys and to spin states and n is shifted by 0.5 because of Berry phase of π. So the filling factor is:

$$\nu = g(n + 0.5), g = 4 \tag{38}$$

and n can get negative value and then the current carrier change from electrons to holes.

6. *the capacitor*

in the experiment we have a metal that connected to the sample, to control the voltage applied as part of the Hall configuration. a simple model of the capacitor is: for some gate voltage V_g the charge Q of a capacitor with a constant C is:

$$Q = CV_g \tag{39}$$

so if the area of the graphene is A:

$$Q = Aen_e \tag{40}$$

$$n_e = \frac{C}{Ae} V_g$$

II. EXPERIMENT MAIN MEASUREMENT DEVISE - "THE LOCK-IN AMPLIFIER"

A. Noisy measurements in the lab

In every experiment there is a level of "Noise" that interrupt the clear data observation from the physical phenomena one wants to measure. In electronics devises and fields dependence phenomena the problem is not trivial. in every lab we have electronics that radiates field from there components. If one wants to measure a phenomenon that has a low signal to measure, that correspond to an EM fields it's become difficult to made a good observation without being "Blind" from the noise, where the real phenomena signal is "hiding" behind the noise. to demonstrate the problem in simple and visual terms we will show a pure sin signal with random noise with magnitude of 5% of the main sin, and a noise with 20%, for the noise we will add a White Gaussian noise which correspond to the distribution of the noise from electronics in many cases for example. the Signal to noise ratio as chosen as the noise is maximum half of the signal, but this is not necessary the case, the noise can be at the same scale or larger than the signal that one wants to measure in the lab. We need to distinguish between a small signal relative to the measurement devises, and a small signal relative to the noise. If the signals are only small relative to the measurement devise, an amplifier can amplify the signal and the problem solved. A signal that small relative to the noise can be big enough to read with the measurement devises but "hiding" behind the noise.

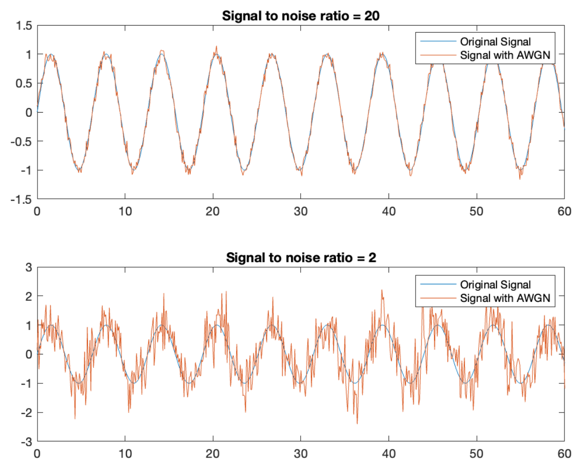

Figure 5. Sin and WGN with SGN ratios of 20/2

fig 5 show that the sin signal starts to be less clear with the ratio of signal-to-noise. the visual logically correspond to the measurement of the phenomena, it will be harder to measure the wanted signal if the cause to the phenomena or the measurement in the scale of the noise.

B. Lock in amplifier

To solve the problem from the last subsection the "Lock in amplifier" device invented. the device can measure a noisy signal even when the noise is larger with a large scale over the signal.
We present the device components and frequency response to periodic signals to understand how the device able to give the experimenter a clean of noise as possible measurements.
Let signal S_1 be a sine signal with frequency ω_1, phase ψ and amplitude A.

$$S_1 = Asin(\omega_1 t + \psi) \tag{41}$$

Let signal S_2 be a reference signal to S_1 with unit amplitude no phase and ω_2 frequency.

$$S_2 = sin(\omega_2 t) \tag{42}$$

these 2 signals is the input to a mixer component, that multiply the signals, so the output of the mixer is:

$$Asin(\omega_1 t + \psi)sin(\omega_2 t)$$

and from trig identity, the output is:

$$\frac{A}{2}cos([\omega_1 - \omega_2]t + \psi) - \frac{A}{2}cos([\omega_1 + \omega_2]t + \psi) \tag{43}$$

now, using a Low pass filter we can remove the sum frequency cosine term we can build a simple RC parallel circuit for example with the transfer:

$$H(s) = \frac{R}{1 + RCs} \tag{44}$$

And the bode magnitude plot:

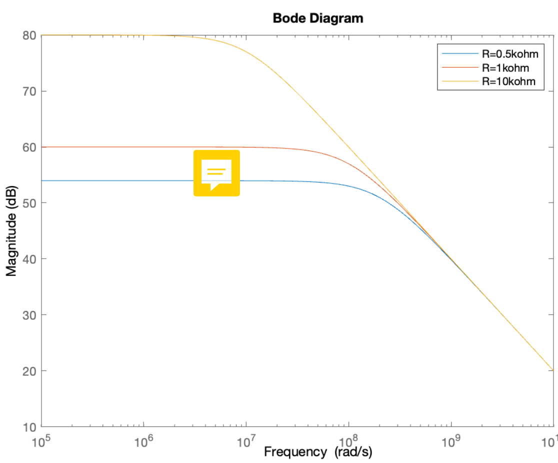

Figure 6. Bode magnitude plot H(s) for $R = (0.5, 1, 10)[k\Omega]$

fig 6 show how we can control the cut off frequency (-3db from the gain in bandwidth) and filter the frequencies adobe with a simple potentiometer. this example is relevant for the capacitor size also (where for this example take to constant 10pF).

with the LPF we can eliminate the element in the signal that has the frequency $\omega_1 + \omega_2$ with a correct chose of the R/C sizes. The output of the LPF is now:

$$Gain(R, C) * \frac{A}{2}cos([\omega_1 - \omega_2]t + \psi) \tag{45}$$

If the reference signal as a frequency $\omega_2 \rightarrow \omega_1$ we say the signals are "Lock" the "Lock in amplifier" can "Lock" the signals by matching the frequencies. the Signal S_1 can be the measurements, we have an input port to the devise, and a reference input port to the device. it's important to use correctly the sensitive of the device to overcome overload and under load of the measurements amplitude and see in the scale on the device that we operate in a range that far enough from that limit. the device control panel has a notification panel that indicate to overload and to un-lock of the reference signal. we can choose the RC time constant to control the acquire time/periods of the signals and the cut off frequency as required. With an analog output, digital and output port we can get the results, show the result, and know if the output is DC.

with the device "lock-in" functionally as mention we know that we have a DC signal from eq' 45 with a "lock" ($\omega_2 \ \rightarrow \omega_1$) we can see that now the signal is DC. now if we have a Noise signal, periodic, from Fourier we know that a signal like that can be represent as a sum of periodic signals, with setting a bias point of zero this kind of signals averaging over time to zero. now we have a DC measurement that we want and noise that spread over zero in random or a sum of AC signals noise that not "lock", Averaging over time will live the measurement WITHOUT noise. Now we can amplify the signal as much as we want.

For a good measurement one need to choose a RC time constant of the filter such that enough periods of signal will be averaging to zero, the signal can be complex and average to zero in a long period of time, averaging to zero for N go to infinity periods.
the Sensitivity is important, measuring a range of Voltages amplitudes signals must include a transition in the sensitivity of the device

III. RESULTS

For the first step in the experiment, we need to set an initial Fermi that gives a range of many level transfers as we are able. for that we used the measurement of $\rho_{xx}(V_g)$ with $B = 0[T]$. the plot shows the voltage where the Fermi level is in the middle between the levels that correspond to holes to the electron also shown in 9 the change in the sign. from this "position" of the Fermi Level the system synchronized to transfer throe the range that let the experimenter as many level transfer as he can, and also indicate the direction to sweep, to the electrons levels or holes without mixing when no needed.

A plot of the measurements data has been made, showing the results of the measurements to ρ_{xx}, ρ_{xy} as a function of the gate Voltage with a constant magnetic field 8,9.

And for magnetic field sweep ρ_{xx},ρ_{xy} with a constant gate voltage correspond to the ideal found in 7

The next data plot 10 that help in the experiment boundaries for V_g, and B_z. is the plot that represent the magnitude of R_{xx} as a function of B and V_g. the plot

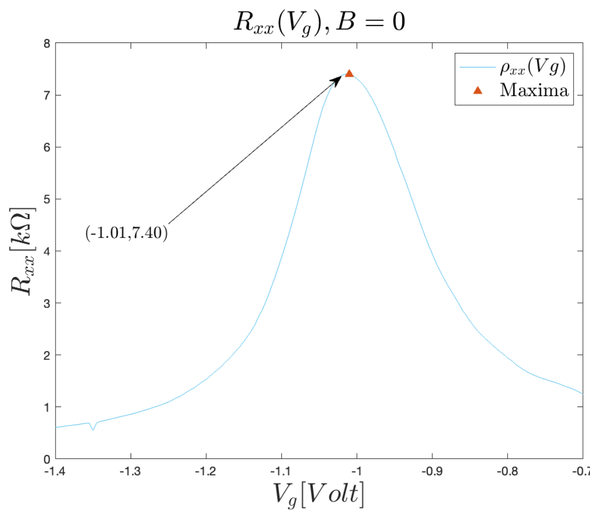

Figure 7. $R_{xx}(V_g), Maxima\ point = (-1.01, 7.40)$

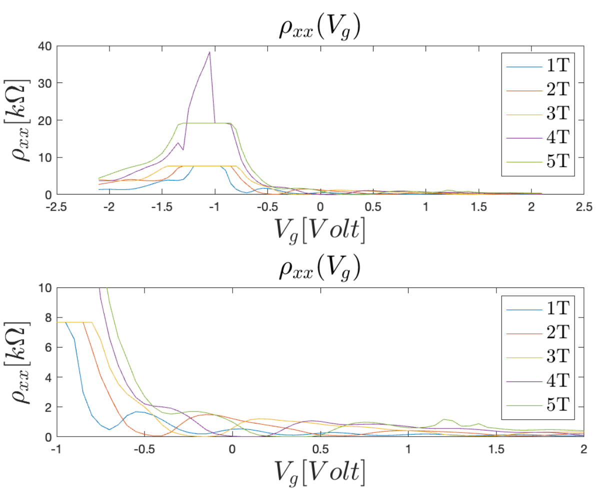

Figure 8. ρ_{xx} for different magnetic fields, as V_g gets bigger ρ_{xx} goes to 0 between landau levels

show lines of peaks that correspond to the maxima and minima of $R_{xx}(B, V_g)$. in the next section it will show how the lines correspond to filling factor, from the plot we also can witness the change in the peaks magnitude and spread with respect to the magnitude of magnetic field. An experimenter can use this map to plan an experiment that sweep over range of fields that will "give" as much landau levels passage with the help of the plot. the lines in the plots represent maxima and minima of R_{xx} from theory we know that this point corresponds to Fermi level pass landau level, a horizontal or orthogonal line in the map will show how a sweep of V_g in a magnetic field will deliver the number of crosses that required and vise-versa.
This map also indicate the behavior of carrier density and mobility with respect to both gate voltage and magnetic field, as will shown in the next fits, from specific fields,

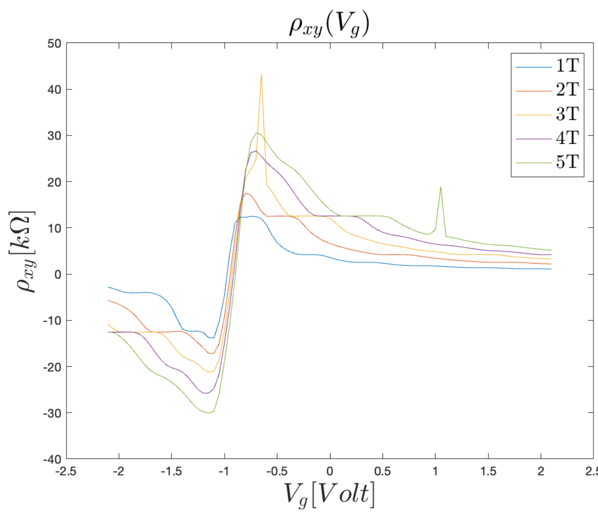

Figure 9. QHA for different magnetic fields, when ρ_{xy} is negative the current carrier change from electrons to holes

after understanding the fits one can return to the map and get insight on the behavior of all this properties from the heat map with respect to both V_g and B.

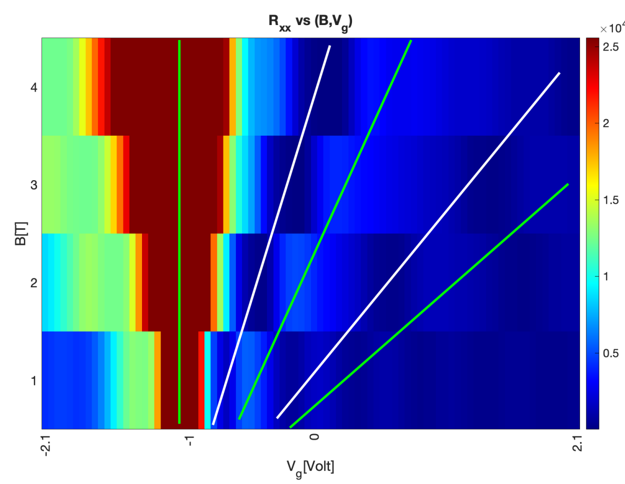

Figure 10. $R_{xx}(B, V_g)$,Maxima-green line,Minima-white line.

A. Drude Model results

To estimate the Drude Model region for $R_{xy}(B)$ a range of points as been take from the low field measurement and a linear fit has been made, The method to find the Drude region from the residuals plot, in the region that has a good Drude model approximation the residuals plot 12 should be WITHOUT a trend, with a good spread, and when the region was not in a good approximation to Drude model (too large) the residuals was with a periodic trend that corresponds to a quantities measurement, as we expect from a higher field and not in the range that corresponded to Drude model.

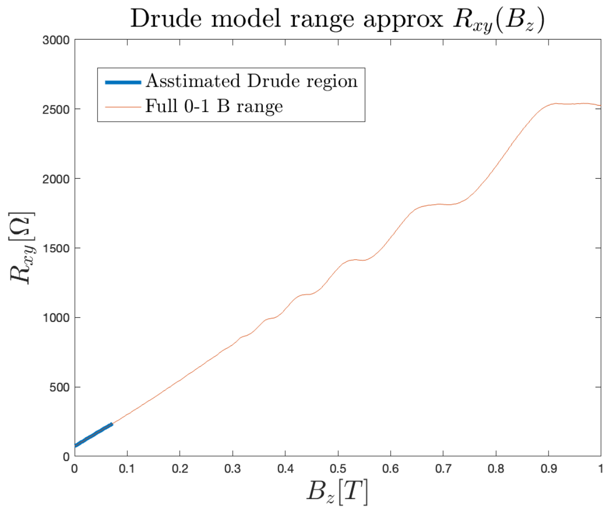

Figure 11. Drude region estimation $R_{xy}(B)$

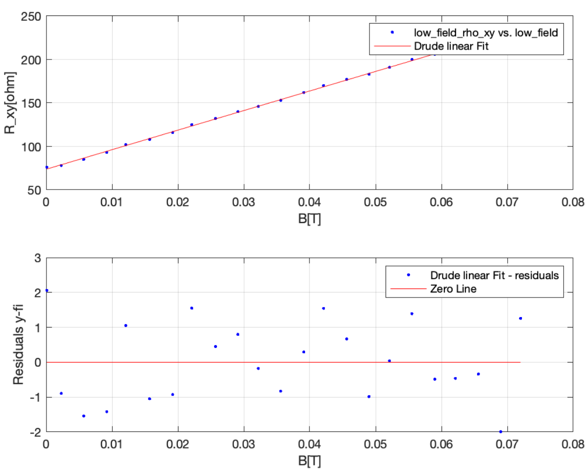

Figure 12. $R_{xy}(B)$ low field linear fit statistics: I

sse	701.75
rsquere	0.047
adjsquare	0.025

Table I. $R_{xy}(B)$ fit statistics

To estimate the Drude Model region for $R_{xx}(B)$ the method of R_{xy} didn't deliver a good approximation, a trend was witnessed for every range of sweep. to approximate the region a mean of ranges as bean made in the low field region, the range that taken was optimized to correspond to enough measurement data, and small deviation as possible from the mean value of the region that been under test, the region shown in 13

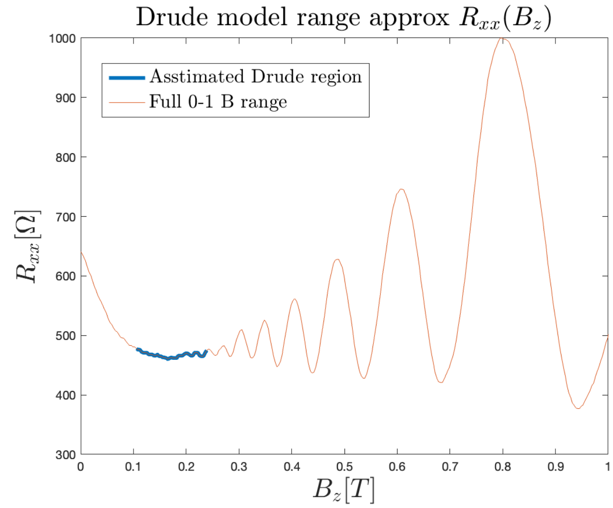

Figure 13. Drude region - R_{xx}

From the estimation of the Drude region $R_{xx,D}$ as taken as the mean of the points that measured in the region for R_{xx}. From the estimation of the Drude for $R_{xy}(B)$ the slope - $\frac{dR_{xy}}{dB}$ as taken from the linear fit 12. from I B

$$n_D = 2.77 \times 10^{11} \pm 2.76 \times 10^{9} [cm^{-2}]$$

$$\mu = 1.59 \times 10^{5} \pm 0.53 \times 10^{3} [cm^2/V]$$

B. Quantum Hall affect

From the measurements of R_{xy} we extract the quantities values from the theory value of R_{vk}. from the measurement of R_{xy} *vs* B. The Plautus are correspond to the position of Fermi level between the landaus levels as mention in theory, by extract the values of R_{xy} at the Plautus 14 we can extract the quantities factor of ν. The points where $\frac{dR_{xy}}{dB} = 0$ is the Plautus by definition.

ν	R_{xy} at Plautus $[k\Omega]$	Relative error to closest integer [%]
5.18	4.98	(5)3.5
6.12	4.22	(6)2.0
10.18	2.53	(10)1.8
14.24	1.81	(14)1.7
18.41	1.39	(18)2.2
22.37	1.15	(22)1.7
26.30	0.98	(26)1.1
30.34	0.85	(30)1.1
34.22	0.75	(34)0.6
42.01	0.61	(42)0.02

Table II. ν, R_{xy} *values* at Plautus,relative error to closest integer

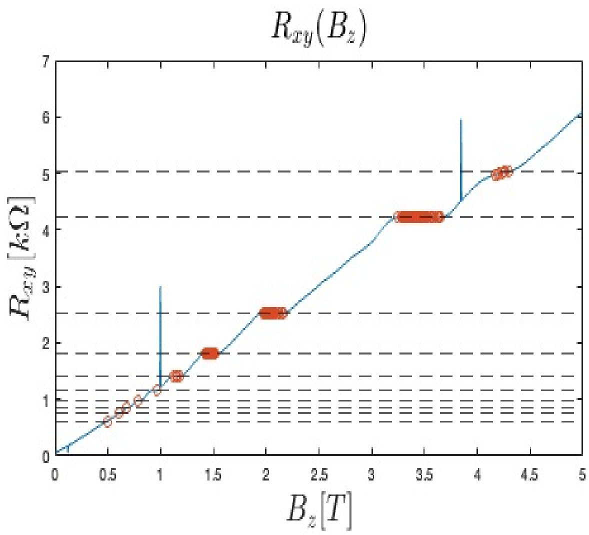

Figure 14. $\frac{dR_{xy}}{dB} = 0$ points

The results for ν shown in table II show the Relative Error Test for the values to the closest integer as we expect from theory for ν be from integer type. The $\nu = 5$ correspond to a unexpected Plautus and will discuss in IV, a value of $\nu = 38$ is missing, and also discuss in IV, (5) measurement are dismissed for the follow fits.
Now we take the Integer correspond to ν and linear fit 15 $R_{xy}(\frac{1}{\nu})$, in that fit we can "transfer" the misfit from theory to the Resistance measured in the experiment, by that we can see the physical lab errors in the measurements, un-ideal conductors, passive impedance - controllable variables, not like ν that the error in is value are likely to be result from voltage or resistance measurements.

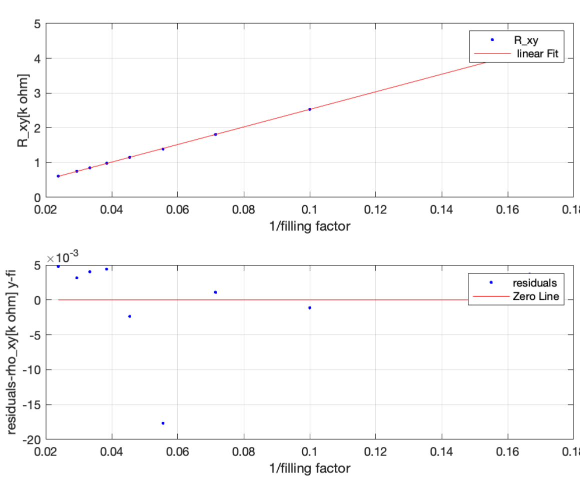

Figure 15. $R_{xy}(\frac{1}{\nu})$ linear fit

The linear fit is -

$$Y = a * X + b$$

sse	2.05e-14
rsquere	1
adjsquare	1

Table III. $R_{xy}(\frac{1}{\nu})$ fit statistics

a expected value is R_{vk} and b neglectable. the fit results are:

$$a = 25.28 \pm 0.14[k\Omega]$$

$$b = 0.0034 \pm 0.0105[k\Omega]$$

As R_{vk} being one of precise measurements existed we can now get a good vision of the Errors in the lab measurements from N_σ Test.

$$N_\sigma = 3.8$$

Now using the results for ν the degeneracy can be found, as discussed in theory the degeneracy in the Graphene 2D material is a special and important property. We assume $\nu = g * N$ with N is Float, so from $\Delta\nu_{(n+1)-(n)}$ we can extract the degeneracy g IV

g	Relative error to closest integer [%]
4.06	(4)1.48
4.06	(4)1.48
4.37	(4)8.47
3.75	(4)6.67
3.93	(4)1.78
4.03	(4)0.75
3.88	(4)3.09

Table IV. g results

from ν and g results we can verify the N as the expected is $\frac{2n+1}{2}$ V

N	Relative error to closest integer [%]
1.5	(3/2)0
2.49	(5/2)0.40
3.48	(7/2)0.57
4.55	(9/2)1.10
5.47	(11/2)0.55
6.43	(13/2)1.09
7.42	(15/2)1.08
8.36	(17/2)1.67

Table V. N results

C. Mobility and electron density

from IE4 and $R_{xx}(B)$ the electrons (holes) density can be extract, using a find peaks method(see appendix) we can find the peaks magnetic field values for 2 Gate voltages.16,17

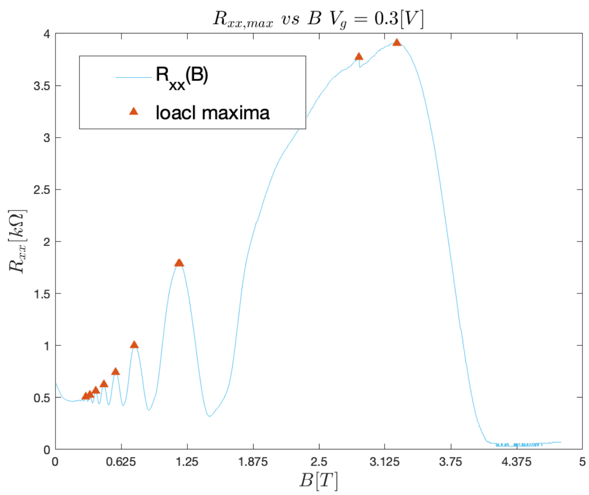

Figure 16. Peaks find in $R_x x$ mesurmant for magnetic field sweep, for $V_g = 0.3[V]$

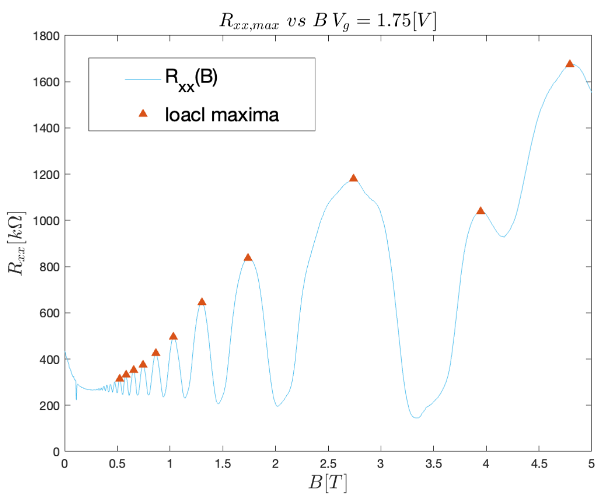

Figure 17. Peaks find in $R_x x$ mesurmant for magnetic field sweep, for $V_g = 1.75[V]$

from the results for the carrier density the mobility can be extract as described in I B

from n_e vs V_g linear fit we can extract the slope, with is a proportional constant that represent the capacitance per unit area per coulomb of the system, the linear fit made by "hand" because the fitting software problematic with a small sample of measurements (Eddington and MATLAB) . (See appendix for full development).

$$slope = \frac{\Delta n_e}{\Delta V_g} \quad (46)$$

$$\Delta slope = \frac{1}{V_{g1} - V_{g2}} \sqrt{(\Delta ne_1^2 + \Delta ne_2^2) + \Delta V_g^2 slope^2} \quad (47)$$

$n_e[cm^{-2}]$	Δn_e	$V_g[V]$
5.9e10	9.2e8	0.3
1.3e11	4.0e9	1.75

Table VI. ne(Vg) results

Slope- $\frac{C}{eA}$
A - surface Area

results:

$$\frac{C}{eA} = 4.65 \times 10^{10}[V^{-1}Cm^{-2}]$$

$$\Delta\frac{C}{eA} = 8.95 \times 10^{9}[V^{-1}Cm^{-2}]$$

from $n_e(V_g)$ and the measurement for σ_{xx} from $R_{xx}(B = 0)$ 7. the mobility extract as follow, from $R_{xx}(B = 0)$ we have the conductivity as a function of gate voltage, and from the linear fit from adobe we have $n_e(V_g)$. than from theory we have

$$\mu = \frac{\sigma_{xx}}{en} \quad (48)$$

i.e $\mu(V_g)$18

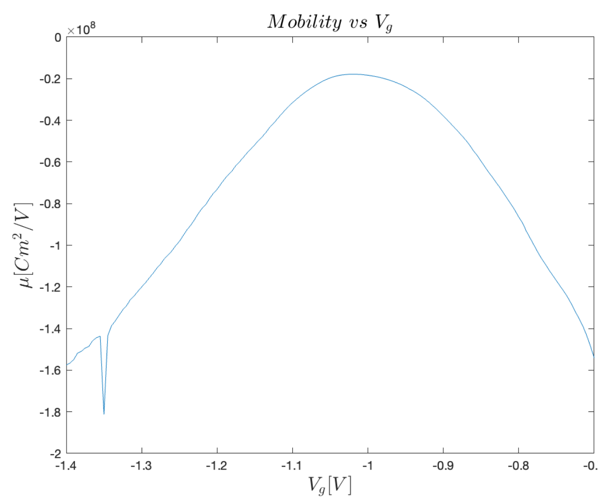

Figure 18. Mobility(V_g)

IV. CONCLUSIONS

From the Plots of the resistance that measured its visible that R_{xx} and R_{xy} are quantities and behave as we expect from the theory and the "move" of Fermi level with respect to landau levels.

Drude range approximation and fit for n_D and μ_D

deliver results that correspond to the order of magnitude that was measured in relevant papers.

the filling factor ν results has a results with a systematic error that indicate the expected values.fit $R_{xy}(\frac{1}{\nu})$ linearly deliver a R_{vk} extract that has a mis-match to the theory with bad statistics. The fact that R_{vk} is known with a high level of confidence can give the experimenter a good measurement to the systematic error in the Resistance measurements of the lab machinery, because R_{vk} is a global unit for Resistance a lab that has the possibility to measure a phenomena involved R_{vk} can calibrate the system with the help of measurements as we take to get a high quality of measurements for every another experiment that measure Resistance not necessary same as this. the N_σ test shows a value higher from 3 and the statistics shown a misfit, from the residuals plot and value it shown a very small error calculation that expected since we use only the digital resolution, also a systematic error is shown, from the filling factor itself and reflected to R_{vk}. In lab terms, systematic error in the Resistance measurements that shown in R_{vk} fit, and in the filling factor that extracted from Resistance measurements

The filling factor $\nu = 34$ as not extracted because of the resolutions, the derivatives has been calculated with MATLAB algorithm that use nearest neighbors to calculate the derivative, the measurements in the area of 34 delivered one point and the resolution made it not present in the measurement

The filling factor $\nu = 5$ is not part of the phenomena that this experiment theory explained, also seen in R_{xx} measurements, as a task for this paper we need to answer the question if this can be explained by Zeeman effect: Zeeman effect is the a splitting of energy levels caused by the interactions of the spin of electrons and a magnetic field. in our description of QHA we neglected the interaction of the spins with the magnetic field, that interaction splits the landau level in very high fields and causes every landau level with a filling factor of ν to split to two different levels with a filling factor of ν. Because the graphene's filling factor is 4, the lowest level other then $\nu = 5$ is $\nu = 6$ and the Zeeman splitting will cause a new level at $\nu = 4$ and $\nu = 2$ and not $\nu = 5$. Since it is not the case, we have to assume that something else split one transverse mode and that was not the Zeeman effect.

the degeneracy g result is 4 with a good statistic. as expected from theory for Graphene what makes this measurement a good indicator that the 2D material is a Graphene because it's a special property of the material.

N results with a good statistics as a halt odd integer $N = \frac{2n+1}{2}$, as expected from theory, this measurement can conclude this conclusion part - to verify the 2D material quantum basic properties with the method of quantum hall effect in Graphene.

the carrier's density in the Drude model as found to be in the same scale as in the quantum range. in the quantum range it has a difference of a factor correspond to the degeneracy of the graphene, the model for the Carrier density is dependent on the movement of the carrier in the sample and the observation is from that movement, the degeneracy than is expected to be smaller by factor correspond to the degeneracy.

The mobility in the quantum observation found to have be parabolic dependents in the gate voltage. as we can learn from the results for $R_{xx}(V_g)$ for $B = 0$ and from $\mu(V_g)$ comparison, we can see that the mobility has its lowest value (absolute) that correspond to the largest Resistance. the mobility parabolic dependents correspond to the position of the Fermi level in the energy scale with respect to landau level, as the position change with V_g. It's not a trivial result, as the gate voltage is proportional to the carrier density, as seen in 18 we can see a parabolic behavior of $\mu(V_g/n_e)$ and not linear or exponential (with saturation), show that the quantum effect and the energy levels model is also shown in the mobility results that can't be explained in classic "tools".

V. APPENDIX

A. Data Analyses

1. *Errors*

As we get the measurement From Tau Lab C course, and not from a measurement made with a data from the experiment devices by us, the errors knowledge that we have in hand is the digital resolutions of the data has shown in the data digital representation, the errors has calculated with that errors.

2. *Raw Data*

the raw data was organized with the magnetic and gate voltage sweep ranges, where it was needed the data has been "smooth" and a problematic measurement has been neglected.
A set of Data of different temperature has found to be not sufficient in volume has such we didn't able to extract a observation with respect to temperature difference of the experiment.

3. *fitting,extraction,peaks find*

the main fitting, extraction, find peaks work has done with MATLAB that has a function that design to this

work.
points that found to be a "peak" from the jump in the measurements do to a resolution change as neglected

B. Linear fit

$$\Delta a = \sqrt{(\frac{\Delta y_1}{X_1 - X_2})^2 + (\frac{\Delta y_2}{X_1 - X_2})^2 + (\frac{\Delta x_1 (y_1 - y_2)}{(X_1 - X_2)^2})^2 + (\frac{\Delta x_2 (y_1 - y_2)}{(X_1 - X_2)^2})^2}$$

$$= \frac{1}{X_1 - X_2}\sqrt{(\Delta y_1^2 - \Delta y_2^2) + \Delta X_1^2 a^2}$$