Convert a given rotation matrix to axis-angle representation

Question

In this function, you need to convert the rotation matrix R into axis-angle form

- Your final solution for the vector in axis-angle representation of R must be stored in vec
- Your final solution for the angle in axis-angle representation of R must be stored in theta

A rotation matrix R can have two sets of solutions:

- (vec, theta) or (-vec, -theta). These solutions are equivalent.
- In this task, we will only test for theta in [0, pi], so you can ignore the other solution unless otherwise specified.

Singularities

You also need to handle two singularity cases, theta = 0 and theta = +pi.

- theta = 0 represents no rotation, and there are infinite solutions for this. So vec must be [NaN NaN NaN] in this case.
- theta = pi has two equivalent solutions. In this case, vec must be 2x3 matrix containing both solutions.
- For all other cases, vec will be a 1x3 matrix containing a single solution.

Input Format

- R will be a valid 3x3 rotation matrix with det(R) = 1

Output Format

- vec will be a 1x3 matrix containing a single solution if only one solution exists in [0, pi]
- vec will be a 2x3 matrix containing two solutions, if two solutions exist in [0, pi]
- vec will be a 1x3 matrix of the form [NaN, NaN, NaN] if infinite solutions exist in [0, pi]
- Every row in vec is of the form [x y z] if the vector is x i + y j + z k
- theta is always a 1x1 matrix, and in radians
- The final output is then to be stored in 'axang' as a 1x4 or 2x4 matrix where each row is of the form [x y z theta]

Code

- Write your code in the space provided within the function, as per the comments.

Helpful Notes

- The general form of the equation is:

$$\theta = acos(\frac{trace(R) - 1}{2})$$

$$\vec{r} = \frac{1}{2sin\theta}[(r_{32} - r_{23})\hat{i} + (r_{13} - r_{31})\hat{j} + (r_{21} - r_{12})\hat{k}]$$

- However, it can be seen that the above equation for r is not valid when sin(theta) = 0
- These are the singularity cases at theta = 0, theta = pi, theta = -pi which need to be handled separately
- When theta = 0, we have infinite solutions, as it represents no rotation. So r can be any arbitrary vector

$$
R_{\vec{r},\theta} =
[r_x^2 v_\theta + c_\theta, r_x r_y v_\theta - r_z s_\theta, r_x r_z v_\theta + r_y s_\theta
r_x r_y v_\theta + r_z s_\theta, r_y^2 v_\theta + c_\theta, r_y r_z v_\theta - r_x s_\theta
r_x r_z v_\theta - r_y s_\theta, r_y r_z v_\theta + r_x s_\theta, r_z^2 v_\theta + c_\theta]
$$

where $c_\theta = cos(\theta)$, $s_\theta = sin(\theta)$, $v_\theta = 1 - cos(\theta)$