

Assignment - IV

CHRISTOPHER OHARA (31459079)

cao36@njit.edu

May 8, 2019

Motor-Driven Pendulum - PEN 5

Pole Placement: $s = -6 \pm 4j$; $\Theta = 0$

```

1 Transfer Function with Gain:
2           1
3 -----
4      2
5 s  + (G2 + 4) s + G1 - 12

```

```

1 Transfer Function:
2           1
3 -----
4      s^2 + 12 s + 52

```

```

1 Gain =
2      64.0000      8.0000

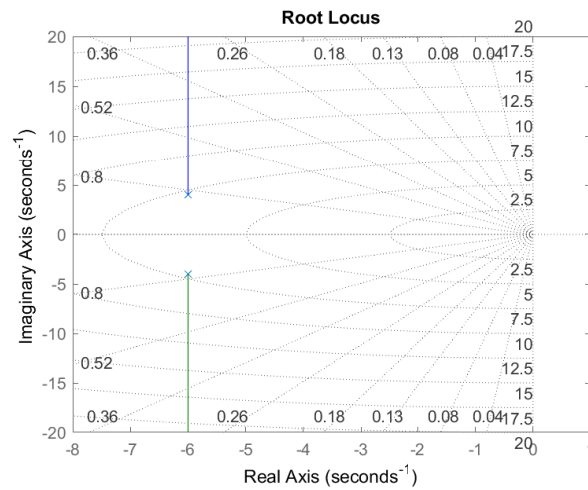
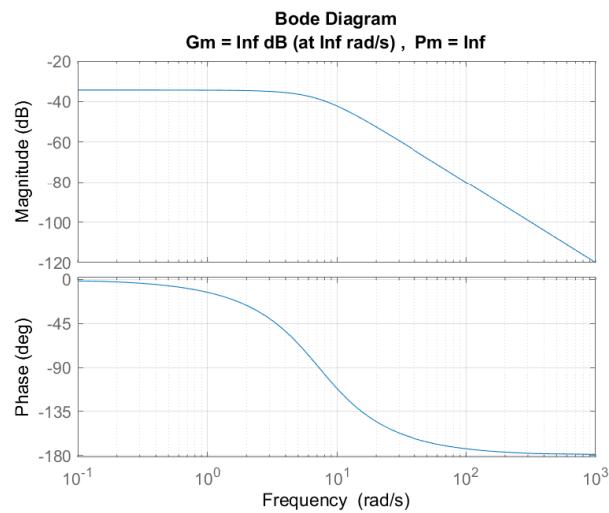
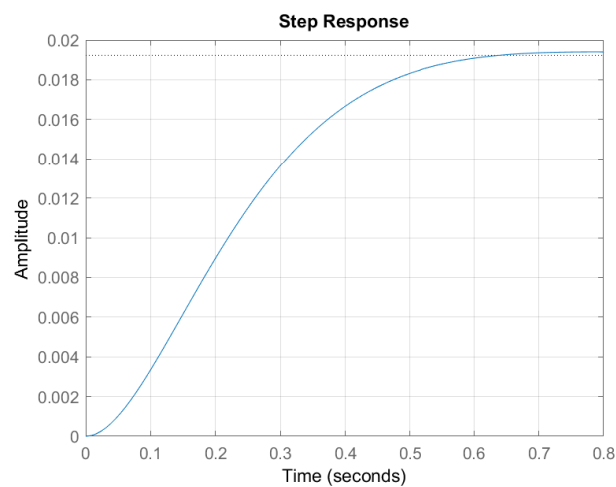
```

Listing 1: PEN 5; $s @ -6 \pm 4j$; $\Theta @ 0$

```

1 %% Theta = 0; s = -6 +/- 4j
2 syms s G1 G2;
3 A = [0 1; 12 -4];
4 B = [0; 1];
5 C = [1 0];
6 D = 0;
7
8 %% Place Poles and Gain
9 poles = [-6+4*j, -6-4*j];
10 P = place(A, B, poles)
11
12 G = [G1 G2]
13 Ac = A-B*G
14 Det = inv(s*eye(2)-Ac);
15 deltaC = det(Det);
16 DeltaC = collect(deltaC, s)
17 pretty(DeltaC)
18 Ac2 = A-B*P
19 [a,b] = ss2tf(Ac2, B, C, D)
20 H = tf(a,b)
21
22 %% Root Locus
23 figure(1)
24 plot(1,2)
25 rlocus(H), grid
26
27 %% Nyquist
28 %figure(2)
29 %plot(1,2)
30 %nyquist(H), grid
31
32 %% Bode
33 figure(3)
34 plot(1,2)
35 margin(H), grid
36
37 %% Step
38 figure(4)
39 plot(1,2)
40 step(H), grid

```

Figure 1: PEN5 - Root Locus - $s = -6 \pm 4j$; Theta = 0Figure 2: PEN5 - Bode - $s = -6 \pm 4j$; Theta = 0Figure 3: PEN5 - Step Response - $s = -6 \pm 4j$; Theta = 0

Pole Placement: $s = -8, -10$; $\Theta = 0$

```

1 Transfer Function with Gain:
2      1
3  -----
4      2
5  s  + (G2 + 4) s + G1 - 12

```

```

1 Transfer Function:
2      1
3  -----
4  s^2 + 18 s + 80

```

```

1 Gain =
2      92.0000      14.0000

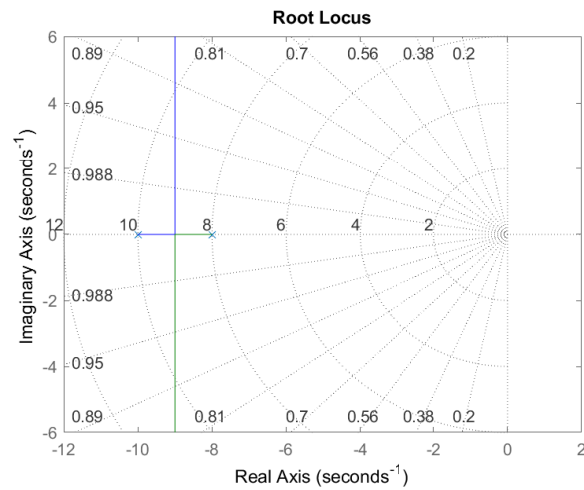
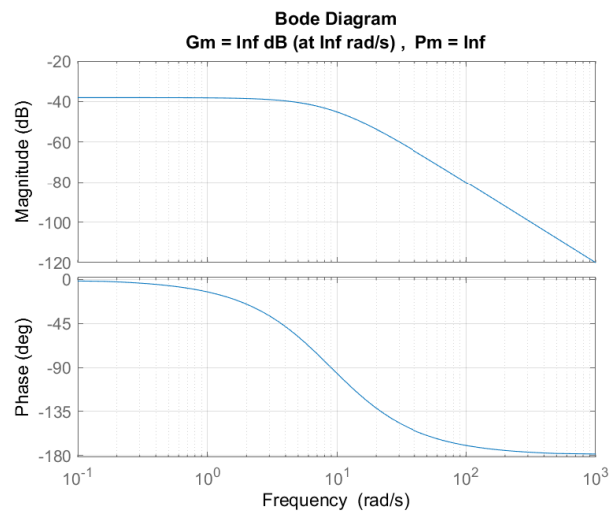
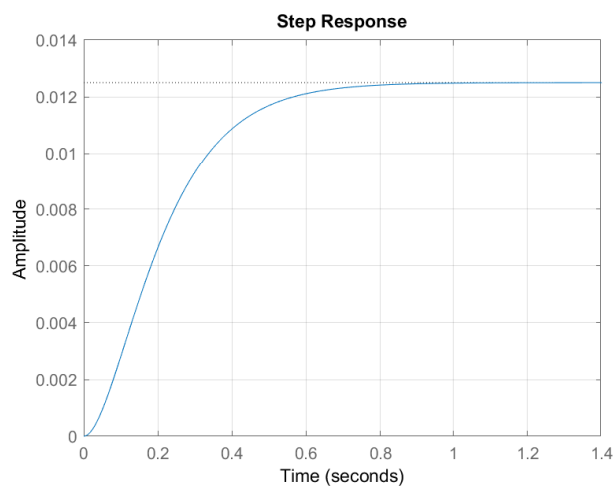
```

Listing 2: PEN 5; $s @ -8, -10$; $\Theta @ 0$

```

1 %% Theta = 0; s = -8,-10
2 syms s G1 G2;
3 A = [0 1; 12 -4];
4 B = [0; 1];
5 C = [1 0];
6 D = 0;
7
8 %% Place Poles and Gain
9 poles = [-8, -10];
10 P = place(A, B, poles)
11
12 G = [G1 G2]
13 Ac = A-B*G
14 Det = inv(s*eye(2)-Ac);
15 deltaC = det(Det);
16 DeltaC = collect(deltaC, s)
17 pretty(DeltaC)
18 Ac2 = A-B*P
19 [a,b] = ss2tf(Ac2, B, C, D)
20 H = tf(a,b)
21
22 %% Root Locus
23 figure(1)
24 plot(1,2)
25 rlocus(H), grid
26
27 %% Nyquist
28 %figure(2)
29 %plot(1,2)
30 %nyquist(H), grid
31
32 %% Bode
33 figure(3)
34 plot(1,2)
35 margin(H), grid
36
37 %% Step
38 figure(4)
39 plot(1,2)
40 step(H), grid

```

Figure 4: PEN5 - Root Locus - $s = -8, -10$; $\Theta = 0$ Figure 5: PEN5 - Bode - $s = -8, -10$; $\Theta = 0$ Figure 6: PEN5 - Step Response - $s = -8, -10$; $\Theta = 0$

Pole Placement: $s = -6 \pm 4j$; Theta = 180

```

1 Transfer Function:
2       1
3  -----
4  s^2 + 12 s + 52

```

```

1 Gain =
2  40.0000    8.0000

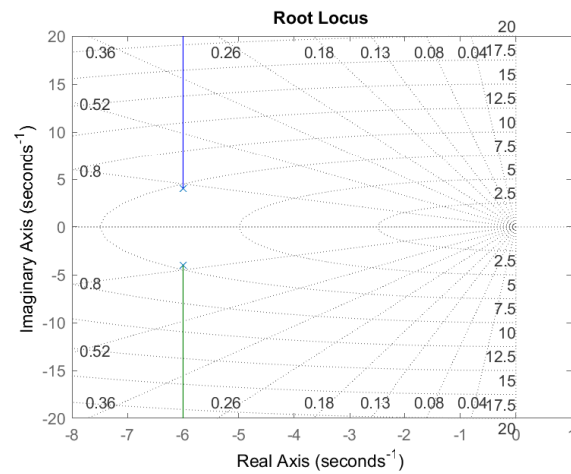
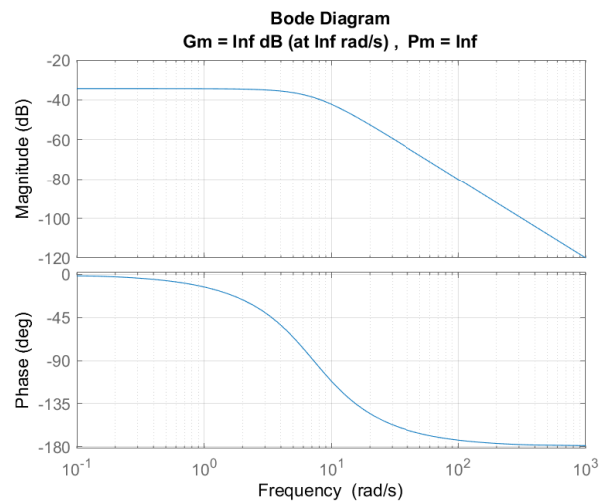
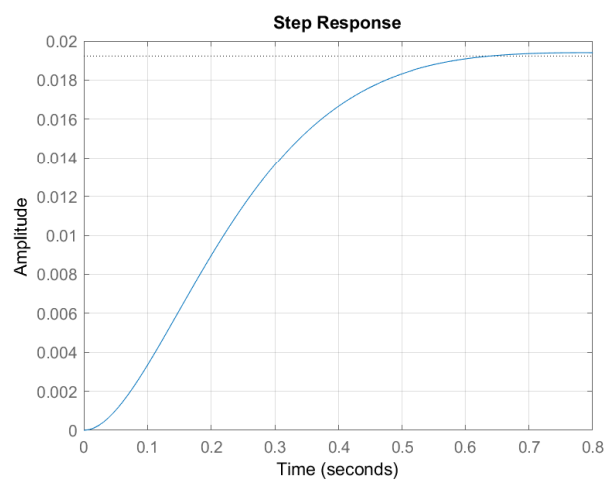
```

Listing 3: PEN 5; $s @ -6 \pm 4j$; Theta @ 180

```

1 %% Theta = 180; s = -6+/-4j
2 syms s G1 G2;
3 A = [0 1; -12 -4];
4 B = [0; 1];
5 C = [1 0];
6 D = 0;
7
8 %% Place Poles and Gain
9 poles = [-6+4*j, -6-4*j];
10 G = place(A, B, poles)
11
12 Ac = A-B*G
13 [a,b] = ss2tf(Ac, B, C, D)
14 H = tf(a,b)
15
16 %% Root Locus
17 figure(1)
18 plot(1,2)
19 rlocus(H), grid
20
21 %% Nyquist
22 %figure(2)
23 %plot(1,2)
24 %nyquist(H), grid
25
26 %% Bode
27 figure(3)
28 plot(1,2)
29 margin(H), grid
30
31 %% Step
32 figure(4)
33 plot(1,2)
34 step(H), grid

```

Figure 7: PEN5 - Root Locus - $s = -6 \pm 4j$; Theta = 180Figure 8: PEN5 - Bode - $s = -6 \pm 4j$; Theta = 180Figure 9: PEN5 - Step Response - $s = -6 \pm 4j$; Theta = 180

Pole Placement: $s = -8, -10$; $\Theta = 180$

```

1 Transfer Function:
2       1
3  -----
4  s^2 + 18 s + 80

```

```

1 Gain =
2  68.0000  14.0000

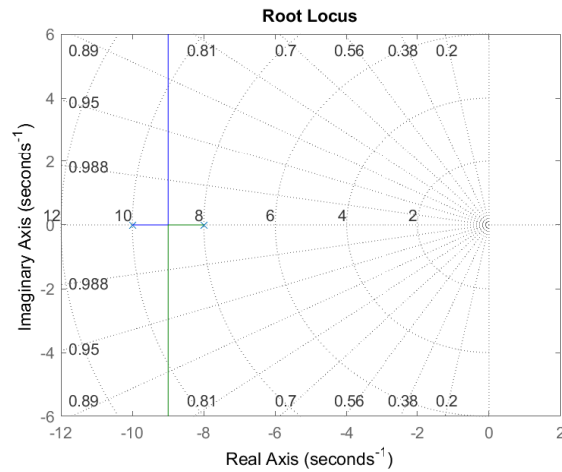
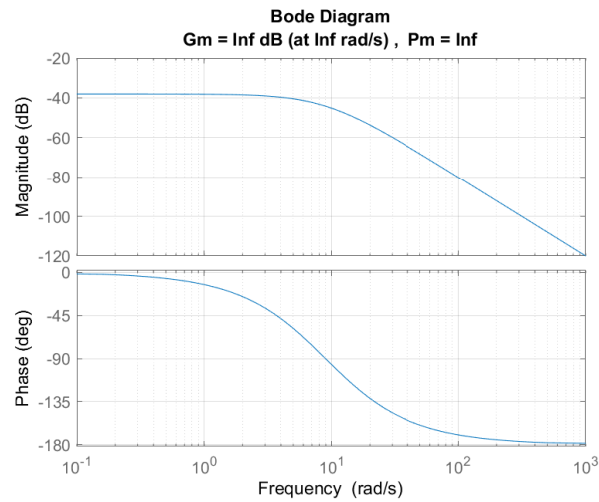
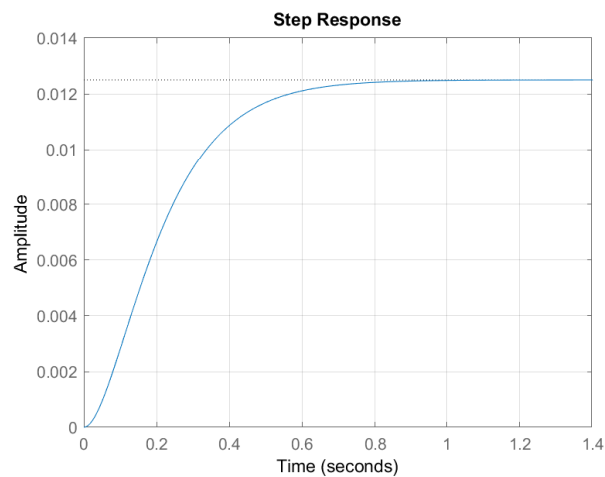
```

Listing 4: PEN 5; $s @ -8, -10$; $\Theta @ 180$

```

1 %% Theta = 180; s = -8,-10
2 syms s G1 G2;
3 A = [0 1; -12 -4];
4 B = [0; 1];
5 C = [1 0];
6 D = 0;
7
8 %% Place Poles and Gain
9 poles = [-8, -10];
10 G = place(A, B, poles)
11
12 Ac = A-B*G
13 [a,b] = ss2tf(Ac, B, C, D)
14 H = tf(a,b)
15
16 %% Root Locus
17 figure(1)
18 plot(1,2)
19 rlocus(H), grid
20
21 %% Nyquist
22 %figure(2)
23 %plot(1,2)
24 %nyquist(H), grid
25
26 %% Bode
27 figure(3)
28 plot(1,2)
29 margin(H), grid
30
31 %% Step
32 figure(4)
33 plot(1,2)
34 step(H), grid

```

Figure 10: PEN5 - Root Locus - $s = -8, -10$; Theta = 180Figure 11: PEN5 - Bode - $s = -8, -10$; Theta = 180Figure 12: PEN5 - Step Response - $s = -8, -10$; Theta = 180

Pole Placement: $s = -6 \pm 4j$; $\Theta = 0$, $r_{\theta} = 0.5$

```

1 Transfer Function (with values equated):
2      1
3      -----
4      2
5      s  + 68 s - 4

```

```

1 Gain =
2      64.0000      8.0000

```

Listing 5: PEN 6; $s @ -6 \pm 4j$; $\Theta @ 0$; $r_{\theta} @ 0.5$

```

1 %% Theta = 0; s = -6 +/- 4j, ref_theta = 0.5
2 syms s G1 G2;
3 ref_theta = 0.5;
4 A = [0 1; 12 -4];
5 B = [0; 1];
6 C = [1 0];
7 D = 0;
8
9 %% Place Poles and Gain
10 poles = [-6+4*j, -6-4*j];
11 P = place(A, B, poles);
12
13 Ac0 = A-B*P;
14 Det0 = inv(s*eye(2)-Ac0);
15 deltaC0 = det(Det0);
16 DeltaC0 = collect(deltaC0, s);
17 pretty(DeltaC0)
18
19 %% Exo
20 G = [G1 G2];
21
22 Ac = A-B*G;
23 Det = inv(s*eye(2)-Ac);
24 deltaC = det(Det);
25 DeltaC = collect(deltaC, s);
26 pretty(DeltaC)
27
28 %% Equating Values
29 G2 = [8 64]; %Analytical Compared
30
31 Ac2 = A-B*G2;
32 Det2 = inv(s*eye(2)-Ac2);
33 deltaC2 = det(Det2);
34 DeltaC2 = collect(deltaC2, s);
35 pretty(DeltaC2)
36
37 %% Control Theory Version
38 %G0 = inv((C*inv(A)*B))*C;
39 %G0 = 12
40 %Ac2 = A-B*G0;
41
42 %Det2 = inv(s*eye(2)-Ac2);
43 %deltaC2 = det(Det2);
44 %DeltaC2 = collect(deltaC2, s);
45 %pretty(DeltaC2)
46 %Ac2 = A-B*P
47
48 %% Transfer Function
49 [a,b] = ss2tf(Ac2, B, C, D);
50 H = tf(a,b); % Get Denominator for sisotool
51
52 %% Simulation
53 stateSpace = ss(Ac2, B, C, D);
54 t = 0:0.05:100;
55 u = ref_theta*ones(size(t));
56
57 lsim(stateSpace,u,t)
58 axis([0 10 0 .1])
59

```

```

60 %% Root Locus
61 figure(1)
62 plot(1,2)
63 rlocus(H), grid
64
65 %% Nyquist
66 %figure(2)
67 %plot(1,2)
68 %nyquist(H), grid
69
70 %% Bode
71 figure(3)
72 plot(1,2)
73 margin(H), grid
74
75 %% Step
76 figure(4)
77 plot(1,2)
78 step(H), grid
79
80 %% Stable Gain Range
81 %s=tf('s');
82 %Gp= 1/(s^2 + 68*s - 4);
83 %sisotool(Gp)
84 % Unstable

```

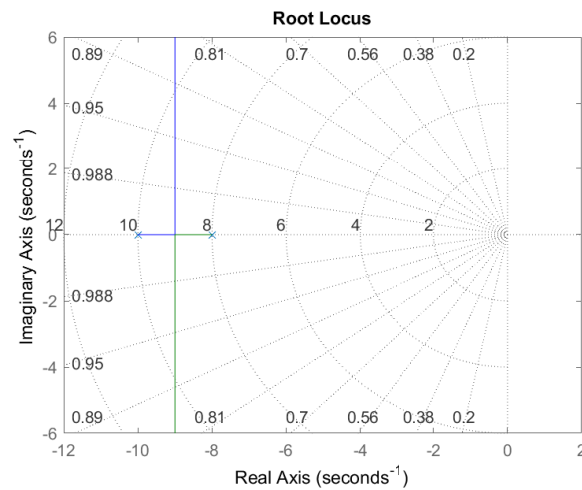
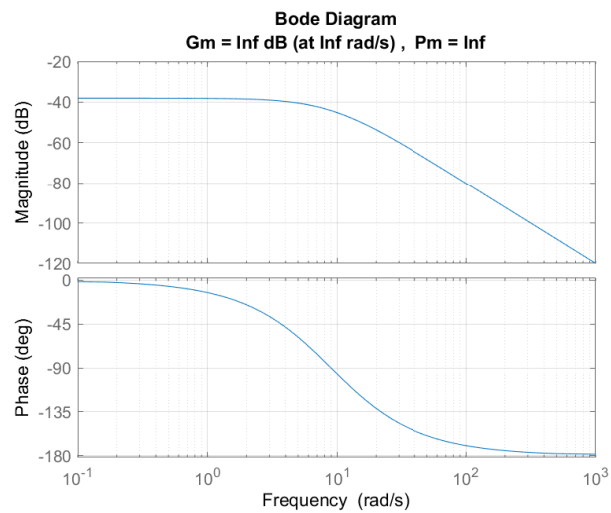
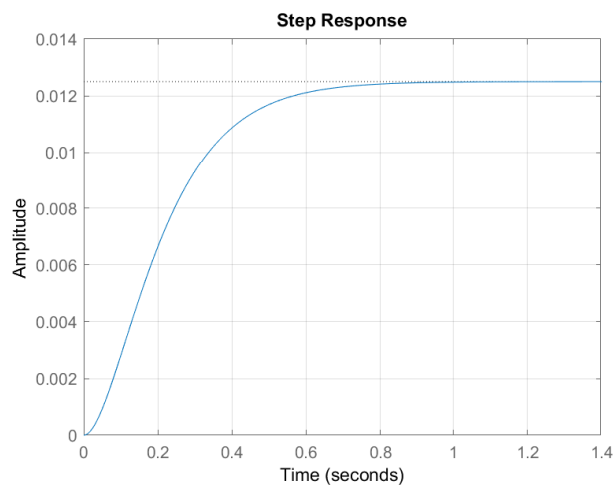
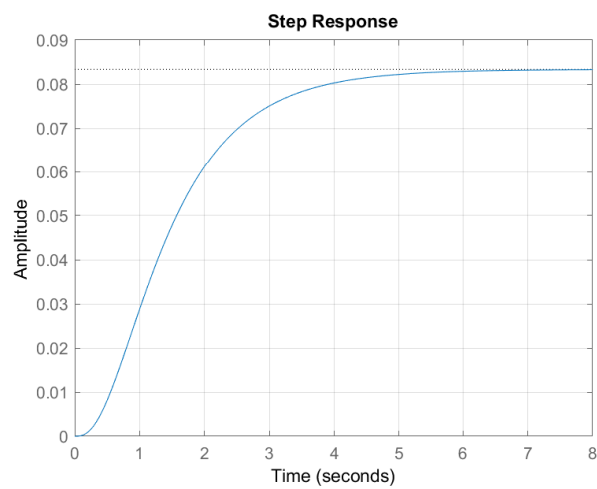


Figure 13: PEN6 - Root Locus - $s = -6 \pm 4j$; $\Theta = 0$

Figure 14: PEN6 - Bode - $s = -6 \pm 4j$; Theta = 0Figure 15: PEN6 - Step Response - $s = -6 \pm 4j$; Theta = 0Figure 16: PEN6 - Linear Simulation - $s = -6 \pm 4j$; Theta = 0

Pole Placement: $s = -8, -10$; $\Theta = 0$, $r_{\theta} = 0.5$

```

1 Transfer Function:
2       1
3  -----
4       2
5  s  + 96 s + 2

```

```

1 Gain =
2  92.0000  14.0000

```

Listing 6: PEN 6; $s @ -8-10$; $\Theta @ 0$

```

1 %% Theta = 0; s = -8,-10, ref_theta = 0.5
2 syms s G1 G2;
3 ref_theta = 0.5;
4 A = [0 1; 12 -4];
5 B = [0; 1];
6 C = [1 0];
7 D = 0;
8
9 %% Place Poles and Gain
10 poles = [-8, -10];
11 P = place(A, B, poles);
12
13 Ac0 = A-B*P;
14 Det0 = inv(s*eye(2)-Ac0);
15 deltaC0 = det(Det0);
16 DeltaC0 = collect(deltaC0, s);
17 pretty(DeltaC0)
18
19 %% Exo
20 G = [G1 G2];
21
22 Ac = A-B*G;
23 Det = inv(s*eye(2)-Ac);
24 deltaC = det(Det);
25 DeltaC = collect(deltaC, s);
26 pretty(DeltaC)
27
28 %% Equating Values
29 G = [14 92]; %Analytical Compared
30
31 Ac2 = A-B*G2;
32 Det2 = inv(s*eye(2)-Ac2);
33 deltaC2 = det(Det2);
34 DeltaC2 = collect(deltaC2, s);
35 pretty(DeltaC2)
36
37 %% Control Theory Version
38 %G0 = inv((C*inv(A)*B))*C;
39 %G0 = -12
40 %Ac2 = A-B*G0;
41
42 %Det2 = inv(s*eye(2)-Ac2);
43 %deltaC2 = det(Det2);
44 %DeltaC2 = collect(deltaC2, s);
45 %pretty(DeltaC2)
46 %Ac2 = A-B*P
47
48 %% Transfer Function
49 [a,b] = ss2tf(Ac2, B, C, D);
50 H = tf(a,b); % Get Denominator for sisotool
51
52 %% Simulation
53 stateSpace = ss(Ac2, B, C, D);
54 t = 0:0.05:100;
55 u = ref_theta*ones(size(t));
56
57 lsim(stateSpace,u,t)
58 axis([0 10 0 .1])
59

```

```

60 %% Root Locus
61 figure(1)
62 plot(1,2)
63 rlocus(H), grid
64
65 %% Nyquist
66 %figure(2)
67 %plot(1,2)
68 %nyquist(H), grid
69
70 %% Bode
71 figure(3)
72 plot(1,2)
73 margin(H), grid
74
75 %% Step
76 figure(4)
77 plot(1,2)
78 step(H), grid
79
80 %% Stable Gain Range
81 %s=tf('s');
82 %Gp= 1/(s^2 + 96*s + 2);
83 %sisotool(Gp)
84 %Stable

```

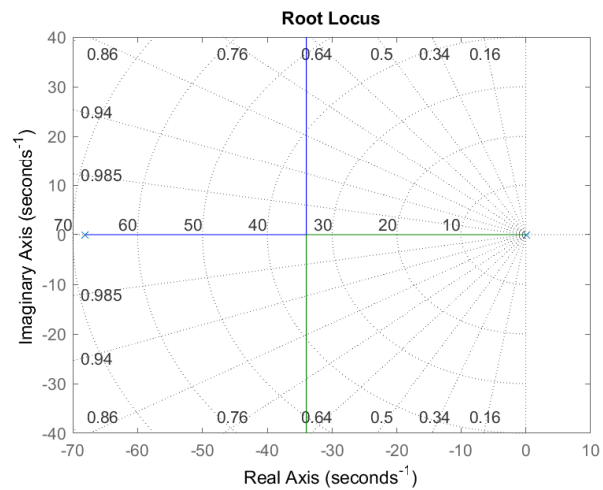


Figure 17: PEN6 - Root Locus - $s = -8, -10$; $\Theta = 0$, $r_{\theta} = 0.5$

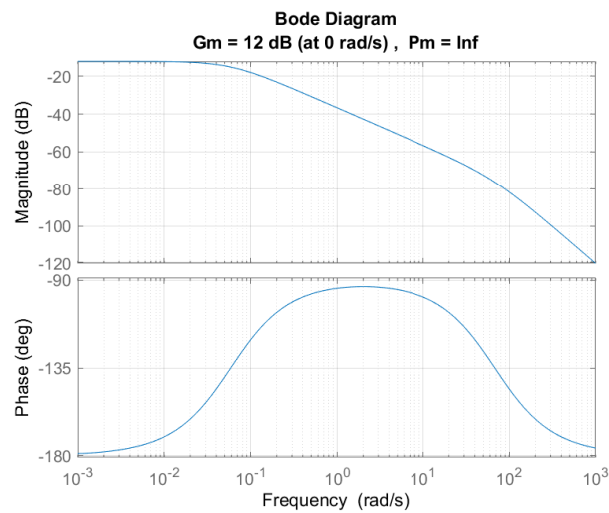


Figure 18: PEN6 - Bode - $s = -8, -10$; $\Theta = 0$, $r_{\theta} = 0.5$

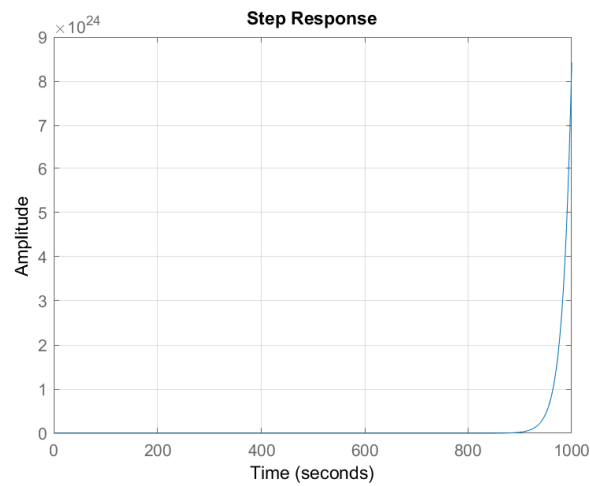


Figure 19: PEN6 - Step Response - $s = -8, -10$; $\Theta = 0$, $r_{\theta} = 0.5$

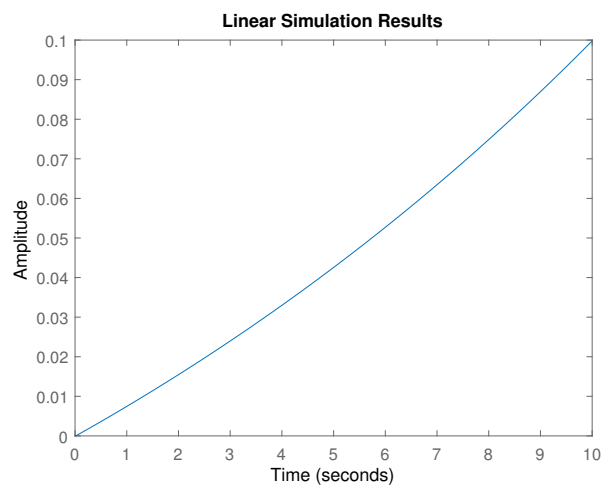


Figure 20: PEN6 - Linear Simulation - $s = -8, -10$; $\Theta = 0$, $r_{\theta} = 0.5$

Pole Placement: $s = -6 \pm 4j$; $\Theta = 180$, $r_\theta = 0.5$

```

1 Transfer Function (with values equated):
2      1
3  -----
4      2
5  s  + 44 s + 20

```

```

1 Gain =
2      40.0000      8.0000

```

Listing 7: PEN 6; $s @ -6 \pm 4j$; $\Theta @ 180$

```

1 %% Theta = 0; s = -6+/-4j, ref_theta = 0.5
2 syms s G1 G2;
3 ref_theta = 0.5;
4 A = [0 1; -12 -4];
5 B = [0; 1];
6 C = [1 0];
7 D = 0;
8
9 %% Place Poles and Gain
10 poles = [-6+4*j, -6-4*j];
11 P = place(A, B, poles);
12
13 Ac0 = A-B*P;
14 Det0 = inv(s*eye(2)-Ac0);
15 deltaC0 = det(Det0);
16 DeltaC0 = collect(deltaC0, s);
17 pretty(DeltaC0)
18
19 %% Exo
20 G = [G1 G2];
21
22 Ac = A-B*G;
23 Det = inv(s*eye(2)-Ac);
24 deltaC = det(Det);
25 DeltaC = collect(deltaC, s);
26 pretty(DeltaC)
27
28 %% Equating Values
29 G2 = [8 40]; %Analytical Compared
30
31 Ac2 = A-B*G2;
32 Det2 = inv(s*eye(2)-Ac2);
33 deltaC2 = det(Det2);
34 DeltaC2 = collect(deltaC2, s);
35 pretty(DeltaC2)
36
37 %% Control Theory Version
38 %G0 = inv((C*inv(A)*B))*C;
39 %G0 = 12
40 %Ac2 = A-B*G0;
41
42 %Det2 = inv(s*eye(2)-Ac2);
43 %deltaC2 = det(Det2);
44 %DeltaC2 = collect(deltaC2, s);
45 %pretty(DeltaC2)
46 %Ac2 = A-B*P
47
48 %% Transfer Function
49 [a,b] = ss2tf(Ac2, B, C, D);
50 H = tf(a,b); % Get Denominator for sisotool
51
52 %% Simulation
53 stateSpace = ss(Ac2, B, C, D);
54 t = 0:0.05:100;
55 u = ref_theta*ones(size(t));
56
57 lsim(stateSpace,u,t)
58 axis([0 10 0 .1])
59

```

```

60 %% Root Locus
61 figure(1)
62 plot(1,2)
63 rlocus(H), grid
64
65 %% Nyquist
66 %figure(2)
67 %plot(1,2)
68 %nyquist(H), grid
69
70 %% Bode
71 figure(3)
72 plot(1,2)
73 margin(H), grid
74
75 %% Step
76 figure(4)
77 plot(1,2)
78 step(H), grid
79
80 %% Stable Gain Range
81 %s=tf('s');
82 %Gp= 1/(s^2 + 44*s + 20);
83 %sisotool(Gp)
84 %Stable

```

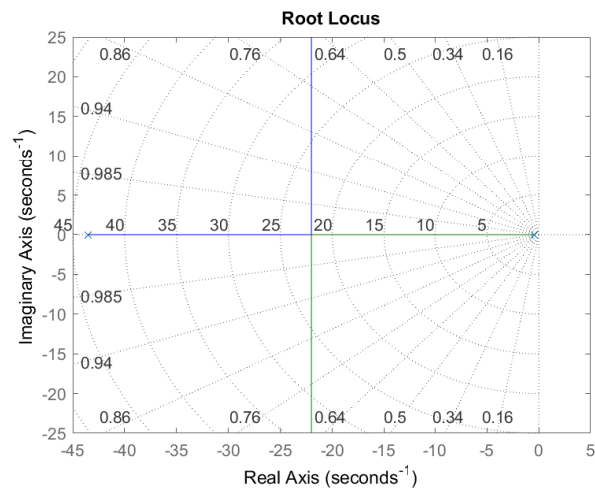
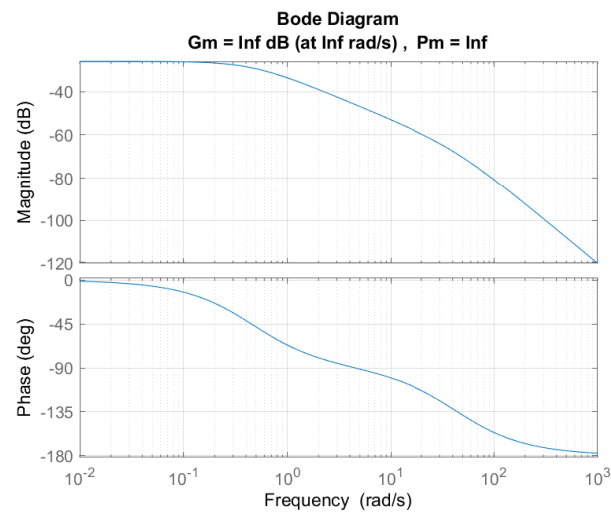
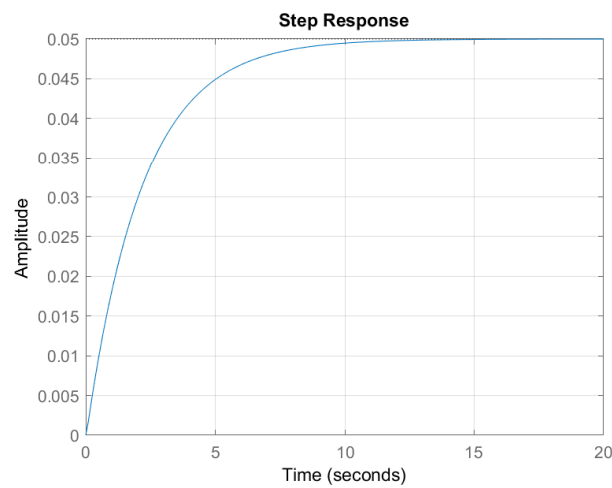
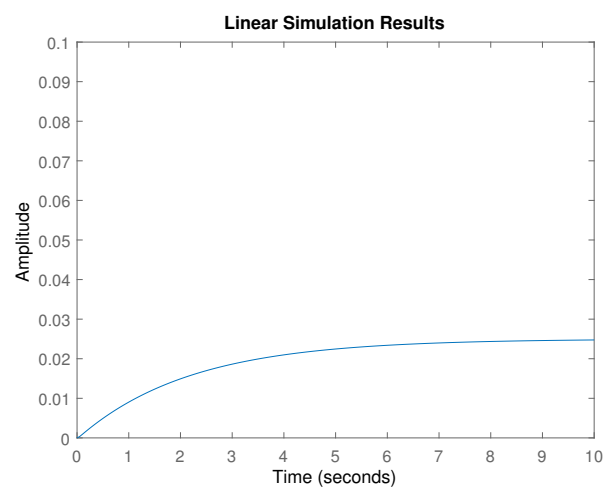


Figure 21: PEN6 - Root Locus - $s = -6 \pm j4$; $\Theta = 180$

Figure 22: PEN6 - Bode - $s = -6 \pm 4j$; Theta = 180Figure 23: PEN6 - Step Response - $s = -6 \pm 4j$; Theta = 180Figure 24: PEN6 - Linear Simulation - $s = -6 \pm 4j$; Theta = 180

Pole Placement: $s = -8, -10$; $\Theta = 180$, $r_{\theta} = 0.5$

```

1 Transfer Function (with values equated):
2      1
3  -----
4      2
5  s  + 96 s + 2

```

```

1 Gain =
2      92.0000    14.0000

```

Listing 8: PEN 6; $s @ -8-10$; $\Theta @ 180$

```

1 %% Theta = 180; s = -8,-10, ref_theta = 0.5
2 syms s G1 G2;
3 ref_theta = 0.5;
4 A = [0 1; -12 -4];
5 B = [0; 1];
6 C = [1 0];
7 D = 0;
8
9 %% Place Poles and Gain
10 poles = [-8, -10];
11 P = place(A, B, poles);
12
13 Ac0 = A-B*P;
14 Det0 = inv(s*eye(2)-Ac0);
15 deltaC0 = det(Det0);
16 DeltaC0 = collect(deltaC0, s);
17 pretty(DeltaC0)
18
19 %% Exo
20 G = [G1 G2];
21
22 Ac = A-B*G;
23 Det = inv(s*eye(2)-Ac);
24 deltaC = det(Det);
25 DeltaC = collect(deltaC, s);
26 pretty(DeltaC)
27
28 %% Equating Values
29 G2 = [14 68]; %Analytical Compared
30
31 Ac2 = A-B*G2;
32 Det2 = inv(s*eye(2)-Ac2);
33 deltaC2 = det(Det2);
34 DeltaC2 = collect(deltaC2, s);
35 pretty(DeltaC2)
36
37 %% Control Theory Version
38 %G0 = inv((C*inv(A)*B))*C;
39 %G0 = 12
40 %Ac2 = A-B*G0;
41
42 %Det2 = inv(s*eye(2)-Ac2);
43 %deltaC2 = det(Det2);
44 %DeltaC2 = collect(deltaC2, s);
45 %pretty(DeltaC2)
46 %Ac2 = A-B*P
47
48 %% Transfer Function
49 [a,b] = ss2tf(Ac2, B, C, D);
50 H = tf(a,b); % Get Denominator for sisotool
51
52 %% Simulation
53 stateSpace = ss(Ac2, B, C, D);
54 t = 0:0.05:100;
55 u = ref_theta*ones(size(t));
56
57 lsim(stateSpace,u,t)
58 axis([0 10 0 .1])
59

```

```

60 %% Root Locus
61 figure(1)
62 plot(1,2)
63 rlocus(H), grid
64
65 %% Nyquist
66 %figure(2)
67 %plot(1,2)
68 %nyquist(H), grid
69
70 %% Bode
71 figure(3)
72 plot(1,2)
73 margin(H), grid
74
75 %% Step
76 figure(4)
77 plot(1,2)
78 step(H), grid
79
80 %% Stable Gain Range
81 %s=tf('s');
82 %Gp= 1/(s^2 + 72*s + 26);
83 %sisotool(Gp)
84 % Stable

```

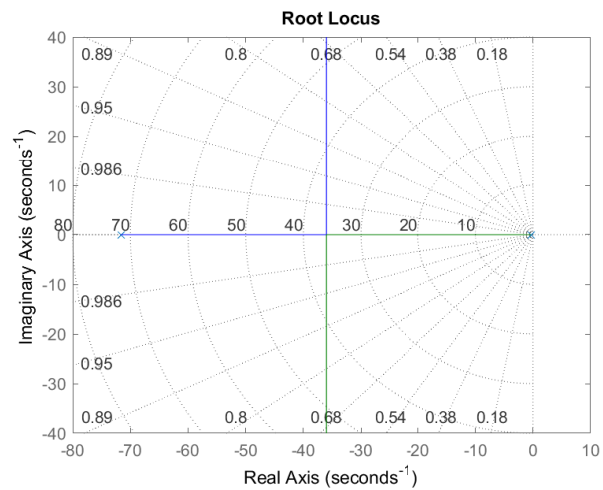


Figure 25: PEN6 - Root Locus - $s = -8, -10$; $\Theta = 180$, $r_{\theta} = 0.5$

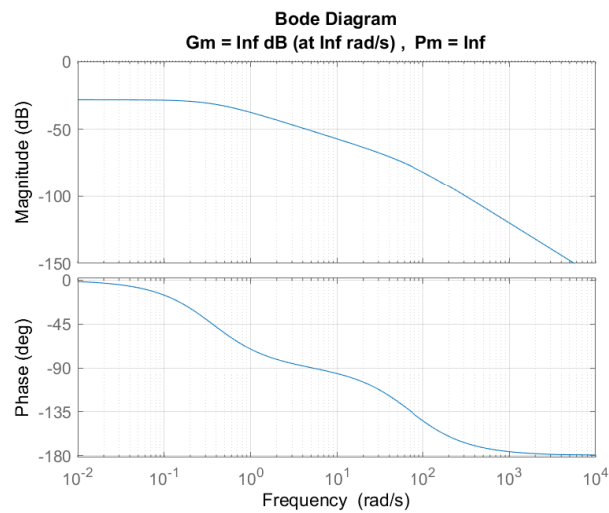


Figure 26: PEN6 - Bode - $s = -8, -10$; $\Theta = 180$, $r_{\theta} = 0.5$

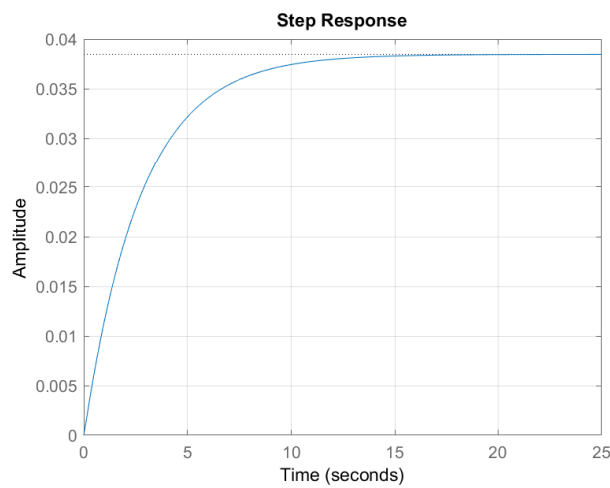


Figure 27: PEN6 - Step Response - $s = -8, -10$; $\Theta = 180$, $r_{\theta} = 0.5$

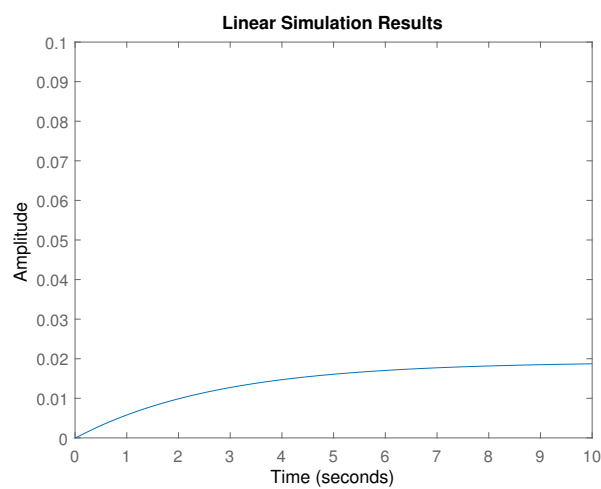


Figure 28: PEN6 - Linear Simulation - $s = -8, -10$; $\Theta = 180$, $r_{\theta} = 0.5$

Third-Order Heat Conduction - TH3-4

Using Rowth-Hurwitz Criterion $\rightarrow \frac{(G_P s + G_I)}{s^4 + 8s^3 + 19s^2 + (12 + G_P)s + G_I} = 0$

s^4	1	19	G_I
s^3	8	$12 + G_P$	0
s^2	$\frac{140 - G_P}{8}$	G_I	0
s^1	$\frac{(140 - G_P)(12 + G_P) + 64G_I}{G_P - 140}$		
s^0	G_I		

As previously derived with *sisotool* in the previous application: $\rightarrow G_P < 140 \wedge G_I > 0$

Using *sisotool* to plot various gain values:

```

1 Transfer Function (with values equated):
2
3
4

```

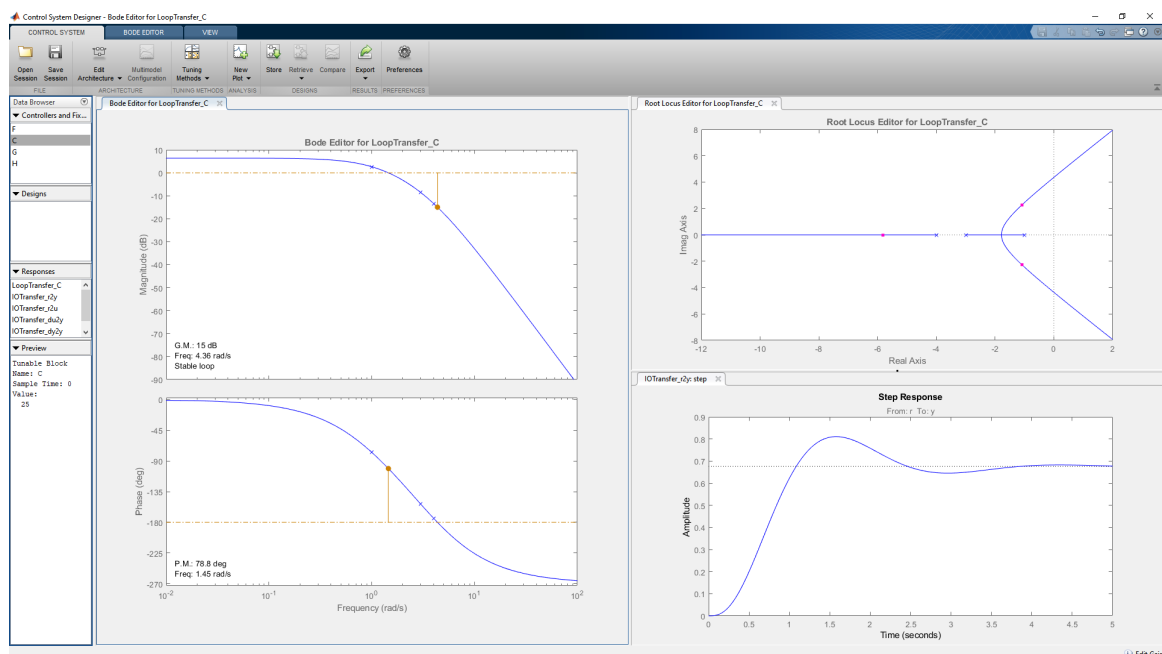
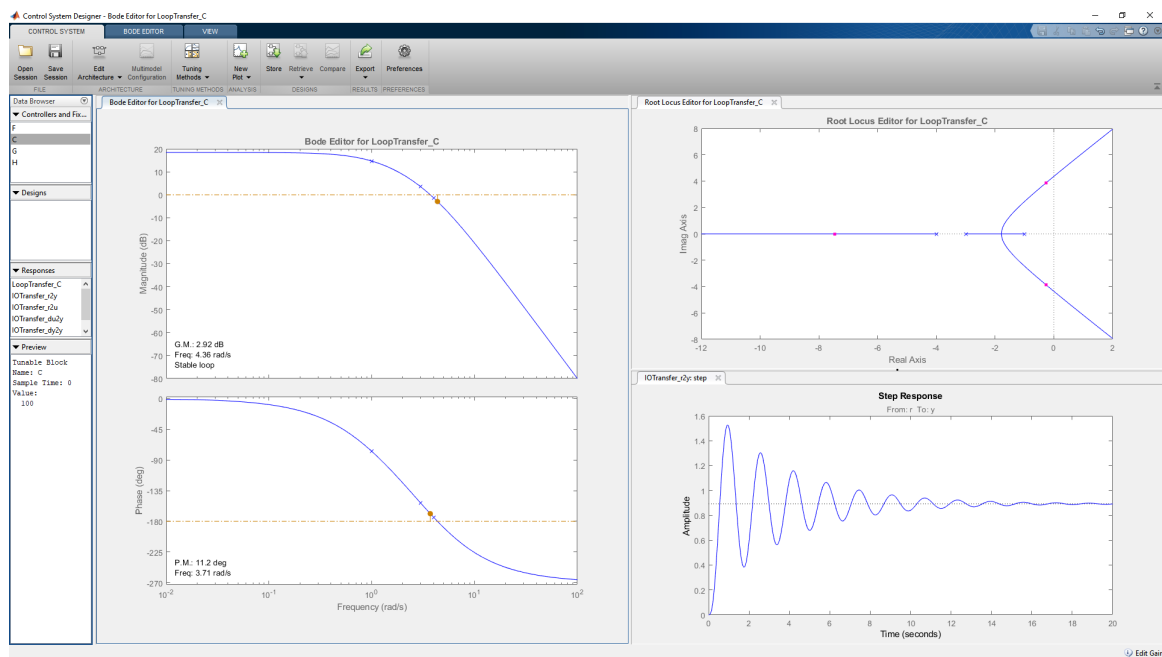
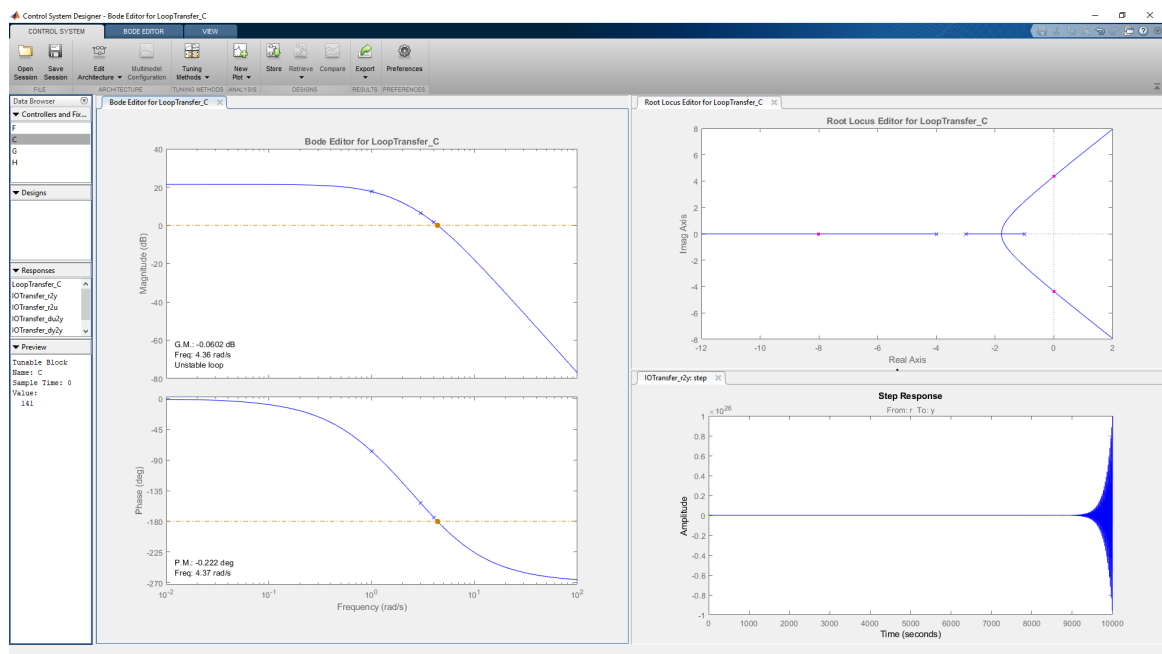
$$\frac{1}{s^3 + 8s^2 + 19s + 12}$$


Figure 29: TH3 4 - $G_P, G_I = 25$

Figure 30: TH3 4 - $G_P, G_I = 100$ Figure 31: TH3 4 - $G_P, G_I = 141$

```

1 %% Stable Gain Range
2 s=tf('s')
3 Gp= 1/(s^3 + 8*s^2 + 19*s + 12)
4 sisotool(Gp)
5 % Stability Analysis from Root Locus:
6 % Scanning the imag axis when the real axis is 0 and -8, values between 0 and 140 are
   stable

```

Listing 9: TH3

```

1 %% Control Law
2 % U(s) = -(Gp+Gi/s)*(Y(s)-Yr(s))
3
4 A = [-3 1 0; 1 -2 1; 0 1 -3];
5 B = [1; 0; 0];
6 C = [0 0 1];
7 D = 0;
8 eig(A)
9
10 H = ss(A,B,C,D)
11 tf(H)
12 %Gm = margin(H)
13
14 %% Stable Gain Range
15 s=tf('s')
16 Gp= 1/(s^3 + 8*s^2 + 19*s + 12)
17 sisotool(Gp)
18 % Stability Analysis from Root Locus:
19 % Scanning the imag axis when the real axis is 0 and -8, values between 0 and 140 are
    stable

```

Pendulum on Cart - PCA 5

Control Law: $u = -G_1x_1 - G_2x_2$

Listing 10: PCA

```

1 syms s a b m M g L G1 G2
2 %A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
3 %B = [0;0;b;-b/L];
4 %a = 4; b = 1; M = 1; m = 0.4; g = 9.81; L = 0.25*M;
5
6 %% Control Law
7 % u = -G1*x1-G2*x2
8
9 %% Previously calculated ss values
10 A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 54.88 16 0];
11 B = [0 0 1 -4]';
12
13 G = [G1 G2 0 0];
14 Ac = A-B*G;
15
16 Delta_c = det(s*eye(4)-Ac);
17 detDelta_C = collect(Delta_c,s);
18 pretty(detDelta_C)

```

```

1
2      4      3      /      1372 \ 2      784 s      196 G1
3 s  + 4 s  + | G1 - 4 G2 - ---- | s  - ---- - ----
4      \      25 /      5      5

```

$$\Rightarrow s^4 + 4s^3 + (G_1 - 4G_2 - \frac{1372}{25})s^2 + \frac{784}{5}s - \frac{196G_1}{5}$$

Since the lead coefficients are alternate, this control law cannot stabilize the system (regardless of arbitrary assignments to $G_1 \wedge G_2$).

References

- [1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.
- [2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)