Assignment - V

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Third-Order Heat Conduction - TH3

Compensator Using a Reduced-Order Observer (TH3-8)

```
G =
   27.0000   271.0000   915.0000
```

```
G =
   27.0000   271.0000   915.0000  -960.0000   330.0000
```

```
K' =
   1.0e+04 *
   -5.6935    -4.2654     0.0077    -0.0000     4.5000
```

```
plant =

  A =
       x1   x2   x3
   x1   -3    1    0
   x2    1   -2    1
   x3    0    1   -3

  B =
       u1
   x1    1
   x2    0
   x3    0

  C =
       x1   x2   x3
   y1    0    0    1

  D =
       u1
   y1    0
```

```
comp =

  A =
               x1          x2          x3          x4          x5
   x1         -30        -270   5.602e+04         960        -330
   x2           1          -2   4.266e+04           1           0
   x3           0           1         -80          -3           1
   x4           0           0   2.728e-12           0           0
   x5           0           0    -4.5e+04           0           0

  B =
               u1
   x1   -5.694e+04
   x2   -4.265e+04
   x3           77
   x4   -2.728e-12
   x5      4.5e+04

  C =
         x1     x2     x3     x4     x5
   y1    27    271    915   -960    330
```

```

  D =
       u1
   y1   0
```

H is the feedback response of the contributions from the compensator with the plant:

```
H =

  A =
                  x1           x2           x3           x4           x5           x6
                          x7           x8
   x1            -30         -270    5.602e+04          960         -330            0
        0   -5.694e+04
   x2              1           -2    4.266e+04            1            0            0
        0   -4.265e+04
   x3              0            1          -80           -3            1            0
        0           77
   x4              0            0    2.728e-12            0            0            0
        0   -2.728e-12
   x5              0            0     -4.5e+04            0            0            0
        0      4.5e+04
   x6            -27         -271         -915          960         -330           -3
        1            0
   x7              0            0            0            0            0            1
       -2            1
   x8              0            0            0            0            0            0
        1           -3

  B =
       u1
   x1   0
   x2   0
   x3   0
   x4   0
   x5   0
   x6   1
   x7   0
   x8   0

  C =
          x1     x2     x3     x4     x5     x6     x7     x8
   y1     27    271    915   -960    330      0      0      0

  D =
       u1
   y1   0
```

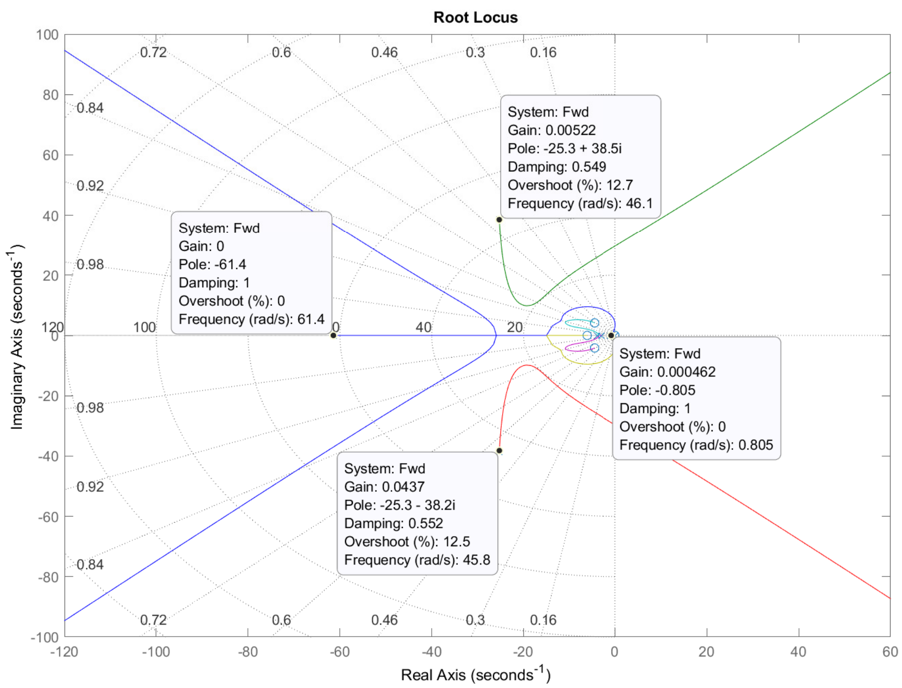

Figure 1: TH3 8 - Root Loci with Gain Locations

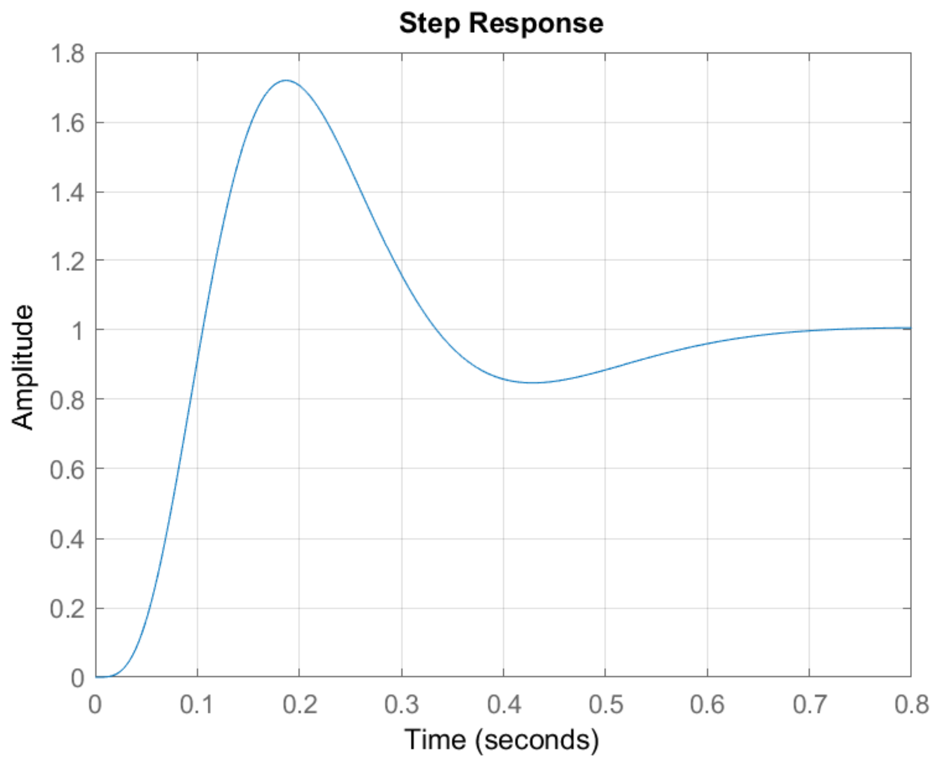

Figure 2: TH3 8 - Step Response

Listing 1: TH3 8

```
syms s
%% Previously calculated ss values
A = [-3 1 0; 1 -2 1; 0 1 -3];
B = [1; 0; 0];
C = [0 0 1];
D = 0;

%% Full-State Feedback
poles = [-10+5j, -10-5j, -15];
G = place(A,B,poles)

E = [0 0; 1 0; -3 1];
Ac = A-B*G;
M = inv(Ac);
N = inv(C*M*B);


G0 = N*C*M*E;
GG = [G G0]
AA = [A E; 0 0 0 0 0; 0 0 0 0 0];
BB = [B; 0; 0];
CC = [C 0 0];

%% Observer and Compensator

poles2 = [-20+10j, -20-10j, -30, -15, 0]
kt=place(AA',CC',poles2)
K=kt'

Ach = AA-BB*GG-K*CC
Rc = inv(s*eye(5)-Ach);
Ds = GG*Rc*K;
Ds = collect(Ds,s);
pretty(Ds)

plant=ss(A,B,C,0)
comp=ss(Ach,K,GG,0)
zc=zero(comp);
pc=pole(comp);

Fwd=comp*plant
H=feedback(Fwd,1)
zero(H);
pole(H);

%% Root Loci
figure(1)
rlocus(Fwd), grid

%% Step Response
figure(2)
step(H),grid
```

Pendulum on Cart - PCA 9

Linear Quadratic Control

```
K =
   1.0e+03 *
   -3.1623   -4.2630   -1.6600   -0.4557
```

```
sys =

  A =
                x1          x2          x3          x4
   x1            0           0           1           0
   x2            0           0           0           1
   x3         3162        4259        1656       455.7
   x4   -1.265e+04    -1.7e+04       -6624       -1823

  B =
        u1
   x1   0
   x2   0
   x3   1
   x4   -4

  C =
        x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   1   0   0

  D =
        u1
   y1   0
   y2   0
```

Next, the step response was approximated (i.e., without offset) to find the values for Q.

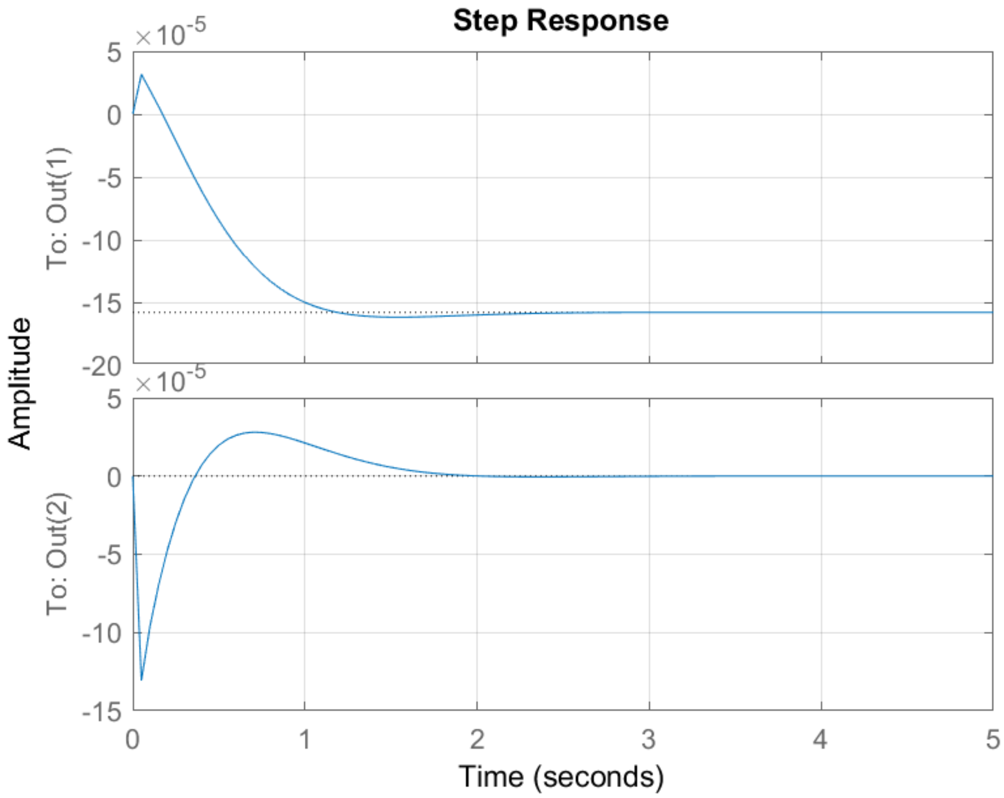

Figure 3: PCA 9 - Simulated LQR Response

The values for Q were found experimentally (trial-and-error).

```
Q = diag([10000000 10000000 0 0]);
```

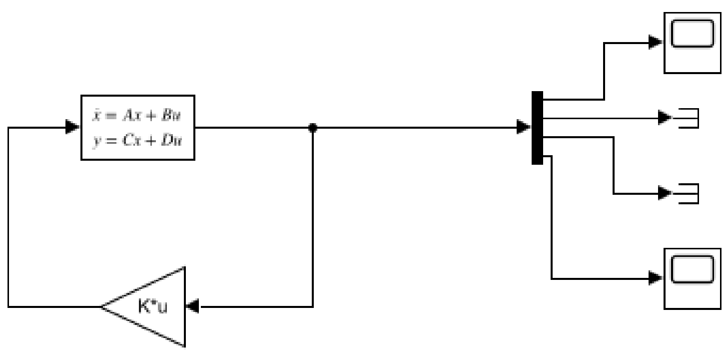

Figure 4: TH3 - Simulink Model

Then, the offset was check with a Simulink file to ensure that the settling time was less than two seconds ($x1$) and the and was within $0.5rad$ ($x2$).

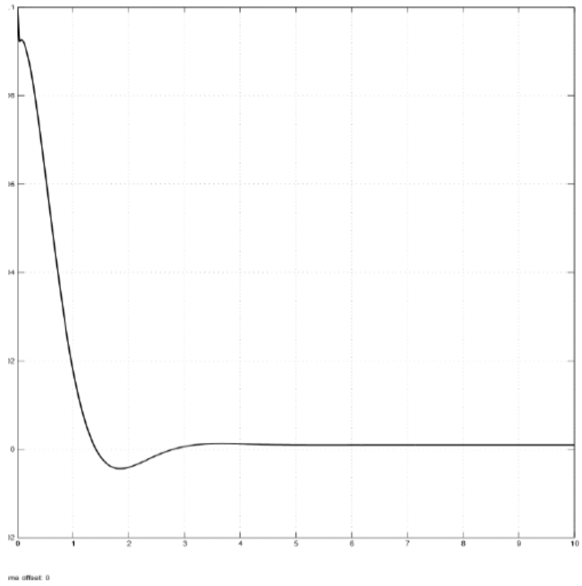

Figure 5: PCA - $x1$ over time.

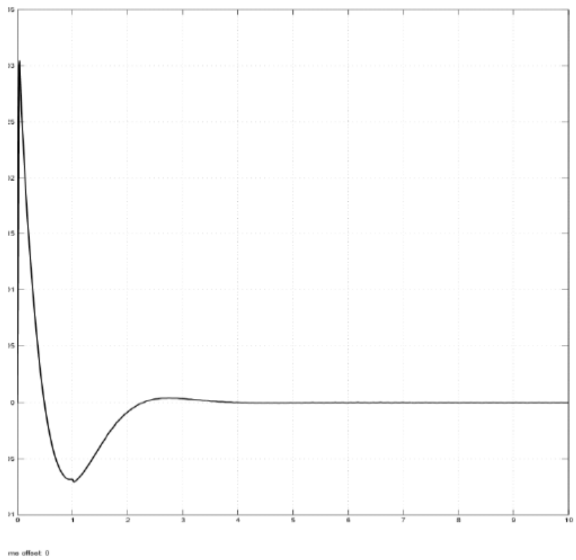

Figure 6: PCA - $x2$ over time.

Listing 2: PCA

```
syms s a b m M g L G1 G2
%A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
%B = [0;0;b;-b/L];
%a = 4; b = 1; M = 1; m = 0.4; g = 9.81; L = 0.25*M;

%% Previously calculated ss values
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 54.88 16 0];
B = [0 0 1 -4]';
C = [1 0 0 0; 0 1 0 0];
D = [0; 0];

Q = diag([10000000 10000000 0 0]);
R = 1;
[K,S,e] = lqr(A,B,Q,R);

%% Step Response
sys = ss(A-B*K,B,C,D);
t=0:0.05:5;
step(0.5*sys,t) % Active suspension step response with gain K
grid on
```

References

[1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.

[2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)