

Assignment - II

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NB: Using the MATLAB TF function versus iteratively calculating $G(s)$ yield different results. Within the LISTINGS, the TF is reported.

DC Servomotor - DCS 2

The transfer function is:

$$G(s) = \frac{1}{(s^2 + 4s + 1)}$$

```

1 sys =
2
3 A =
4     x1  x2
5     x1   0   1
6     x2   0  -4
7
8 B =
9     u1  u2
10    x1   0   0
11    x2   1   0
12
13 C =
14     x1  x2
15    y1   1   0
16    y2   0   0
17
18 D =
19     u1  u2
20    y1   0   0
21    y2   0   0
22
23 Continuous-time state-space model.
24
25
26 Gs =
27
28 [ 1/(s^2 + 4*s + 1), 0]
29 [          0, 0]
```

Listing 1: DCS

```

1 syms s;
2 sI = [s 0; 0 s];
3 A = [0 1; 0 -4];
4 B = [0 0; 1 0];
5 C = [1 0; 0 0];
6 D = [0 0; 0 0];
7 S = sI-A;
8 inv_s = [((s+4)/(s^2+4*s+1)) 1/(s^2+4*s+1); 0 s/(s^2+4*s+1)];
9 sys = ss(A,B,C,D)
10 Gs = C*inv_s*B
11 %pretty(Gs);
12 %step(sys)
```

Motor-Driven Pendulum - PEN 2

The transfer function at when $\theta = 0$ is:

$$G(s) = \infty$$

```
1
2 sys =
3
4 A =
5     x1  x2
6     x1   1   0
7     x2  -4   0
8
9 B =
10    u1  u2
11    x1   0   0
12    x2   1   0
13
14 C =
15     x1  x2
16    y1   1   0
17    y2   0   0
18
19 D =
20    u1  u2
21    y1   0   0
22    y2   0   0
23
24 Continuous-time state-space model.
25
26
27 Gs =
28
29     NaN     NaN
30     NaN     NaN
```

Motor-Driven Pendulum - PEN2

The transfer function at when $\theta = \pi$ is:

$$G(s) = 0$$

```

1 sys =
2
3
4 A =
5           x1      x2
6 x1      1      0
7 x2     -4  1.47e-15
8
9 B =
10      u1  u2
11 x1      0  0
12 x2      1  0
13
14 C =
15      x1  x2
16 y1      1  0
17 y2      0  0
18
19 D =
20      u1  u2
21 y1      0  0
22 y2      0  0
23
24 Continuous-time state-space model.
25
26
27 Gs =
28
29      0      0
30      0      0

```

Listing 2: PEN

```

1 %syms theta
2 theta_1 = 0;
3 %theta_1 = pi;
4 A = [1 0; -4 12*sin(theta_1)];
5 B = [0 0; 1 0];
6 C = [1 0; 0 0];
7 D = [0 0; 0 0];
8 S = sI-A;
9 norm = 1/(12*sin(theta_1));
10 AA = [12*sin(theta_1) 0; 4 1];
11 inv_s = norm*AA;
12 %Gs2 = C.*inv(S).*B %for checking
13 sys = ss(A,B,C,D)
14 Gs = C.*inv_s.*B
15 TF=tf(ss(A,B,C,D))

```

Third-Order Heat Conduction - TH3 2

The transfer function is:

$$G(s) = \frac{1}{(s+2)(s+3)}$$

```

1 sys =
2
3
4 A =
5     x1  x2  x3
6     x1  -3   1   0
7     x2   1  -2   1
8     x3   0   1  -3
9
10 B =
11     u1  u2  u3
12     x1   1   0   0
13     x2   0   0   0
14     x3   0   1   0
15
16 C =
17     x1  x2  x3
18     y1   0   0   1
19     y2   0   0   0
20     y3   0   0   0
21
22 D =
23     u1  u2  u3
24     y1   0   0   0
25     y2   0   0   0
26     y3   0   0   0
27
28 Continuous-time state-space model.
29
30 ans =
31
32 [ 0, 1/((s + 2)*(s + 3)), 0]
33 [ 0, 0, 0]
34 [ 0, 0, 0]
35
36 TF =
37
38 From input 1 to output...
39           1
40 1:  -----
41     s^3 + 8 s^2 + 19 s + 12
42
43 From input 2 to output...
44           s^2 + 5 s + 5
45 1:  -----
46     s^3 + 8 s^2 + 19 s + 12

```

Listing 3: TH3

```

1 A = [-3 1 0;1 -2 1;0 1 -3];
2 B = [1 0 0;0 0 0;0 1 0];
3 C = [0 0 1; 0 0 0; 0 0 0];
4 D = [0 0 0; 0 0 0; 0 0 0];
5 S = sI-A;
6 norm = 1/((s + 2)*(s + 3)^2);
7 AA = [s+3 1 0; -1 s+2 1; 0 1 s+3];
8 inv_s = norm*AA;
9 Gs = C.*inv_s.*B
10 sys = ss(A,B,C,D)
11 TF=tf(ss(A,B,C,D))

```

Pendulum on Cart - PCA 2

The transfer function is:

$$G(s) = \frac{-1}{s^4 + 4s^3 + \frac{(1372s^2)}{25} + \frac{(784s)}{5}} \quad \text{or} \quad \frac{s^2 + 1.11e - 15s + 70.56}{s^4 + 4s^3 + 54.88s^2 + 156.8s}$$

```

1
2 sys =
3
4 A =
5      x1      x2      x3      x4
6      x1      0      0      1      0
7      x2      0      0      0      1
8      x3      0     -3.92     -4      0
9      x4      0    -54.88    -16      0
10
11 B =
12      u1  u2  u3  u4
13      x1  0  0  0  0
14      x2  0  0  0  0
15      x3  1  0  0  0
16      x4 -4  0  0  0
17
18 C =
19      x1  x2  x3  x4
20      y1  1  0  0  0
21      y2  0  0  0  0
22      y3  0  0  0  0
23      y4  0  0  0  0
24
25 D =
26      u1  u2  u3  u4
27      y1  0  0  0  0
28      y2  0  0  0  0
29      y3  0  0  0  0
30      y4  0  0  0  0
31
32 Continuous-time state-space model.
33
34
35 ans =
36
37 [ -1/(s^4 + 4*s^3 + (1372*s^2)/25 + (784*s)/5), 0, 0, 0]
38 [                                     0, 0, 0, 0]
39 [                                     0, 0, 0, 0]
40 [                                     0, 0, 0, 0]
41
42      s^2 + 1.11e-15 s + 70.56
43 1:  -----
44      s^4 + 4 s^3 + 54.88 s^2 + 156.8 s

```

Listing 4: PCA

```

1 A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 -54.88 -16 0];
2 B = [0 0 1 -4; 0 0 0 0; 0 0 0 0; 0 0 0 0]';
3 C = [1 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
4 D = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
5 S = sI-A;
6 %det = det(S)
7 norm = 1/(s^4 + 4*s^3 + (1372*s^2)/25 + (784*s)/5)
8 AA = [0 0 -1 0; 0 -4 0 -1; 0 3.92 0 0; 0 54.88 16 0];
9 inv_s = norm*AA;
10 %Gs2 = C.*inv(S).*B %for checking
11 sys = ss(A,B,C,D)
12 Gs = C.*inv_s.*B
13 TF=tf(ss(A,B,C,D))

```

References

- [1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.
- [2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)