Assignment - II

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NB: Using the MATLAB TF function versus iteratively calculating $G(S)$ yield different results. Within the LISTINGS, the TF is reported.

DC Servomotor - DCS 2

The transfer function is:

$$G(s) = \frac{1}{(s^2 + 4s + 1)}$$

```
sys =

  A =
        x1   x2
    x1   0    1
    x2   0   -4

  B =
        u1   u2
    x1   0    0
    x2   1    0

  C =
        x1   x2
    y1   1    0
    y2   0    0

  D =
        u1   u2
    y1   0    0
    y2   0    0

Continuous-time state-space model.


Gs =

[ 1/(s^2 + 4*s + 1), 0]
[                  0, 0]
```

Listing 1: DCS

```
syms s;
sI = [s 0; 0 s];
A = [0 1; 0 -4];
B = [0 0;1 0];
C = [1 0;0 0];
D = [0 0;0 0];
S = sI-A;
inv_s = [((s+4)/(s^2+4*s+1)) 1/(s^2+4*s+1); 0 s/(s^2+4*s+1)];
sys = ss(A,B,C,D)
Gs = C*inv_s*B
%pretty(Gs);
%step(sys)
```

Motor-Driven Pendulum - PEN 2

The transfer function at when $\theta = 0$ is:

$$G(s) = \infty$$

```

sys =

  A =
        x1   x2
   x1    1    0
   x2   -4    0

  B =
        u1   u2
   x1    0    0
   x2    1    0

  C =
        x1   x2
   y1    1    0
   y2    0    0

  D =
        u1   u2
   y1    0    0
   y2    0    0

Continuous-time state-space model.


Gs =

   NaN   NaN
   NaN   NaN
```

Motor-Driven Pendulum - PEN2

The transfer function at when $\theta = \pi$ is:

$$G(s) = 0$$

```

sys =

  A =
                x1          x2
    x1           1           0
    x2          -4    1.47e-15

  B =
         u1   u2
    x1    0    0
    x2    1    0

  C =
         x1   x2
    y1    1    0
    y2    0    0

  D =
         u1   u2
    y1    0    0
    y2    0    0

Continuous-time state-space model.


Gs =

     0      0
     0      0
```

Listing 2: PEN

```
%syms theta
theta_1 = 0;
%theta_1 = pi;
A = [1 0; -4 12*sin(theta_1)];
B = [0 0; 1 0];
C = [1 0; 0 0];
D = [0 0; 0 0];
S = sI-A;
norm = 1/(12*sin(theta_1));
AA = [12*sin(theta_1) 0; 4 1];
inv_s = norm*AA;
%Gs2 = C.*inv(S).*B %for checking
sys = ss(A,B,C,D)
Gs = C.*inv_s.*B
TF=tf(ss(A,B,C,D))
```

Third-Order Heat Conduction - TH3 2

The transfer function is:

$$G(s) = \frac{1}{(s+2)(s+3)}$$

```

sys =

  A =
       x1  x2  x3
   x1  -3   1   0
   x2   1  -2   1
   x3   0   1  -3

  B =
       u1  u2  u3
   x1   1   0   0
   x2   0   0   0
   x3   0   1   0

  C =
       x1  x2  x3
   y1   0   0   1
   y2   0   0   0
   y3   0   0   0

  D =
       u1  u2  u3
   y1   0   0   0
   y2   0   0   0
   y3   0   0   0

Continuous-time state-space model.

ans =

[ 0, 1/((s + 2)*(s + 3)), 0]
[ 0,                   0, 0]
[ 0,                   0, 0]

TF =

  From input 1 to output...
                  1
   1:  -----------------------
       s^3 + 8 s^2 + 19 s + 12

  From input 2 to output...
            s^2 + 5 s + 5
   1:  -----------------------
       s^3 + 8 s^2 + 19 s + 12
```

Listing 3: TH3

```
A = [-3 1 0;1 -2 1;0 1 -3];
B = [1 0 0;0 0 0;0 1 0;];
C = [0 0 1; 0 0 0; 0 0 0];
D = [0 0 0; 0 0 0; 0 0 0];
S = sI-A;
norm = 1/((s + 2)*(s + 3)^2);
AA = [s+3 1 0; -1 s+2 1; 0 1 s+3];
inv_s = norm*AA;
Gs = C.*inv_s.*B
sys = ss(A,B,C,D)
TF=tf(ss(A,B,C,D))
```

Pendulum on Cart - PCA 2

The transfer function is:

$$G(s) = \frac{-1}{s^4 + 4s^3 + \frac{(1372s^2)}{25} + \frac{(784s)}{5}} \texttt{ or } \frac{s^2 + 1.11e - 15s + 70.56}{s^4 + 4s^3 + 54.88s^2 + 156.8s}$$

```

sys =

  A =
            x1       x2       x3       x4
   x1        0        0        1        0
   x2        0        0        0        1
   x3        0    -3.92       -4        0
   x4        0   -54.88      -16        0

  B =
        u1   u2   u3   u4
   x1    0    0    0    0
   x2    0    0    0    0
   x3    1    0    0    0
   x4   -4    0    0    0

  C =
        x1   x2   x3   x4
   y1    1    0    0    0
   y2    0    0    0    0
   y3    0    0    0    0
   y4    0    0    0    0

  D =
        u1   u2   u3   u4
   y1    0    0    0    0
   y2    0    0    0    0
   y3    0    0    0    0
   y4    0    0    0    0

Continuous-time state-space model.


ans =

[ -1/(s^4 + 4*s^3 + (1372*s^2)/25 + (784*s)/5), 0, 0, 0]
[                                             0, 0, 0, 0]
[                                             0, 0, 0, 0]
[                                             0, 0, 0, 0]

           s^2 + 1.11e-15 s + 70.56
   1:  ---------------------------------
       s^4 + 4 s^3 + 54.88 s^2 + 156.8 s
```

Listing 4: PCA

```
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 -54.88 -16 0];
B = [0 0 1 -4; 0 0 0 0; 0 0 0 0; 0 0 0 0]';
C = [1 0 0 0; 0 0 0 0 ; 0 0 0 0; 0 0 0 0];
D = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
S = sI-A;
%det = det(S)
norm = 1/(s^4 + 4*s^3 + (1372*s^2)/25 + (784*s)/5)
AA = [0 0 -1 0; 0 -4 0  -1; 0 3.92 0 0; 0 54.88 16 0];
inv_s = norm*AA;
%Gs2 = C.*inv(S).*B %for checking
sys = ss(A,B,C,D)
Gs = C.*inv_s.*B
TF=tf(ss(A,B,C,D))
```

References

[1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.

[2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)