Recursive Counting

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Outline

Number of Paths

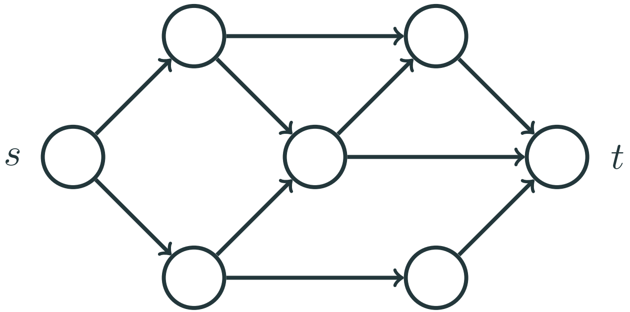
Rule of Product

Back to Recursive Counting

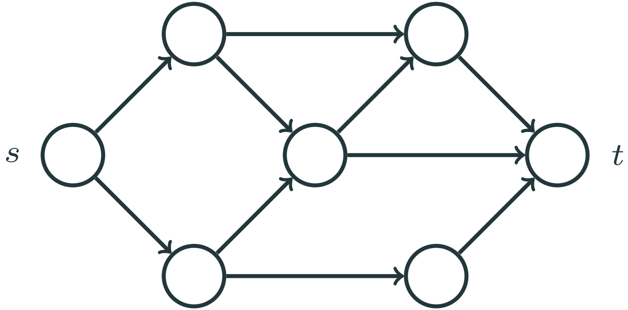
Number of Paths

Problem

Suppose there are several points connected by arrows. There is a starting point s (called source) and a final point t (called sink). How many different ways are there to get from s to t?



Number of Paths



- There are several various paths; how not to miss anything when counting?

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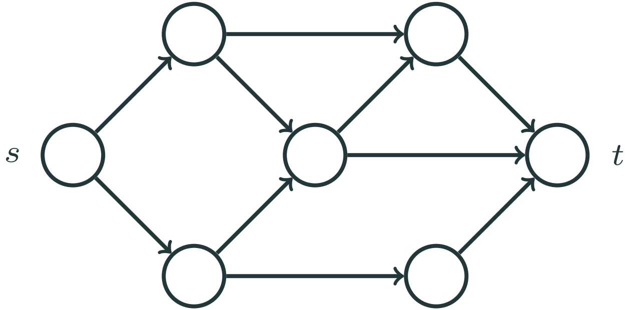

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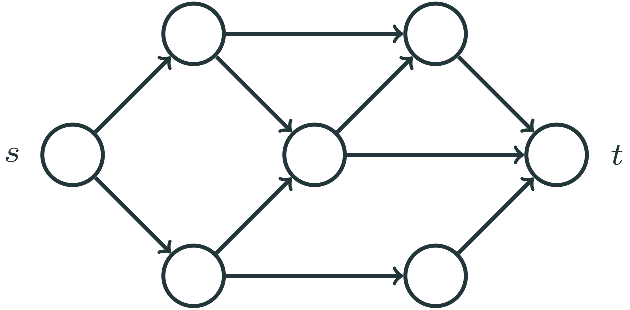

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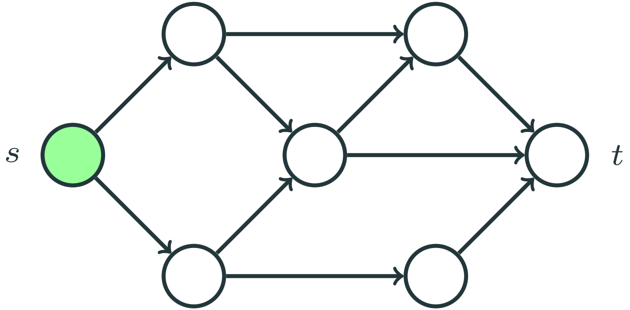

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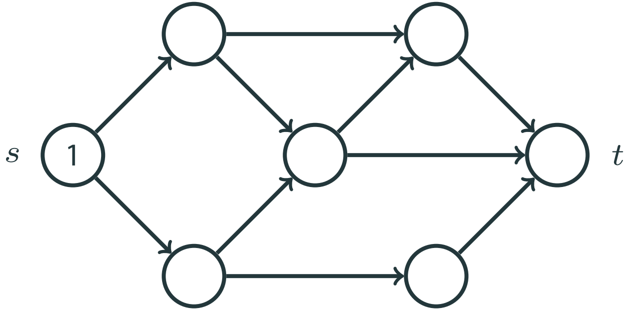

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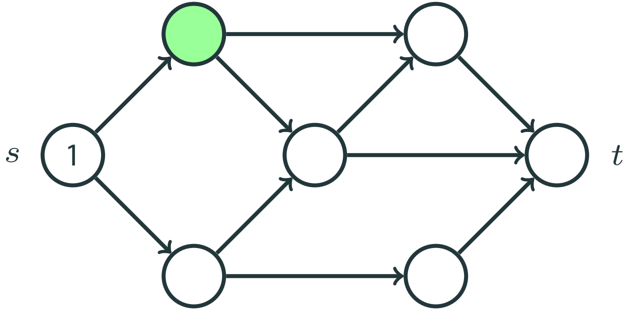

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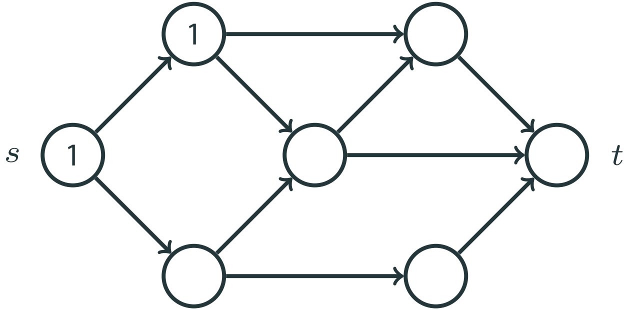

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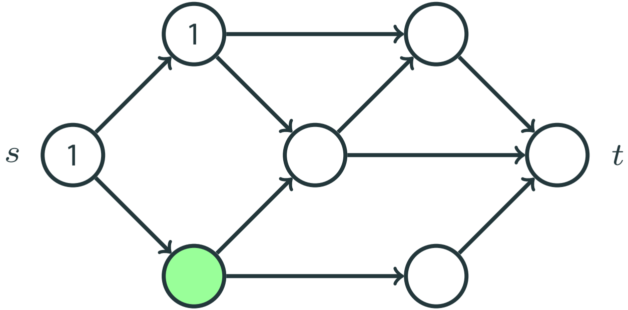

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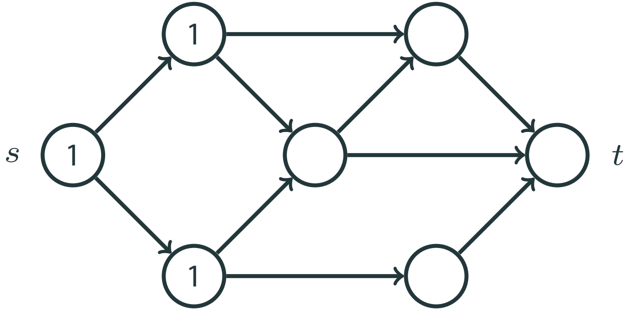

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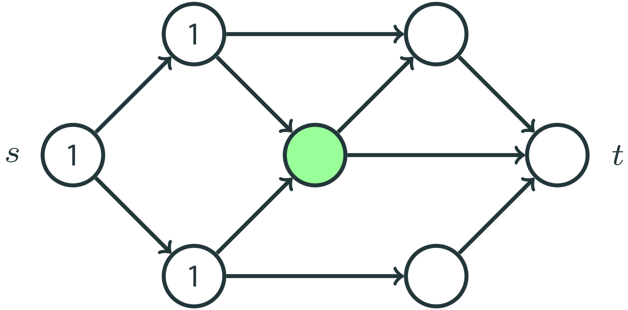

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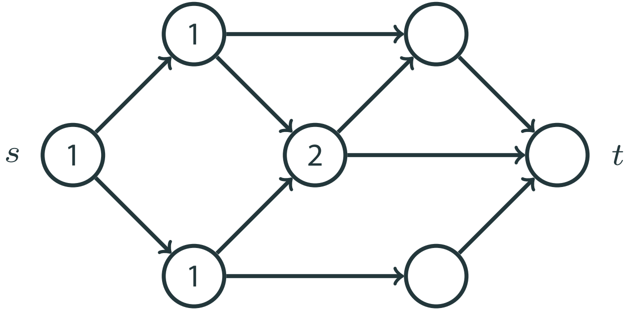

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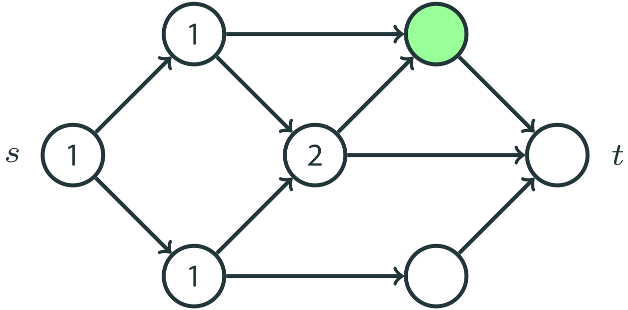

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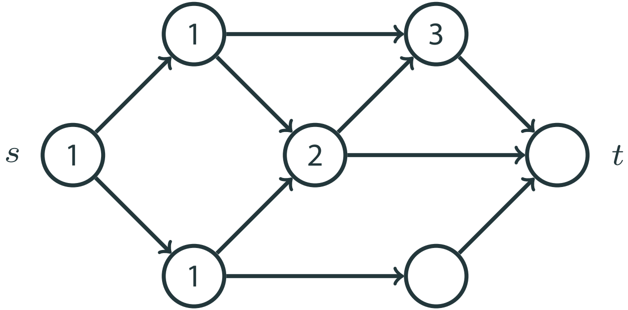

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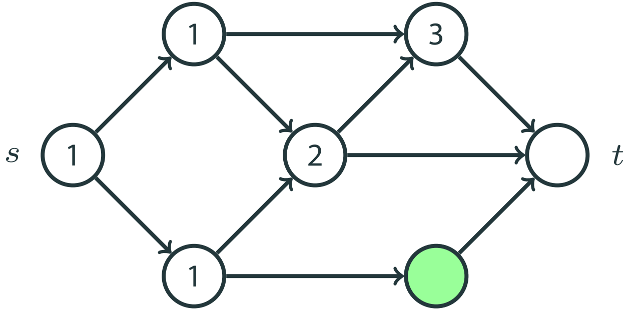

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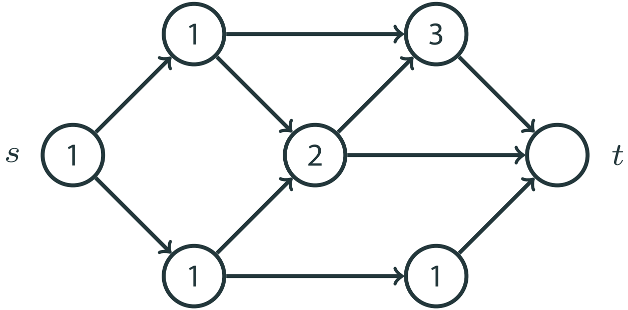

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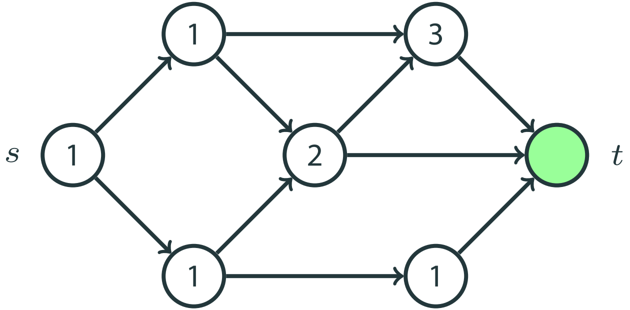

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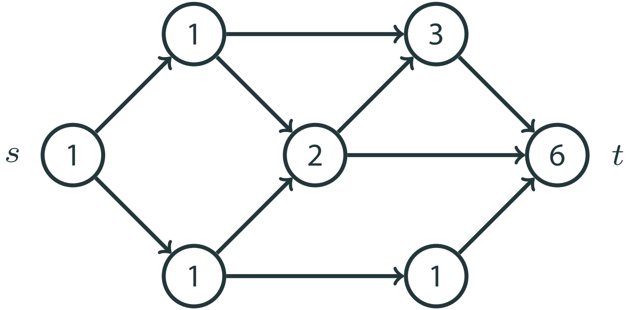

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If there are k object of the first type and there are n object of the second type, then there are $k \times n$ pairs of objects, the first of the first type and the second of the second type

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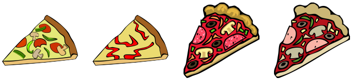
Pizza options



Soda options

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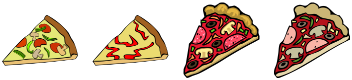
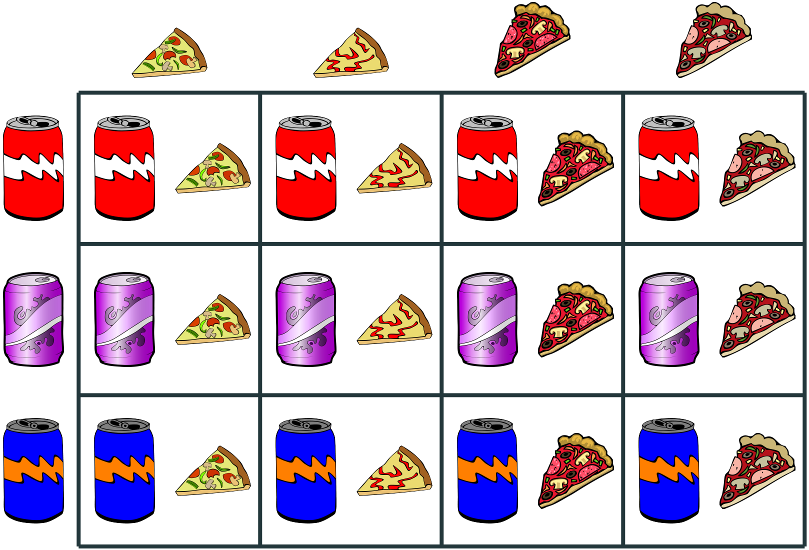


Soda options

$4 \times 3 = 12$ combo options

List of All Combo Options



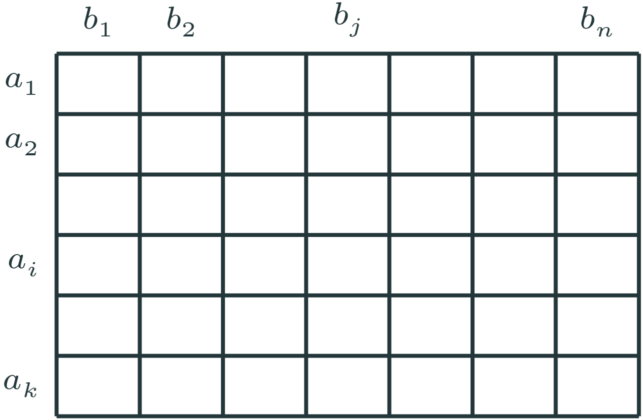
Rule of Product in the Set Language

Rule of Product

If there is a finite set A and a finite set B, then there are $|A| \times |B|$ pairs of objects, the first from A and the second from B

Why the Rule of Product is True?

$A = \{a_1, \dots, a_k\}$
$B = \{b_1, \dots, b_n\}$



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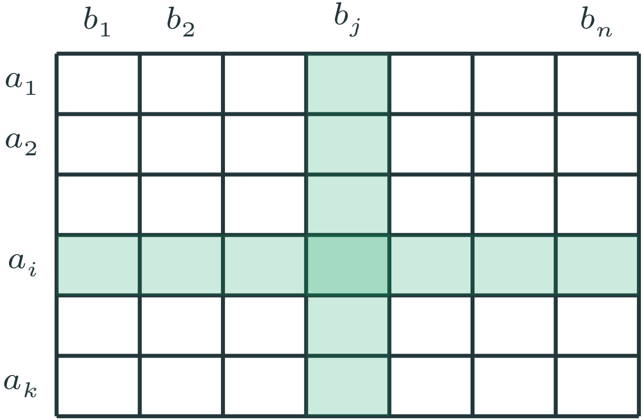

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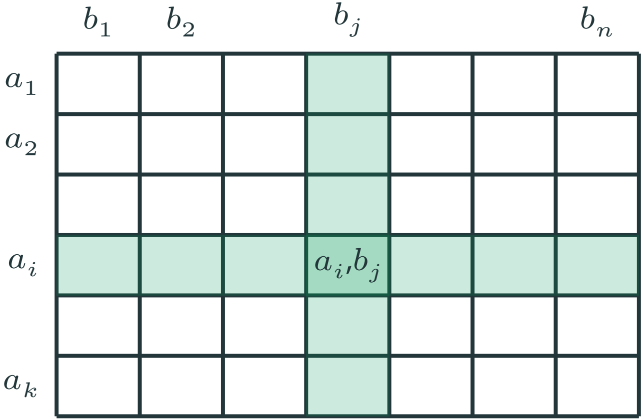

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	b_1	b_2		b_j			b_n
a_1							
a_2							
a_i				a_i, b_j			
a_k							

There are as many pairs as cells in this table

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Can we express this counting rule in terms of counting the number of paths?

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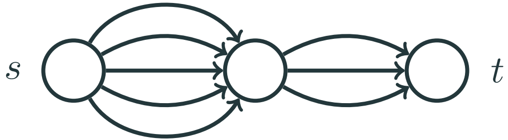

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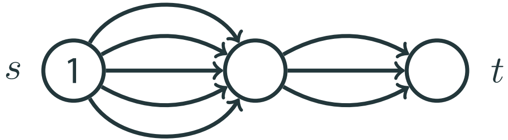

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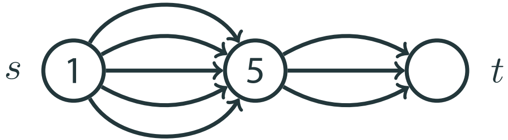

$$1 + 1 + 1 + 1 + 1 = 5$$

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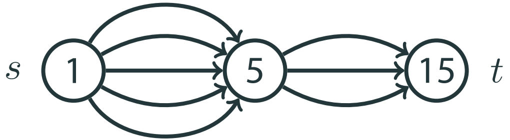

$$5 + 5 + 5 = 3 \times 5 = 15$$