Nyquist-Shannon Sampling Theorem   Sampling (statistics)   +3

What is the intuitive meaning of the Nyquist–Shannon sampling theorem?

Why is it that the sampling frequency has to be greater than twice of the message signal? What is the physical meaning and significance of over and under sampling?

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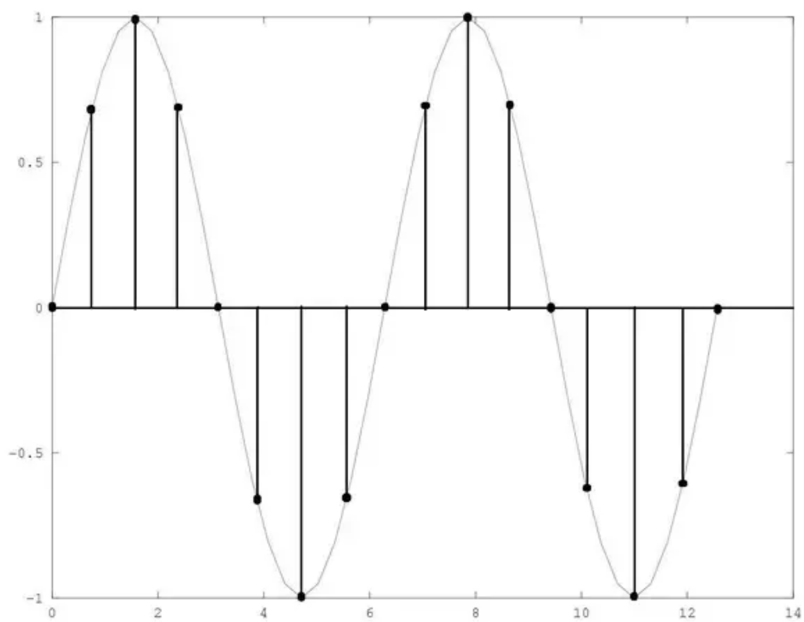
6 Answers

Aneesh Kulkarni, Engineer at Qualcomm
Updated Apr 7, 2015

For me, the most intuitive way to understand the sampling theorem has been to visualize the sampling process in time domain. This way, the problem gets reduced to **'connecting the dots'.**

Consider a sine wave, being sampled at a rate eight times its frequency.



We have eight samples per cycle of the wave. Do these samples contain enough information for us to reconstruct the original wave? Yes, for **if we were to draw a 'sine wave' passing through these eight 'dots', the reconstructed waveform would be exactly the same as the original wave.** Physically, this reconstruction can be done with the help of a low pass filter. With the cut-off frequency adjusted appropriately, a low pass filter will extract the fundamental from the sampled waveform.

Next, let us try to reduce the number of samples (*dots*) per cycle.

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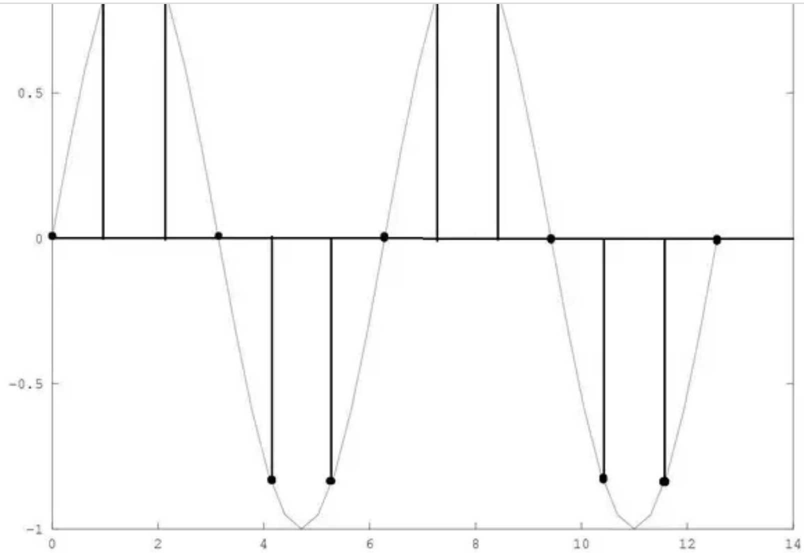
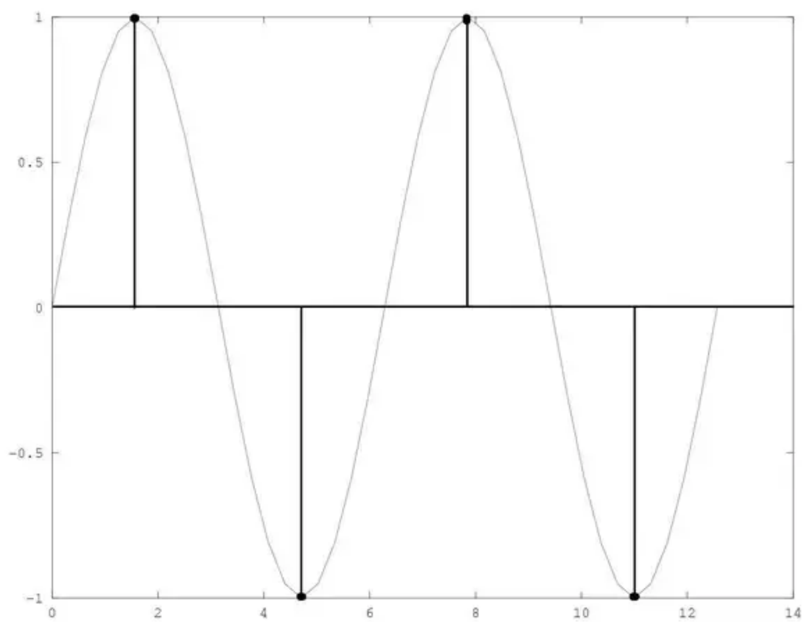
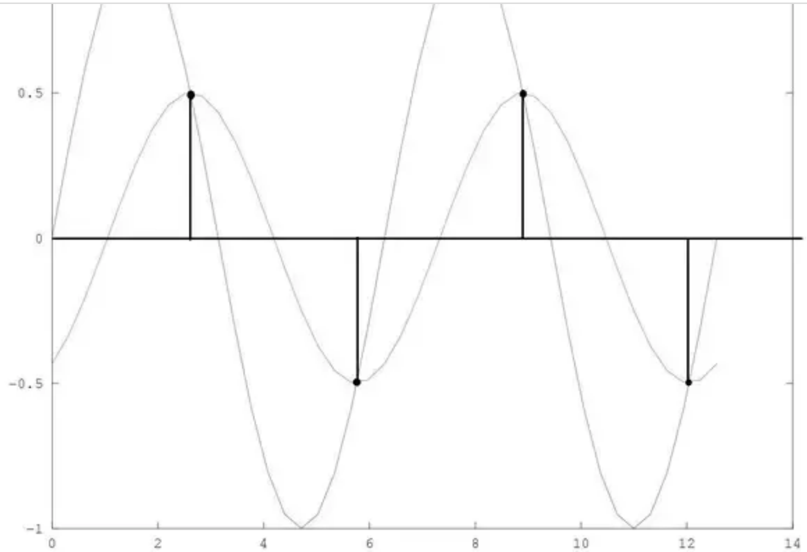


Is there a lower limit to the number of dots per cycle? Clearly, we would need at least **two** dots, ***one in the positive lobe and the other in the negative lobe***, so that we may join them to reconstruct the original wave.



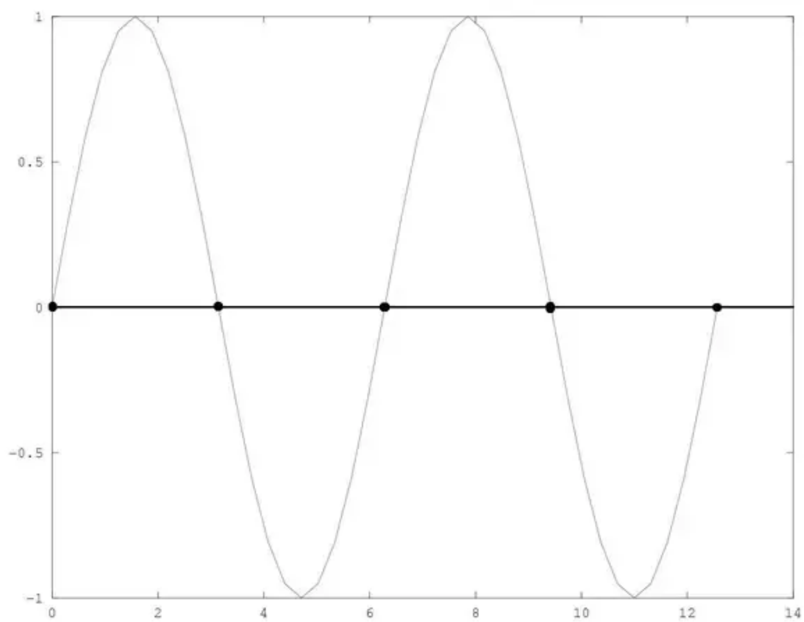
This is how the number **'two'** crops up in the sampling theorem.

But there's a catch here - in the above figure, we have assumed that we are sampling right at the peaks. What if we were sampling slightly off the peaks?



We still have two samples per cycle. Surprisingly, although the wave formed by joining the dots has exactly the same frequency as the original wave, it differs in amplitude and phase.

Extending this logic even further, what if we were to sample exactly at the *zero crossings*?

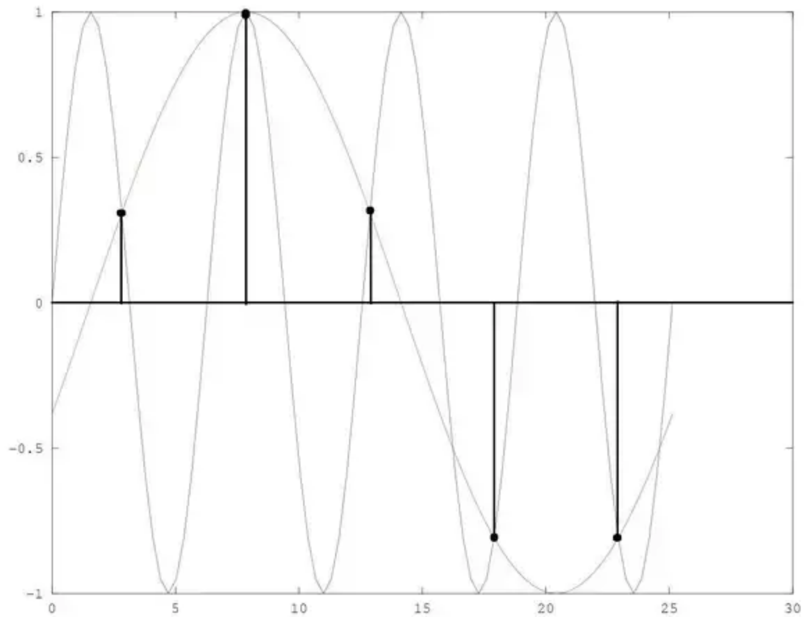


Again we have two samples per cycle, *but they are all zeroes!* There is no way we can reconstruct the original wave from a series of zeroes.

Thus we see that sampling at double the frequency may not lead to correct to reconstruction in all cases. This is the reason why the theorem contains a ***strict inequality***. That is, the sampling frequency must be *greater than* (not *equal to*) twice the frequency of the sampled wave.

Here we considered the signal to be a single sine wave. Real world signals can be thought to be made up of a number of sinusoidal components. Thus the sampling frequency must be greater than twice the highest frequency component.

Undersampling occurs when the sampling frequency is less than the critical frequency. An undersampled signal, on reconstruction, gives a signal whose frequency is lower than the original signal. This is called ***aliasing***.



In the above figure, we have one sample per cycle of the original wave. The reconstructed wave, formed by joining the dots, has a lesser frequency than the original wave. These two waves are said to be the ***aliases*** of each other.

Andrea Baisero
Superb and clear visual explanation! Imho much better than the prison/windo...

Kedar Tatwawadi, works at Indian Institute of Technology, Bombay
Answered Jan 5, 2015

Instead of giving formal arguments, which have already been pointed out quite well already, I will try to point out real life examples of situations where we get a Shannon-Nyquist criteria.

1) The Stationery Helicopter

For the uninformed this seems pretty bizarre! But, for a student of EE, this is merely an extreme case of non-nyquist sampling.

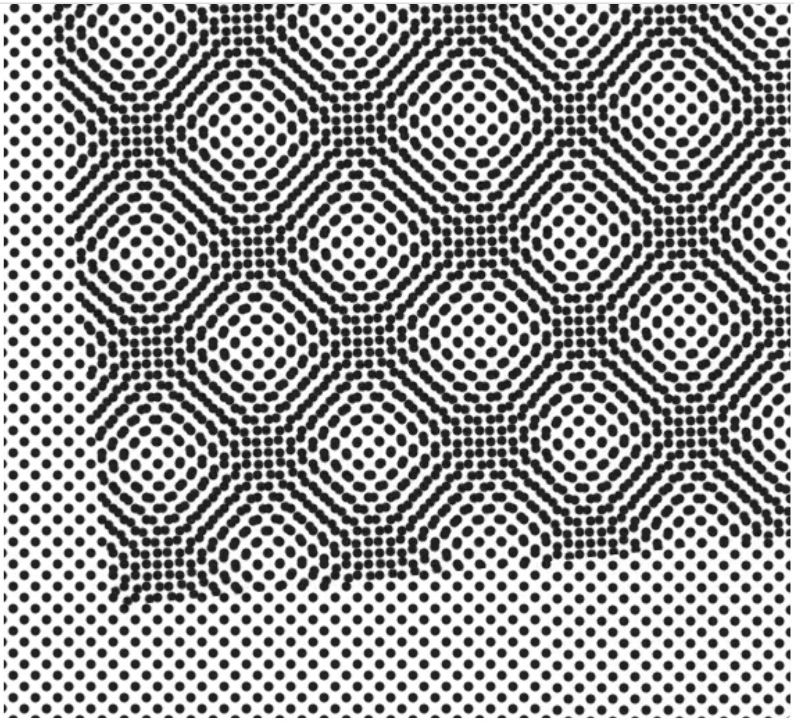
What is happening here?
The camera video feature, is basically taking *samples* at a *frequency (*Frame rate) of 24 frames per second.
If the helicopter blades rotate at multiples of this rate, (say 48 fps), this is what you will observe!

How is it related to the Shannon-Nyquist Criteria?
In signal processing terms, we are trying to capture a *signal* (the rotation of blades in this case) by a much smaller sampling rate, due to which there is no unique reconstruction from the samples.
Thus, our brain tries to pick up the *simplest reconstruction,* and falters at that, thanks to the Shannon-Nyquist criteria, which simply states that the sampling frequency should be twice the frequency of the signal.

A more educative example of the same phenomenon, which I am sure most of us would have expericed is given below:

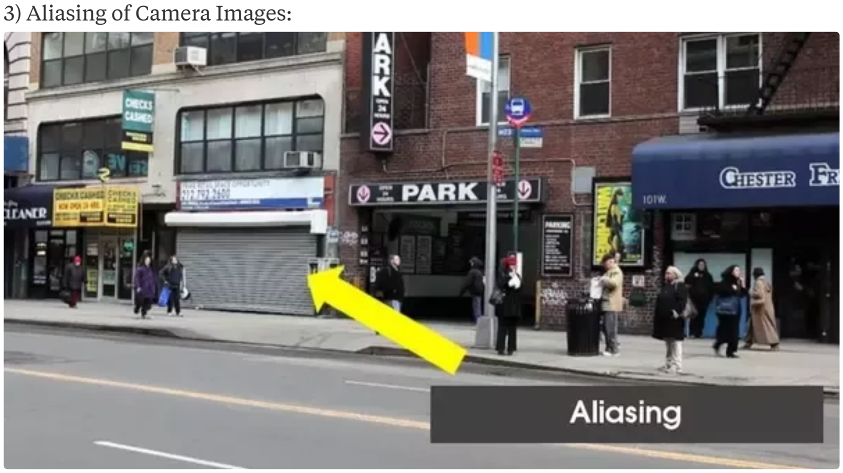
Apart form the example above, I also love a few others, which showcase how wrong reconstruction of signal can occur due to insufficient sampling rates:

2) Moire Patterns:



Moiré pattern

3) Aliasing of Camera Images:



Jaidev Deshpande, works at Enthought
Answered Jun 6, 2013

Think about it this way...

You are a prisoner. You were born and raised in a dungeon. You have never seen the sun or the sky. Then, one day, you were shifted to a cell with a single window, which you can open and close at will. However, the window is fully opaque. Opening it will let the sunlight in and closing it will block it out entirely. You have been promised that if you can figure out the period of time (in hours) between sun crossing the horizon (now you know what the sun is, since you have a window), you will be set free. However, every time you open the window, you are penalized in some way. Let's say you are whipped every time you open the window.

The Sampling Problem

You need to figure out how often you should open and close the window, in order to correctly guess the duration between sunrise and sunset, while at the same time minimizing the frequency of opening the window (you want to be whipped as less as possible).

Assumptions:

1. All days are like the equinoxes, i.e. the durations of day and night are exactly equal.
2. The sun rises at 0600 hours.

The Solution

We know that 12 hours pass between sunrise and sunset. So this is a periodic 'signal' with a period of 12 hours. Sampling this particular signal consists of opening the window, noting if it is day or night, and closing the window. Since this signal repeats every 12 hours, the Nyquist sampling theorem tells us that you must open and close the window at (at least) twice it's frequency, i.e. every 6 hours. So the problem reduces to this:

1. How do we arrive at the figure of six hours when we don't know anything about sunrise and sunset?
2. How do we ensure that this is indeed the *minimum* frequency? (You don't want to get whipped unless it is absolutely necessary.)

The Proof

- Consider an extreme approach:

You want to be set free so badly that you keep the window open all the time and keep watching the sky for 24 hours and correctly record the answer. But there's a heavy penalty. You'll probably be flogged to death. Similarly, higher than necessary sampling rates can make your signal processing application die slowly and painfully. Moreover, they're just unnecessary! This is called ideal sampling.

- Another case: you open the window every hour.

of these sightings of the sun, the only time you get any useful information is when the sun is rising and setting. The rest of the information is redundant, since the sun is in the sky for eleven straight samples and then the sky is dark during eleven straight samples. After making this observation, you conclude that you can easily afford to open and close the window less frequently.

- Open the window every three hours.

The sun rises when you first open the window (0600 hrs), remains in the sky for the next three samples (0900 hrs, 1200 hrs, 1500 hrs), sets during the next sample (1800 hrs), there sky is dark for the next three samples (2100 hrs, 0000 hrs, 0300 hrs), and the cycle repeats. Now, between sunrise and sunset, there are three samples which between them give you no information. So you can further reduce the sampling rate.

- Open the window every six hours (you don't yet know that this is the correct answer).

There will be four samples. The sun rises (0600 hrs), it is up in the sky (1200 hrs), it sets (1800 hrs), the sky becomes dark (0000 hrs), and the cycle repeats. Since you know that these samples are equally spaced in time, you can again calculate the right time as twelve hours. But you're getting too confident, and you decide to decrease your frequency further.

- Open the window every eight hours. (This is where it gets tricky!)

You see the sunrise (0600 hrs), you see the sun in the sky (1400 hrs), you see the sky dark (2200 hrs), and the cycle repeats. But hold on! You never saw the sunset! The sun could have set any time between 1400 hrs and 2200 hrs! You have no way of knowing *when* sunset happened, and your calculations go wrong. This is called undersampling.

Inferences
Since you know that sampling every eight hours is not sufficient (undersampling), you go back to sampling every six hours (Nyquist sampling). But you want to leave no room for error, so you increase the sampling rate slightly (oversampling).

For an interactive demo of the Nyquist theorem, see this:
"Sampling Theorem " from the Wolfram Demonstrations Project
Wolfram Demonstrations Project
(Demo contributed by: Carsten Roppel)

Adam Liss
This is a wonderfully clear and intuitive explanation, but with a fatal problem: t...