Monty Hall Paradox

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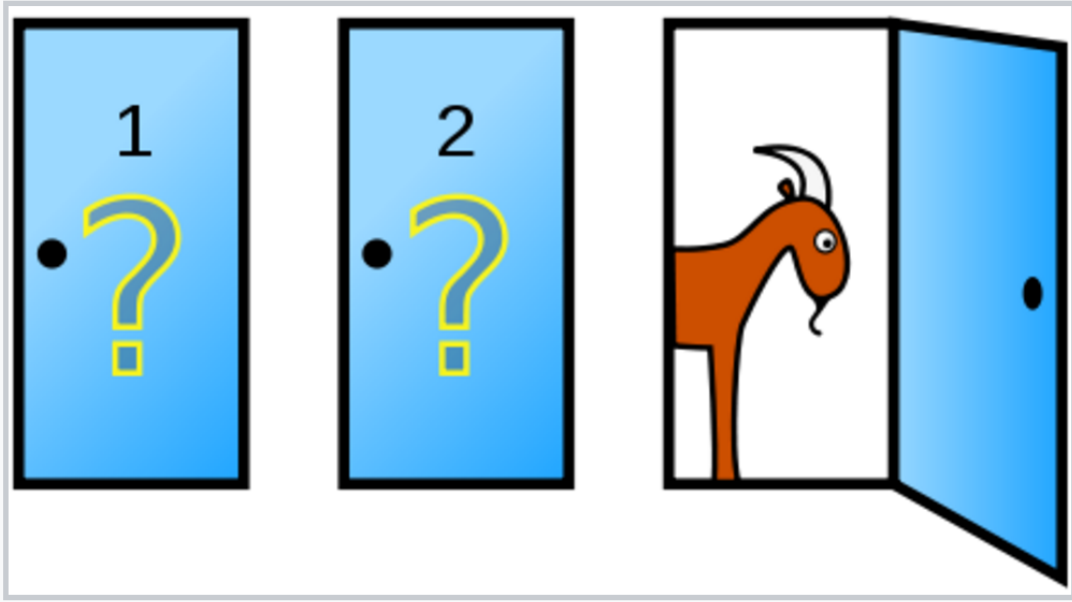
Outline

Three “convincing” argument

Our Position

Monty Hall Problem in Wikipedia



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- door opens and prize is given (if the final guess is correct)

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- why to make a new random guess among two doors: now there are two doors where the prize can be; we do not know where it is, so we can only make a random guess
- why to change the door: the first door has the prize with probability $1/3$. so the other one has it with better probability $2/3$

Which argument convinces you more?

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You do not need to *prove* something to influence the probabilities; the information may be indecisive, but still valuable.

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There are several wrong assumptions in this argument. In fact, if you want to guess the result of a random process with known probability, you should *not* imitate this process. If you have a coin that gives ‘head’ in 70% of cases, you should always bet on ‘head’.

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When we speak about the probability, we have in mind some experiment. Opening the empty door changes the experiment — why does not it change the probabilities?

Spoiler Follows: Stop Here!

We strongly encourage you to stop here before we explain the correct answer (or ‘our position on what is the correct answer’).

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- In our elaborated setting the third solution is correct: changing the door increases the chances of winning by factor 2

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- ‘choose random’ wins in $1/2$ cases

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- obviously in this setting the suggestion to change the guess indicates that it was correct: keep it!