Assignment - III

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DC Servomotor - DCS 3

Stability

Closed-loop proportional control: $u = -G_1(\theta - \theta_r)$

The transfer function is:

$$H(s) = \frac{1}{(s^2 + 4s + G_1)}$$

Using Rowth-Hurwitz Criterion $\rightarrow (s^2 + 4s + G_1) = 0$

S^2	1	G_1
S^1	4	0
S^0	G_1	0

Therefore, the system is stable for all values of $G_1 > 0$ NB: For the remainder of the stability problems, the root locus diagram will be analyzed from the `sisotool` in the control systems design toolbox and GUI.

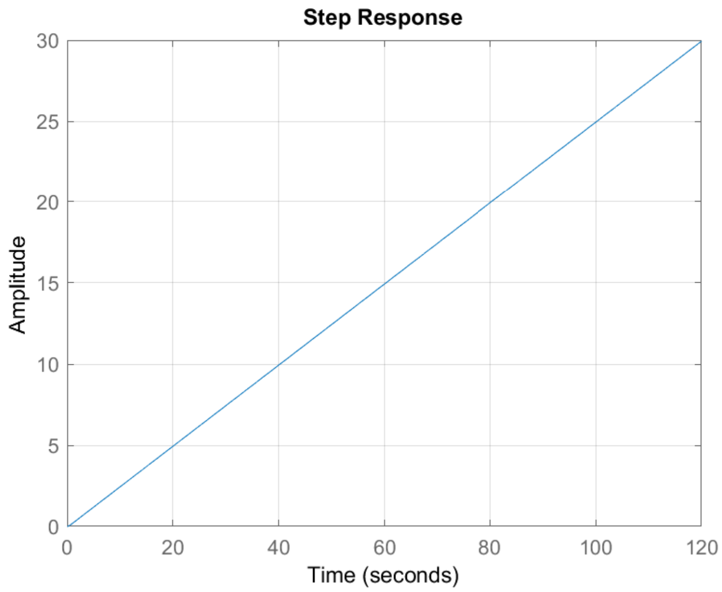

Figure 1: DCS - Step Response

Nyquist

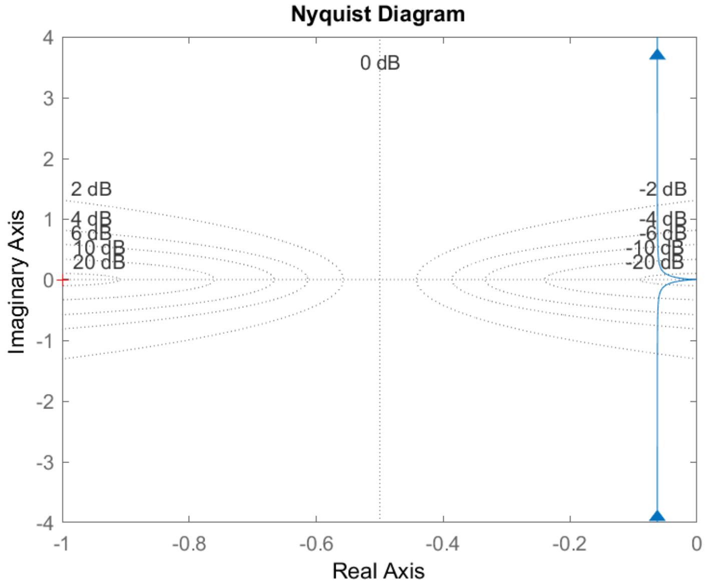

Figure 2: DCS - Nyquist

Bode

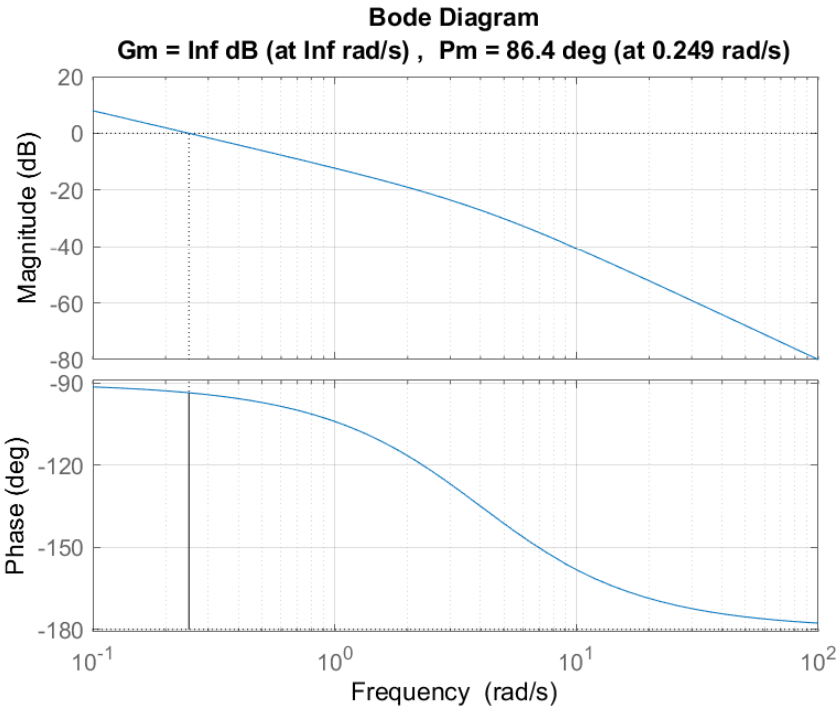

Figure 3: DCS - Bode

Root Locus

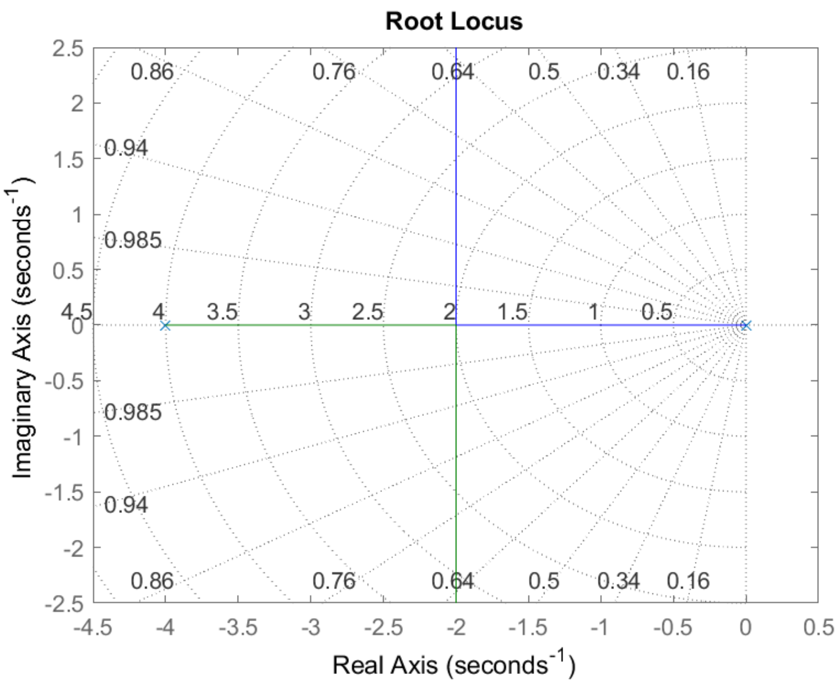

Figure 4: DCS - Root Locus

Listing 1: DCS

```
A = [0 1;0 -4];
B = [0;1];
C = [1 0];
D = 0;
eig(A)

H=ss(A,B,C,D)
tf(H)

Gm = margin(H)

%% Root Locus
figure(1)
plot(1,2)
rlocus(H), grid

%% Nyquist
figure(2)
plot(1,2)
nyquist(H), grid

%% Bode
figure(3)
plot(1,2)
margin(H), grid

%% Step
figure(4)
plot(1,2)
step(H), grid

%% Stable Gain Range
s=tf('s')
Gp= 1/(s^2+4*s)
sisotool(Gp)
% Stability Analysis from Root Locus:
```

```
% Scanning the real axis, the gain is stable between values of 0 and -4
% Scanning the imag axis when the real axis is at -2, all values are stable
```

Motor-Driven Pendulum - PEN3-I

Stability

```
%% Stable Gain Range
s=tf('s')
Gp= 1/(s^2+4*s+12)
sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the imag axis when the real axis is at -2, all values are stable
```

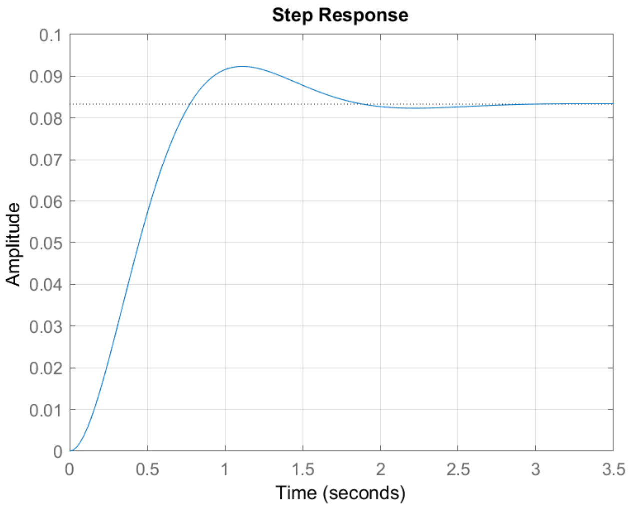

Figure 5: PEN 3-I - Step Response

Nyquist

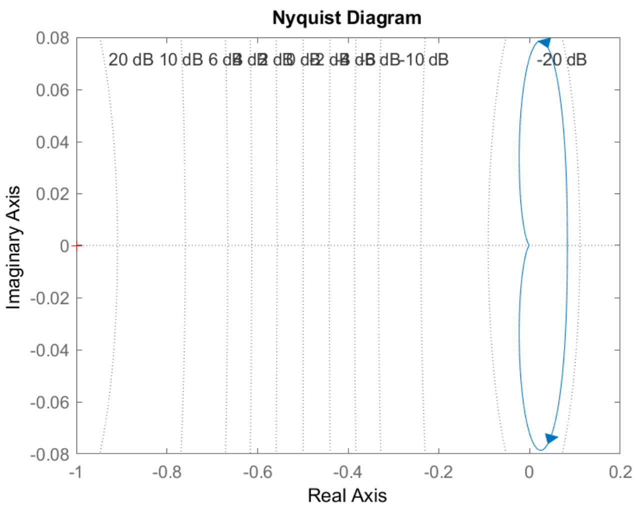

Figure 6: PEN 3-I - Nyquist

Bode

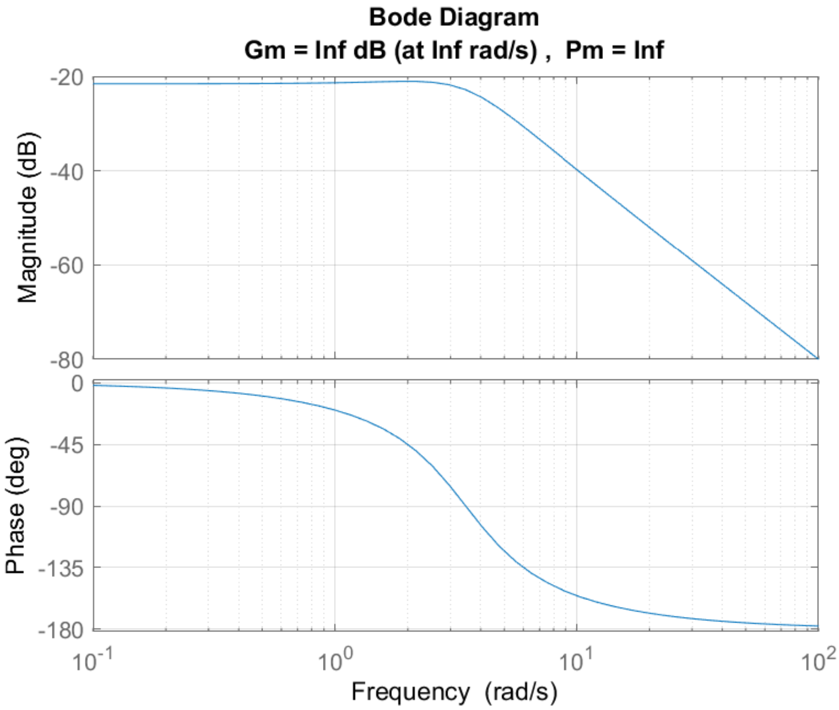

Figure 7: PEN 3-I - Bode

Root Locus

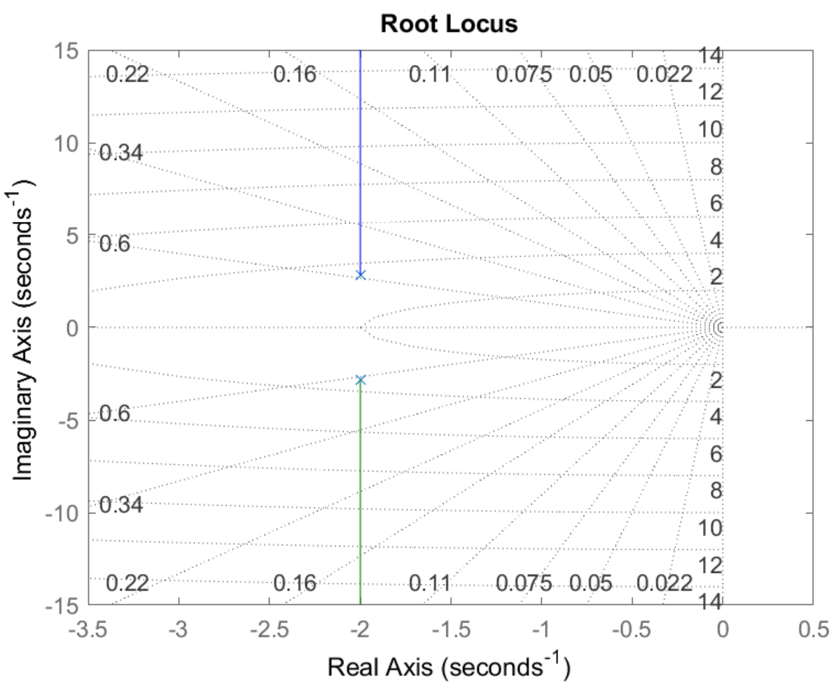

Figure 8: PEN 3-I - Root Locus

Listing 2: PEN 3-I

```
A = [0 1;-12 -4];
B = [0;1];
C = [1 0];
D = 0;
eig(A)

H = ss(A,B,C,D)
tf(H)
Gm = margin(H)


%% Root Locus
figure(1)
plot(1,2)
rlocus(H), grid

%% Nyquist
figure(2)
plot(1,2)
nyquist(H), grid

%% Bode
figure(3)
plot(1,2)
margin(H), grid

%% Step
figure(4)
plot(1,2)
step(H), grid

%% Stable Gain Range
s=tf('s')
Gp= 1/(s^2+4*s+12)
sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the imag axis when the real axis is at -2, all values are stable
```

Motor-Driven Pendulum - PEN 3-II

Stability

```

%% Stable Gain Range
s=tf('s')
Gp= 1/(s^2+4*s-12)
sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the real axis, the gain is stable between values of 12 and 16
% Scanning the imag axis when the real axis is at -2, all values are stable
```

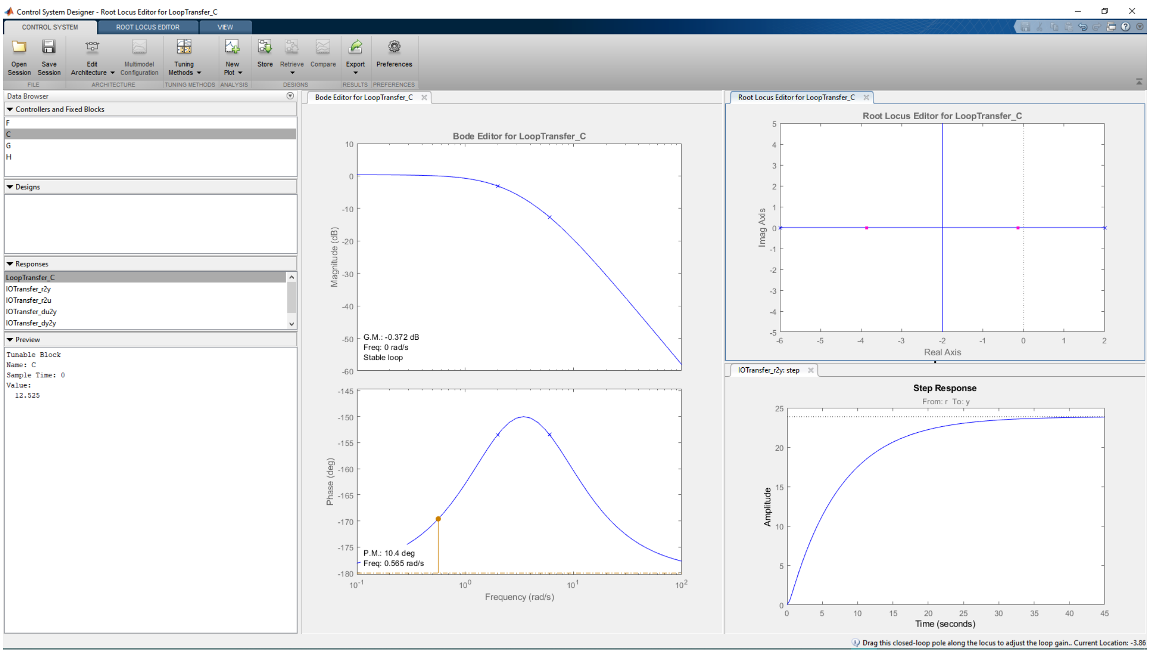

Figure 9: PEN 3-II - Example of Control System Design Toolbox GUI for Gain Range

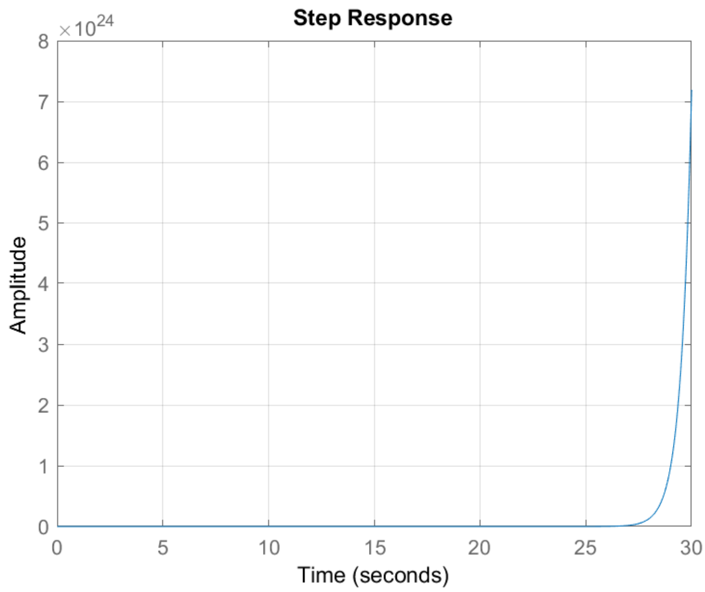

Figure 10: PEN 3-II - Step Response, note the system is generally unstable

Nyquist

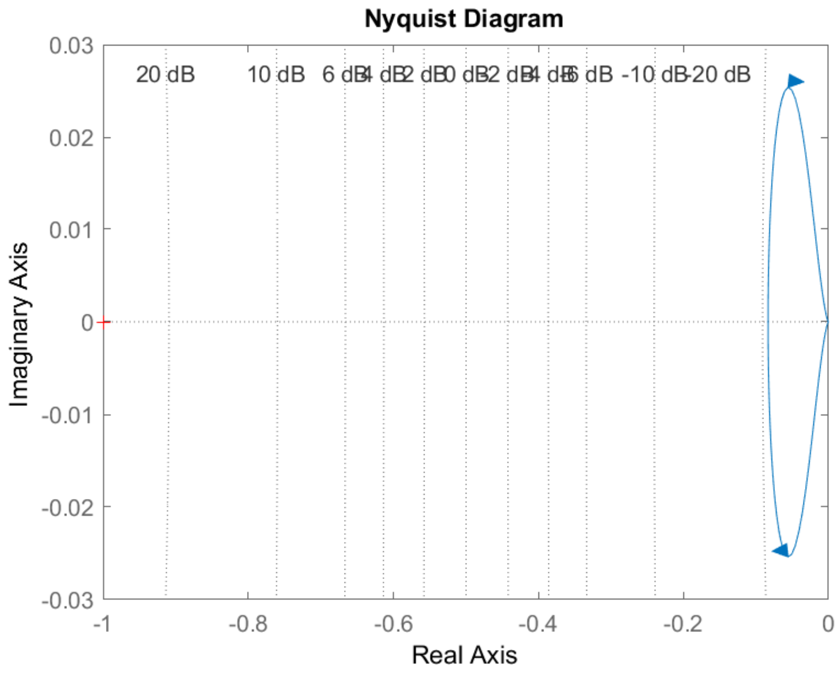

Figure 11: PEN 3-II - Nyquist

Bode

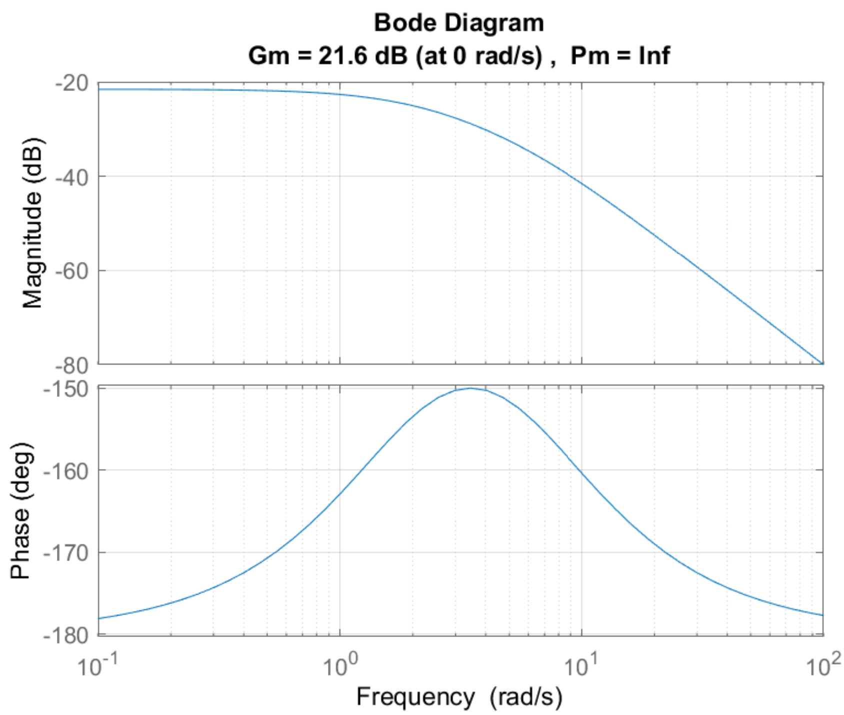

Figure 12: PEN 3-II - Bode

Root Locus

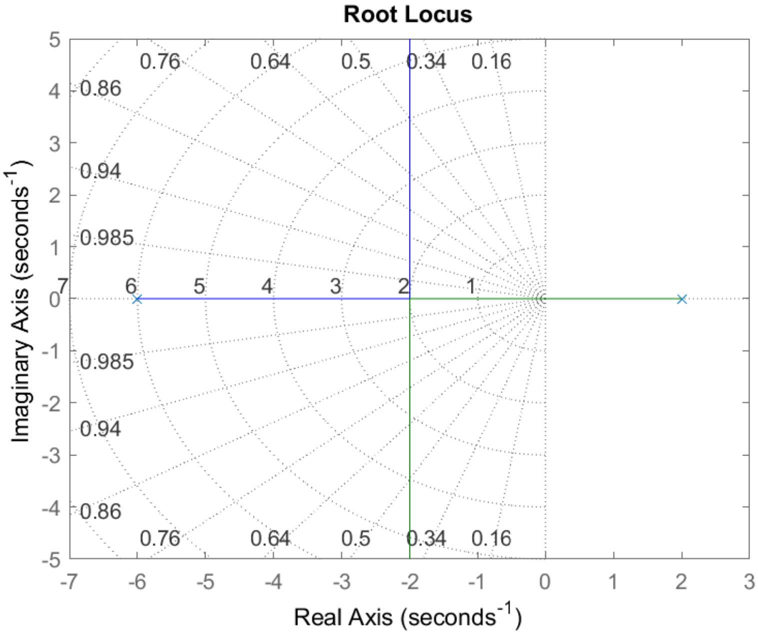

Figure 13: PEN 3-II - Root Locus

Listing 3: PEN 3-II

```
A = [0 1;12 -4];
B = [0;1];
C = [1 0];
D = 0;
eig(A)

H = ss(A,B,C,D)
tf(H)
Gm = margin(H)


%% Root Locus
figure(1)
plot(1,2)
rlocus(H), grid

%% Nyquist
figure(2)
plot(1,2)
nyquist(H), grid

%% Bode
figure(3)
plot(1,2)
margin(H), grid

%% Step
figure(4)
plot(1,2)
step(H), grid

%% Stable Gain Range
s=tf('s')
Gp= 1/(s^2+4*s-12)
sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the real axis, the gain is stable between values of 12 and 16
% Scanning the imag axis when the real axis is at -2, all values are stable
```

Third-Order Heat Conduction - TH3 3

Stability

```
%% Stable Gain Range
s=tf('s')
Gp= 1/(s^3 + 8*s^2 + 19*s + 12)
%sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the imag axis when the real axis is 0 and -8, values between 0 and 140 are
    stable
```

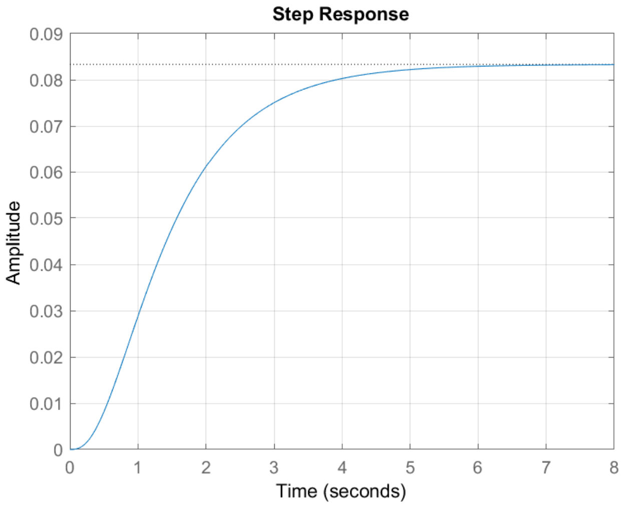

Figure 14: TH3 - Step Response

Nyquist

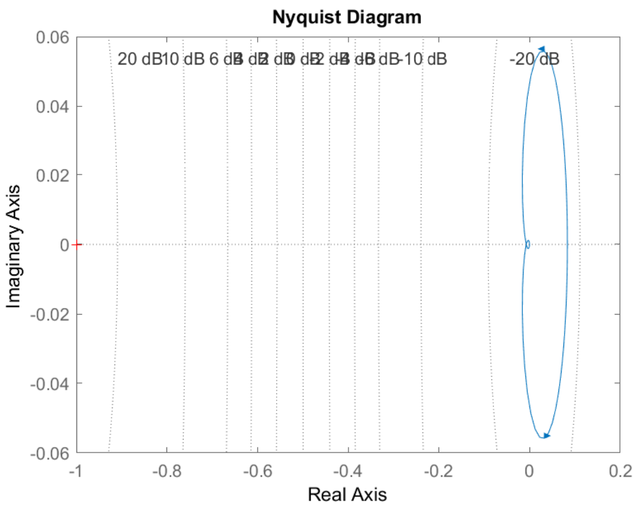

Figure 15: TH3 - Nyquist

Bode

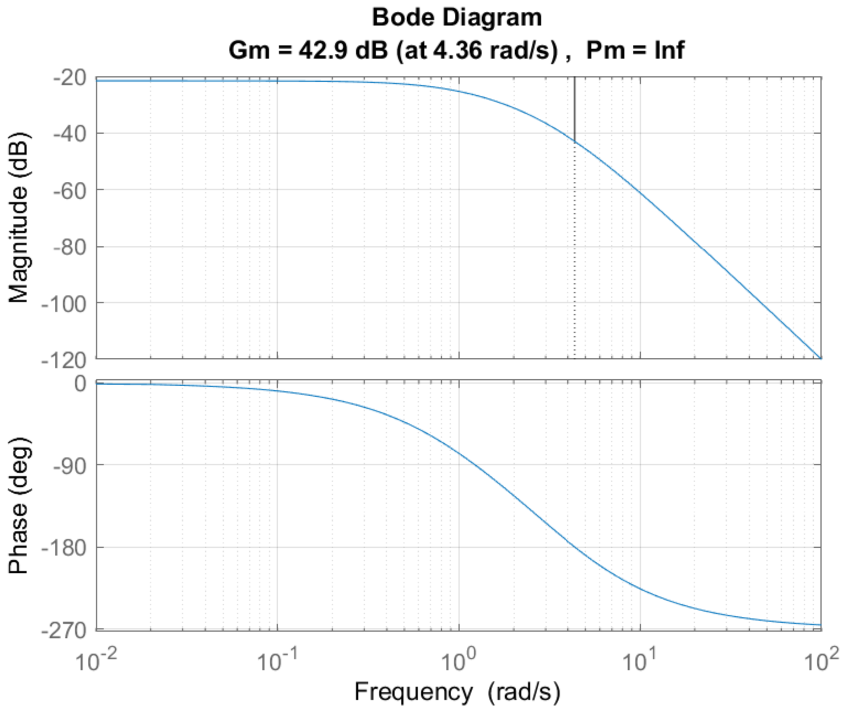

Figure 16: TH3 - Bode

Root Locus

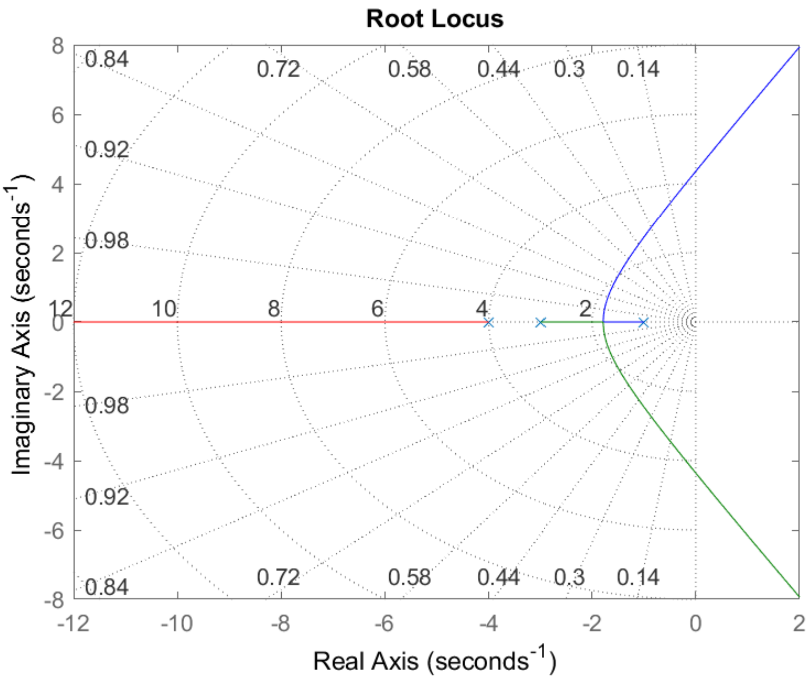

Figure 17: TH3 - Root Locus

Listing 4: TH3

```
A = [-3 1 0;1 -2 1;0 1 -3];
B = [1; 0; 0];
C = [0 0 1];
D = 0;
eig(A)

H = ss(A,B,C,D)
tf(H)
%Gm = margin(H)


%% Root Locus
figure(1)
plot(1,2)
rlocus(H), grid

%% Nyquist
figure(2)
plot(1,2)
nyquist(H), grid

%% Bode
figure(3)
plot(1,2)
margin(H), grid

%% Step
figure(4)
plot(1,2)
step(H), grid

%% Stable Gain Range
s=tf('s')
Gp= 1/(s^3 + 8*s^2 + 19*s + 12)
sisotool(Gp)
% Stability Analysis from Root Locus:
% Scanning the imag axis when the real axis is 0 and -8, values between 0 and 140 are
    stable
```

Pendulum on Cart - PCA 3

Listing 5: PCA

```
syms s a b m M g L G1 G2
%A = [0 0 1 0; 0 0 0 1; 0 -m*g/M -a 0; 0 (M+m)*g/(M*L) a/L 0];
%B = [0;0;b;-b/L];
%% Previously calculated ss values
A = [0 0 1 0; 0 0 0 1; 0 -3.92 -4 0; 0 -54.88 -16 0];
B = [0 0 1 -4]';

G = [G1 G2 0 0];
Ac = A-B*G;
Delta_c = det(s*eye(4)-Ac);
detDelta_C = collect(Delta_c,s);
pretty(detDelta_C)
```

```

 4      3    /             1372 \   2    / 784           \      1764 G1
s  + 4 s  + | G1 - 4 G2 + ---- | s  + | --- - 32 G2 | s + -------
             \              25  /       \  5           /       25
```

$$\Rightarrow s^4 + 4s^3 + (G_1 - 4G_2 + \frac{1372}{25})s^2 + (\frac{784}{5} - 32G_2)s + \frac{1764G_1}{25}$$

Since all of the lead coefficients are positive, this control law can stabilize the system (given $G_1 > 4G_2$ and $G_2 < 4.9$).

References

[1] B. Friedland, Observer-Based Control System Design Lecture Notes for ECE660.

[2] B. Friedland, Control System Design: An Introduction to State Space Methods, McGraw-Hill, 1985. ISBN:0070224412 (Reprinted by Dover Publications May 2005, ISBN: 0-486-44278-0.)