Additional Reading

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Basic Rotations:

You can use the below formulae to get a rotation matrix that represents a rotation of θ along either the x, y, or z-axis.

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composition of Rotations:

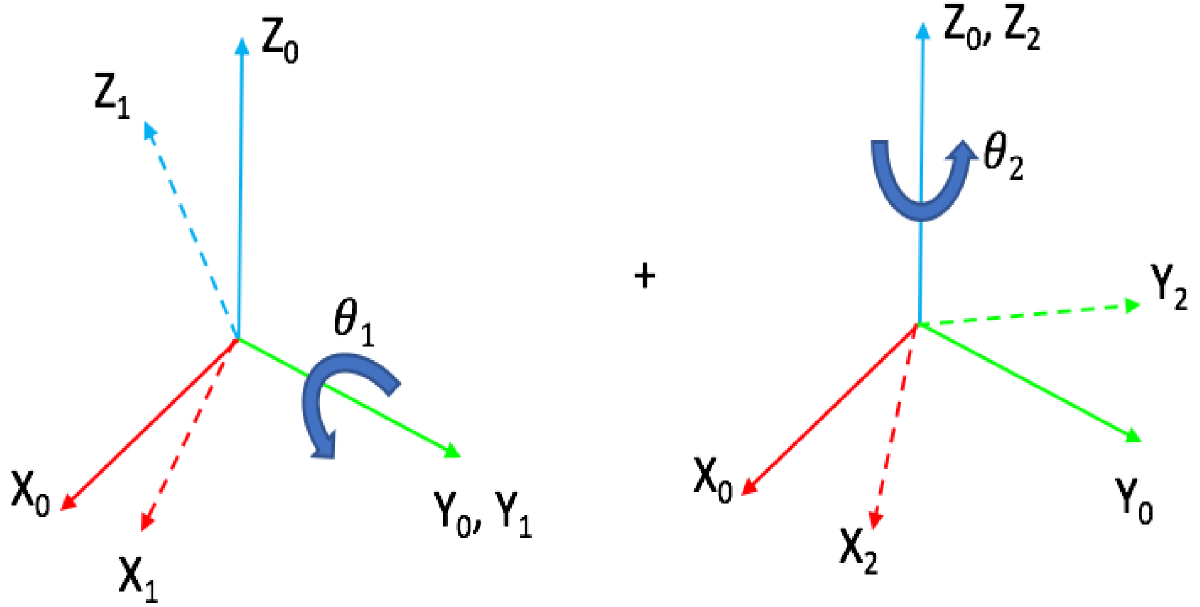
Suppose we have two different rotations, and we want to find the effective combined rotation, R, which results from the two rotations. This can be computed based on how the two rotations occurred.

- Rotating around a fixed frame

  The figure below shows two rotations. The first rotation is by θ_1 around Y_0 and the second rotation is by θ_2 around Z_0. As you can see, both these rotations happen with respect to the same fixed frame, frame 0.

Composition of rotations in the fixed frame is done by pre-multiplying each successive rotation about the fixed frame. So in this case, the effective rotation is given by

$$R = R_{z_0,\theta_2} R_{y_0,\theta_1}$$

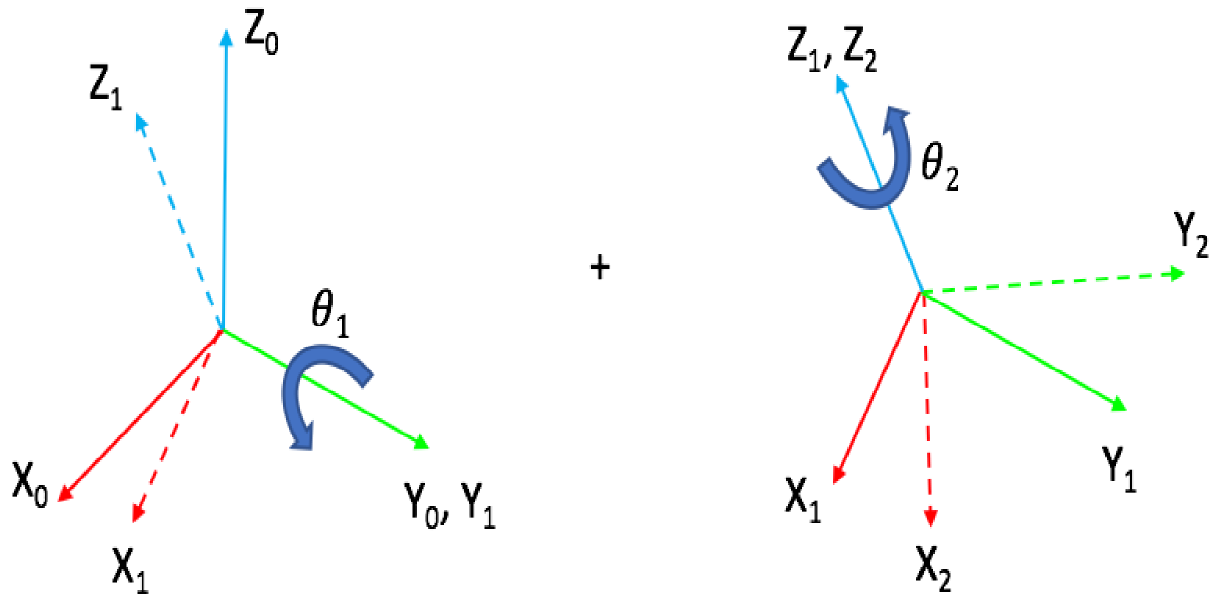


- Rotating around an intermediate frame

The figure below shows two rotations. The first rotation is by θ_1 around Y_0 and the second rotation is by θ_2 around Z_1. Here, the first rotation happens about frame 0, and the second rotation happens about frame 1, which is the intermediate or current frame obtained after the first rotation.

Composition of rotations in the intermediate frame is done by post-multiplying each successive rotation about the intermediate frames. So in this case, the effective rotation is given by

$$R = R_{y_0,\theta_1} R_{z_1,\theta_2}$$



Composition of homogeneous transformations also follow the same rules for pre-multiplication and post-multiplication based on whether the transformations are about fixed frames or intermediate frames.

Representing a Rotation:

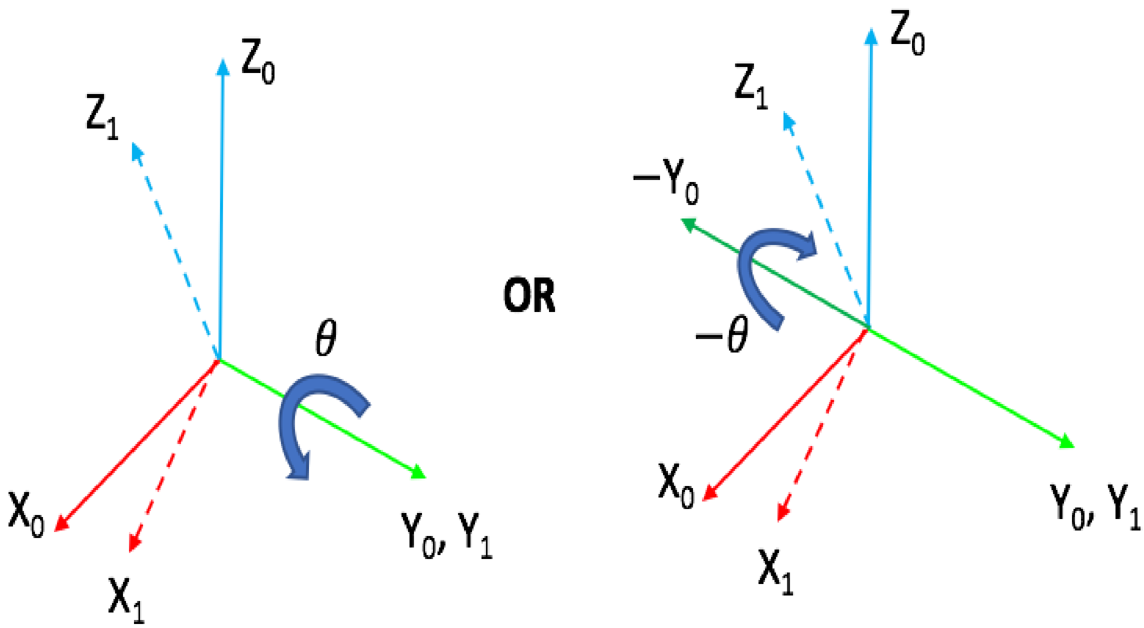
In this project, we explore multiple ways to represent rotations, and how we can convert from one form to another. Below we take a look at these representations by considering a simple example of rotation of θ along the y axis.

- The first way is to use a rotation matrix. As noted in the 'Basic Rotations' section above, the rotation matrix for this scenario would be:

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

- Second way of representing this rotation is using axis-angle notation. So the same rotation can be thought of as rotating by an angle of θ around the axis $\hat{w} = 0\hat{\imath} + 1\hat{\jmath} + 0\hat{k}$. Note that this is also equivalent to rotating by an angle of $-\theta$ around the axis $\hat{w} = 0\hat{\imath} - 1\hat{\jmath} + 0\hat{k}$.

This is true whenever we convert a rotation represented by a rotation matrix to axis-angle form. There will be two equivalent solutions. A rotation of θ around the axis $\hat{w}$ or a rotation of $-\theta$ around the axis $-\hat{w}$.



- The third way of representing the same rotation is using quaternions. The quaternion form of a rotation is

$$q = \cos\frac{\theta}{2} + \hat{w}_x \sin\frac{\theta}{2}\hat{i} + \hat{w}_y \sin\frac{\theta}{2}\hat{j} + \hat{w}_z \sin\frac{\theta}{2}\hat{k}$$

OR

$$q = [\cos\frac{\theta}{2}, \hat{w}\sin\frac{\theta}{2}]$$

where θ and $\hat{w}$ are as defined in axis-angle representation above. Again, note that q and -q represent the same rotation. To see how this is true, replace $\hat{w}$ by $-\hat{w}$ and θ by $2\pi - \theta$ (which is equivalent to $-\theta$). This gives -q. We know that a rotation of θ around the axis $\hat{w}$ or a rotation of $-\theta$ around the axis $-\hat{w}$ are equivalent, therefore q and -q must also be equivalent.