Assignment - V

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Third-Order Heat Conduction - TH3

Pole Placement with FSFB (TH3-5)

First, a Simulink model based on the course was created for the TH3 implementation combined with the previous gains and the poles at $[-10+5j, -10-5j, -15]$.

```
Transfer Function =

   -2
  -----
  s + 1
```

```
Gain =
   27.0000   271.0000   915.0000
```

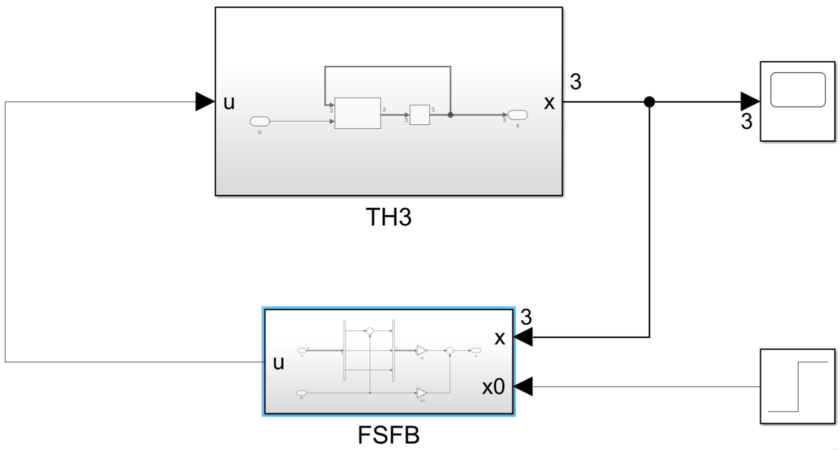

Figure 1: TH3 5 - Full Simulink Model

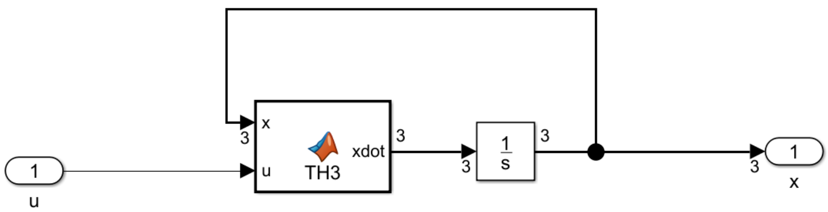

Figure 2: TH3 5 - PCA Simulink Model

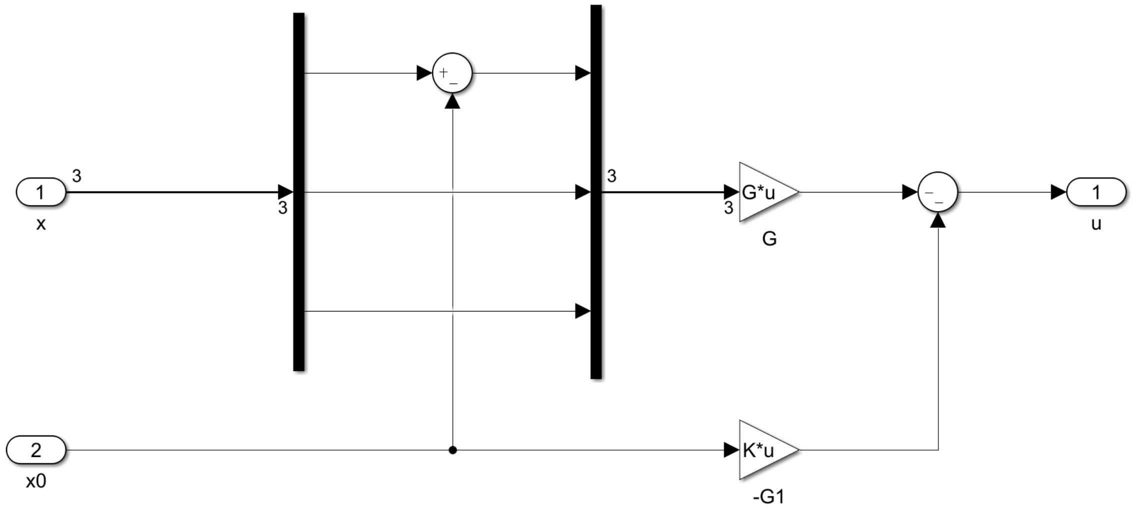

Figure 3: TH3 5 - FSFB Simulink Model

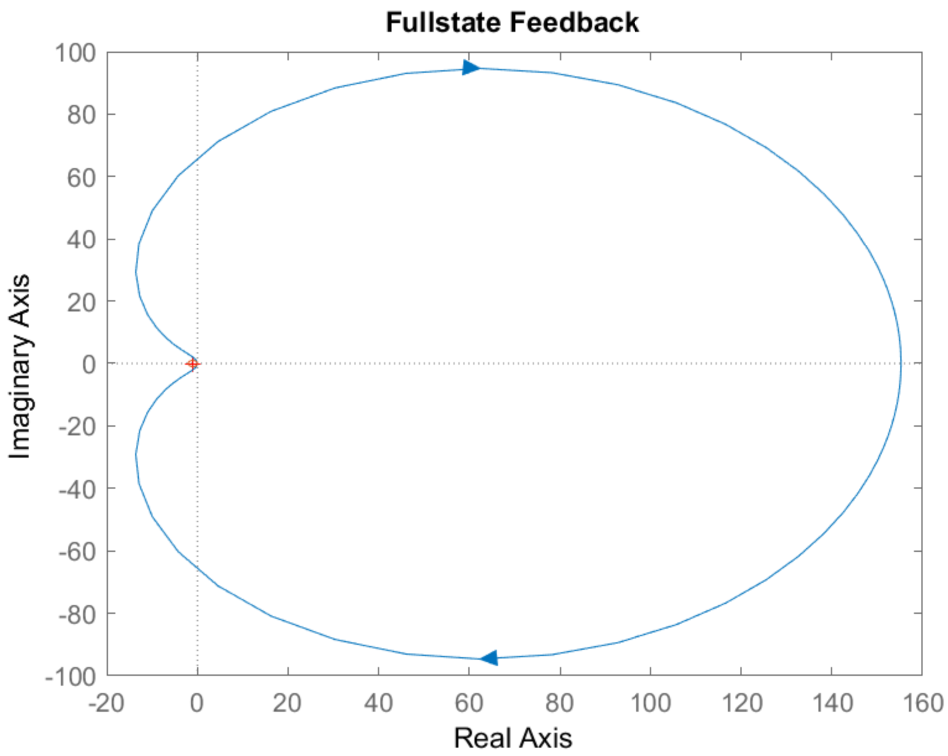

Figure 4: TH3-5 - Effective FSFB Response

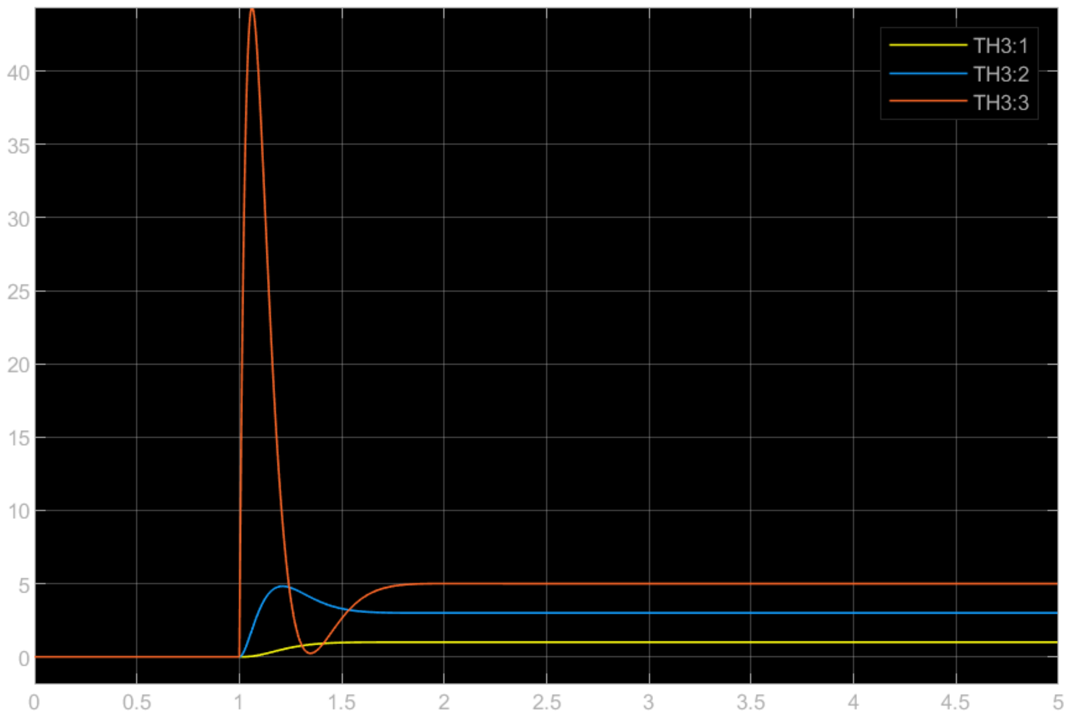

Figure 5: TH3-5 - Effective FSFB Response

Listing 1: TH3 5

```
%% Previously calculated ss values
A = [-3 1 0; 1 -2 1; 0 1 -3];
B = [1; 0; 0];
C = [0 0 1];

%% Renumbered Matrices
T = [0 0 1; 0 1 0; 1 0 0];
A = T*A*inv(T);
B = T*B;
C = C*T;
plant = ss(A,B,C,0);

%% Fullstate Feedback
poles = [-10+5j, -10-5j, -15];
G = place(A, B, poles)
Ac = A-B*G;
Bnum = inv(C*inv(Ac)*B)*C*inv(Ac);
E = [-3; 1; 0];
G0 = Bnum*E;
Fwd = ss(A,B,G,0);
circ = tf(-2,[1 1]);
figure(1)
nyquist(Fwd,circ),title('Fullstate Feedback')
```

Pole Placement with FSFB and FOO (TH3-6)

Next, the poles were moved to $[-20+10j, -20-10j, -30]$ and a Full-Order Observer (Kalman) was added to check the overlap of the circles.

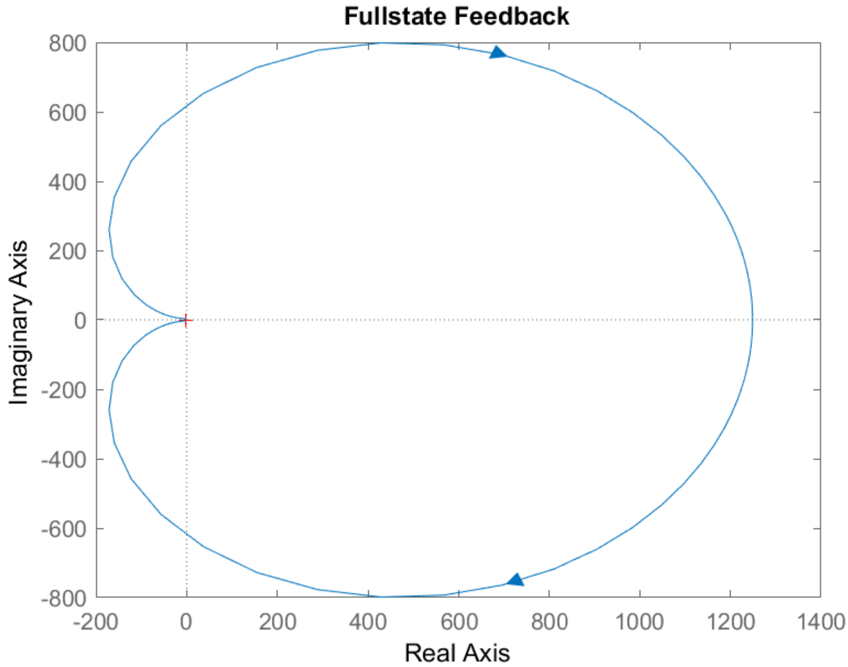

Figure 6: TH3-5 - FSFB

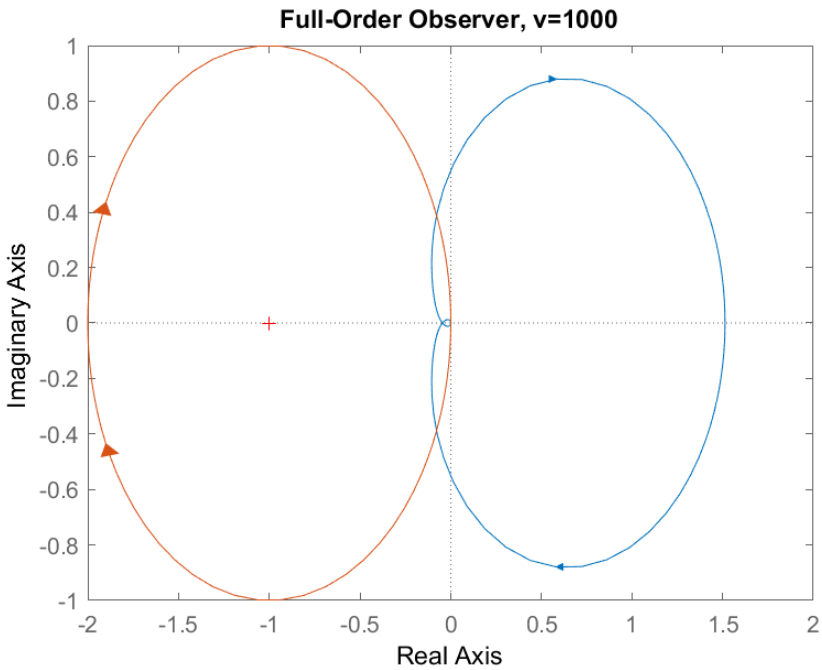

Figure 7: TH3-5 - FOO, v=1000

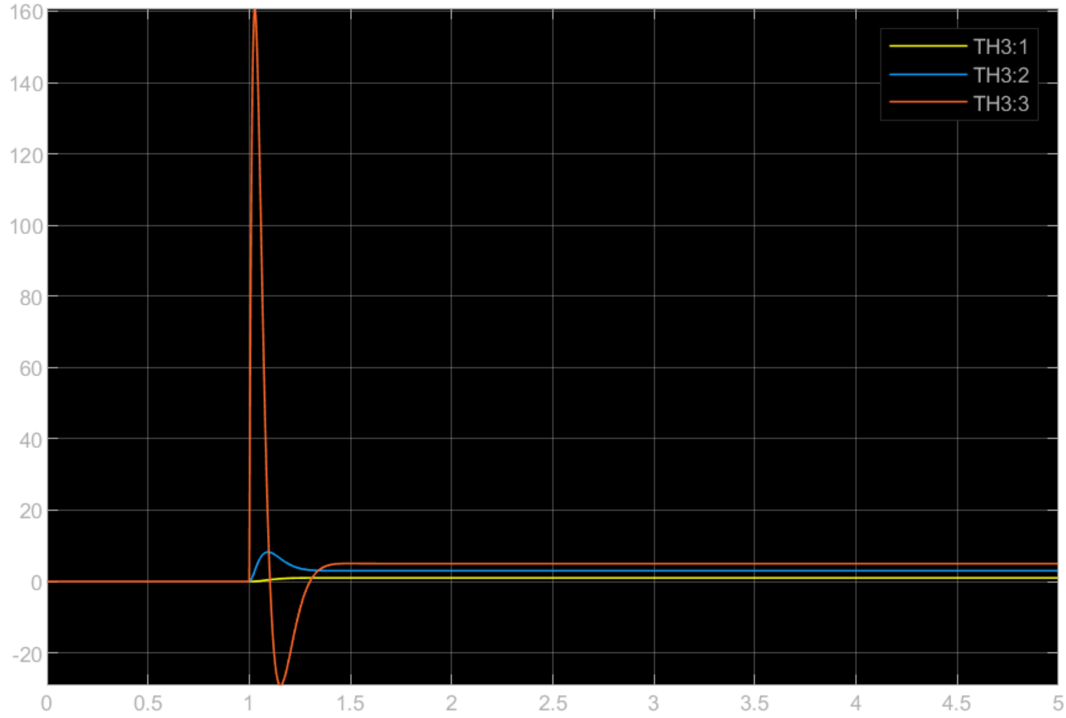

Figure 8: TH3-5 - FSFB with FOO Response

Listing 2: TH3 6

```
syms s G1 G2 G3
A = [-3 1 0; 1 -2 1; 0 1 -3];
B = [1; 0; 0];
C = [0 0 1];

%% Renumbered Matrices
T = [0 0 1; 0 1 0; 1 0 0];
A = T*A*inv(T);
B = T*B;
C = C*T;
plant = ss(A,B,C,0);

%% Fullstate Feedback
poles = [-20+10j, -20-10j, -30];
G = place(A, B, poles)
Ac = A-B*G;
Bnum = inv(C*inv(Ac)*B)*C*inv(Ac);
E = [-3; 1; 0];
G0 = Bnum*E;
Fwd = ss(A,B,G,0);
circ = tf(-2,[1 1]);
figure(1)
nyquist(Fwd,circ), title('Fullstate Feedback')

%% Full-Order Observer
v=1000;
K = lqe(A,B,C,v,1);
Ach = A-B*G-K*C;
comp = ss(Ach,K,G,0);
Fwd = plant*comp;
figure(2)
nyquist(Fwd,circ), title('Full-Order Observer, v=1000')
```

Pendulum on Cart - PCA 6

FSFB with offset

$s = 10 \pm 5j, 20 \pm 10j$

```
Symbolic Transfer Function:
                                25
-----------------------------------------------------------------
    4                           3         2
25 s  + (25 G2 - 100 G1 + 100) s  - 1372 s  + (- 980 G2 - 3920) s
```

Initial Conditions:

```
Transfer Function:
                        50
----------------------------------------------------
    4           3           2
50 s  + 404995 s  + 104746 s  + 2124052 s + 317912
```

```
Gain =
G = 1.0e+03*[-2.2959 -1.0877 -0.5780 -0.1622];
```

Initially, a Simulink model was created to simulate the behavior. The results are in line with the initial assignment (01) but are not very meaningful. Therefore, a simplified state space was created to demonstrate the effective fullstate feedback.

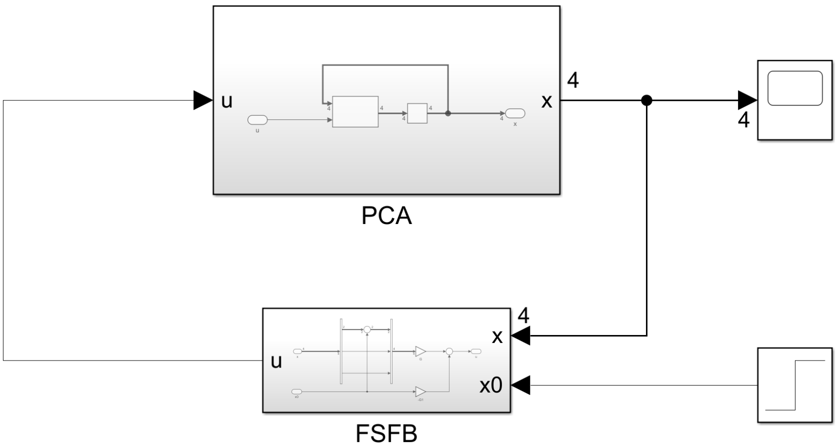

Figure 9: PCA 6 - Full Simulink Model

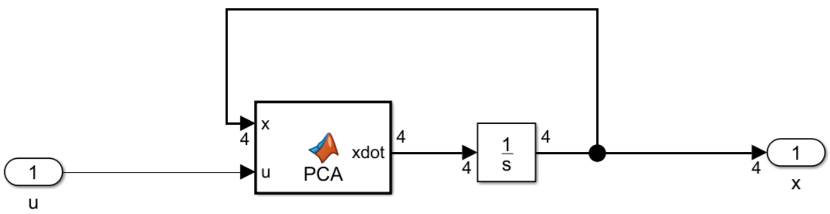

Figure 10: PCA 6 - PCA Simulink Model

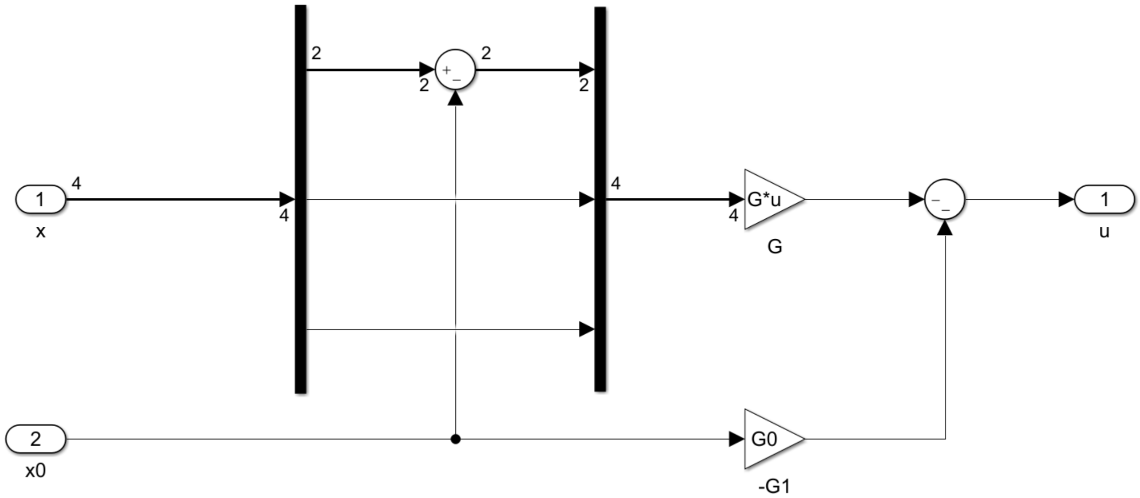

Figure 11: PCA 6 - FSFB Simulink Model

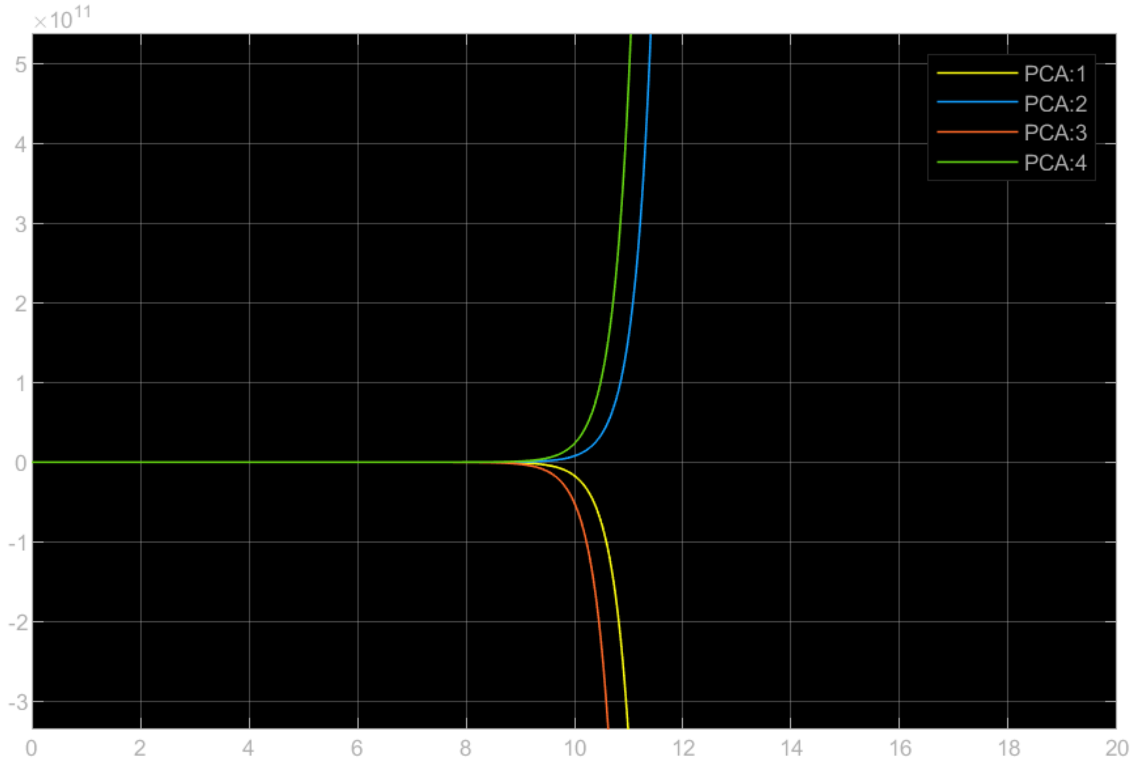

Figure 12: PCA 6 - FSFB Simulink Output ($s = -10, -15, -20, -30$)

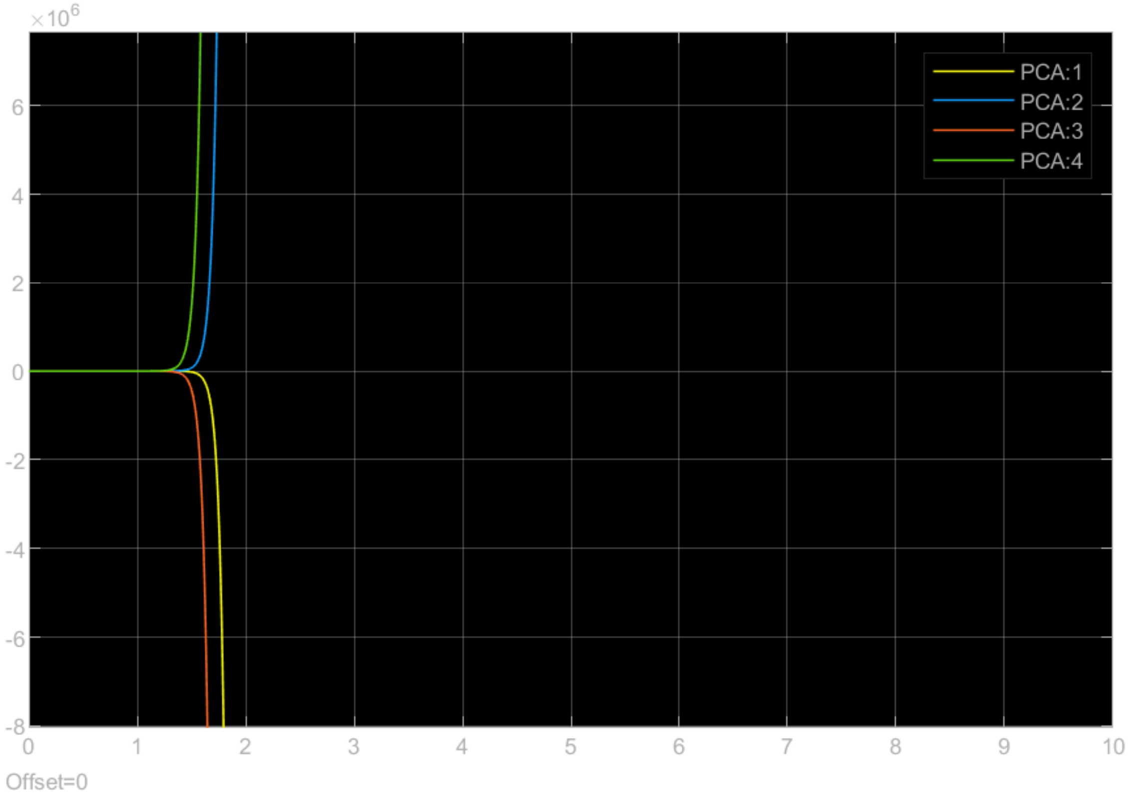

Figure 13: PCA 6 - FSFB Simulink Output, $G = 1.0e + 03 * [-6.4032 - 2.2650 - 1.0265 - 0.2756]$

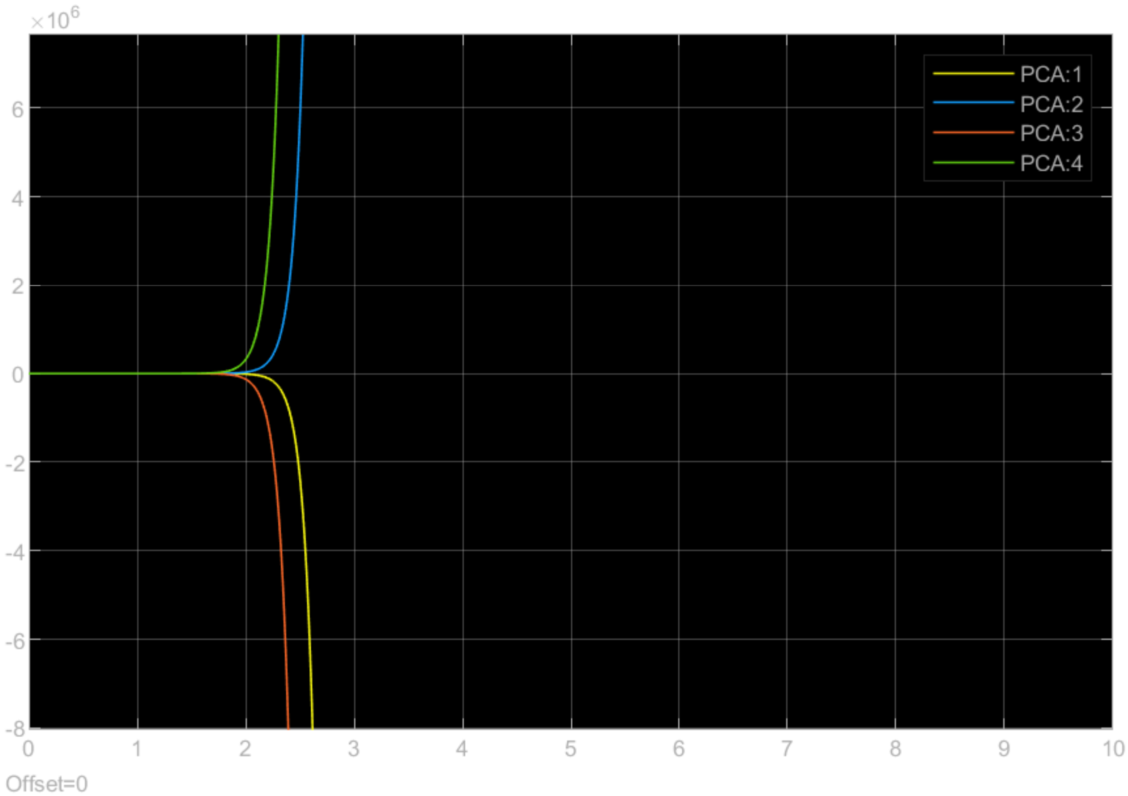

Figure 14: PCA 6 - FSFB Simulink Output, $G = [-583.5842 - 392.7435 - 197.6926 - 60.9732]$

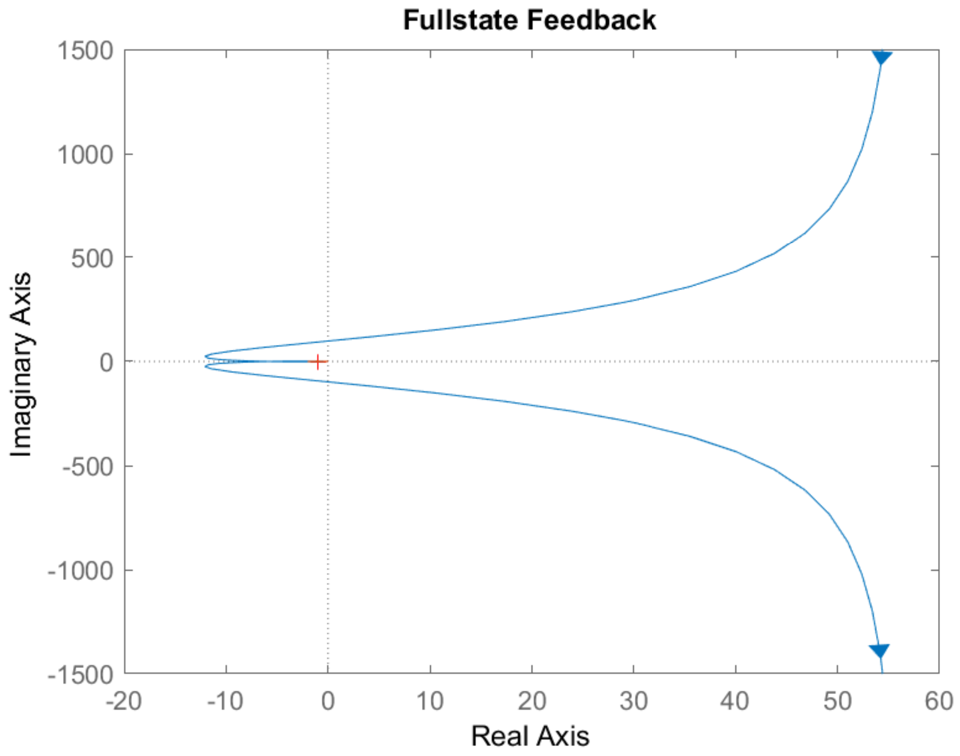

Figure 15: PCA 6 - FSFB ($s = -10, -15, -20, -30$)

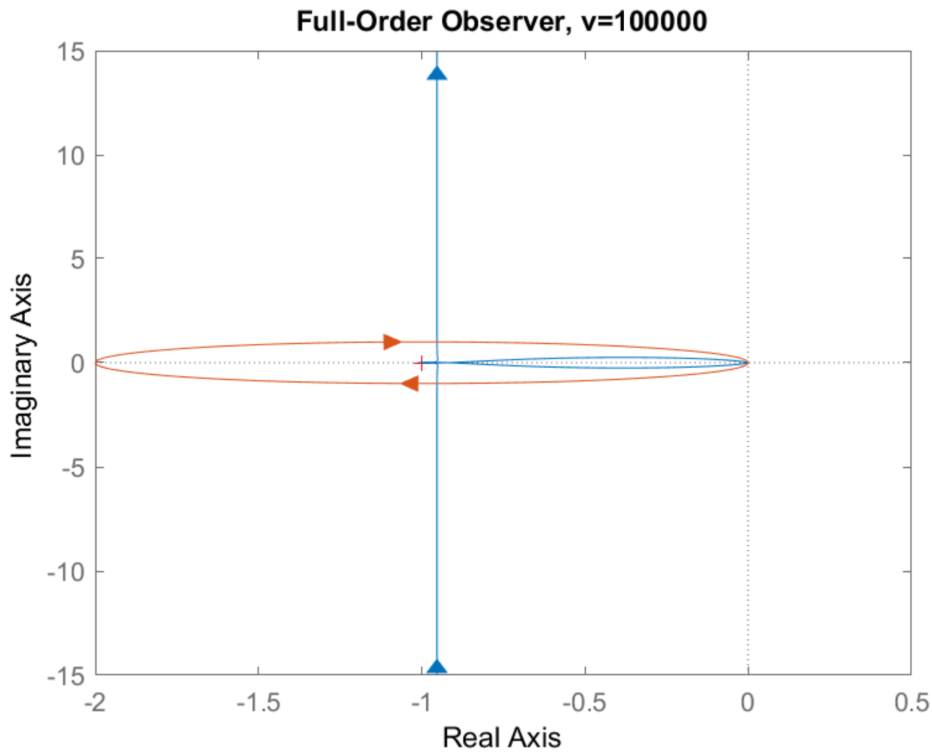

Figure 16: PCA 6 - FOO ($s = -10, -15, -20, -30$)

Effective FSFB Simulation

To simulate FSFB behavior, the following Simulink model was created:

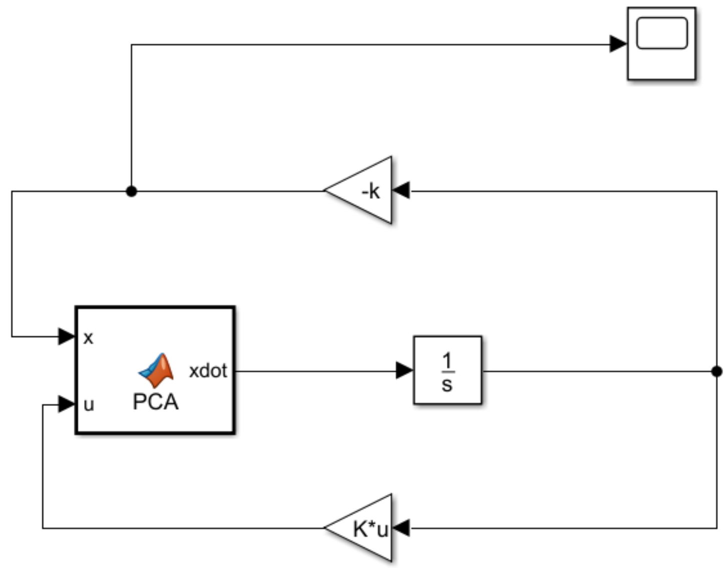

Figure 17: PCA 6 - Simulink Model

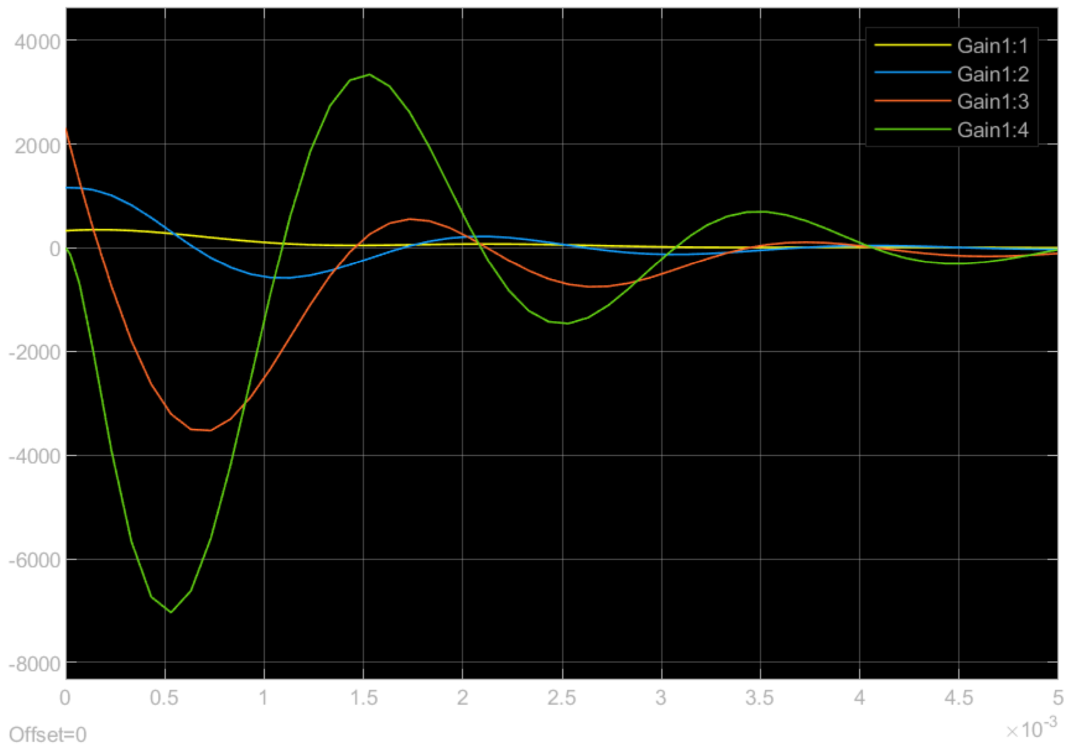

Figure 18: PCA 6 - $poles = [-10+5j, -10-5j, -20+10j, -20-10j]$

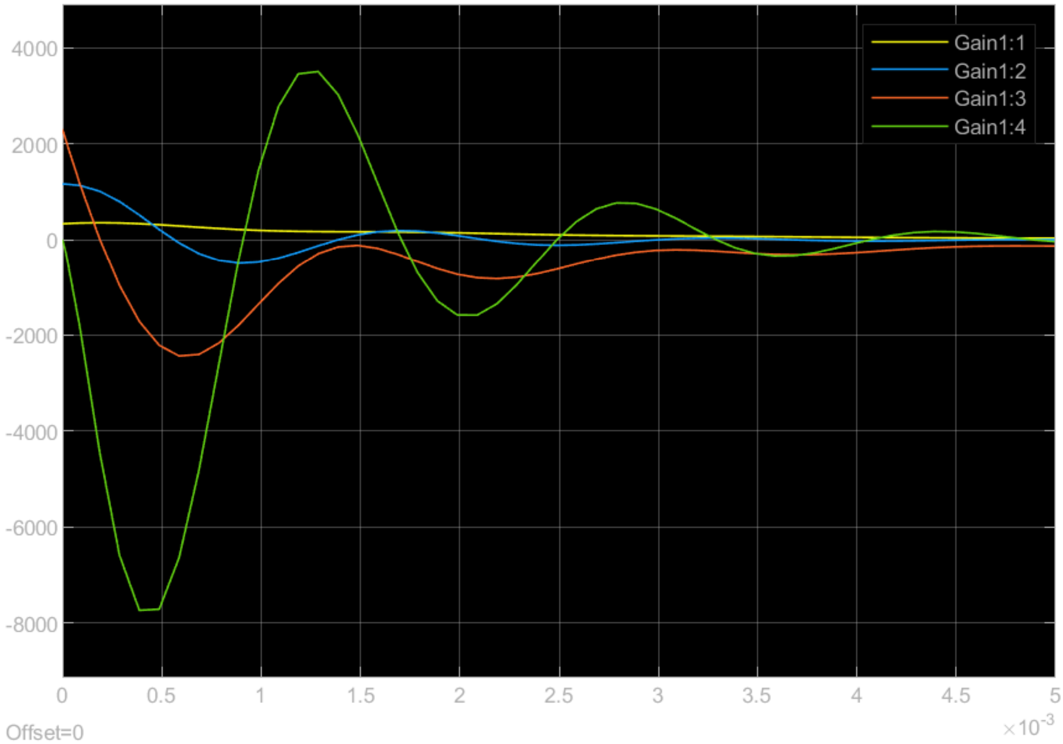

Figure 19: PCA 6 - $poles = [-10, -15, -20, -30]$

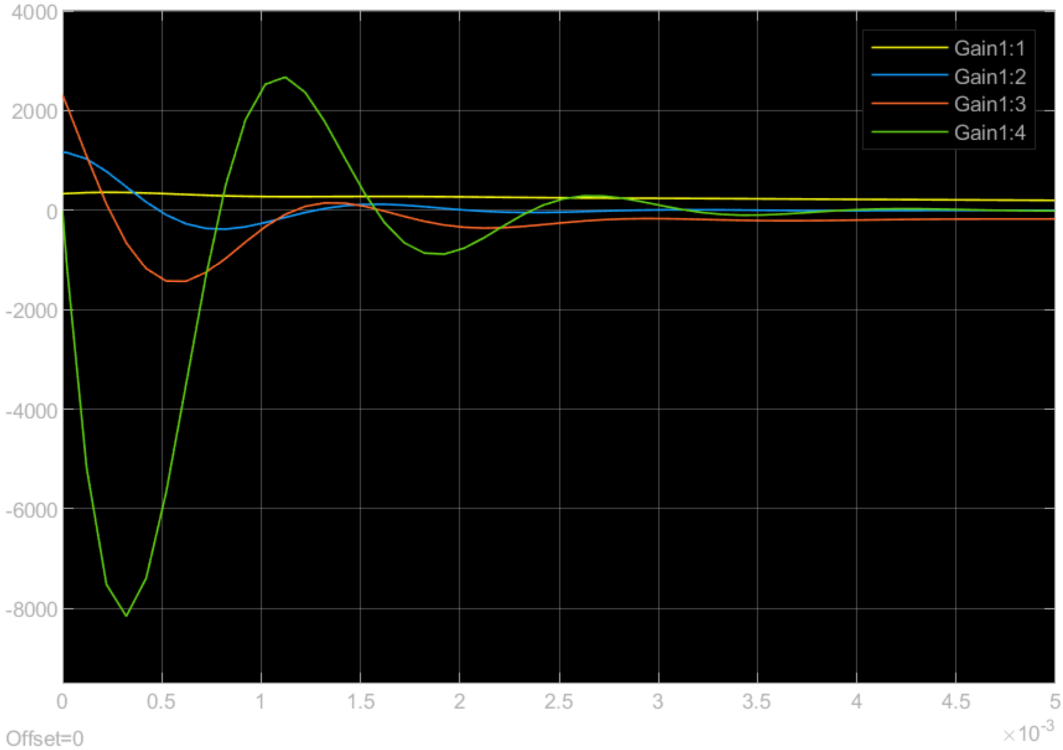

Figure 20: PCA 6 - poles = [-10+5.1j, -10-5.1j, -20+10j, -20-10j],
G = [-401.8176 -276.9269 -132.0663 -42.0166]

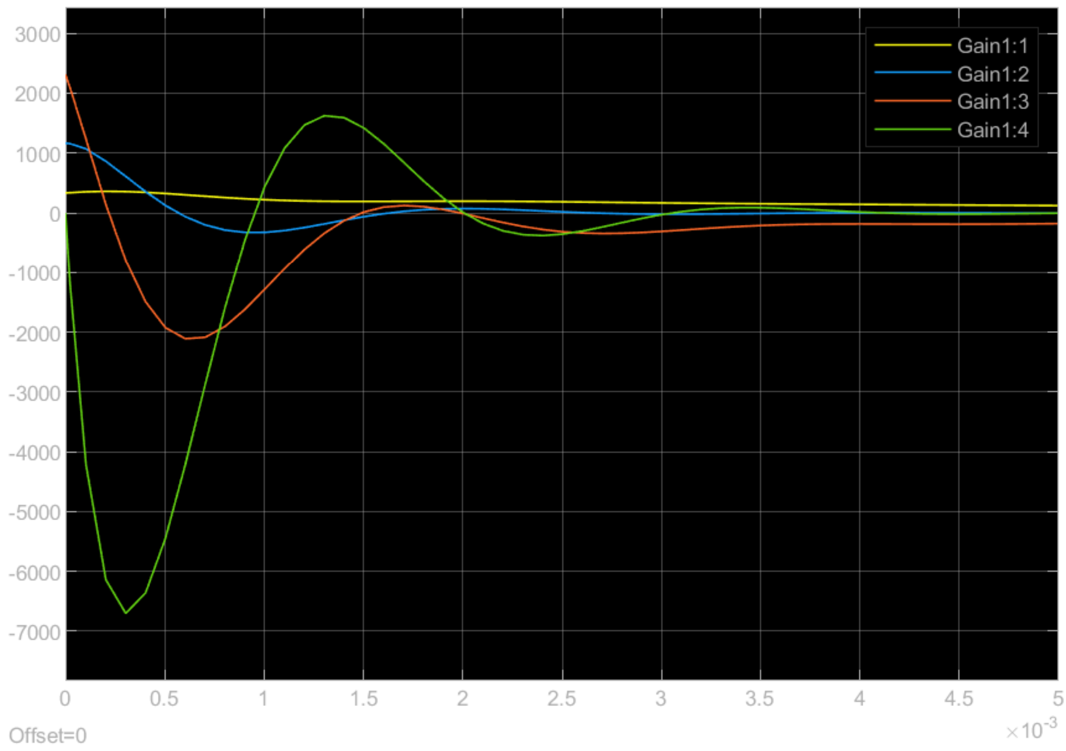

Figure 21: PCA 6 - poles = [-10.1, -15.1, -20, -30], G = [-583.5842 -392.7435 -197.6926 -60.9732]

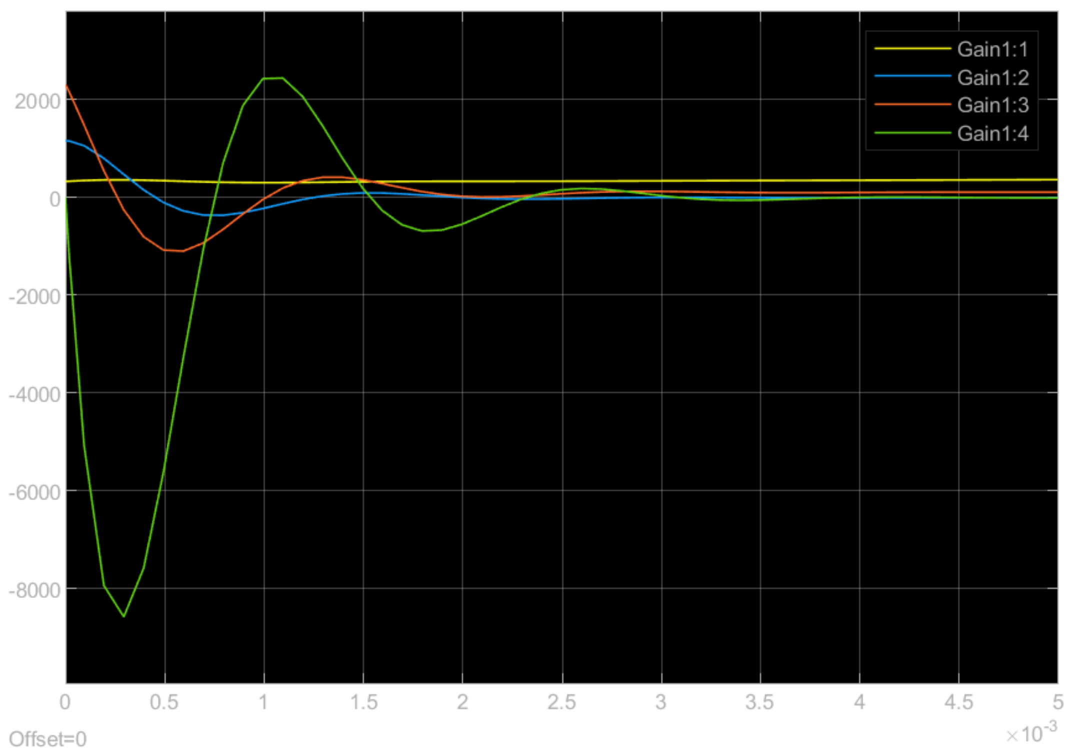

Figure 22: PCA 6 - poles = [-10+5j, -10-5j, -20+10.1j, -20-10.1j], G = [-6.4032 -2.2650 -1.0265 -0.2756]

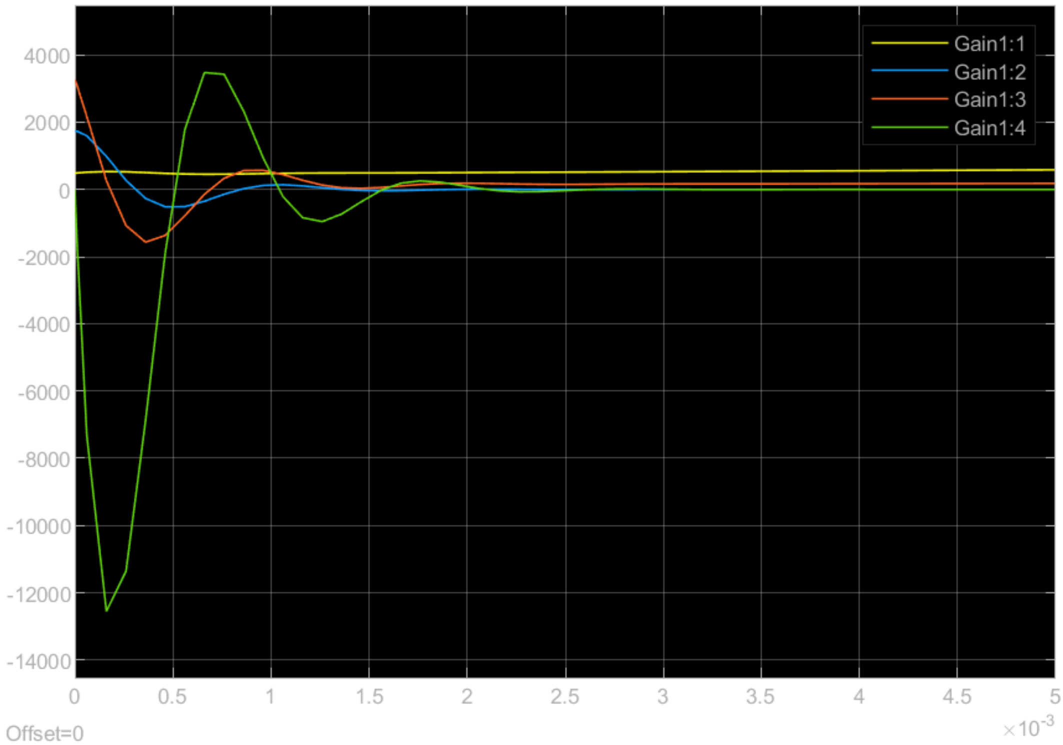

Figure 23: PCA 6 - poles = [-10, -15, -20.1, -30.1], G = [-9.2604 -3.2576 -1.5441 -0.4101]

Listing 3: PCA Model for Simulink Method

```
syms s a b m M g L G1 G2
%A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
%B = [0;0;b;-b/L];
%a = 4; b = 1; M = 1; m = 0.4; g = 9.81; L = 0.25*M;

%% Previously calculated ss values
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 54.88 16 0];
B = [0 0 1 -4]';
C = [1 0 0 0];

T=[0 0 0 1;0 0 1 0; 0 1 0 0; 1 0 0 0];
A=T*A*inv(T);
B=T*B;
C=C*T;
plant = ss(A,B,C,0);


poles = [-10.1, -10.1, -15, -15.1];
%G = place(A, B, poles)
G = 1.0e+03 *[-6.4032   -2.2650   -1.0265   -0.2756];

Ac = A-B*G;
Bnum = inv(C*inv(Ac)*B)*C*inv(Ac);
E = [0; 0; 0; 1];
G0 = Bnum*E;
Fwd = ss(A,B,G,0);
circ = tf(-2,[1 1]);
figure(1)
nyquist(Fwd,circ), title('Fullstate Feedback')

%% Full-Order Observer
v=100000;
K = lqe(A,B,C,v,1);
Ach = A-B*G-K*C;
comp = ss(Ach,K,G,0);
Fwd = plant*comp;
figure(2)
```

```
nyquist(Fwd,circ), title('Full-Order Observer, v=100000')
```

Listing 4: PCA poles @ -10; -15; -20; -30

```
syms s a b m M g L G1 G2
%A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
%B = [0;0;b;-b/L];
%a = 4; b = 1; M = 1; m = 0.4; g = 9.81; L = 0.25*M;

%% Previously calculated ss values
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 54.88 16 0];
B = [0 0 1 -4]';
C = [1 0 0 0];

T=[0 0 0 1;0 0 1 0; 0 1 0 0; 1 0 0 0];
A=T*A*inv(T);
B=T*B;
C=C*T;

%% Fullstate Feedback
%R=100
%Q=diag([10000,0,0,0])
%G=lqr(A,B,Q,R)%note LQR failed to stabilize due to ctrl ^ weights
%plant=ss(A,B,C,0);
%Bnum=inv(C*inv(Ac)*B)*C*inv(Ac);
%E = [0;0;1;0];
%G0=Bnum*E;

%% Place Poles and Gain
poles = [-10, -15, -20, -30];
%G = place(A, B, poles)

G = [G1 G2 0 0];
%G = 1.0e+03*[-2.2959 -1.0877 -0.5780 -0.1622];

Ac = A-B.*G;
Det = inv(s*eye(4)-Ac);
deltaC = det(Det);
DeltaC = collect(deltaC,s);
pretty(DeltaC)

%% Test Gain
k2 = place(Ac, B, poles)
%k2 = [-0.0184 0.0004 0.0204 -0.0449]
```

Listing 5: PCA poles @ -10+5j; -10-5j; -20+10j; -20-10j

```
syms s a b m M g L G1 G2
%A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
%B = [0;0;b;-b/L];
%a = 4; b = 1; M = 1; m = 0.4; g = 9.81; L = 0.25*M;

%% Previously calculated ss values
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 54.88 16 0];
B = [0 0 1 -4]';
C = [1 0 0 0];

T=[0 0 0 1;0 0 1 0; 0 1 0 0; 1 0 0 0];
A=T*A*inv(T);
B=T*B;
C=C*T;

%% Fullstate Feedback
%R=100
%Q=diag([10000,0,0,0])
%G=lqr(A,B,Q,R)%note LQR failed to stabilize due to ctrl ^ weights
%plant=ss(A,B,C,0);
%Bnum=inv(C*inv(Ac)*B)*C*inv(Ac);
%E = [0;0;1;0];
%G0=Bnum*E;


%% Place Poles and Gain
```

```
poles = [-10+5j, -10-5j, -20+10j, -20-10j];
k = place(A, B, poles)

G = [G1 G2 0 0];
%G = 1.0e+03*[-1.5944 -0.7686 -0.3867 -0.1107];

Ac = A-B.*G;
Det = inv(s*eye(4)-Ac);
deltaC = det(Det);
DeltaC = collect(deltaC,s);
pretty(DeltaC)

%% Test Gain
k2 = place(Ac, B, poles)
%k2 = [0.0122 0.0331 0.0469 0.0367]
```

References

[1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.

[2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)