Application Studies for Control System Design

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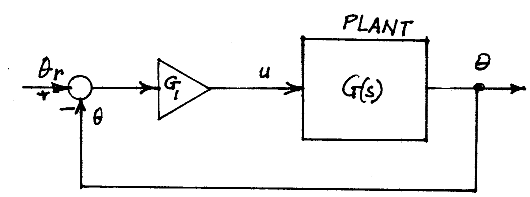
DCS – DC Servomotor

Dynamics

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\alpha\omega + \beta u$$
$$y = \theta$$



Let

$$x_1 = y, \quad x_2 = \omega, \quad \alpha = 4, \quad \beta = 1$$

Application studies

1. Simulate open-loop system
2. Calculate plant transfer function $G(s) = C(sI - A)^{-1}B$
3. Closed-loop proportional control: $u = -G_1(\theta - \theta_r)$ (See block diagram above.)
   (a) Analytically show that closed-loop system is stable for all values of G_1.
   (b) Obtain Nyquist plot using Matlab.
   (c) Obtain Bode plot using Matlab.
   (d) Sketch Root locus and verify using Matlab.
   (e) Develop closed-loop simulation and simulate performance for several values of G_1.

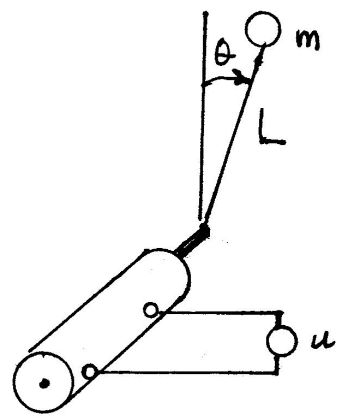
PEN – Motor-Driven Pendulum

Dynamics

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\alpha\omega + (g/L)\sin\theta + \beta u$$
$$y = \theta$$

Let

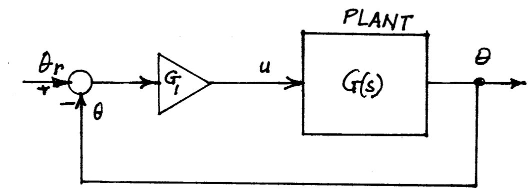
$$x_1 = y, \quad x_2 = \omega, \quad \alpha = 4, \quad \beta = 1, \quad g/L = 12$$



Application studies

1. Simulate open-loop system
2. Obtain the plant transfer functions $G(s) = C(sI - A)^{-1}B$ for
   - $\theta = 0$
   - $\theta = \pi$

   Do this analytically and using the Matlab function linmod
3. Closed-loop proportional control for $\theta = \pi$ and $u = -G_1(\theta - \theta_r)$
   (a) Analytically show that closed-loop system is stable for all values of G_1.
   (b) Obtain Nyquist plot using Matlab
   (c) Obtain Bode plot using Matlab
   (d) Sketch Root locus and verify using Matlab.
   (e) Develop closed-loop simulation and simulate performance for several values of G_1.



4. Closed-loop proportional control for $\theta = 0$ and $u = -G_1(\theta - \theta_r)$
   (a) Find the range of G_1 for which the system is stable.
   (b) Obtain Nyquist plot.
   (c) Obtain Bode plot.
   (d) Sketch Root locus and verify using Matlab.
   (e) Develop closed-loop simulation and simulate performance for several values of G_1.

5. Full-state feedback, $\theta_r = 0$, Pole placement For each equilibrium state ($\theta = 0, \theta = \pi$ determine the full state feedback gains that place the closed-loop poles at
   (a) $s = -6 \pm 4j$
   (b) $s = -8, -10$

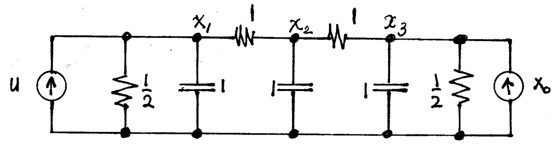
   For each equilibrium state and set of control gains, simulate the closed loop behavior, and comment on the results.

6. Full-state feedback with reference input. For $\theta_r = 0.5$ compute the gain G_0 for the exogenous input for each equilibrium state in Part 5, and simulate the performance. Verify that the steady-state error goes to zero.

TH3 – Third-Order Heat Conduction

Dynamics

$$\begin{aligned}
\dot{x}_1 &= -3x_1 + x_2 + u \\
\dot{x}_2 &= x_1 - 2x_2 + x_3 \\
\dot{x}_3 &= x_2 - 3x_3 + x_0 \\
y &= x_3
\end{aligned}$$

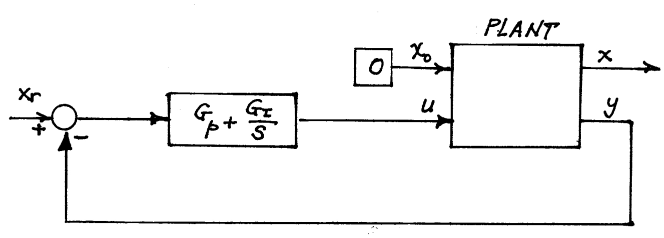


Application Studies

1. Simulate open-loop system
2. Calculate plant transfer function $G(s) = C(sI - A)^{-1}B$
3. Closed-loop proportional control: $u = -G_3(x_3 - x_r)$
    (a) Find the range of G_3 for which the closed-loop system is stable.
    (b) Obtain Nyquist plot using Matlab
    (c) Obtain Bode plot using Matlab.
    (d) Sketch Root locus and verify using Matlab.
    (e) Develop a closed-loop simulation and simulate performance for several values of G_3.
    (f) Determine the steady-state error for a unit step reference input as a function of G_3.
4. Closed-loop proportional + integral control For the closed-loop control law

$$U(s) = -(G_p + G_I/s)(Y(s) - Y_r(s))$$

    (a) Show that the steady state error is zero for $G_I > 0$
    (b) Find the region in the G_p, G_I plane for which the system is stable.
    (c) Develop a closed-loop simulation and simulate performance for several values of G_P, G_I within the range of stability.



$G_I = 0$ for Part 3.

5. Design a full-state feedback control law that places the closed-loop poles at $s = -10 \pm 5j, -15$ and accounts for a non-zero nonzero reference value, and for a nonzero exogenous constant input. Simulate closed-loop performance.

6. Design a full-order observer for $x_0 = 0$ (and for $y = x_3$) with poles at $s = -20 + 10j, -30$. Using this observer with the gain matrix of Part 5, design the full-order compensator and simulate performance

7. Augment the observer of Part 6 by the additional state variable to represent the reference and exogenous inputs.

8. Design (and simulate performance) of a compensator using a reduced-order observer, with poles at $s = -20 \pm 10j$

9. Design (and simulate performance) of a compensator with gains designed for a linear-quadratic regulator with

   $$Q = \mathrm{diag}[0, 0, q_3], \quad R = 1$$

   and a full-order Kalman filter using

   $$F = B, \;\; V = v, , \;\; W = 1$$

   Choose q_3 and v to make the step response five times faster than open-loop with overshoot less than 5% . This is to be done by simulation.

PCA – Pendulum on Cart

Linearized Dynamics

For an inverted pendulum on a cart, driven by a d-c motor, the linearized dynamics are

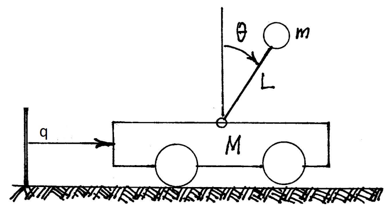
$$\ddot{q} = -\alpha\dot{q} - (mg/M)\theta + \beta u$$
$$\ddot{\theta} = (\alpha/L)\dot{q} + ((M+m)g/ML)\theta - (\beta/L)u$$

The state and output variables are

$$x_1 = q, \;\; x_2 = \theta, \;\; x_3 = \dot{q}, \;\; x_4 = \dot{\theta}, \;\; y_1 = x_1, \;\; y_2 = x_2$$

Numerical values of the parameters are:

$$\alpha = 4, \;\; \beta = 1, \;\; L = 0.25\,meters, \;\; m = 0.4\,kilogram, \;\; M = 1\,kilogram, g = 9.81\,meters/sec^2$$



Application Studies

1. Develop a simulation of the system

2. Calculate the transfer functions from the input u to the outputs y_1 and y_2. (Do this analytically–with the aid of Matlab's Symbolic Toolbox, if necessary, and verify using the Control System Toolbox.)

3. Find the open loop poles and the zeros of each transfer function.

4. Determine whether the system can be stabilized by the control law

$$u = -G_1 x_1$$

5. Determine whether the system can be stabilized by the control law

$$u = -G_1 x_1 - G_2 x_2$$

6. Determine the full state feedback gains that place the closed-loop poles at

   (a) $s = -10 \pm 5j, -20 \pm 10j$

   (b) $s = -10, -15, -20, -30$

   Simulate the transient response using this gain matrix. (Investigate whether the poles farther from the origin are very important.)

   (a) For an initial position offset ($x_1 = .1$), all other state variables initially zero, for both set of gains.

(b) For an initial angular error ($x_2 = .1$), all other state variables zero, for both sets of gains.

7. Design a full-order observer having poles at $s = -40 \pm 30j, -60 \pm 20j$. Simulate the performance with this observer in place and using the gain matrix determined in Part 6. Compare the actual state and the state estimate produced by the observer.

8. Design a reduced-order observer with poles at $s = -40 \pm 30j$. Simulate the performance with this observer in place and using the gain matrix determined in Part 6. Compare the actual state and the state estimate produced by the observer.

9. Design a full-state feedback control law using quadratic optimization (LQR) with matrices given by

$$Q = \mathrm{diag}[q_1, q_2, 0, 0], \quad R = 1$$

   Choose q_1 and q_2 to meet the following design goal (if possible). For an initial cart position error of .1 m, and with the pendulum vertical,

   - Reduce the position error to zero in about 2 sec with overshoot $<$.01 m
   - Keep the pendulum angle $< .05$ rad.

   The values of q_1 and q_2 are to be determined by simulation.

10. Design a compensator by use of the gains obtained in Part 9 and the reduced-order observer of Part 8, and simulate performance.

11. Determine the gains of a Kalman-filter based full-order observer. Use $F = B$ and $W = I_2$ (2 by 2 identity matrix). To pick the value of V, try to match the pole locations of Part 6.