

## Building Towers (Green Worksheet)

10 points possible

+4/10

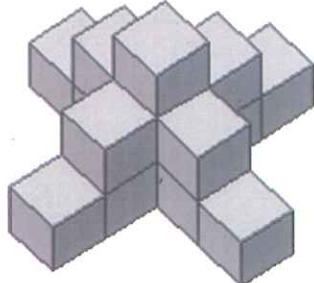
Name: Megan LaCalameto

Date: \_\_\_\_\_

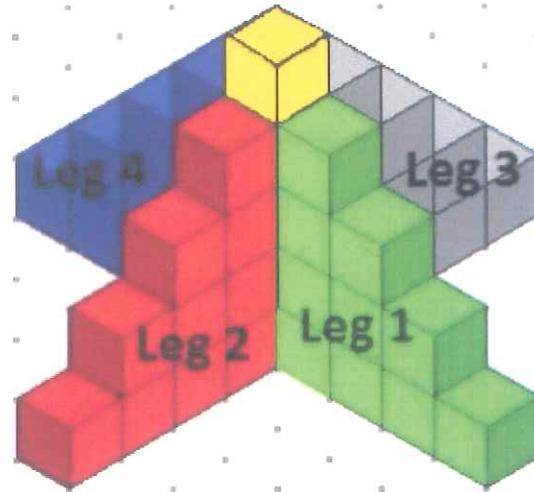
**Directions:** Consider the cube towers illustrated below, then answer each of the 4 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



Height 1



Height 3



Height 5

1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

1. The building is increasing by odd numbers.

2. Each number in the center column is increasing by 2 each time.

$$1 + 2 = 3 \quad 3 + 2 = 5$$

OK, but what about # of cubes? Is there a pattern there?

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) (0.5 points) How many cubes are in the **center column**?

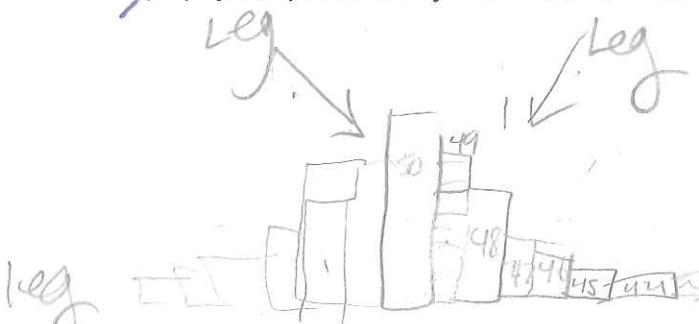
50 cubes

b) (1 point) How many cubes are in **one leg**? Show your work.

If the height of 5 has 10 cubes in one leg then 50 should have 100 cubes in one leg.

$$5 \cdot 2 = 10 \quad 50 \cdot 2 = 100$$

c) (1 point) How many **total cubes** are in the tower? Show your work.



how can  
there only be 100  
cubes in each  
leg, but 4900  
total?

$$1225 \cdot 4 =$$

$$1 \quad (4,900)$$

+50

what  
about the  
center  
column?

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c). Write them below.

1. Adding every number down from 50 and then multiplying by 4. remember to add middle column!

$$2. 49 \cdot 50 = 2,450 (2) = 4,900$$

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height n**, where n can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. Show all work.

$$4+3+2+1=10(4)=40+5=45$$

What about a general formula?

One that we can plug 300 in for n and find the number of cubes?

## Building Towers (Green Worksheet)

10 points possible

8/10

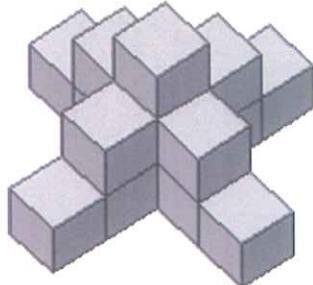
Name: Taylor Wilkinson

Date: 3-5-14

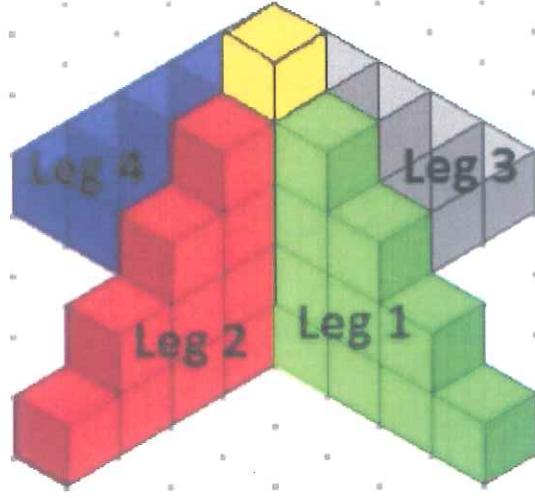
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Height 1



Height 3



Height 5

1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	$0+1$	$1+5=6$
2	2	$1+2$	$4+9=13$
3	3	$3+2$	$15+13=28$
4	4	$6+3$	$38+13=51$
5	5	$10+4$	$45+17=62$

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

• As the height of the tower increases by 1, the total number of cubes increases by 4 from the previous amount.

A little confusing...  
The difference in cubes increases by 4

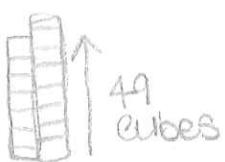
• The Number of cubes in one leg increases by 1 each time from the previous amount.

Again, the difference in cubes from one height to the next increases by one.

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) **(0.5 points)** How many cubes are in the **center column?** 50

b) **(1 point)** How many cubes are in **one leg?** Show your work.



$$1,225 \quad \text{Did you really add all those numbers?} \\ (49 + 48 + 47 + 46 \dots)$$

c) **(1 point)** How many **total cubes** are in the tower? Show your work.

$$1,225 \cdot 4 + 50 = 4,950$$

3. **(0.5 points)** With your group, describe **two different methods** to find the cubes in problem 2(c). Write them below.

• Multiply the number of cubes in one leg by four + add the number of cubes in the center column.  
good!

1. **(5 points - see rubric)** Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

$$t = 4l + n$$

$$45 = 4(10) + 5 = 45$$

$l = \text{number of cubes in one leg}$  } how would you find  $l$ ?

$t = \text{total number of cubes}$

We want 1 variable "n"!  
Can  $l$  be written in terms of "n"?

$n = \text{height of tower}$

Building Towers (Green Worksheet)  
10 points possible

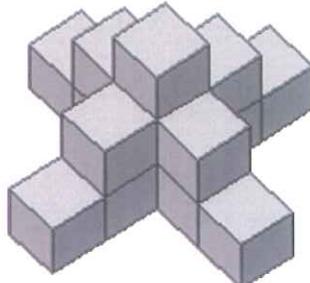
3.5/10

Name: Chris Compton  
Date: \_\_\_\_\_

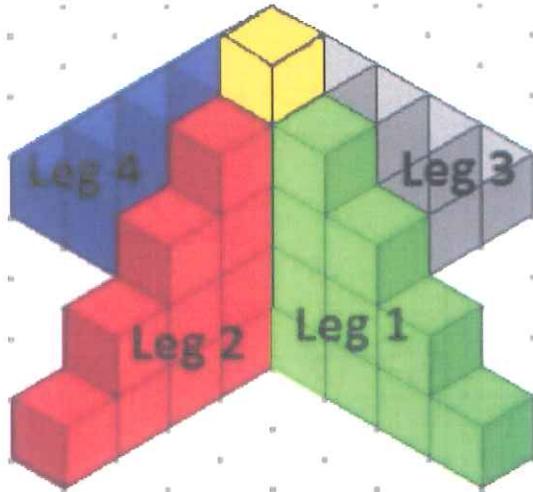
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Height 1



Height 3



Height 5

1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

- The total number of cubes goes up by 4 then multiplied by the height of the tower
- The number of cubes in center column is multipled by continual numbers (0, 1, 2, 3, ...) starting with 0.  
OK, so #cubes in center column multiplied by (0, 1, 2, ...) to get #cubes in one leg.  
Awesome observation!

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) (0.5 points) How many cubes are in the **center column**?  $50$

b) (1 point) How many cubes are in **one leg**? Show your work.

$$1225 = \frac{(50)(49)}{2}$$

$100$

Where did you  
get 100?

c) (1 point) How many **total cubes** are in the tower? Show your work.

$50 \cdot 5 = 250$  total cubes

$$4(\# \text{ of cubes in one leg}) + (\# \text{ cubes in center column})$$

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c). Write them below.

multiply height of tower by 5  
why?

What is another method? Could it be  
similar to a method used on the  
front to fill out the table?

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

-A

$$n+4(n) = \text{Total number of cubes}$$

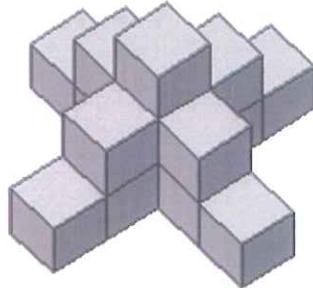
HS

what does  $n$  stand for?  
where did 45 come from?  
does your equation work  
with  $n=1$ ?  $n=3$ ?

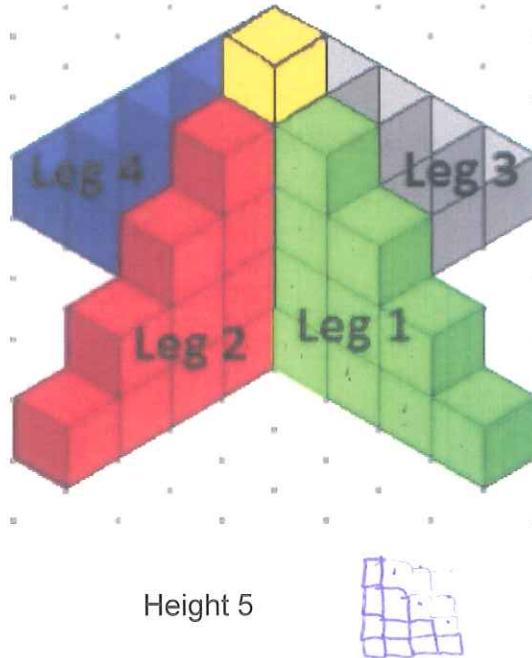
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Height 1



Height 3



Height 5



1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

① the higher the tower the higher (more) the number of cubes on one leg, ② The number of center column cubes & the height of the tower is the same.  
Is this important to know?

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) (0.5 points) How many cubes are in the **center column**? **50**

b) (1 point) How many cubes are in **one leg**? Show your work.

good!

$$50 + 49 + 48 + 47 + 46 + 45 + 44 + 43 + 42 + 41 + 40 + 39 + 38 + 37 + 36 + 35 + 34 + 33 \\ + 32 + 31 + 30 + 29 + 28 + 27 + 26 + 25 + 24 + 23 + 22 + 21 + 20 + 19 + 18 + 17 + 16 + 15 + 14 + 13 + 12 \\ + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \boxed{1225}$$

c) (1 point) How many **total cubes** are in the tower? Show your work.

$$1225 \times 4 = 4900 + 50 = \boxed{4950 \text{ cubes}}$$

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c).

Write them below.

①  $\begin{matrix} 1225 \\ | \\ 1 \text{ leg} \end{matrix} \times 4 + \begin{matrix} 50 \\ | \\ \text{cubes in center column} \end{matrix} = \begin{matrix} 4950 \\ | \\ \text{total} \end{matrix}$

② COUNT

↓ wasn't adding 40 to 1 like counting?  
Is there an easier way to "count"

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height n**, where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

-2

$$L \times 4 + \textcircled{n} = T$$

$$10 \times 4 + 5 = \boxed{45} \quad \checkmark$$

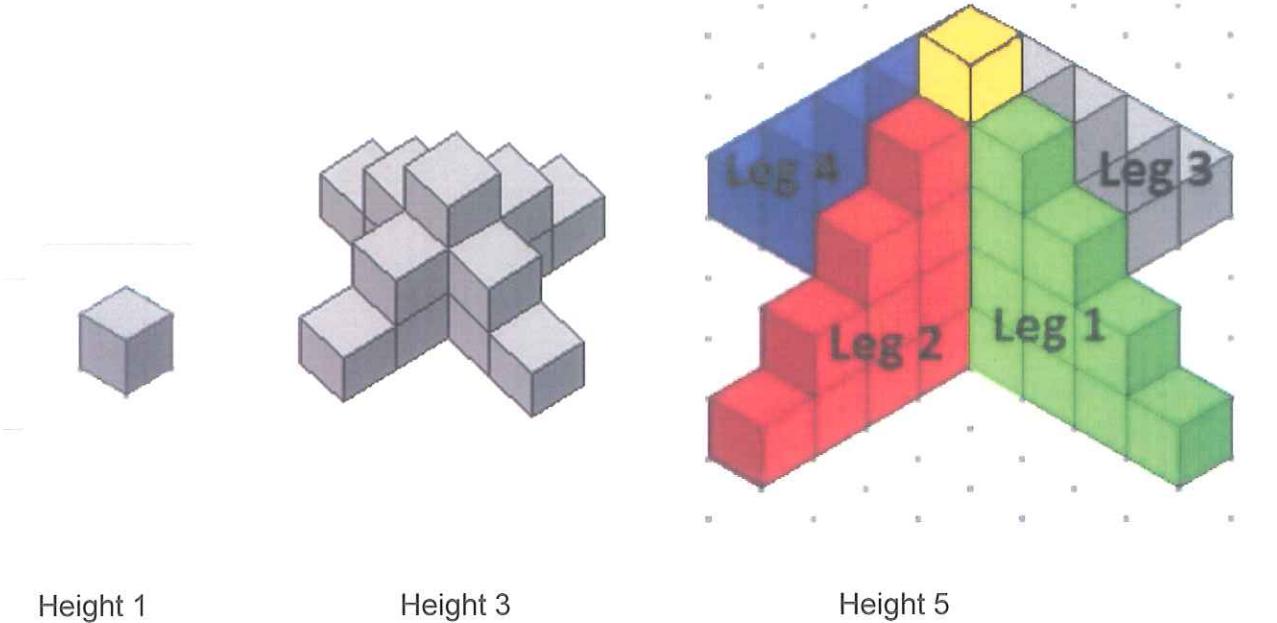
$L = \text{leg cubes}$   
 ~~$T = \text{total cubes}$~~

how would  
you find L?

We just want  
one variable, n!

~~$$\begin{aligned} 49 \cdot 49 \div 2 &= 1200.5 \times 4 = \\ 4802 + 50 &= \end{aligned}$$~~

**Directions:** Consider the cube towers illustrated below, then answer each of the 4 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

- 1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

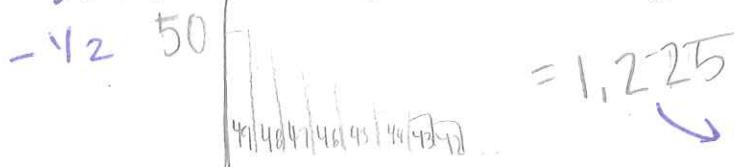
15  
3  
45

The number of blocks is tripled, and the number of cubes in the center column was the same as the height of the tower. good!  
do you think these are important patterns?

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) (0.5 points) How many cubes are in the **center column**? **50**

b) (1 point) How many cubes are in **one leg**? Show your work.



→ where did this number come from?

c) (1 point) How many **total cubes** are in the tower? Show your work.

$$\begin{array}{r} 1225 \\ \times 4 \\ \hline \end{array}$$

$$4,900 \text{ Total cubes}$$

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c). Write them below.

1. adding every number down from 50, and then multiplying that by 4.

2.  $49 \cdot 50 = 2,450$  (2) = 4,900

How did you know that this would work?

+50 (for center column)

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of **height 5**. Show all work.

$$(4+3+2+1) = 10(4) = 40 + 5$$

Could you use this method to write a formula?

45

What about the generic formula?

## Building Towers (Green Worksheet)

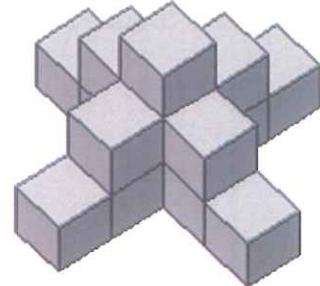
10 points possible

+750/10  
+8

Name: Clove Hess

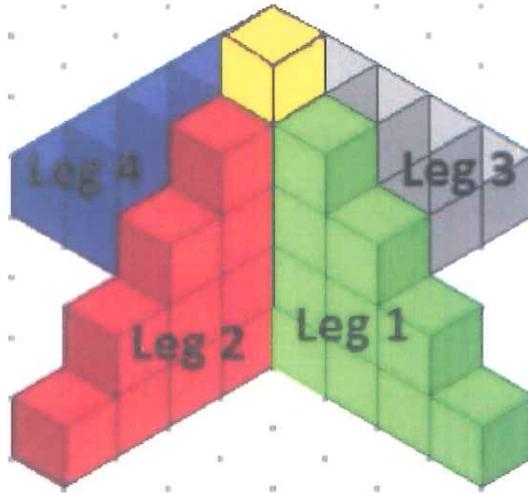
Date:

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Height 1

Height 3



Height 5

1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

The # of cubes in the center column increases by 2. The height of the tower is equal to the # of cubes in the center column.

technically correct based on table but the height towers given are random

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) **(0.5 points)** How many cubes are in the **center column?** 50

b) **(1 point)** How many cubes are in **one leg?** Show your work.



- .5

$$49 + 48 + 47 + 46, \dots = 1225$$

50 cubes

100 cubes

sum of

how did you add the first 49 integers?



c) **(1 point)** How many **total cubes** are in the tower? Show your work.



100 cubes

400 cubes

$$1225 \times 4 = 4900 \quad 450 \text{ total cubes}$$

$$4900 + 50 = 4950 \text{ total cubes}$$

3. **(0.5 points)** With your group, describe **two different methods** to find the cubes in problem 2(c). Write them below.

add all ~~#~~  $(49 + 48 + 47, \dots)$ , then multiply it by 4 because it's the # of legs then add the height.

- .25

4. **(5 points - see rubric)** Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

$p = \text{previous numbers}$  ?

$$(4+3+2+1) \cdot 4 + 5$$

$$10 \cdot 4 + 5$$

40 + 5

45

3.75

What is previous number,  $p$ ?  
we just want one variable  $n$ !  
Is there a relationship between the height and the # of cubes in a leg?

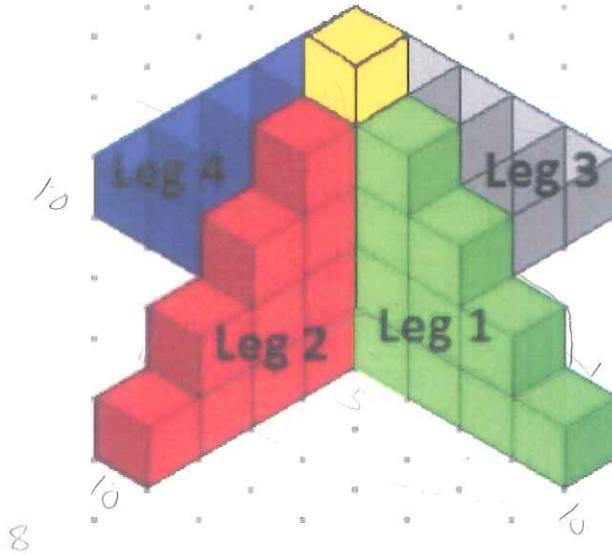
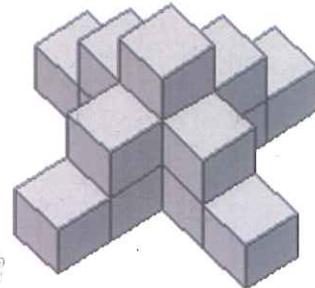
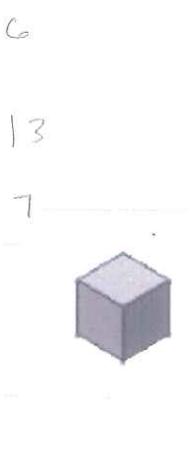
**Building Towers (Green Worksheet)**  
10 points possible

+3/10

Name: Savannah Arno  
Date: 3/5/14

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- H1) 1  
H2) 6  
H3) 15  
H4) 28  
H5) 45



Height 1

Height 2

Height 3

Height 4  
(4 tall)

Height 5

$$4+3+2+1=10$$

$$10 \times 4 = 40 + 5 = 45$$

1. (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3	15
5	5	10	45

2. (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

Pattern #1: Duplicate the model and then add 1 to each of the legs.  
Ex) Height 1 → You would duplicate it  (2 blocks tall) and then add one to each of the legs  which totals 6, the number of blocks in Height 2. And same with Height 2 to Height 3,  you start with 6, duplicate  and then add one to each of the legs  and then add two so it would look like:  and get a total of 15.  
The rule for this pattern is duplicate the center column then add one (the number of the height) and then add an increasing number to each leg as shown in my example.

Pattern #2: Do the Height (Ex: Height 5 - 1 = 4) and do the number plus each decreasing number (Ex: 4 + 3 + 2 + 1 = 10) →

Pattern 2 (cont.): the third step is that you multiply by the number of legs (Ex: 4) and then add the height (Ex: Height 5) I used the Height 5 model and did  $4+3+2+1=10$

2. Consider a tower of height 50 cubes. Answer the questions (a) - (c) below.

then mult. by the legs(4)  
and then added the height(5)  
totaling in the correct answer of 45.

a) (0.5 points) How many cubes are in the **center column**?

50 cubes are in the center column

b) (1 point) How many cubes are in **one leg**? Show your work.

1225 cubes are in each leg. The way

I found this is by doing 1225 mult. by 4 (for each leg) and add the height(50) = 4950 which is the total

$$1225 \times 4 = 4900 + 50$$

$$4950$$

c) (1 point) How many **total cubes** are in the tower? Show your work.

$$49+48+47+46+\text{etc.} = 1225 \times 4 = 4900 + 50 = 4950$$

Does this go in (c)? how do you know what this is?

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c).

Write them below.

Method 1:  $49+48+47+46+45+44+43+42+41+40+39+38+37+36+35+34+33+32+31+$   
 $30+29+28+27+26+25+24+23+22+21+20+19+18+17+16+15+14+13+12+11+$   
 $10+9+8+7+6+5+4+3+2+1 = 1225 \times 4 = 4900 + 50 = 4950$

OK. Subtract 1 from the height 50 - 1 = 49 and add all of the numbers together then mult. by the number of legs (4) and then add the height (+50)

Method 2: Duplicate the model previously and add the previous number to each one of the legs.

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

$P = \text{previous number}$  (4)  $P+3+2+1 = 10 \times 4l = 40+n = 45$

$l = \text{legs}$

$n = \text{height}$

$$P = 4 + n$$

where do you get this "previous number"?  
what is previous number?  
is it related to  $n$ ?

Building Towers (Green Worksheet)  
10 points possible

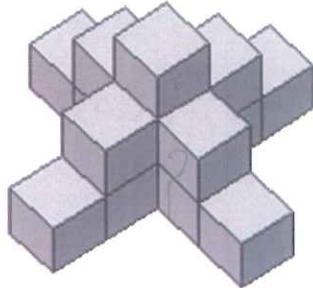
6.5/10

Name: Brent Haugen  
Date: 3.5.14

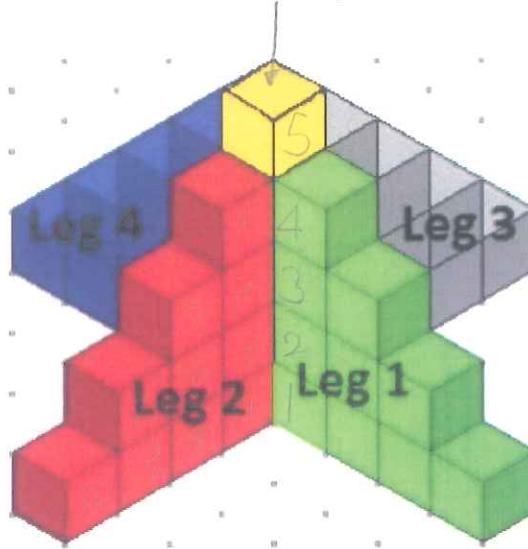
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Height 1



Height 3



Height 5

- (1 point) Use the figures above and the table below to help you determine how many cubes are needed to build these and similar figures.

Height of Tower	Number of Cubes in Center Column	Number of cubes in one leg	Total number of Cubes
1	1	0	1
3	3	3 $3 \times 3 = 15$	15
5	5	10 $10 \times 4 = 40 - 15 = 45$	45

- (1 point) List at least two patterns that you see from the images and/or the table above (0.5 pts. for each pattern).

Stair case... each level is one block higher. the closer it gets to the center which is always the highest point.

The longer the Picture gets - the longer it becomes!  
Confusing!

2. Consider a tower of **height 50 cubes**. Answer the questions (a) - (c) below.

a) (0.5 points) How many cubes are in the **center column**? 50

b) (1 point) How many cubes are in **one leg**? Show your work. 1,275

$$\underline{1,225}$$

i think you  
added the center  
column of 50 too  
soon!

c) (1 point) How many **total cubes** are in the tower? Show your work.

$$-12 \quad (1,225 \cdot 4) + 50 \quad \text{Right idea, wrong numbers!}$$

$$5,100 + 50$$

$$5,150 \text{ in the tower}$$

3. (0.5 points) With your group, describe **two different methods** to find the cubes in problem 2(c).

Write them below.

$n$  = Number of cubes in one leg

$w$  = Number of cubes in center column

In order to find 1,275 you could have done  $50+49+48+47+46\dots$

Why should we include 50?

4. (5 points - see rubric) Use the information you found above to create an equation (function) to calculate the total number of cubes in a tower with **height  $n$** , where  $n$  can be any positive integer. Use your function to calculate the number of cubes in a tower of height 5. **Show all work.**

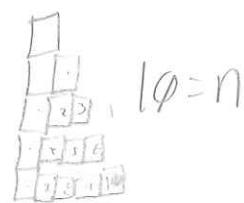
$$nA+w$$

$$nA+5$$

$$10 \cdot 4 + 5$$

$$45$$

Is there a formula for "n"?  
Does your formula work for  $n=1$ ?

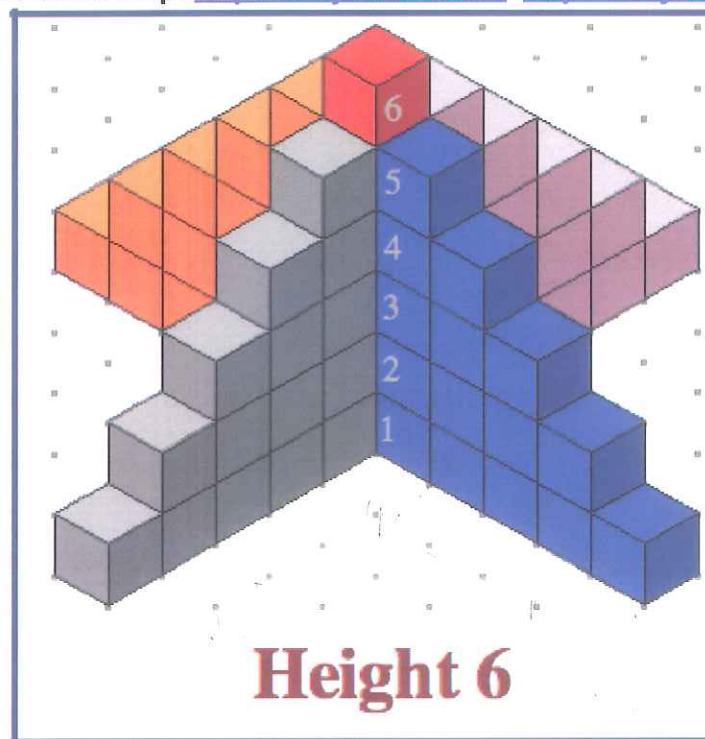


Building Towers (Blue Worksheet)  
10 points possible

95/10

Name: Stephen Quay  
Date:

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$6 \times 4 + 3 + 2 + 1 = 36$$

$$36 + 4 + 3 + 3 + 2 + 2 + 1 + 1 = 56$$

1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$\begin{array}{r} 15 \times 4 = 60 \\ \downarrow \\ 60 + 6 = 66 \end{array}$$

*What is this?  
Explain !!*

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$\begin{array}{r} 6 \times 4 = 24 \\ + 4 \\ \hline 28 \end{array}$$



~~$$4 \times 3 + 3 + 2 + 2 + 1 + 1 = 16$$~~

~~$$16 + 3 + 2 + 2 + 2 + 1 + 1 + 1 + 1 = 28$$~~

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\left( \frac{150(150-1)}{2} \right) (4) + 150 = x$$

$$1125(4) + 150 = 44850$$

$$149,148 \dots$$

$$\left( \frac{n(n-1)}{2} \right) \cdot 4 + h = x$$

*does h = n?*

E  
good!

4. (1 point) With your group, describe **two different methods** to find the cubes in problem 3. Write them below.

1) add up all the numbers then multiply them by 4  
 then add the original height of the object to it

2)  $\left(\frac{n(n-1)}{2}\right)(4) + h = x$

Formulae

How did you get this fancy formula?

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$$\left(\frac{n(n-1)}{2}\right)(4) + n = x$$

Check your answer ☺

$n$  = # of cubes high,  $n-1$  = the top height of the wings/sides.  
 then divide it by 2 for the upper missing half of the sides then multiply by 4 (in this case), then add the original height for the row in the middle

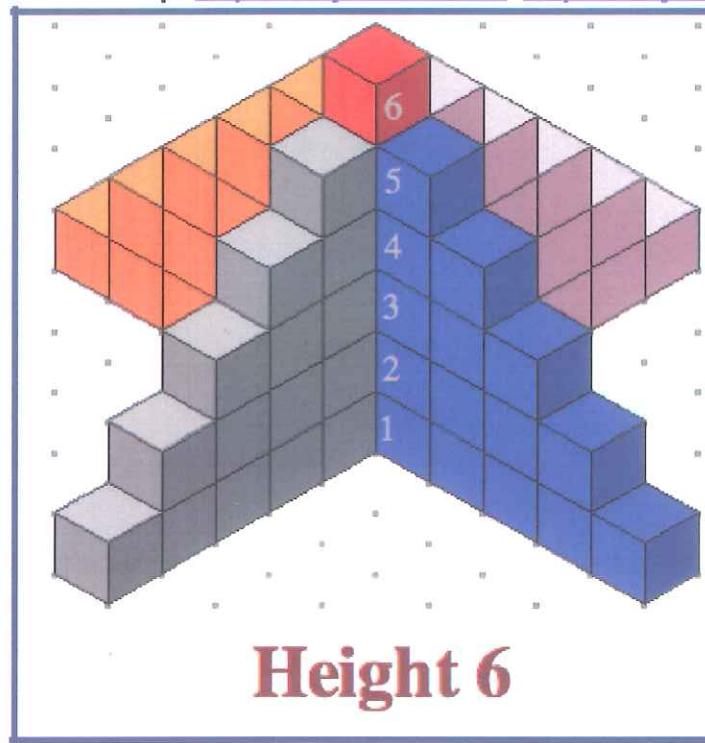
**Building Towers (Blue Worksheet)**  
10 points possible

9.5/10

Name: Buzz Berleman

Date:

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$15 \times 4 = 60 + 6$$

↑  
Outside blocks  
6 red  
blocks in  
the middle

66 cubes needed

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$6 \times 4 = 24 + 4 = 28$$

How did you get 24?

28 cubes needed

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$x = 4 \left( \frac{n(n-1)}{2} \right) + 150$$

what does this mean?  
 $n = 150$

$$x = 4 \left( \frac{150(150-1)}{2} \right) + 150$$

$$4 \left( \frac{150(150-1)}{2} \right) + 150 = x$$

$$44700 + 150 = x$$

$$44850 = x$$

good!

44850 cubes needed

4. (1 point) With your group, describe **two different methods** to find the cubes in problem 3. Write them below.

1<sup>st</sup> method

$$\text{a formula: } x = 4\left(\frac{h(h-1)}{2}\right) + h$$

2<sup>nd</sup> method

ha! brute force, count the blocks on a leg, times it by 4 and add the middle column.

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

method 1

-1/2  
awesome!

a formula: You times the height of the tower ( $n$ ) to the height of it minus one. Then you divide it by two and times the whole thing by four. Finally, you then just add the height ( $n$ ) to that to get your answer.

$$x = 4\left(\frac{n(n-1)}{2}\right) + n$$

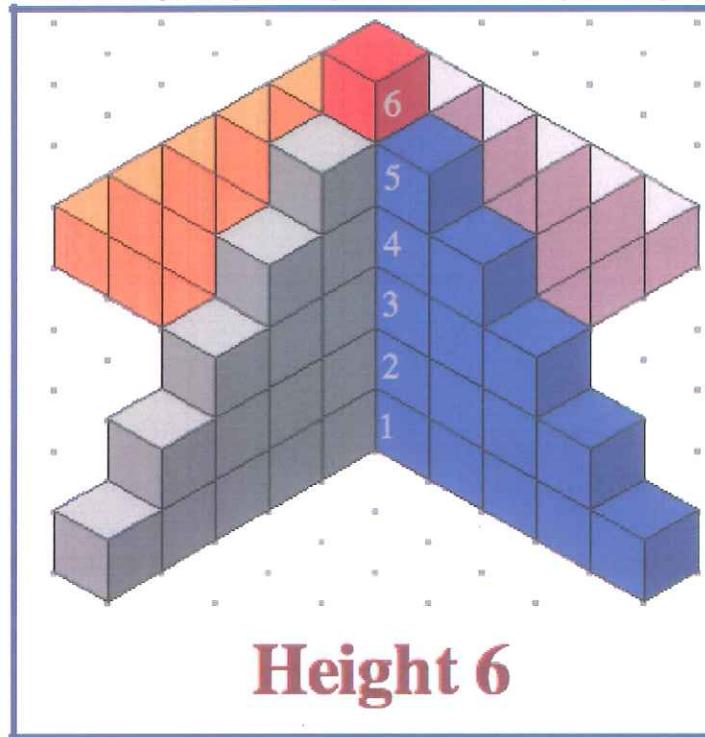
Check answer with example from front.  
we want to make sure it works ☺

Building Towers (Blue Worksheet)  
10 points possible

9.5/10

Name: Tanya C.  
Date: \_\_\_\_\_

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{array}{r} 10 \times 9 \\ - 5 \\ \hline 95 \end{array}$$

$$\begin{array}{r} 1 \times 1 \\ 2 \times 3 \\ 3 \times 5 \\ 4 \times 7 \\ 5 \times 9 \\ 6 \times 11 \end{array}$$

1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

Explain where  
your numbers  
come from

$$\begin{array}{r} 15 \times 4 = 60 \\ + 6 \\ \hline 66 \end{array}$$

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$\begin{array}{r} 6 \times 4 = 24 \\ + 4 \\ \hline 28 \end{array}$$



3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\left( \frac{(n)(n-1)}{2} \right) \cdot 4 + n = X$$

Show more  
work!

$$X = 44,850$$

4. (1 point) With your group, describe **two different methods** to find the cubes in problem 3. Write them below.

1) add all numbers together multiply by 4  
add height number

$$2) \left( \frac{(n)(n-1)}{2} \right) \cdot 4 + n = X$$

What does this  
mean in terms  
of the tower?

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$$\left( \frac{(n)(n-1)}{2} \right) \cdot 4 + n = X$$

$n$  represents number of cubes high.  $n-1$  is for top height of the wings. I divided it by two for the missing top half of wings, then multiplied by 4 for the wings. Lastly I added by the height number again for the middle row. good!  
(column)

Loved your explanation!

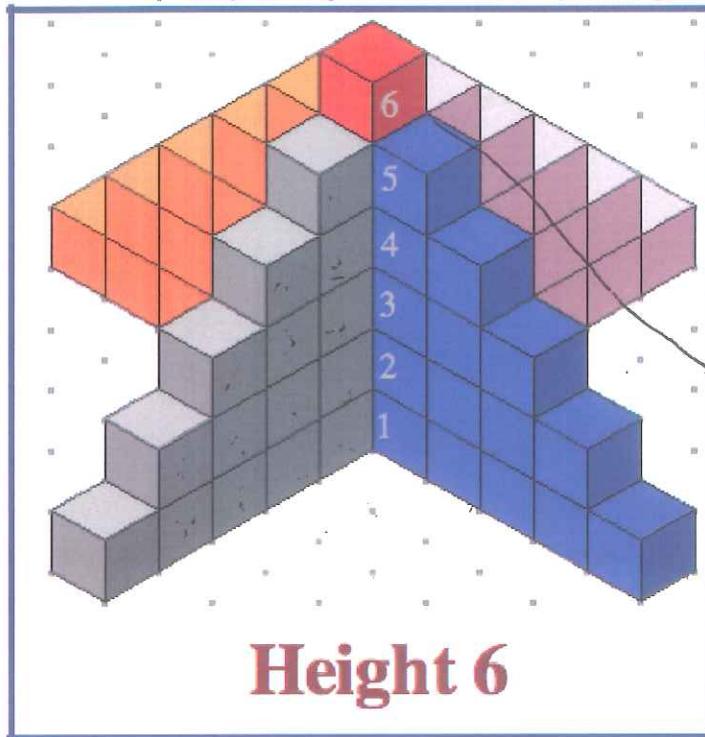
**Building Towers (Blue Worksheet)**  
10 points possible

3.5/10

Name: Paxton Robinson  
Date: 3-5-14

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

$$\begin{array}{r} 15 \\ \times 4 \\ \hline 60 \\ + 6 \\ \hline 66 \\ \text{---} \\ + 5 \end{array}$$



1. (1 point) How many cubes are needed to make the figure above? Show your work.

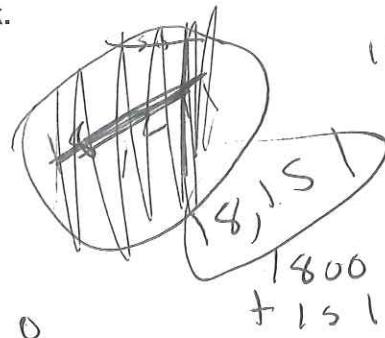
-12  
66

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? Show your work.

-18  
-1  
45

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. Show your work.

$$\begin{array}{r} 150 \\ \times 3 \\ \hline 450 \\ \times 4 \\ \hline 1600 \end{array}$$



11,354,600

44,850

+ 151

1951

11,355  
x 4  
45,420  
45,420

4. (1 point) With your group, describe two different methods to find the cubes in problem 3. Write them below.

$$\begin{array}{r} \cancel{5} \quad \cancel{5\sqrt{150}} \\ \times 30 \\ \hline 150 \end{array}$$

~~5~~

5 is 30 or 150

? So I just multiplied  
The answers of one side  
of 5 times 30 & multi  
that by 4 address

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$$\frac{n(4(n^2+2))}{2} \quad n(2n-1) = 2n^2-n \quad \checkmark$$

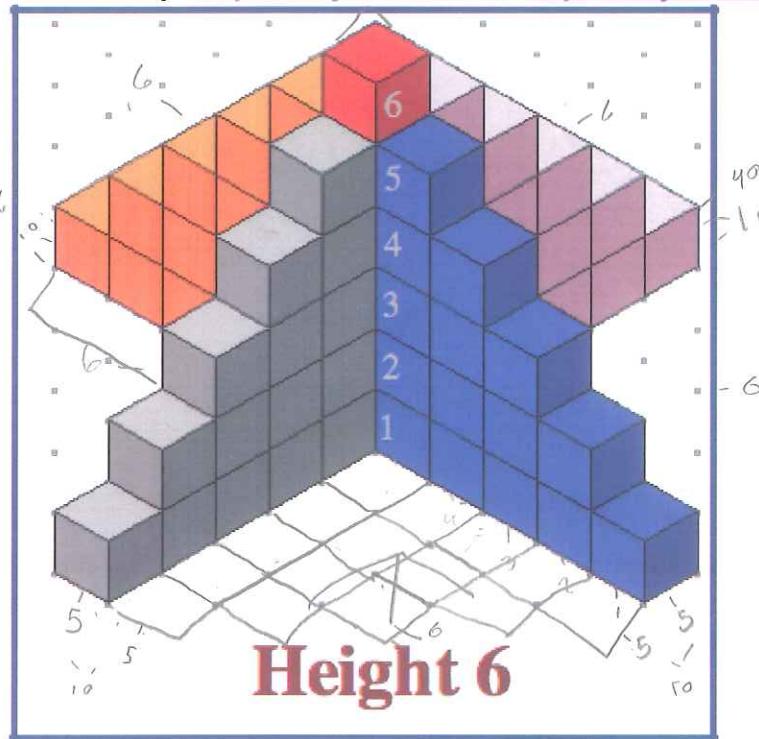
Check formula!  
what does  $n$  stand for?

Building Towers (Blue Worksheet)  
10 points possible

8/10

Name: Rody DeLeon  
Date:

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$6 \times 4 = 24$$

$$5 \times 8 = 40$$

1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

How did you get this?

$$(6 \times 4 = 24)$$

$$5 \times 8 = 40$$

64 cubes  
66

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$\begin{array}{r} 6 \times 4 = 24 \\ + 4 \\ \hline 28 \end{array}$$

OK

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\left( \frac{150 \times (150-1)}{2} \right)$$

$$\times 4 + 150 = 14,850$$

$$\underline{(150-1)} \times (4) + 150 = x$$

good!

$$\frac{n^2 - n}{2} \cdot 4 + n = x$$

Awesome formula! how did you get it?

$$\left( \frac{n(n-1)}{2} \right) \cdot 4 + n = x$$

4. (1 point) With your group, describe **two different methods** to find the cubes in problem 3. Write them below.

Add up all the numbers then multiply  
then by 4 then up original height of the figure

?  $\left(\frac{n(n-1)}{2}\right)(4)+n=x$

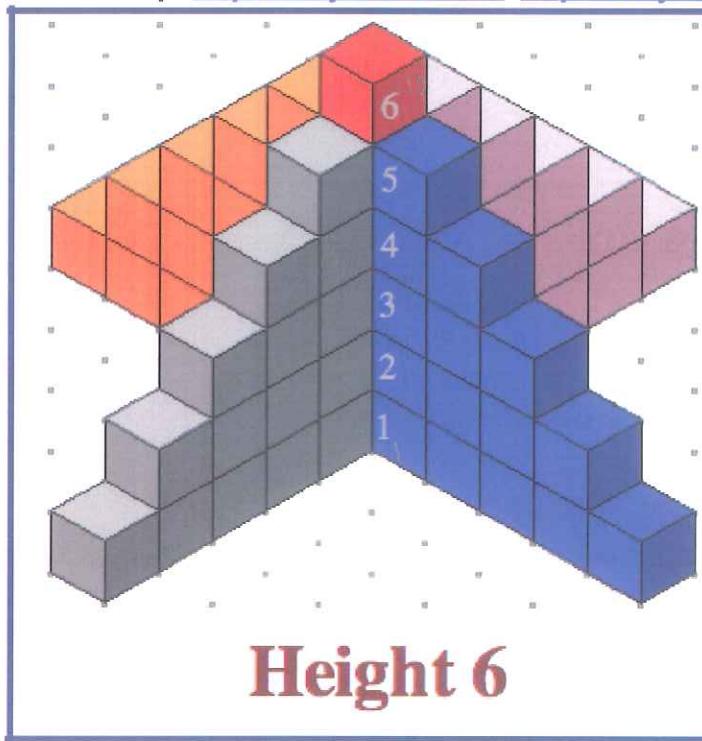
5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

-1  
 $\left(\frac{n(n-1)}{2}\right)(4)+n=x$

$n=h$  of cubes high.  $n-1$  = the top height of the sides.  
then divide it by 2 for the upper missing half of the  
sides then multiply by 4 then add the original  
height for the row in the middle  
yes!

Never forget to check your  
formula!

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$\underline{15 \cdot 4}$$

$$\underline{60 + 6}$$

$$\underline{\circlearrowleft 66}$$

What do these numbers represent?

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**



$$\underline{6 \cdot 4}$$

$$\underline{24 + 4}$$

$$\underline{\circlearrowleft 28}$$

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\underline{n(4) + 150}$$

$$\begin{array}{r} 11174(4) = 44700 \\ + 150 \\ \hline 44850 \end{array}$$

**44,850 Cubes**

**we added up**

**9955**

**all the #'s**

**11325**

**How?**

$$\begin{array}{r} 150 \cdot 119 \\ \hline 2 \\ \cdot 4 \\ + 150 \end{array}$$

**11175**

**+ 150**

4. (1 point) With your group, describe **two different methods** to find the cubes in problem 3. Write them below.

- Adding them all together, multiplying by 4 & adding 150 Anthon

- $\frac{150 \cdot (59-1)}{2} \cdot 4 + 150$

$n$  = height of  
OK cubes

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

-1  
more descriptive!  
correct answer,  
but what does  
each term represent?

Where did this come from?  
What does it mean?

$$\frac{n \cdot (n-1)}{2}$$

- $4 + 16$

$$\frac{6 \cdot (6-1)}{2} \cdot 4 + 16$$

$$\frac{6 \cdot 5}{2} \cdot 4 + 16$$

$$\frac{30}{2} \cdot 4 + 16$$

$$15 \cdot 4 + 16$$

$$60 + 16$$

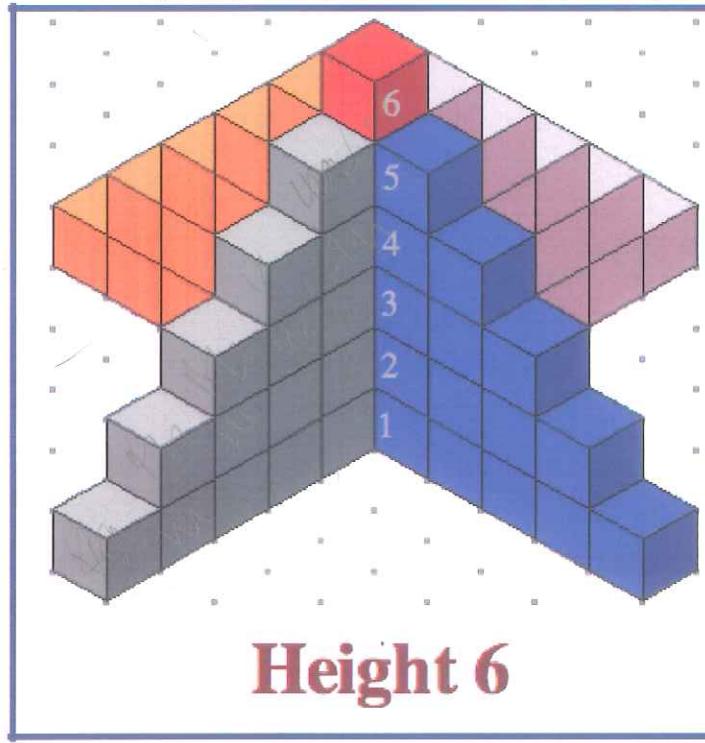
66 cube

Building Towers (Blue Worksheet)  
10 points possible

+9/10

Name: Casey Senay  
Date: 3/15

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$15 \cdot 4 + 6 = 66$$

What does each mean?

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$6 \cdot 4 + 4 = 28$$

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$(n \cdot 4 + 150) = \frac{n(n+1)}{2} \cdot 4 + n$$

we added all the numbers up to 150 up = 11325

44,850 cubes  
manually?  
or used a formula?

$$\begin{array}{r} 11325 \\ \times 4 \\ \hline 45250 \end{array}$$

$$\frac{150 \cdot 149}{2} \cdot 4 + 150 = 44,850 \text{ cubes}$$

4. (1 point) With your group, describe two different methods to find the cubes in problem 3. Write them below.

1) by adding them all then multiplying by 4, then adding 150       $4n + 150 = x$

$$2) 150 \cdot \frac{(150-1)}{2} \cdot 4 + 150$$

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$\frac{n \cdot (n-1)}{2} \cdot 4 + n$

1 leg  
 $\overbrace{\hspace{10em}}$   
 $n \cdot (n-1)$   
 $\overbrace{\hspace{10em}}^2$

$n = \text{height of cubes}$   
 $n$   
center column

be more descriptive!  
what does each term represent?

$$6 \cdot \frac{(6-1)}{2} \cdot 4 + 6$$

$$\frac{6 \cdot 5}{2}$$

$$15 \cdot 4 = 60$$

$$60 + 6 = 66$$

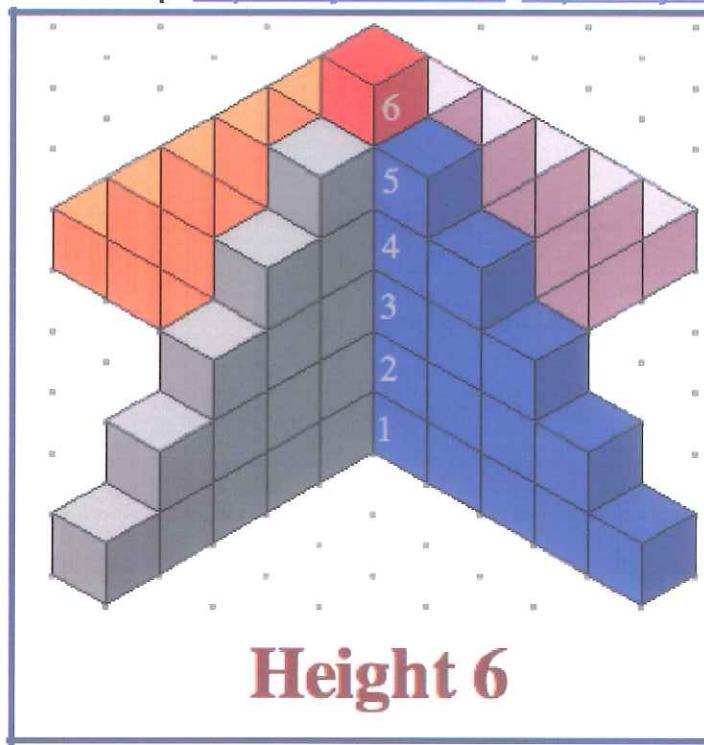


Building Towers (Blue Worksheet)  
10 points possible

+9/10

Name: Sonja Vidovic  
Date: March 5, 2014

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$6 + 15 + 15 + 15 + 15 = 66$$

66 blocks

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$4 + 6 + 6 + 6 = 28$$

where is the 9th leg?

28 blocks

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\underline{150 \cdot 149} \cdot 4 + 150 = 44850 \text{ blocks}$$

2  
How did you come up with this?

4. (1 point) With your group, describe two different methods to find the cubes in problem 3. Write them below.

$$1) 4C + n$$

or

$$n + (n-1) + (n-2) + (n-3) \dots + \text{so on until } n - (n-1)$$

n = height

C = # cubes in leg

2)

$$\frac{n \cdot (n-1)}{2} \cdot 4 + n =$$

n = height

way to be descriptive!

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$n=b$

$$16 \cdot 5 = 30$$

$\rightarrow$  n = height

$\rightarrow$  base  $\times$  height of one leg to get rectangle

$$30 \div 2$$

$\rightarrow$  divide by 2 b/c only half of the rectangle is shown

$$15 \cdot 4$$

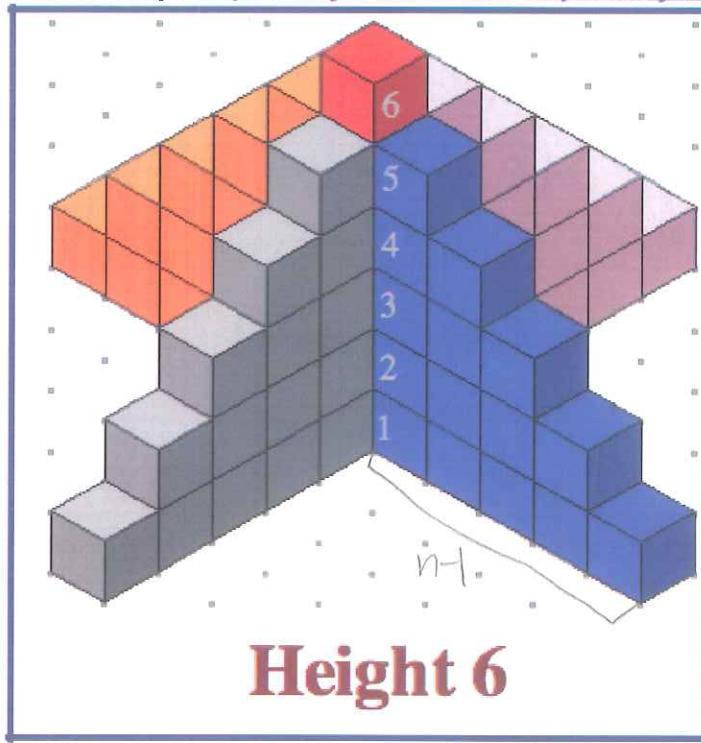
$\rightarrow$  multiply by 4 b/c that's how many legs there are

$$60 + b$$

$\rightarrow$  add  $b$  b/c that is the center column

66 cubes

Directions: Work on question 1 and 2 individually, then get into groups and complete the worksheet together.  
See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



1. (1 point) How many cubes are needed to make the figure above? **Show your work.**

$$6 + 16 + 15 + 16 + 15$$

66 cubes

2. (2 points) Suppose you built a similar figure 4 cubes high. How many cubes are needed to make this figure? **Show your work.**

$$4 + (6 \cdot 4)$$

28 cubes

3. (1 point) Using the methods you came up with above, find the amount of cubes in a figure with height 150. **Show your work.**

$$\frac{160 \cdot 149}{2} \pm 1175 \cdot 4 = 44700 + 160$$

why did you  
decide to do  
this?

44860 cubes

✓

4. (1 point) With your group, describe two different methods to find the cubes in problem 3. Write them below.

long way:  $4c + h$

$c = \text{the counted number}$   
 $\text{of cubes in one leg}$   
 $n = \text{height}$

*Love that you're specifying variables!*

OR

$$n + (n-1) + (n-2) + (n-3) + \dots + n - (n-1)$$

*oh!*

until you get to

short way:  $\frac{n \cdot (n-1)}{2} + n$

$n = \text{height}$

$\uparrow \text{OK}$

5. (5 points - see rubric) Generalize one of the above methods to determine a formula for the number of cubes in a similar tower  $n$  cubes high. (Hint: Try to use a table to organize your thoughts). Using the formula that your group came up with, choose a figure from the front side of the worksheet and check your answer.

$$n = 6$$

i'm assuming this is your formula?  
 $6 \cdot 5 = 30$

$$\frac{30}{2} = 15$$

$n = \text{height}$

► base  $\times$  height of one leg to get rectangle

► divide by 2 b/c only half of the rectangle is shown

► multiply by 4 b/c there are 4 legs

$$15 \cdot 4$$

$$60 + 6$$

66 cubes total

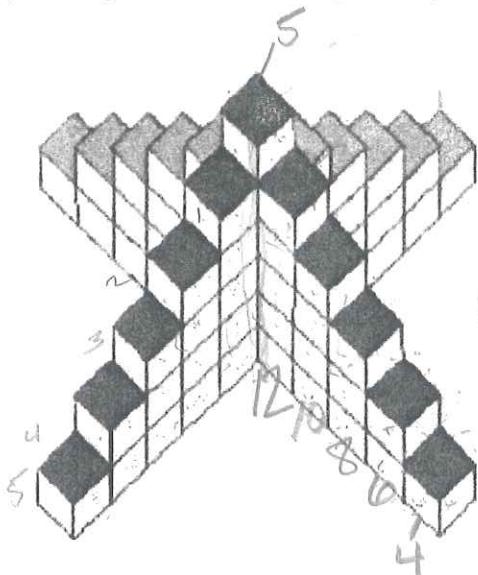
► add 6 because there are 6 cubes in the center column.

Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: Jackson Tinsley  
Date: 3/6/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



2 staircases = 1 rectangle

$$n-1 + n-2 + n-3 + n-4 + n-5$$

$$4(n-1)$$



$$\frac{1}{2}n(n+1)$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 The tower is composed of four "staircases" with 15 blocks each. There is an additional tower in the middle between all four staircases.  $60 + 6 = 66$  total blocks

- 2) Reference the diagram shown above:

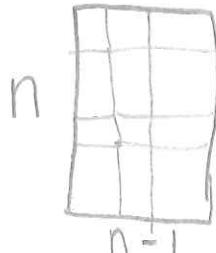
- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1 There are four identical staircases with 15 blocks each plus an extra 6 blocks in the middle of the four staircases.  $15 \times 4 = 60 + 6 = 66$  total blocks

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 Your strategy would work but it isn't the most efficient. Yes, you know the highest number of the staircase is one less than the height. If each staircase is 299 then you can add  $299 + 298 + 297 + \dots$  then multiply by 4 then add 300. This would not work easily for n.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.



Good labeling

2 rectangles

$$n(n-1) + 2(n^2 - n)$$

$$2n^2 - 2n + n = 2n^2 - n$$

+1.5  
Check your formula w/  
 $n=6$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$\text{Formula} \rightarrow 2n^2 - n$$

$n = \text{height}$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

The height is 100 cubes.

$$\frac{1 \pm \sqrt{1+4(2)(-19,900)}}{2(2)}$$

$$\frac{1 \pm \sqrt{159201}}{4}$$

$$\frac{1 \pm 399}{4}$$

$$\frac{400}{4} = 100 \quad 100 = n \quad \frac{400}{4}$$

~~-398~~

I set the equation equal to zero to use the quadratic equation.

The negative answer does not work because height cannot be negative.

Good Work!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

w/bottom  
painted

$$5+7+9+11+13 = 45$$

$$4 \text{ staircases} \rightarrow 4(45) = 180$$

Each Stair on Staircase has 5 painted faces. The lone block on top of the tower face in the middle of the tower that is not a part of any staircase.

$$5+1=6 \quad 180+6$$

$$186 \text{ units}^2$$

+2

Area of n:

Good Job!

Building Towers (Yellow Worksheet)

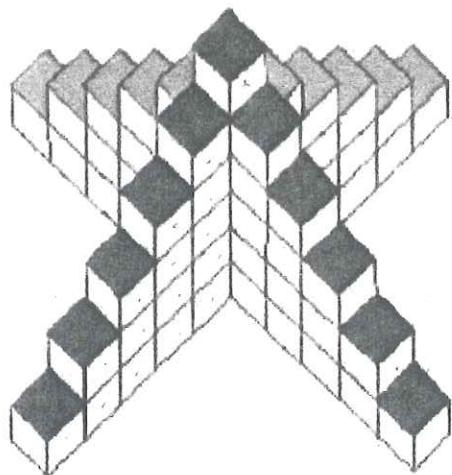
10 points possible

11.5 / 10

Name: Aaron Beglin

Date:

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$2h+1$$

$$(2h+1)h(2h-1)^2$$

$$4h(h-1) + 4(2h-1) + 1$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

In the center there is a one by six tower of blocks

four staircase structures attach to the center on four of its sides one block shorter than the center tower. The staircase cases decrease by one block at a time as it travels away from the center.

+1

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\text{base layer} = 0$$

$$\text{height from base layer} = 1$$

$$[(5-h) \times 4] + 1 =$$

# of blocks per layer

$$5 \times 4 + 1 = 21$$

$$4 \times 4 + 1 = 17$$

$$3 \times 4 + 1 = 13$$

$$2 \times 4 + 1 = 9$$

$$1 \times 4 + 1 = 5$$

$$6 \times 0 \times 4 + 1 = 1$$

$$21$$

$$17$$

$$13$$

$$9$$

$$5$$

$$= 66$$

(5-h) = length of the staircase from center

$\times 4$  = # of staircases

+1 = one block per layer in the center

+1

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

I4 would work because you could plug 300 in for the 5 in step a) and work up to the top height.

+1 Won't be the easiest but it does work.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

How do you  
know this?

Show work/process

$$h(2h-1) = \text{area of tower}$$

$$6(2(6)-1) = 66$$

\*2

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

41.5

I would like to  
see more written  
explanation. Your  
math and work is  
sound.

$$19,900 = h(2h-1)$$

$$2h^2 - h - 19,900$$

$$\frac{1 \pm \sqrt{1-4(2)(-19,900)}}{2(2)}$$

$$\frac{1 \pm 399}{4}$$

100 or (-49.5)

Showing work  
allows the ~~reader~~  
reader to understand  
your process.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2 presuming the bottom isn't part of surface area.

165

$$[4h(h-1)] + [4(2h-1)] + 1$$

$$\underline{4h^2 - 4h + 8h - 4 + 1}$$

$$4h^2 - 4h + 8(h-1) + 5$$

$$4h^2 - 4h + 8h - 8 + 5 + 8h - 3$$

$4h^2 + 12h - 12$

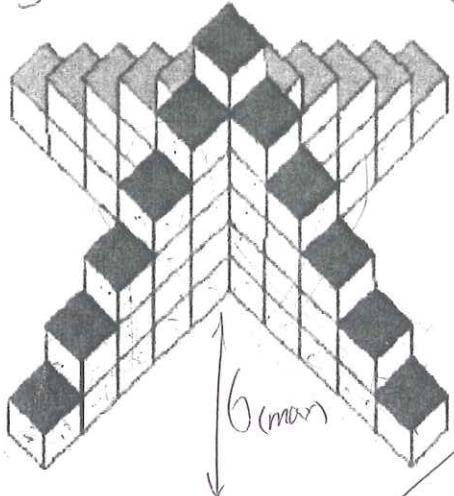
10.5/10  
Building Towers (Yellow Worksheet)  
10 points possible

3(h-1)  
 $4n(h-1) + 4(h-1) + 1$

Name: Connor Miller  
Date:

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

4.5



$$\text{Total blocks} = 8(h-1) + 5, \quad 4(h-1) + 1$$

12 13

$$n=5 \quad 20(4) \\ 80$$

$$3 \cdot 4 \cdot 2 = 24$$

$$6 \cdot 4 \cdot 1 = 24$$

$$3 \cdot 2 \cdot 1 = 6$$

$$4 \cdot 2 \cdot 3 \cdot 1 = 24$$

$$-3$$

$$\frac{24}{2} = 12$$

$$-2 - 1 = 0$$

- +1 1) (1 point) Describe what you notice about the structure of the tower above.

It has symmetry in 3 dimensions (thus radial symmetry). Could be considered to contain 5, 3-dimensional coordinate points of blocks, each one perceived as a "unit block", in which the 4 external points connect to the center one while ever increasing a single unit ~~in all three axes, 3, 2, etc.~~ in two directions but spanning 3 axes, until they reach the center point, or collide with each other.

- +1 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Break the figure into 5 parts, the center and each of the 4 congruent steps. The middle tower is  $6 \times 1 \times 1$  based upon the steps containing increase in height. Each step is ~~the~~ a sequence  $5 + 4 + 3 \dots$  5 being replaced by the highest point of a step system excluding the center  $6 \times 1 \times 1$  tower piece. That equals 15, multiply by 4, get 60 and add the six middle blocks to get 66.

- +1 b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Possibly depending upon ones definition of "easily", it is very easy to do  $x + x-1 + x-2 + \dots$  but time consuming. ( $x$  being 1 less than the height). A possible equation could be determined with it but

where  $x-1 > 0$

- Very true point  
3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\frac{1}{4}h(h-1)^2$$

Show work/  
Process

$$\frac{1}{4}(6)(2 \cdot 6 - 1)^2$$

$$\frac{1}{4}(12-1)^2 \\ 12 \cdot 11 = 11^2 =$$

$$\frac{1}{4}h(h-1)^2 \\ h(2h-1)(2h-1)$$

$$\frac{h(2h-1)}{2h^2-h}$$

+1.5

Didn't check

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+4.5  
Would like more explanation.

$$2h^2 - h - 19,900 = 0$$

Quadratic  $\rightarrow \frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{2(2)}$

$$\frac{1 \pm 399}{4} =$$

100 or -99.5

Sharing Work will help the ~~reader~~ understand your process and understand your logic.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+1.5  
explain

\* presuming bottom is not included SA

$$4h^2 - 4h \quad \checkmark$$

sides

$$\begin{aligned} & 4h(h-1) + 4(h-1) + 1 \\ & 4h(h-1) + 4(2h-1) + 1 \\ & 4h^2 - 4h + 8h - 4 + 1 \\ & 4h^2 + 4h - 3 \end{aligned}$$

$8(h-1) + 5$   
steps

Provide work.

165

$$\frac{[4h^2 - 4h] + 8h - 8 + 5}{8h - 3}$$

4h^2 + 4h - 3

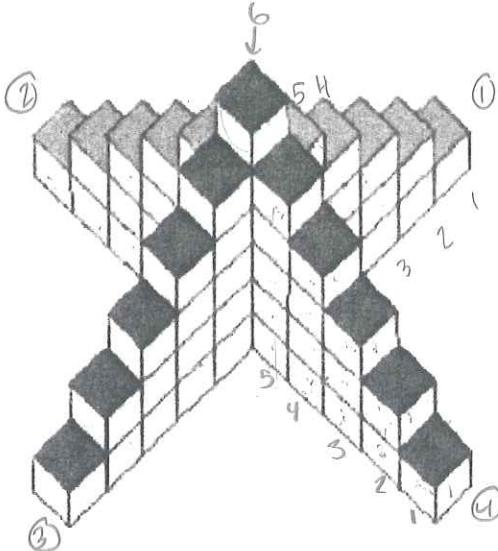
Building Towers (Yellow Worksheet)  
10 points possible

5.5

10

Name: Isabelle Tessier  
Date: 3/6/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower is symmetrical, one cube is added to each step,  
66 cubes used,

$$1+2+3+4+5 = 15$$

$$15(4) = 60 + 6 = 66$$

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1  
1+2+3+4+5 = 15(4) = 60 + 6 = 66 cubes used  
There are four sides, each with 15 cubes, & one column of 6 cubes.  
There is one column of 6 cubes, 4 columns of 5 cubes, 4 columns of 4 cubes, 4 columns of 3 cubes, 4 columns of 2 cubes, and 4 columns of 1 cube.  $1(4) + 2(4) + 3(4) + 4(4) + 5(4) + 6 = 66$  cubes.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1  
It would not be easy because you would have to add a lot of numbers and then multiply by 4. There is a lot of room for error. ← Great Point!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$n$	#ubes
1	1
2	6
3	15
4	28
5	45
6	66

$n$	Base
1	1
2	6
3	15
4	28
5	45
6	66

$$4n^2 - 3$$

Generalized Equations should work for all entries  
Check n=1, n=2, n=3...  
 $1 + (n-1)(4n-3)$



?

+0.5

If  $n=6$ , this equation won't work.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

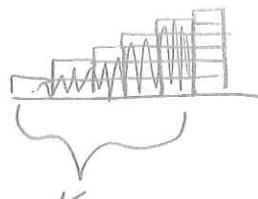
(+0)

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

(+2)

$$15(8) + 2(20) + 5 = 165 + \underbrace{11 + 10}_{\text{SA Base}} = 186 \text{ units}^2$$

↑  
SA of one side (8 sides) + 2 sides (20 cubes) + SA top cube



8 ↑ + 2 sides for each top cube + 5 sides of the Top cube

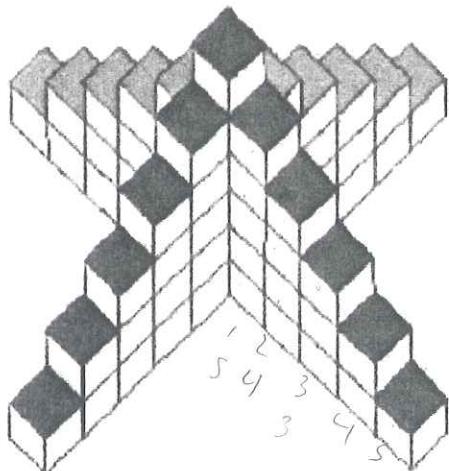
Good Work!

Building Towers (Yellow Worksheet)  
10 points possible

5/10

Name: Priya Thomas  
Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

symmetric, increasing height  $\rightarrow$  less cubes per layer,  
resembles staircase

Good Observation!

- 2) Reference the diagram shown above:

+1

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

n	# of cubes total
1	1
2	5
3	9
4	13
5	17

$$6 + (5 \cdot 4) + (4 \cdot 4) + (3 \cdot 4) + (2 \cdot 4) + (1 \cdot 4) = 66$$

Good Equation

66 cubes

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, the work is tedious & arduous without a formula.  
Manual labor is taxing. There are many components to this way I solved it.

+1

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

ratio of blocks per layer w/ full filled layers  
+4

$$n + ((n-1) \cdot 4) + ((n-2) \cdot 4) + ((n-3) \cdot 4) + ((n-4) \cdot 4) + ((n-5) \cdot 4) = 6 + n + (n-1)(4n-20) = 6n - 16$$

$$n + 4n - 4 + 4n - 8 + 4n - 12 + 4n - 16 + 4n - 20 = 6n - 4n - 3$$

$$1 + (n-1)(4n-3)$$

$$1 + (n-1)(4n-3) = 1 + (4n^2 - 4n)$$

13	9	5
49	25	9
7	5	3
n=4	n=3	n=2

Generalized formula should work for all n's.

Only works for n=6

+1 check n=1, n=2, n=3...

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+10

If you have questions, ask Mr. Drumm or a classmate.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$(15 \times 8) + 2(20) + 5 = 165 + 21 \rightarrow \text{SA of bottom}$$

186 units

Show your  
work and process  
to help the reader  
follow your steps.

Building Towers (Yellow Worksheet)

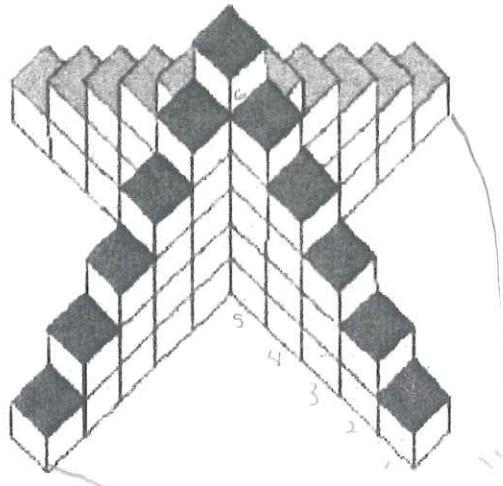
10 points possible

11 / 10

Name: Ansley Layman

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$8(2n-1) + 4(2n-1) + 5$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

it's staircase-like on its 4 points,

it goes up by 1 for each step,

all of the staircases meet at the highest step.

Good Observations.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$1+2+3+4+5 = 15$  cubes for 1 staircase

$15 \times 4 = 60$  cubes for the 4 staircases

$60 + 6 = 66$  cubes total

the 4  
Staircases

↑  
the tallest stack of cubes

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

no because it would involve adding each step  
 $(1+2+3+\dots+299)$  and that is efficient, but it takes a while. Great finding.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1.5

I would like to

see how you found  $n(2n-1)$ ,  
but this is correct.

$$n(2n-1)$$

$$6(2(6)-1)$$

$$6(11)$$

$$66$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = n(2n-1)$$

$$19900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19900$$

$$0 = (2n+199)(n-100)$$

$n = \cancel{-199}$        $n = 100$   
can't be negative

The height is 100 cubes.

Written explanation helps the reader follow your process/understanding.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$15 \cdot 8 = 120 + 10 \cdot 4 = 160 + 5$$

Surface area of 1 side      # of sides      ↑      ↑      ↑  
↑      ↑      ↑  
4 sides      4 sides      Surface area of top

Surface area is 165 cubes

Nice labels & description

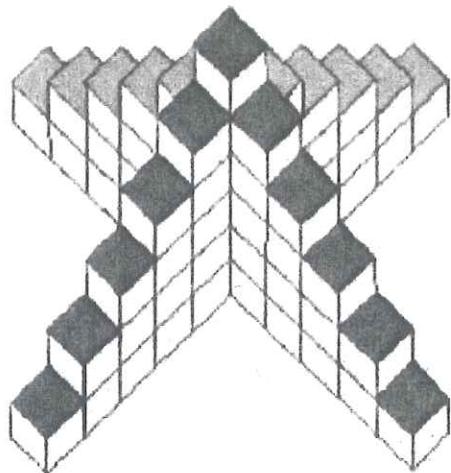
6. additional bonus: find a way to find the surface of a tower of height  $n$ .

Building Towers (Yellow Worksheet)  
10 points possible

10/10

Name: Annie Münser  
Date: 3/10/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 X shaped base with steps building up towards the center, tallest in center then 1 less in each step out in 4 directions

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{c} \uparrow \\ 6 \\ \downarrow \end{array} + \begin{array}{c} 5 \\ 6 \\ 5 \\ \downarrow \\ 4 \\ \downarrow \\ 3 \\ \downarrow \\ 2 \\ \downarrow \\ 1 \end{array} + \begin{array}{c} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array} = 60 \text{ blocks}$$

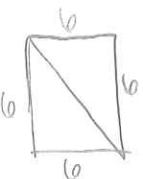
Good Diagram.

I looked at the structure as if I was above it & determined there was one stack of 6, 4 stacks of 5, 4 stacks of 4, etc. So I added 1-5 & multiplied each # by 4

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 No, it would take too long to add every number counting upto 300 (multiplied by 4). A formula would be easier

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.



$$2(300) - 6 = 600$$

$$2(300)^2 - 300 = 179700$$

+1.5

Provide more explanation of how you got  $2n^2 - n$

$$4\left(\frac{1}{2}n^2\right)$$

$$2n^2 - n$$

Showing work allows the reader to understand your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = 19900$$

$$2n^2 - n - 19900 = 0$$

Started by using # of blocks formula on #3 & set equal to 19,900

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-19900)}}{2(2)}$$

$$\frac{1 \pm \sqrt{1 + 159200}}{4}$$

$$\frac{1 \pm 399}{4} \quad \frac{400}{4} = 100 \text{ cubes tall}$$

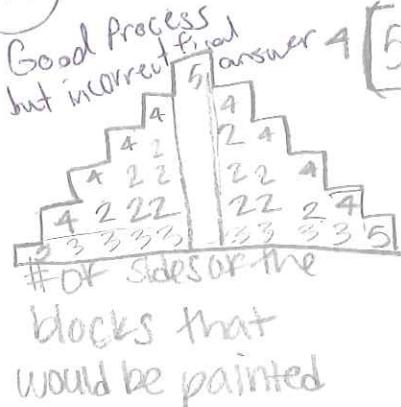
+4.5

Show more explanation

Label your steps, so the reader knows what you are doing.

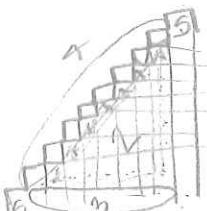
5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

(+1)



$$4(n^2 + n) + 6$$

I believe you forgot the other two legs.



~~$$3(n-2) + 4(n-2) + 11(n-3)(n^3)$$~~

$$3n - 6 + 4n - 8 + n^2 - 6n + 9$$

~~$$4(n^2 + n - 5 + 5) + 6$$~~

## Building Towers (Yellow Worksheet)

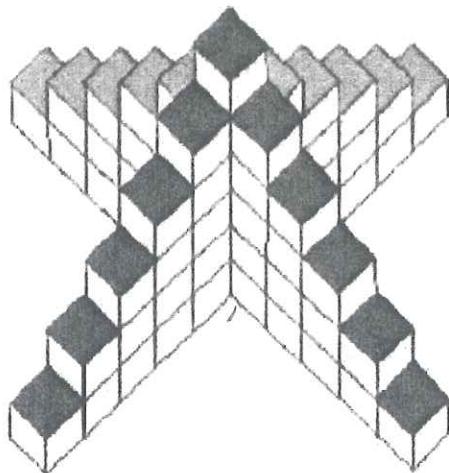
10 points possible

9.5  
—  
10

Name: Lauren Reeder

Date: 3/6/2014

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1) It is made of 4 right triangles, each w/ a base & height of 5 cubes. There's also 1 tower in the middle.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

(+1)  $n = 4(5 + 4 + 3 + 2 + 1) + 6 = 66 \text{ cubes}$   
Found each triangle, added heights.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1) No, the addition would be too tedious. ← Good Point.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

(+1.5)  
Would like more explanation on how you found  $2n^2 - n$

$$4\left(\frac{1}{2}n^2\right) - n$$

$$[2n^2 - n]$$

$$2(6)^2 - 6 = 66$$

Showing your work allows the reader to follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 2n^2 - n$$

$$2n^2 - n - 19900 = 0$$

$$n = \frac{(-1 \pm \sqrt{(-1)^2 - 4(2)(-19900)})}{2(2)}$$

$$\boxed{n = 100 \text{ cubes}}$$

+4.5

More explanation

Great work, but a written explanation would help support your work better.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+0.5

You started with the right process, but you should check the # of sides showing on 1 triangle.

# of sides showing  
on 1 triangle: 32

$$32 \cdot 4 = 128$$

Add 5 for top

cube + 1 for

bottom

$$\boxed{134}$$

Building Towers (Yellow Worksheet)

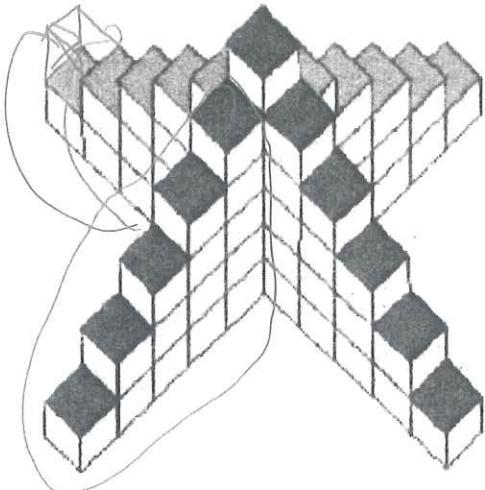
10 points possible

10/10

Name: Adam Sparks

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{aligned} & 8(2n-1) + 4(2n-1) + 5 \\ & 16n - 8 + 8n - 4 + 5 \\ & 24n - 7 \end{aligned}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1) The structure has a pyramid-like shape. It is six units tall, and 11 units from one corner to another.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

(+1)  $6 + 4(5) + 4(4) + 4(3) + 4(2) + 4(1)$

(66)

$$\begin{array}{cccccc} 1 & 6 & 15 & 28 & 45 & 66 \\ 5 & 9 & 13 & 17 & 21 & \end{array}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1) It would work easily, but it is lengthy to do.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1.5 more explanation to work to find  $n(2n-1)$

$$\begin{aligned} & n(2n-1) \\ & 6(12-1) \\ & 6(11) \end{aligned}$$

Work will help the reader understand your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+4.5 more explanation

$$19900 = n(2n-1)$$

$$2n^2 - n - 19900 = 0$$

$$(2n+199)(n-100)$$

$$2n = -199 \quad n = 100$$

How do you  $n = -99.5$  or  $100$

know

can't be negative

$$2n^2 - n - 19900 = (2n+199)(n-100) \quad h = 100 \text{ units}$$

$$\begin{array}{r} 4975 \\ 4 \sqrt{19900} \\ \hline 1990 \end{array}$$

$$\begin{array}{r} 498 \\ 50 \sqrt{19900} \\ \hline 1990 \end{array}$$

$$\begin{array}{r} 331 \\ 60 \sqrt{19900} \\ \hline 1990 \end{array}$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+1 Right

answer but  
no work.

$$\begin{array}{l} 8(2(6)-1) + 4(2(6)-1) \\ 80 + 45 \end{array}$$

$$\cancel{BBN} \quad 165$$

additional bonus: find surface of a tower of height  $n$ .

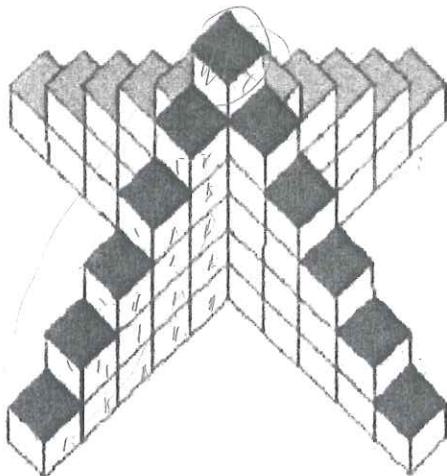
$$12(2n-1)\sqrt{5}$$

Building Towers (Yellow Worksheet)  
10 points possible

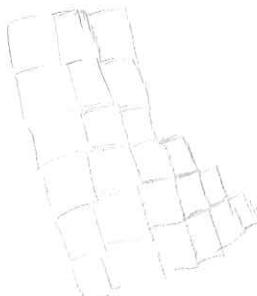
11.5  
10

Name: Kaetlin McVaugh  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



6<sup>2</sup>



- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1) The above tower is in the shape of a pyramid, resembling a square pyramid. It has a Stair Step design, making it not so much like a typical pyramid.  
Good Observation.

- 2) Reference the diagram shown above:

a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

So really, each "wing" of the pyramid is five high. So because the pyramid has the stair step thing going on, I know it goes 5 in a row, 4 in a row, 3 in a row etc. So I end up with  $5+4+3+2+1 = 21$  in one wing. I multiply that by four to get 60 and then I added 6 to get 66 in the tower.

(+5) Good answer, just watch your math.

b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1) I think it would work moderately well. The person would need a calculator and they would do  $300 \cdot 299 + 298 + \dots + 1$ . All the way down to 1. There are probably ways to do it without so much addition, which is why this is only a way that looks moderately well. It's not necessarily the best way.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

(+1.5) How did I find  $n(2n-1)$ ?

$$\begin{aligned} n(2n-1) &= ? \\ (6)(2(6)-1) &= ? \\ 6(11) &= ? \\ 66 &= ? \end{aligned}$$



Show your work will help the reader follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = n(2n-1)$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

$$0 = \frac{1 \pm \sqrt{1 - 4(2 - 19,900)}}{2(2)}$$

$$0 = \frac{1 \pm \sqrt{1 - 4(-39,800)}}{4}$$

$$0 = \frac{1 \pm \sqrt{159,201}}{4}$$

$$\frac{1 \pm 399}{4}$$

$$\frac{400}{4} \text{ or } -\frac{398}{4}$$

100 cubes

I used my formula and plugged in 19,900 for  $x$ . A quadratic equation formed so I used the quadratic formula to solve it & got 100

Good Process.  
Easy for the reader to follow.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

40 on one wing  $\times$  4 for four wings = 160 surfaces needing paint + 5 faces of top cube

The surface area is 165

$$(2n) + n + 1$$

Surface area equation

$$4((2n + n + 1)) \leftarrow \begin{matrix} \text{How did get from this step to} \\ \text{the next.} \end{matrix}$$

$$4(n+4) + 5 = x$$

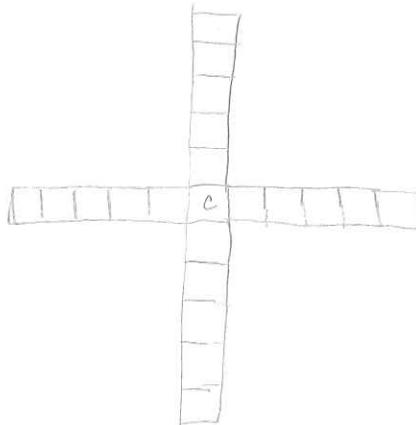
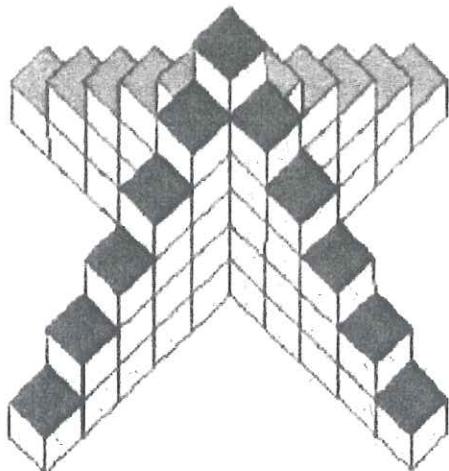
Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: Amy Saunders

Date: 3/6/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1  
 - It is six cubes high in the middle  
 - cross shaped  
 - steps down one cube at a time  
 - unfilled step pyramid

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1  
 bottom  $5 \times 4 = 20$   
 $\downarrow$   
 $4 \times 4 = 16$   
 $3 \times 4 = 12$   
 $2 \times 4 = 8$   
 $1 \times 4 = 4$   
 top ~~excluded~~  
 $= 160 + 16 = 166$  middle

I excluded the center column  
 + multiplied # of cubes per row by Four (Four arms).  
 Good Explanation.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1  
 Not really, you'd get the right answer, but would have to add up 299 steps. However, an equation based on my strategy would help

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1.5  
 $X = n(2n-1)$

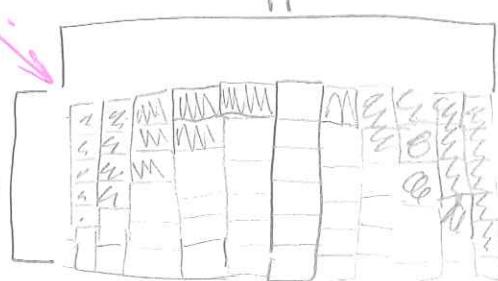
Solid work  
 but provide  
 more steps  
 before  $n(2n-1)$

$$X = 6((2)(6)-1)$$

$$X = 6(11)$$

$$\underline{X = 66 \text{ cubes}}$$

Nice  
 Diagram  
 $\frac{11}{6}$   
 6



4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

(+5)

$$19,900 = n(2n - 1)$$

$$19,900 = 2n^2 - n$$

$$2n^2 - n - 19,900$$

We plugged  
19,900 into the  
equation as  
y and solved  
for N, which  
was 100 cubes.

$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-19,900)}}{2(2)}$$

Great job showing  
your work.  
Very easy to  
follow!

$$\frac{1 \pm \sqrt{1 + 159200}}{4}$$

$$\frac{1 \pm 399}{4}$$

$$\frac{400}{4}$$

$$\cancel{\frac{398}{4}}$$

100 cubes

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

two sides shown

$$(5 \times 2) 4 \quad 40$$

$$(4 \times 2) 4 \quad 32$$

$$(3 \times 2) 4 \quad 24$$

$$(2 \times 2) 4 \quad 16$$

$$(1 \times 2) 4 \quad 8$$

10 faces  
on top - 4 sides

$$10 \times 4 = 40$$

$$\text{top cube} = 5 \quad 40$$

165 square faces

6. Surface area of tower height h

$$x = (2n + 2(n-1))4 + 5$$

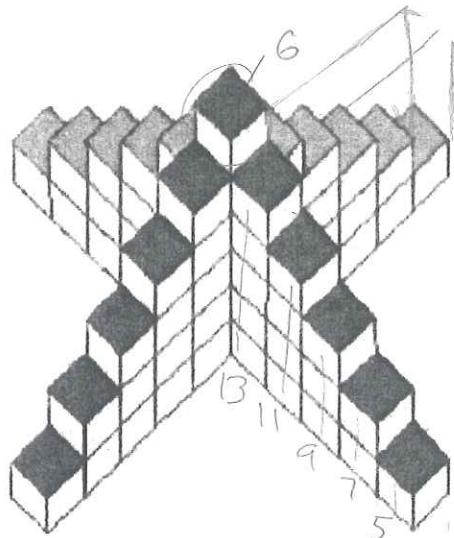
$$2n - 2$$

Building Towers (Yellow Worksheet)  
10 points possible

12/10

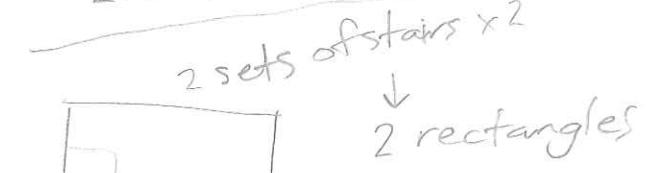
Name: Riley Hunter  
Date: 3/6/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\frac{[2(h-1)+3]}{2h+1} + \frac{[2(h-2)+3]}{2h-1} + \frac{[2(h-3)+3]}{2h-3}$$

5A steps



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

There is a column in the middle, with steps descending outward from each of the four sides of the column.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$4(15) + 6 = 66 \text{ cubes}$  I multiplied the number of cubes in a set of steps by four (there are four sets), and then added that to the number of cubes in the center column.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

It wouldn't work easily because it's too tedious, a formula would be efficient to determine the number of blocks in a tower of any height

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+2

$$\text{height} = h$$

middle column  $(h + 2(h)(h-1))$   
 $h + 2(h^2 - h)$   
 $h + 2h^2 - 2h$

Formula  
 $2h^2 - h$

$$2(6)^2 - 6$$

$$72 - 6$$

$$66 \checkmark$$

Great job!

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 2h^2 - h$$

$$2h^2 - h - 19900 = 0$$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{1 \pm \sqrt{1 - 4(2)(-19900)}}{2(2)}$$

$$h = 100 \text{ or } h = -99.5$$

+5

The height is 100 cubes. I set the equation equal to zero, then solved for "h" using the quadratic formula. I obtained two answers, but the height can't be negative, so only one of them was correct.

Correct reasoning and logic!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$13 + 11 + 9 + 7 + 5 = 45 \cdot 4 = 180 + \underset{\substack{\text{middle} \\ \text{column}}}{6} = 186 \text{ faces covered}$$

+2

The middle column has 5 faces on top, plus one on the bottom. Each of the steps has 45 faces, multiplied by 4, since there are 4 sets of steps. When added together, the number of faces covered with paint will be 186.

Great explanation.

middle → 6 + 4[  
middle column  
SA]

## Building Towers (Yellow Worksheet)

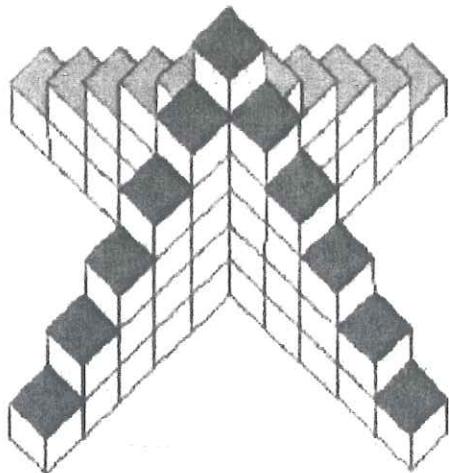
10 points possible

11/10

Name: Will Nguyen

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 It seems to be made completely up of cubes. Using a staircase like grade on each side, (1 cube, then 2 etc) up to 6. There is only one layer of 6, the center layer of 6 by 6.

- 2) Reference the diagram shown above: 4 towers of 5 cubes 4, 3, 2, and 1 cube.

+1 a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

I just multiplied 1 stack of 6 = 6 cubes 4 stacks of 3 = 12  
# of cubes per stack by 4

6 stacks of 2 = 12 cubes 4 stacks of 2 = 8 6 cubes

6 stacks of 1 = 6 cubes 4 stacks of 1 = 4

b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? No, my strategy would not work well, because 300 is a much larger number than 6 and I basically just counted all the cubes for that structure, using the same strategy for 300 would eventually work, but it's inefficient and time-exhausting.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1 Show work of how you found that  
Also, check your equation.

$$n + 2(n(n-1))$$

$n = \text{height}$   
Showing work allows the reader to follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$n + 2(n(n-1)) = 19,900$$

$$2n^2 - n - 19,900 =$$

$$1 \pm \sqrt{1+4(19,900)} \\ \quad \quad \quad 4$$

100 cubes is  
the height

$$1 \pm \sqrt{1+159200}$$

$$1 \pm \frac{394}{4} = \frac{400}{4} = 100$$

Great explanation.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$\underline{12 + 10 + 8 + 6 + 4}$$

We wanted how  
many faces were  
on one stack,  
multiplied by 4

to cover all 4

165 cube faces  
(surface area)

Staircases then odds  
the 5 faces of the  
cube at the top.

.5 faces for top cube always

$$5 + 4[2(n-1) + n(n-1)]$$

times 4 = 4 staircases

$t = \text{top block}$

$2(n-1) = \text{layers or front of each block}$

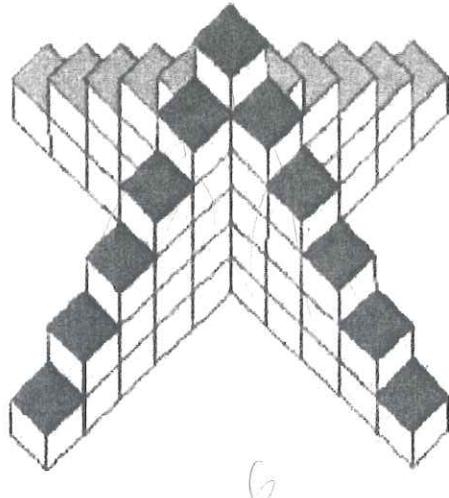
$n(n-1) = \text{area of side view}$

Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: Yash Patel  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



n: height of tower

$$n+4[(n-1) + (n-2) + (n-3) + (n-4) \dots ]$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1)

The tower has a stair-step structure coming in to a central tower of cubes. The tower essentially is a pyramid like structure.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

(+1)

$$6+6+4(5+4+3+2+1) \\ 6+6+4(15) \\ 6+6+60 \\ 66 \text{ cubes}$$

(+1) Take the central tower (6 cubes in height) and add it to one set of steps ( $5+4+3+2+1$ ) multiplied by four, for the four sets of steps.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1)

No it wouldn't be easy to find the number of cubes for a tower of 300 because you would have to find the sum of all the integers from 300 to 1, which isn't all that easy. ← & time consuming.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

n: height of tower

$$n+2(n+1)$$

$$n+2(n)(n+1)$$

Provide more work to show how you got  $n+2(n)(n+1)$

$$6+2(6)(6+1)$$

$$66 \checkmark$$

$$1+2+3+4+5+6 = 21$$

$$(x-1) + (x-1) + \dots + (x-1) = x(x-1)$$

$$x+1+x$$

Work allows the reader to follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$19900 = n + 2(n)(n-1)$$

$$19900 - n = 2n^2 - 2n$$

$$2n^2 + n - 19900 = 0$$

$$\frac{-1 \pm \sqrt{1+4(2)(-19900)}}{2(2)}$$

We set our equation from number 3  
to the # of cubes, 19900.

We got a quadratic, so we used the  
quadratic formula and got  $n=100$ .

The other answer was -99.5, but the  
height can't be negative, so the answer is 100 blocks.

$$\frac{-1 \pm \sqrt{1+4(2)(-19900)}}{2(2)} = \frac{400}{4} = 100$$

$$\begin{cases} n = 100 \\ n = -99.5 \end{cases}$$

Great explanation  
and logic.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$5 + 4[15(2) + 5(2)] = 165 \text{ faces}$$

5 faces for top block

$$5 + 4(2(5) + 10(9))$$

$$A = 5 + 4[2(n-1) + n(n+1)]$$

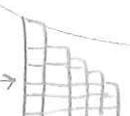


5 = top block

$2(n-1)$  = the layering blocks on  
each staircase

$n(n+1)$  = the area of the blocks  
from a horizontal view →

Take 1 staircase, and  
find the faces on the  
two sides, which is  $15(2)$ ,  
then take the 5 blocks beyond  
the staircase and include  
2 faces for each of those ( $5 \times 2$ )  
and multiply that by 4 for  
each staircase, then add five  
for the very top block.



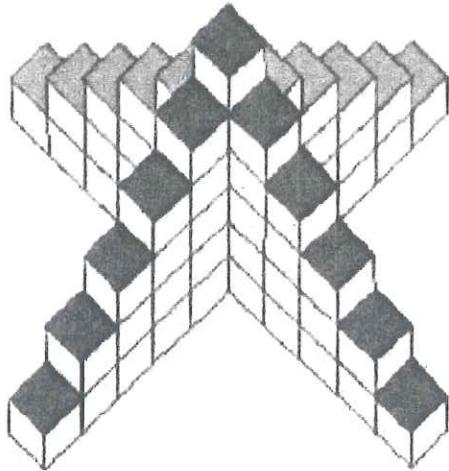
4 = 4 staircases

Building Towers (Yellow Worksheet)  
10 points possible

9/10

Name: Bailey Edeche  
Date: 3-6-14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

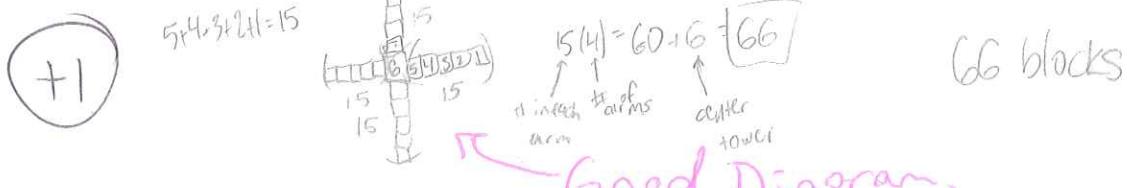


- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 There's a tower of six blocks in the center, and attached to each side of the tower is a "staircase" with five towers of decreasing height

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.



- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 Not really since I had to add 5+4+3+2+1 to find the number of blocks in each arm, which would be a tedious process with a larger height.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

I found the volume of the rectangular prism created by filling the negative space, and the ratio of that volume to the number of blocks was  $V = n(n+(n-1))^2$ . The height plus one less than the height, so the equation is  $\frac{\text{volume}}{n(n-1)} \text{ or } 2n^2-n$ .

$$V = n(n+(n-1))$$

$$\frac{n(2n-1)^2}{2n-1} = n(2n-1)$$

$$(4:196, 28)$$

$$(5:405, 45)$$

Interesting Approach! +2

tower: 15 : 5:1

+1.5

Make sure you check your height

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2n^2 - n$$

+4.5

$$2n^2 - n - 19900 = 0$$

$$(2n+199)(n-100) = 0$$

Provide More explanation.  $2n+199=0 \quad n-100=0$

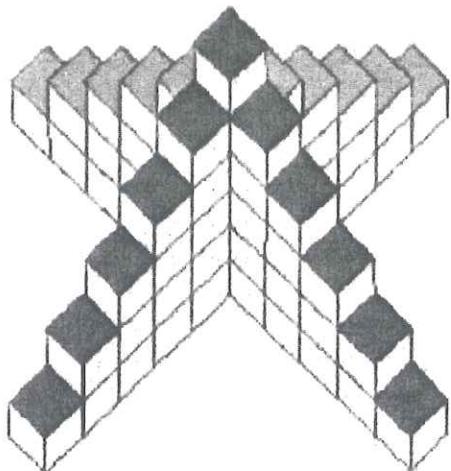
$$n = \frac{-199}{2} \quad n = 100$$

Showing work/written explanation helps the reader understand your process better.

height is  
100 blocks

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

Looking above it is the shape of a cross.

(+1)

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

6-middle column 2(4) I started in the center  
 5(4)- Good job! 1(4) with the tallest of 6 cubes,  
 4(4) then worked my way out  
 3(4) total of 5 to 4 to 3 to 2 to 1 cube  
 66 cubes. in each column and multiplied

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? Most likely not, it would be too much multiplication and too long of a process.

Good job stating why it would be a long process.

4 for each end of cross.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Awesome! ratio volume to # of cubes

Interesting Approach.

$$3 \text{ cubes tall} - 5:1 \quad V = (n + h - 1)^2 n$$

$$\text{CUBES} = \frac{(2n-1)^2 n}{(2n-1)}$$

$$4 \text{ cubes tall} - 7:1 \quad V = (2n-1)^2 n$$

$$5 \text{ cubes tall} - 9:1 \quad x = (2(n-1)) 6$$

$$x = 11(6) = 66$$

$$\checkmark \text{CUBES} = (2n-1)n$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = (2n-1)n$$

$$19,900 = 2n^2 - n$$

$$2n^2 - n + 19,900 = 0$$

$$(2n+199)(n-100) = 0$$

$$2n+199=0 \quad n-100=0$$

$$2n=-199$$

$$\cancel{n=-99.5}$$

the height is  
100 cubes.

+4.5

Provide  
more  
explanation

Showing written  
explanation provides  
extra support for the  
reader.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

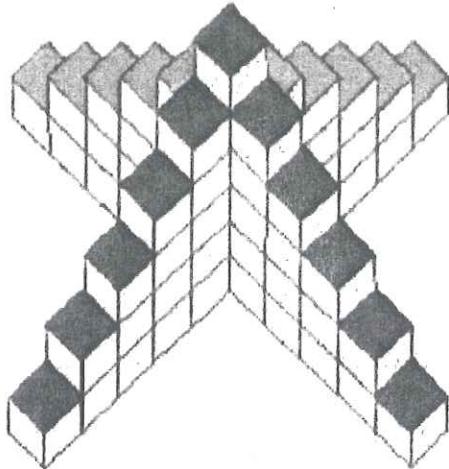
Building Towers (Yellow Worksheet)  
10 points possible

10, 5/10

Name: Emily Ann

Date:

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 It has a center column surrounded by 4 congruent walls composed of columns of decreasing height in relation to their distance from the center column. < Interesting Observation!

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$(6+4)(1+2+3+4+5) = 60$$

+1 it looks like you erased your explanation, but right answer.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 The strategy from part a would be rather cumbersome when used to calculate the number of cubes involved in a tower of 300 cubes. However, it is a relatively simple strategy suitable for smaller numbers.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units. # of cubes =  $y$

$$y = n + 2n(n-1)$$

Show your process on how you got your equation. Also check your formula for  $n=6$ .  
It helps the reader to follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = n + 2n(n-1)$$

$$0 = 2n^2 - 2n + n - 19900$$

$$0 = 2n^2 - n - 19900$$

$$0 = (2n+199)(n-100)$$

$$\cancel{n = 99.5}$$

$$n = 100$$

$$n = 100$$

It would have a height of 100 cubes

Using 19900 cubes as the number of cubes yields a height of 100.

How do you know this can be factored? Did you use a certain process?

+4.5

Provide more step by step explanation.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$1 \text{ wall} = \frac{n(n-1)}{2}$$

$$4 \text{ walls} = 2n(n-1)$$

$$\text{Top} = 5$$

+2

Good Work!

$$SA = 4 + \frac{8n(h-1)}{2} + 12(h-1)$$

$$SA = 4 + \frac{8(6)(6-1)}{2} + 12(6-1)$$

$$SA = 1860 \text{ cubes}^2$$

Building Towers (Yellow Worksheet)

10 points possible

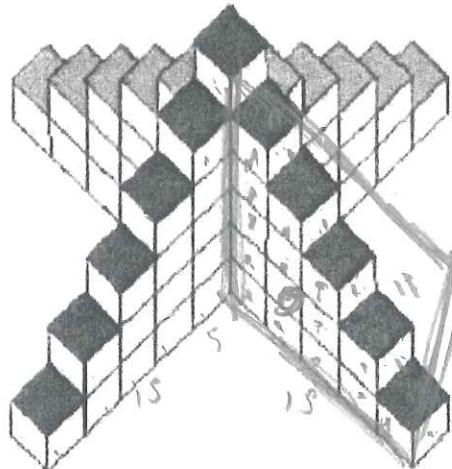
11.5 /

10

Name: Logan Brown

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 a center stack of cubes with four descending sets of five stacks of cubes on each side.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1  $6 + 4(5+4+3+2+1) = 66 \text{ cubes}$  The center is 6 and the four sets each have a total of 15 which I times by 4 since there are four sets.  
 $6 + 4(15)$   
 $6 + 60$   
 $66$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? NO, there would be far too much addition to do simply and efficiently.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$a = n + 4(n-1)^2$$

$$\times a = n + 2(n-1)^2$$

$$a = n + 2n(n-1)$$

$$\cancel{\times 66 = 6 + 2(5)^2}$$

$$a = 6 + 12(5)$$

$$a = 66$$

Good Process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$n = 100$$

$$19900 = n + 2n(n-1)$$

$$19900 = 2n^2 - 2n + n$$

$$19900 = 2n^2 - n$$

111

$$0 = 2n^2 - n - 19900$$

$$0 = (2n+199)(n-100)$$

$$n = \frac{199}{2} \text{ or } 100$$

↑

can't be negative  $\leftarrow$  Nice explanation!

the height is  
100 cubes,  
I simply plugged  
19900 into the  
formula I found  
in #3

How did you do this?

+4.5  
Provides  
more explanation

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2  
5 cubes<sup>2</sup> always on  
top plus the 1 on  
bottom each

$$6 + 4[6(5)]$$

$$6 + 48\left(\frac{h(h-1)}{2}\right)$$

$$\times \quad SA = [6 + 4h(h-1) + 12(h-1)]$$

$$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$$

$$6 + 6(6)(5) = SA$$

$$\begin{array}{r} 6 + 180 \\ \hline 186 = SA \end{array}$$

$$6 + 8h(h-1) = SA$$

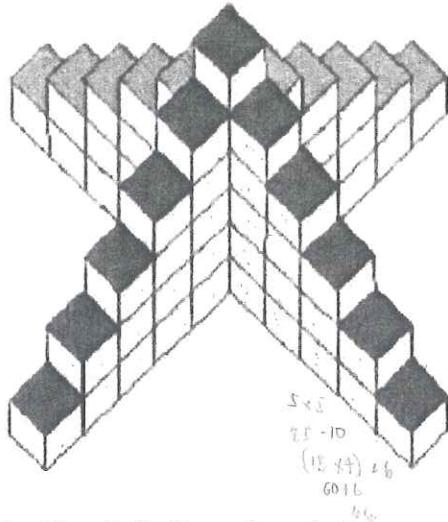
$$6 + 12h(h-1) = SA$$

$$2 + 12(1) = SA$$

$$\begin{array}{r} 2 + 12 \\ \hline 14 \end{array}$$

\* We counted  
the bottom

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1)

The cube tower has equal stacks on each of the four sides and shares a common stack in the middle. I noticed that the number of cubes reduced by one as it rises.

What <sup>↑</sup> reduced by one?

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{aligned} & 6^2 + (6-1)6 \\ & = 6^2 + 5(6) \\ & = 36 + 30 \\ & = 66 \end{aligned}$$

I multiplied the number of rows by the number of columns of one stack. Great work/explanation  
Subtract the total number of rows and columns from the product.  
Multiply the answer by four to find the number of cubes on the other stacks and add 6 to get the middle inclusive.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1)

- My strategy would work easily to find the number of cubes for a tower with height of 300.  
- Following this strategy can lead to finding the answer despite the several steps involved.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$n=6$

$$\begin{aligned} & = n^2 + (n-1)n \\ & = 6^2 + (6-1)6 \\ & = 36 + (5)6 \\ & = 36 + 30 \\ & = 66 \end{aligned}$$

+1.5 Show more work how you got  $n^2 + (n-1)n$

How did you get this? Show work allows the reader to follow your process.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$n^2 + (n-1)n = 19,900$$

$$n^2 + n^2 - n = 19,900$$

$$2n^2 - n = 19900$$

$$n(2n-1) = 19900$$

$$= 2n^2 - n - 19900$$

$$(2n+199)(n-100)$$

$$n = 100$$

+4,5

Provide more explanation.

Written Explanation would be great for the reader. It helps the reader follow your process and help troubleshoot if you went wrong somewhere.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$= 15 \times (8)$$

+0

120,

$$= 120 \cdot (4)$$

$$480 + 36$$

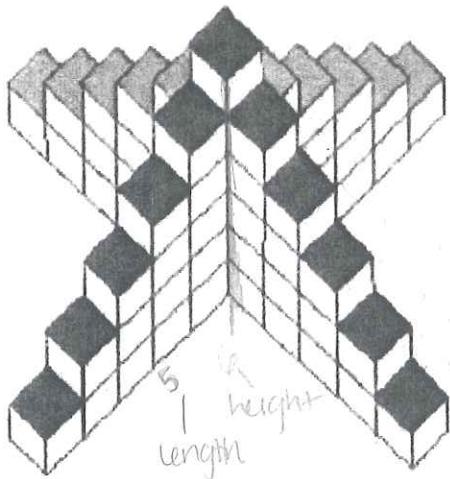
$$= 516$$

**Building Towers (Yellow Worksheet)**  
10 points possible

7/10

Name: Chalaun Lomax  
Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



originally a  $5 \times 5$  but missing 10 cubes  
so, area = 25 units $^2$   
# of cubes is  $n^2 - 2n$   
for each branch  
+ 6 future towers  
in center  
 $4(n^2 - 2n) + 6 = \text{total #}$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

The length of the sides of the structure is one less than the height of the tower (if you exclude the center column of the tower).

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

height - 1 = width  
 $h = h + 1$   
# of cubes

The sides surrounding the tower will have 4 sides created a square with side 5 units long. However, there are 10 cubes missing. So the formula would be  $5 \times 5 - 25$  or  $n^2 - n^2$ . It's missing 10 cubes so  $2(5) = 10$

$$\begin{aligned} & \text{4 sides } \cancel{5} \\ & \text{# of cubes per side is } (n^2 - 2n) \\ & 4(n^2 - 2n) + h = 4(25 - 10) + 6 \\ & \text{height adding 6 (cubes) because if the height of the tower is } 6, \text{ it would be } 10 \text{ cubes needed} \end{aligned}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

The strategy wouldn't work easily because once the tower is so large, it would be more difficult to find the number of cubes needed for each side. Formula only works easily when given a diagram and tower is smaller.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$x$  = length of cubes (measured in cubes)

$$\text{height} = n$$

$$x + 1 = n$$

$$\text{ex. } n = 6$$

$$\checkmark n = x + 1$$

$$6 = x + 1$$

$$5 = x$$

$$4(x^2 - 2x) + n = \# \text{ of cubes}$$

$$4((6)^2 - 2(5)) + 6$$

$$4(25 - 10) + 6$$

$$4(15) + 6$$

$$60 + 6$$

$$66 = \# \text{ of cubes}$$

← Wouldn't work if  $n = 2$

← When creating a generalized formula you should only use 1 variable.

This situation you are looking for # of cubes w.r.t. the height.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\begin{aligned} \text{Total # of cubes} &= 19,900 \\ n = x-1 & \\ 4(x^2 - 2x) + n &= 19,900 \\ 4 \text{ sides of one structure} & \\ = 4975 \text{ cubes} & \end{aligned}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-19900)}}{2(4)}$$

$$x = \frac{7 \pm \sqrt{49 + 318400}}{8}$$

$$x = \frac{7 \pm \sqrt{318449}}{8}$$

$$\begin{aligned} 4(x^2 - 2x) + n &= 19900 \\ 4x^2 - 8x - 19900 &= -n \end{aligned}$$

$$4(x^2 - 2x) + n = \# \text{ of cubes}$$

$$4(x^2 - 2x) + n = 19900$$

$$\frac{4(x^2 - 2x) + (x-1)}{4} = \frac{19900}{4}$$

$$x^2 - 2x + \frac{x-1}{4} = 4975$$

$$\frac{4x^2 - 8x + x - 1}{4} = 4975$$

$$\frac{4x^2 - 7x - 1}{4} = 4975$$

$$4x^2 - 7x - 1 = 19900$$

$$4x^2 - 7x - 19901 = 0$$

(+3)

This should have prompted you to recheck your formula in #3.

Correct strategy and work but the ~~stating~~ equation wasn't correct.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

## Building Towers (Yellow Worksheet)

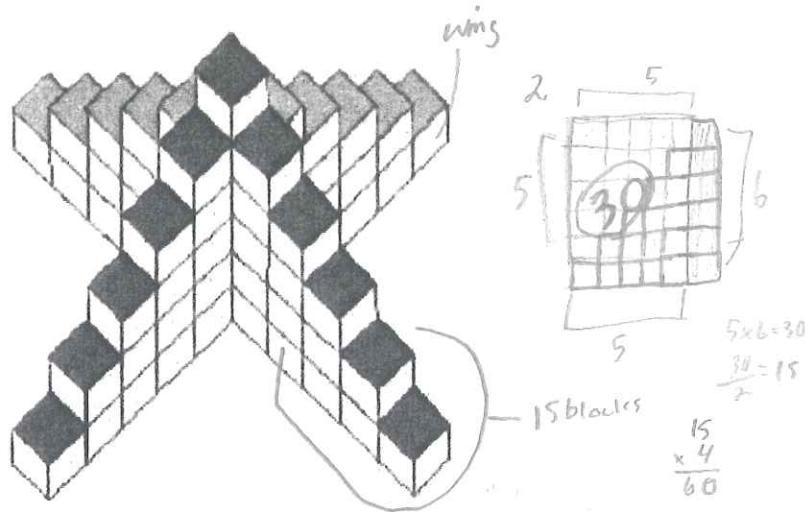
10 points possible

8.5 / 10

Name: Katelyn Beisigopoulos

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

The structure is symmetrical, with four ~~wings~~ branches or segments (wings) off of the center. Each wing has the same amount of blocks and the same structure or shape.

Good Observation

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

I divided the tower into 5 groups, a straight tower of 6 cubes and the 4 remaining sections. Each of the four have a base & height of 5. If one wing of the tower was put on top of another, a rectangle that is a 5x6. A rectangle like this has an area of 30 cubes, so one wing has 15 cubes. Multiply that by 4, and the total of blocks in all the wings is 60. Then the remaining 6 is added.

The total is 66 cubes.

$$\begin{array}{r} 15 \text{ blocks} \\ \times 4 \\ \hline 60 \end{array}$$

Great Explanation!

+1

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? Not necessarily, because it technically doesn't give the base of the "wings". However, it may be assumed the height is always one taller than the length of the wing extensions, so the base could be found and then the rest can be found from that. However, its definitely not an "easy" method time-wise.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1.5

$n$ =height

C=cubes amount

$$\begin{array}{l} n+2((n-1)(n)) = C \\ \text{or} \\ n + 2(n^2 - n) = C \end{array}$$

$$\begin{aligned} 6 + 2(5(6)) &= 66 \\ 6 + 2(30) &= 66 \\ 6 + 60 &= 66 \\ 66 &= 66 \end{aligned}$$

Where did you get this?  
More work.

Show your steps and process will help strengthen your answer.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\begin{array}{l} \text{Cubes } C = 19,900 \\ \text{n-height } n = ? \end{array}$$

$$n + 2(n^2 - n) = C$$

$$n + 2(n^2 - n) = 19,900 *$$

$$n + 2n^2 - 2n = 19,900$$

$$2n^2 - n = 19,900$$

$$2n^2 - n - 19,900 = 0$$

$$(2n+1)(n-100) = 0$$

$$2n+1 = 0 \quad n-100 = 0$$

$$\begin{array}{r} 100,000 : 19,900 \\ \underline{-99} \\ 100,000 \\ \underline{-99,000} \\ 100 \end{array}$$

The method is to use the formula from previous question and then figure out what is known. Using the formula  $n + 2(n^2 - n) = C$  and the variable  $n$ -height + c= amount of cube total, we know the height should be found. The number of cubes is already known to be 19,900, which replaces C while the variable n remains. The formula becomes  $n + 2(n^2 - n) = 19,900$ . Then it is carried out the 2, making it  $2n^2 - 2n = 19,900$ . After that, subtract the 19,900 and factor it out.

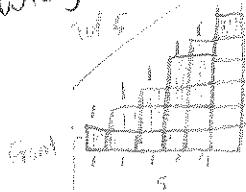


Good start, you are on the right path.

The quadratic equation should help you find the factors.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) ~~excluding the bottom of the base?~~

+1  
Wrong answer, but interesting approach,



$$\begin{array}{r} 5 \\ 5 \\ \hline 25 + 6 = 31 \end{array}$$

$$\begin{array}{r} 15 \\ 2 \\ \hline 30 \end{array} \text{ for each side of middle}$$

$$\begin{array}{r} 30 \\ 5 \\ \hline 150 \\ 150 \times 4 = 600 \end{array}$$

$$144 + 3 = 147$$

$$147 + 21 = 168$$

The original tower has one central tower part of 6 stacked cubes with 4 wings of 15 blocks extending outwards. Based on how they are connected to the middle, only the top and four sides of the middle tower's bottom cube will be painted; a total of 2 faces. Each wing has 15 blocks, so on the side (showing all 15 blocks) there are 15 faces to be painted. This is multiplied by 2 for each side of the wings, with 30 faces. Then on top, 5 faces are showing and one on the front for a total of 5+30. So on each wing, 30 faces are painted. There are 4 wings so 120 faces are painted. This is added to the 5 faces from the top cube and in all, excluding the bottom, 125 faces are painted.

Including the bottom, there are 5 bottom faces on each wing and 20 on all the wings. In addition the middle column has one bottom face. So including the bottom, 160 faces are painted.

Building Towers (Yellow Worksheet)

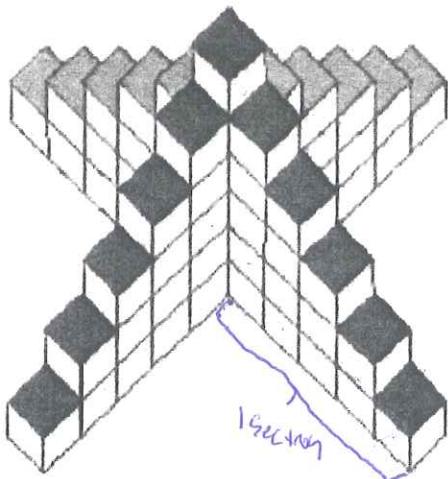
10 points possible

10.5/10

Name: WYC Galletta

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 Each successive layer of the pyramid shaped structure is one more block wider than the last. Starts with 1 block and goes in 6 blocks.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$1+2+3+4+5 = 15 + 4 = 60 + 6 = 66 \text{ blocks}$$

↑  
the  
sections  
with 6 blocks  
of sections

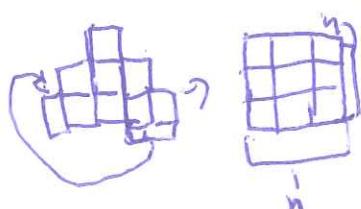
↑  
center  
section

Great equation.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 No because it would take along time to add up the 300 blocks per section.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.



$2n^2 - n$

Other reasoning showing  $2n^2 - n$  due to always

$2(6)^2 - 6$

$2(36) - 6$

$72 - 6 = 66$

be 3 columns in 6 rows

+2

To just be careful  
show more work to  
help show your  
work to practice

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = 19,900 \quad \text{Set } = \text{to # of blocks}$$

~~2n^2 - n - 19,900 = 0~~

↓

$$(2n+1)(n-100)$$

~~(2n+1)(n-100)~~

$n=100, -99$

$n=100$

How did you find this?

+4.5

Provide more support.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)



$$4n + n + 3(n-1)$$

Right Answer  
I don't know how  
you got there

$$4(2(n^2-n))$$

$$4(6) + 6 + 3(4-1)$$

$$24 + 6 + 15$$

(15)

$$8(n^2-n) + 4n + 2(n^2-n)$$



$$(8(n^2-n) + 9n + 3(4-1))$$

$$16S_{\text{surf}} = \text{surface area}$$

$$8(36+6) + 30 + 3(4-1)$$

$$8(30) + 30 + 15$$

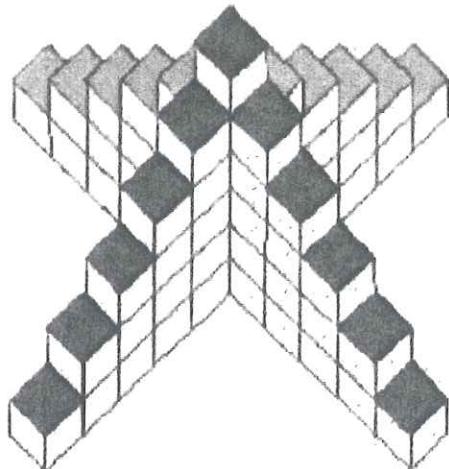
$$240 + 30 + 15$$

Building Towers (Yellow Worksheet)  
10 points possible

W/10

Name: Andy Cress  
Date: March 5<sup>th</sup>, 2014

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{aligned}6 \times 6 &= 36 \\5 \times 6 &= 30\end{aligned}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 There is one stack of cubes in the middle that is 6 cubes tall.

+1 There are four equal sections of cubes surrounding it.

The four sections are built like stairs and contain 15 cubes each.

- 2) Reference the diagram shown above:

a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1  $6 + 4(15)$   
 $6 + 60$   
(66)

It takes 66 cubes to build the tower.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 No. I counted the cubes on one of the sections.  
It would take way longer if it had a height of 300.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n + 4(1+2+3+4+\dots+n) \quad \leftarrow \text{This is a formula but is it the most efficient?}$$

Count by ones until you get to one less than  $n$ .

$$\begin{aligned}6 + 4(1+2+3+4+5) \\6 + 4(15) = 66\end{aligned}$$

+2 Series aren't that efficient. Your equation in #4 would be a better formula.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$n+4(1+2\ldots)n = 19,900$$

$$\begin{aligned} n^2 + (n-1)(n) &= 19,900 \\ n^2 + n^2 - n &= 19,900 \\ 2n^2 - n - 19,900 &= 0 \\ n = 100 \end{aligned}$$

Where did you get this formula?

If you move the cubes, you can make two squares. One will be  $n \times n$  if  $n = \text{height}$ , and the other will be  $(n-1) \times n$ .

Nevermind

How did you get  $n=100$ ? What process did you do?

$$2(100)^2 = 100$$

$$2(10,000) = 100$$

$$20,000 - 100 = 19,900$$

+4

The height of the tower is 100 cubes.

More explanation would help people follow your work and understand.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

The stack in the middle is covered except for 5 faces on top and one on bottom.

Good job!

$$6 + 4(45)$$

There are 45 faces showing on each of the 4 remaining sections.

$$6 + 4(45)$$

$$6 + (180)$$

$$SA = 186$$

Add to Bonus

There will always be 6 faces showing on the middle stack.

$$6 + 4((3n-1) + 2(n-1) + 2 + 2(n-2) + 2 \dots \text{until } n=0)$$

↑                    ↓ bottom rows

4 Sections = to each other

$$+ 2(n-3) + 2 \dots + 2(n-4) + 2$$

bottom row of 4 sections

rest of the rows of each section

$x = \text{number in row}$

$2x + 2 = \text{cubes showing}$

$$3(n-1) + 2$$

$$3n-3 + 2$$

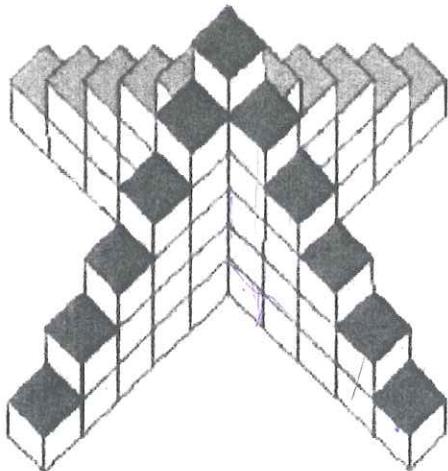
3n-1

Building Towers (Yellow Worksheet)  
10 points possible

3/10

Name: Ricky Ouel  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 I + has 6 levels, all with a height of  
7-n if n is the first level (at the top) and 2n is the  
2<sup>nd</sup> level, etc.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$$\begin{array}{cccccc} 6 & + & 5(4) & + & 4(4) & + 3(4) + 2(4) + 1(4) = 6+20+16+12+8+4=66 \\ \uparrow & & \uparrow & & \uparrow & \uparrow \\ \text{center} & \text{2nd level} & \text{3rd level} & \text{4th level} & \text{5th level} & \text{6th level} \end{array}$$

Nice labeling.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

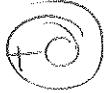
+1 No because it would take forever to write the equation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+0

If you have questions, make sure to ask the teacher or a fellow classmate.

4. **(5 points - see rubric)** Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.



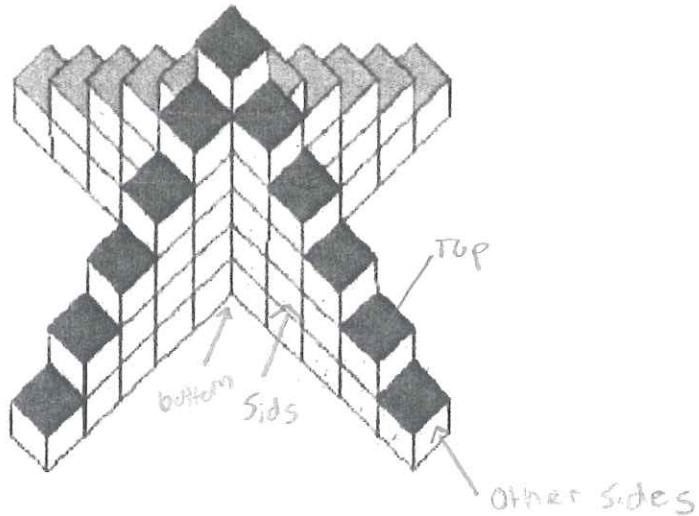
5. **(Bonus - up to 2 points).** Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (*In other words, if I paint the original tower, how many square faces would be covered with paint?*)

Building Towers (Yellow Worksheet)  
10 points possible

10.5 / 10

Name: Corey Denham  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



Good Observation.

- 1) (1 point) Describe what you notice about the structure of the tower above.

(+) The block tower has both horizontal, vertical,  $\frac{1}{2}$  radial symmetry. It follows a continuous pattern of adding one block until it reaches the Peak of the Structure.

- 2) Reference the diagram shown above:

(+) a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Each section of the structure has 15 cubes (excluding the 6 in the middle). Thus  $15 \times 4 = 60$  blocks + the additional 6 = 66 blocks in total.

b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+) No it would not because I found the block numbers through observation. A formula is required to find one with a height of 300 blocks. *Great Point!*

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Are you including the center tower?

Total =  $4\left(\frac{1}{2}(n-1)(n)\right) + 6$  [the area of the branches is  $(n-1) \cdot (n)$  and dividing by 2 will give the amount of blocks.]  
Make sure you check your formula with  $n=6$ . Then multiply by 4 to add the center piece.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

The formula used to determine the total # of blocks can now be used in reverse order.

Then multiply the equation out to solve using quadratic formula. One answer is negative, which is an impossibility within the realm of the real world. Therefore, the answer must be an 100 block high.

$$19900 = 4\left(\frac{1}{2}(n-1)(n)\right) + n$$

$$19900 = 2n^2 - 2n + n$$

$$19900 = 2n^2 - n$$

$$2n^2 - n + 19900 = 0$$

$$(2n+99)(n-100) = 0$$

$$n = 100$$

+5

Good Job!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

0/2 +1

Good Attempt but

didn't provide an answer for height of 6.

The surface area can be found by using height + 1 as a variable.

$$SA = 8\left(\frac{1}{2}(n-1)(n)\right) + (4(n-1)+1) + (4(n-1)+1) + (4(n-1)+4)$$

Sides                      bottom                      Top                      sides (other)



Simplified version:

By multiplying  $(n-1)(n)$   
an area is achieved then  
multiply by 8 to  
account for all the  
sides

$$d=4 \quad \begin{array}{r} 5+1 \\ \hline 6 \quad 1 \\ 15 \quad 3 \end{array} \quad \begin{array}{r} 5+1 \\ \hline 5 \quad 1 \\ 15 \quad 5 \end{array} = \frac{1}{2}(n^2 - 6, 66)$$

### Building Towers (Yellow Worksheet)

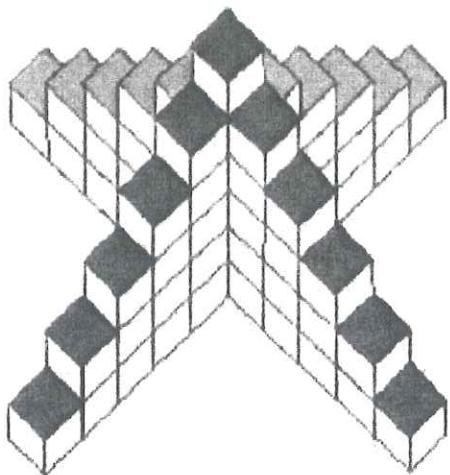
10 points possible

$$4(n-1) + n$$

Name: Mark Lienhardt

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{array}{c|c} 5 & n \\ \hline 1 & 1 \\ 5 & 2-10 \\ 15 & 3-27 \\ 28 & 9-52 \end{array}$$

$$(4n-3)$$

$$n(4-\frac{3}{n})$$

$$x = d(n-1) + n$$

$$n^2(4n-3)$$

$$\cancel{1 + (4(n-1))} \quad \cancel{n(4n-3)}$$

$$2 + n(4n-3) - 1$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1)

- There are four congruent legs to the tower
- Each leg contains 15 blocks
- There is a stack of 6 blocks in the middle of the tower
- The tower has 66 blocks

- 2) Reference the diagram shown above:

(+1)

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Each layer of the tower contains 4 blocks less than the previous. The bottom layer contains 21 blocks (5 on each leg, and 1 in the middle).  $21 + 17 + 13 + 9 + 5 + 1 = 66$  blocks

Good Job!

(+1)

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? Yes, the equation to solve for the total number of blocks is  $n(n+4)$  where  $n$  is the height. You can do this because each layer is a number in an arithmetic series.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

(+0)

For generalized formulas, the formula should work for all  $n$ 's. Check with  $n=1, n=2, \dots$

$C = \# \text{ of cubes}$

$$C = 4n - 3 \leftarrow n=6, \text{ would get}$$

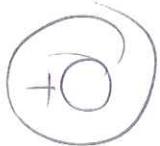
you 21 cubes.

Not 66 cubes.

Also provide work. It helps people see where you might have gone wrong.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

If you have questions, ask Mr. Dumm  
or a classmate.

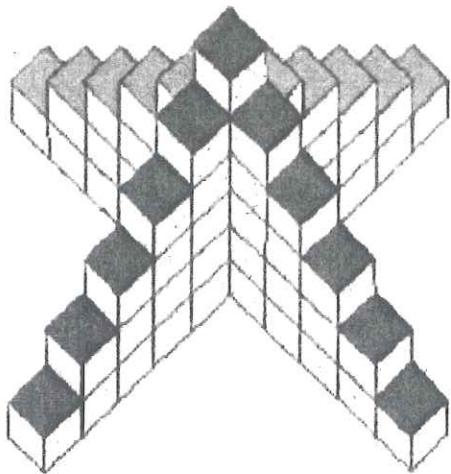


5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

**Building Towers (Yellow Worksheet)**  
10 points possible

Name: Ashley Lee  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



Great  
Observations!

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 Starting from the column of cubes in the very middle (Top cube), the structure changes from a six-cube column all the way to a one-cube column, as it "stretches out". Eliminating the top cube of the six-cube column in the middle, each side of the middle column is bounded by a five-cube column of the "side walls". Again, eliminating the top cube of the middle column, each layer of cubes increases by a number of four as you go from top to bottom.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1 There's 6 layers in total, eliminating the top cube of the middle column, the first layer has 5 cubes. As I wrote above, the # of cubes each layer has increases by four everytime you move down a layer. 6 layers in total

$$\therefore 1 (\text{Topcube}) + 5 (\text{1st layer}) + 9 (\text{2nd}) + 13 (\text{3rd}) + 17 (\text{4th}) + 21 (\text{5th}) = \boxed{66 \text{ (cubes)}}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 I won't be easy to find the # of cubes for a tower that has a height of 300 with my strategy unless I make it into a formula using n as a representation for real numbers.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$n = 1$	$\# = 1$	change
2	6	$\times 1$
3	15	$\times 3$
4	28	$\times 5$

$$\therefore \text{Formula: } n(2n-1) = \#\text{ of cubes}$$

Plug in 6

$$6(2 \times 6 - 1) = 66 \text{ (cubes)} \quad (\checkmark)$$

Good job using the table help you find the formula.

+2

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

Using the formula:  $n(2n-1) = \# \text{ of cubes}$ ,

$$n(2n-1) = 19,900$$

[Plug in 19,900 cubes]

$$2n^2 - n = 19,900$$

[Solve]

$$2n^2 - n - 19,900 = 0$$

[Quadratic Formula]

$$\boxed{n=100} \quad n = -99.5 \text{ (nonreal)} (x)$$

∴ The height of this tower would be 100.

Strategy: Plug Given information into formula

Awesome  
Step by step  
Explanation!

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+0.5

If each side is 1.

Area of 1 side of a cube is  $1^2 = 1$ .

Good Start, but  
Your strategy wasn't complete.  
 $A \text{ of Top cube} = 5$ .

add up side of cubes

$$5 + (12+4) + (4+16).$$

$$5 + (4 \times 3 + 4) + (4 + 4 \times 4) + (4 + 6 \times 4) + (4 + 8 \times 4) + (4 + 10 \times 4)$$

$$= 5 + 16 + 20 + 28 + 36 + 44.$$

$$\boxed{= 149.}$$

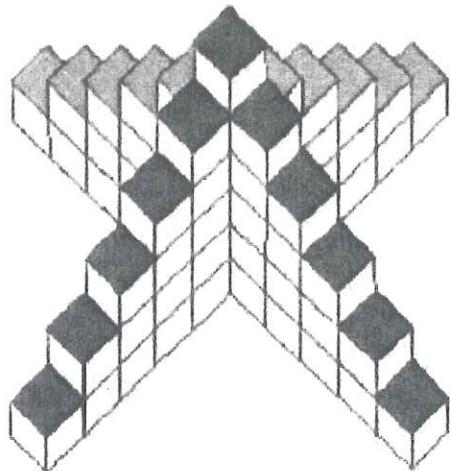
Area of tower is 149.

Building Towers (Yellow Worksheet)  
10 points possible

5, 5/10

Name: Lukas Lenninger  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

The tower above is a single tower of 6 blocks surrounded by four structures made up of 5 blocks; for each block outward, their height is reduced by one.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+0.5

66

$$6 + 4 \left( \frac{1}{2} (5^2 + 5) \right)$$

Please provide your work/process.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? Yes, n+1 could be substituted for 6, and n for 5

+0.5

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.  $n = \text{height}$

+0.5

$$(n+1) + 4 \left( \frac{1}{2} (n^2 + n) \right) \leftarrow \text{Wouldn't work if } A=1 \text{ or } n=6$$

Make sure you check your generalized formula. If your formula is incorrect, 3-5 checks will probably show an incorrect output.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

To find the height

$$19900 = n + 1 + 2n^2 + 2n$$

I simply set

$$19900 = 2n^2 + 3n + 1$$

$$0 = 2n^2 + 3n - 19899$$

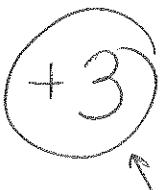
I then equal to my formula and solved for  $n$ , which represents height.

$$\circ (2n+201)(n-99)$$

99 blocks

Used the equation from 3, but it was the incorrect formula.  
If you check  $n=6$ , you wouldn't get 66.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)



**Building Towers (Yellow Worksheet)**

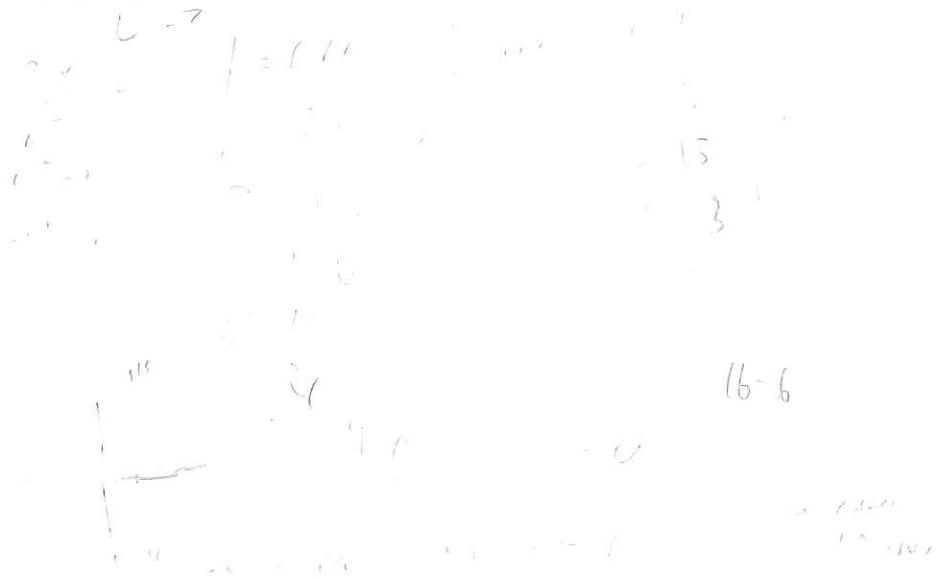
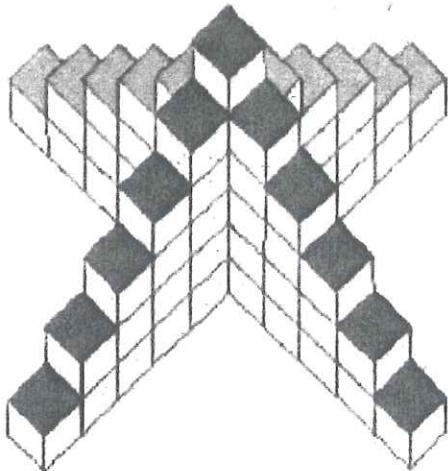
**10 points possible**

11/10

Name: Shawn Liekheit

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 The structure is in a pyramid-like shape and the number of blocks in each row increases by 4 as a new row is observed.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1  $1+5+9+13+17+21=66$  I used my formula to find the number of blocks in the nth row then added the sum of blocks in each row together.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 No, because my strategy works well for finding smaller heights for the number of cubes in a row. ↗ Good Observation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Work helps support your formula. You would need  $2n^2-n$

+1 cubes to construct a tower of height  $n$ . This formula

Show work on a tower of height 6 units.  
the work for  $n=6$ . Also how did you create  $2n^2-n$ ?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$\begin{cases} 0 = 2n^2 - n - 19900 \\ (n-100)(2n+199) = 0 \end{cases}$$

$$n = 100$$

I set my equation equal to

200 using the formula that I created. Then, I used the quadratic formula,  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , to

find two answers,  $\frac{-199}{2}$  and 100. A tower cannot be composed of negative rows, so the tower is 100 rows tall.

Good Job!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) Note: My base is built on the ground and the bottom will not be painted.

+2

The surface area is 165 units<sup>2</sup>. I

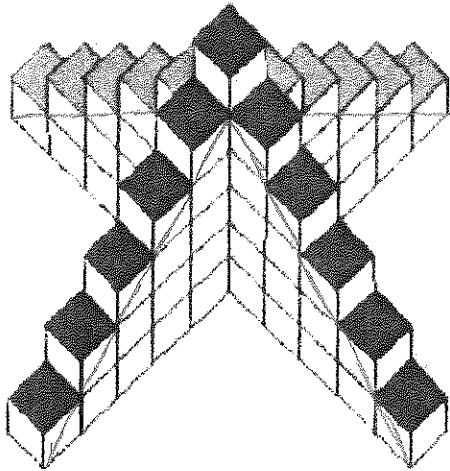
found that the difference of the difference between each row is 8. I used this to find the surface area of a tower with a height of 6.

Building Towers (Yellow Worksheet)  
10 points possible

9/10

Name: Grant Mays  
Date: 3-5-14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 The tower consists of a center  $n$  units tall and four sets of steps, each  $n-1$  units long at the base, connected to the faces of the tower. The steps start one unit shorter than the tower, and become one unit shorter for each unit they extend.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$$6 + 4\left(\left(\frac{(6-1)^2}{2}\right) + \frac{6-1}{2}\right)$$

$$6 + 4\left(\left(\frac{25}{2}\right) + \frac{5}{2}\right)$$

$$6 + 4(15) = 6 + 60 = 66 \text{ cubes}$$

The height of the central column      The number of cubes of one set of steps, if the top cubes had been cut into a slant,

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 Yes. The hardest part would be calculating  $(299)^2$ , but are there easier ways? that can be multiplied fairly easily.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

# cubes =  $n + 4\left(\left(\frac{(n-1)^2}{2}\right) + \frac{n-1}{2}\right)$

+2 See work in

2 a.) but make sure you include work down here

$$\# \text{ cubes} = 6 + 4\left(\left(\frac{(6-1)^2}{2}\right) + \frac{6-1}{2}\right)$$

$$\# \text{ cubes} = 6 + 4\left(\left(\frac{25}{2}\right) + \frac{5}{2}\right)$$

$$\# \text{ cubes} = 6 + 4(15)$$

$$\# \text{ cubes} = 6 + 60$$

$$\# \text{ cubes} = 66$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = n + 4\left(\left(\frac{(n-1)^2}{2}\right) + \frac{n-1}{2}\right)$$

$$19900 = n + 2(n-1)^2 + 2(n-1)$$

$$19900 = n + 2n^2 - 4n$$

$$19900 = 2n^2 - 3n$$

$$0 = 2n^2 - 3n - 19900$$

$$0 = (2n-1)(n- )$$

Be careful  
with your  
math  
operations.

+2

Quadratic Equation  
 $2n^2 - n - 19900$ .

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

Good Job!

$$SA = 8(15) + 4(5) + 4(5) + 5$$

$$SA = 120 + 20 + 20 + 5$$

$$SA = 165$$

Not including the bottom

$$SA = 165 + 21$$

$$SA = 186, \text{ if including the bottom.}$$

The fronts of each step in one set.  
The bottom faces, if included.

$$SA = 8(15) + 4(5) + 4(5) + 5 + (4(5) + 1)$$

One side of one set of steps.  
The tops of each step in one set.

The open faces of the top block.

To find the surface area of a tower with height  $n$ .

4 sets of steps  
2 faces per set

Top 3 of one set

Fronts of one set

The open faces of top block

IF bottom is included.

$$SA = 4\left[2\left(\frac{(n-1)^2}{2} + \frac{n-1}{2}\right) + (n-1) + (n-1)\right] + 5 + (4(n-1) + 1)$$

The area of one side of a set

$$SA = 4(n-1)^2 + 12(n-1) + 5 + (4n-4+1)$$

$$SA = 4n^2 + 4n - 14 + (4n+3) \quad \text{General Equation}$$

Building Towers (Yellow Worksheet)

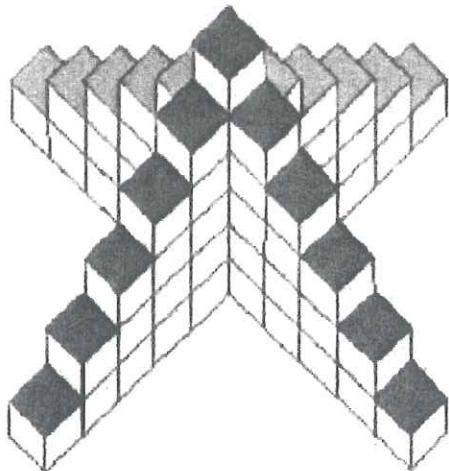
10 points possible

10/10

Name: Nick Zahneis

Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

It is a 3-D structure with radial symmetry as well as vertical & horizontal. It has 4 sections starting with 1 cube, and increases until it reaches a height of 6 cubes, where all 4 sections connect.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$$6+4(5+4+3+2+1)$$

$$6+4(15)$$

$$6+60$$

There is one height of 6, and four heights of 5-1. So, I added 1-5 together (15), multiplied that by the number of heights (4), then added 6.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

would be lots of adding (1-299) before I could get to the answer.

Good Observation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+1.5

$$n(2n-1)$$

$$6(12-1)$$

$$66$$

how did you get this?  
Show work.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

Great Work!

$$\begin{aligned}
 19900 &= n(2n-1) \\
 19900 &= 2n^2 - n \\
 2n^2 - n - 19900 &= 0 \\
 \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} & \\
 \frac{1 \pm \sqrt{1 + 4(2)(19900)}}{4} & \\
 \frac{1 \pm \sqrt{1 + 159200}}{4} & \\
 \frac{1 \pm \sqrt{159201}}{4} & \\
 \frac{1 \pm 399}{4} & \\
 \frac{400}{4} & = 100
 \end{aligned}$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$\begin{array}{lll}
 n=1 & n=2 & n=3 \\
 6 & 26 & 50 \\
 10 & 24 & 28 \\
 20 & 24 & 32 \\
 20+4(n-1) & &
 \end{array}$$

The height is 100.

If the formula previously used results in the number of cubes in the tower, you can set the formula equal to 19,900 and solve for  $n$ . I set one side equal to 0, then used the quadratic formula to reach 100.

Great Explanation.

146 faces. The difference between each height's surface area starts at 20 and increases by 4 from there.

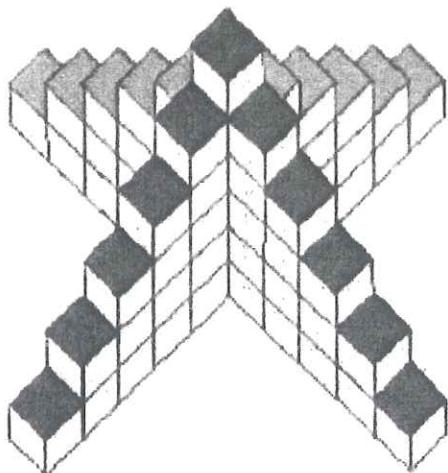
+0.5

Good start of strategy but check again - Not correct solution.

11/10

Name: Meghan Ullrich  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1) The height of the center is 6 cubes with stacks of cubes coming from 4 sides, each decreasing in height by 1 cube.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

(+1)  $6 + 4(5) + 4(4) + 4(3) + 4(2) + 4(1) = 66 \text{ cubes}$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1) No because it would require writing out  $4(x)$  for every integer from 1-299. ← Great Observation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

(+1.5)  $n + 2n(n-1)$  or  $2n^2 - n$

Each side of the tower is half of a  $n \times (n-1)$  rectangle, cut down the diagonal.  $n(n+1)$  is divided in half for the half rectangle and then multiplied by 4 for the 4 sides.  $n$  is added because the center stack wasn't included in any of the rectangles. This creates the formula  $n + 2n(n-1)$ , which can be simplified into  $2n^2 - n$ .

Check with  
 $n=6$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.  $\text{height} = n$ ,  $19,900 = 2n^2 - n$

$$2n^2 - n - 19,900 = 0$$

$$\frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{4}$$

$$\frac{1 \pm 399}{4}$$

$$n = 100$$

$$+ 45$$

Show ~~steps~~  
Process and steps.

Provide explanation. (i.e. ~~quadratic formula~~)

The height is 100 cubes, which comes from setting the formula for the number of cubes, with height  $n$ , equal to the given number of cubes, 19,900.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$8\left(\frac{n(n-1)}{2}\right) + 8n - 3$$

$$165 \text{ square units}$$

Good Job!

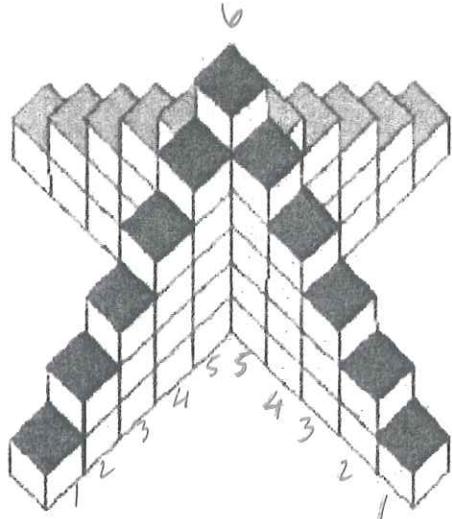
The same half rectangle method was used, this time multiplying by 8 for the 2 lateral sides of the 4 sections. The  $8n - 3$  comes from the 2 exposed surfaces of the top cubes.  $n$  cubes in each section  $\times$  2 exposed surfaces  $\times$  4 sections. 3 is subtracted because the top surface of the center cube was counted 4 times when it should be counted once.

Building Towers (Yellow Worksheet)  
10 points possible

12/10

Name: Linh Do  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



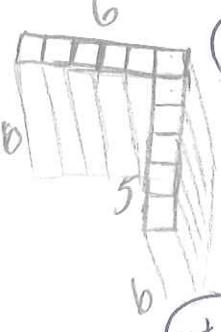
- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

The structure above is a tower with a height of 6. This tower has 4 legs of 15 cubes that creates a stairway of 1 step, 2 steps, 3, 4, 5 steps. to a center of 6 cubes.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.



If combine two legs (of the 4) It will form a perfect square of  $6 \times 6$ . If I do it to the other two I will also get a  $6 \times 6$ , however the two square share 1 row.  
← using the picture on the left. I can multiply  $6 \times 6 = 36$  and a  $6 \times 5 = 30$  and add them together.  $36 + 30 = 66$  cubes

The two legs  
get you a  
 $6 \times 5$  rectangle  
The perfect  
square comes from  
the center  
column

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? yes, with a tower of 300. I

can form two square of  $300 \times 300$  that shares one row.

+1  
Good  
Justification

To find the numbers of cubes I can multiply  $300 \times 300 = 90000$ , and a  $300 \times 299 = 89700$  add them together.  
 $90000 + 89700 = 179700$  cubes

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+2

$$n^2 + n^2 - n$$

$$2n^2 - n = \# \text{ of cubes}$$

You ~~never showed~~  
your work above but  
make sure you  
reference it down  
here to explain  
why

$$(b \cdot b) + (b \cdot b) - b \quad 2(b^2) - b$$

$$b^2 + b^2 - b$$

$$36 + 36 - 6$$

$$66 \checkmark$$

$$2(36) - b$$

$$72 - b$$

$$66 \checkmark$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

using my formula. if  $2n^2 - n = \# \text{ of cubes}$   
 I can substitute 19,900 for the # of cubes and  
 solve for  $n$ .

$$19,900 = 2n^2 - n$$

Reference quadratic formula for this step:

$$\begin{aligned} 2n^2 - n - 19,900 &= 0 \\ (2n - 200)(n + 99.5) &= 0 \\ 2n - 200 &= 0 \quad n + 99.5 = 0 \\ n = 100 & \quad n = -99.5 \end{aligned}$$

+5

Awesome!

check answer

$$\begin{aligned} 2(n^2) - n &= \# \text{ of cubes} \\ 2(100^2) - 100 & \\ 2(10000) - 100 & \\ 20000 - 100 & \\ 19900 & \end{aligned}$$

Excellent observation.

We can eliminate the negative answer because we can't have a negative height. Therefore the answer is 100. (height)

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

ANSWER IS  $165 \text{ units}^2$   
 is the surface area,  
 not including the  
 base. If I include  
 the base, theres

$$(5 \times 4) + 1 = 20 + 1$$

$$5 = 5$$

$$5 \times 4 = 20$$

$$186 \text{ units}^2$$

surface area  
 including the top  
 and bottom.



Surface area of a 6 height tower  
 each leg has two faces, with  
 rows of 5,4,3,2,1 If I add Great  
 them together. I get 15. Explanation  
 each leg has 2 sides so and showing  
 $15 \cdot 2 = 30$  at the top of each thought  
 steps there is a surface for process!  
 each row. and an edge.

$$5 = 5 \text{ surface}, 5 \text{ edges} = 5 \text{ more surface } S(2) = 10$$

each leg has  $(30 + 10) = 40$   
 cube surface.  $40 \times 4$  (b/c there  
 are 4 legs.  $= 160$ . but there  
 is also 4 cube at the center  
 that has one surface cover  
 so 5 surface shown.  $160 + 5 = 165$

Building Towers (Yellow Worksheet)

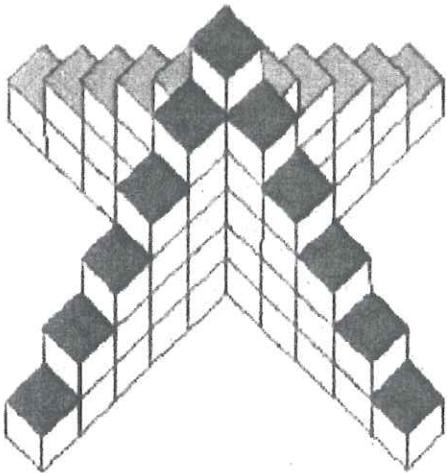
10 points possible

3.5/10

Name: Eddales Horinwall Naumann

Date: 3/5/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



height	total number of cubes
2	6
3	15
4	28
5	45

$$\text{Height } 2: 6 = 2 + 4(2-1)$$

$$\text{Height } 3: 15 = 3 + 4(3-1)(3-2) \quad z = 4x \\ x = 3$$

Height 2: 1 cube on each side

Height 3: 3 cubes on each side

Height 4: 6 cubes on each side

Height 5: 10 cubes on each side

Good job taking time to understand the structure.

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower is built with two congruent towers of blocks that intersect at the middle column, creating a single tower with four points coming off of it,  $90^\circ$  apart.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$1 \text{ column of } 6 = 6 \text{ cubes}$$

$$4 \text{ columns of } 5 = 20 \text{ cubes}$$

$$4 \text{ columns of } 4 = 16 \text{ cubes}$$

$$4 \text{ columns of } 3 = 12 \text{ cubes}$$

$$4 \text{ columns of } 2 = 8 \text{ cubes}$$

$$4 \text{ columns of } 1 = 4 \text{ cubes}$$

$$6+20+16+12+8+4 = 66 \text{ total cubes}$$

I found the total number of cubes by finding the number of columns with a given number of cubes. Once I found the number of cubes in each side column, I added the total number of cubes together to get my answer.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? This would not be an easy way to find the number of cubes for a tower that has a height of 300 because counting all of the cubes and then adding them all together would not be efficient or logical.

Good job seeing how this strategy would be inefficient.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\text{number of cubes} = n + 4(n-1)$$

$$\begin{aligned} & 2+1+1+1+1 \\ & 3+2+2+2+2+1+1+1 \\ & 3+4+2+4+1 \end{aligned}$$

When creating a formula or generalized equation, you should test it to see if it works. Start of the process, but incomplete. Usually you will see an issue after two or three checks. equation.

+0.5

Check with

$n=6$ , you would

only get 26.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\begin{aligned} \text{Number of cubes} &= 19,900 + 4(19,900 - 1) \\ &= 99,406 \text{ cubes} \end{aligned}$$

+ 0



total      Find the height

Be careful to  
read the ~~height~~  
directions.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$2 + 4n = 17$$

Height of 2 cubes:  $2 + 4 \cdot 1 = 6$

3. Height of 3 cubes:  $3 + 4 \cdot 2 = 10$

$$6 + 4 = 10 + 4 = 14$$

Height of 4 cubes:  $4 + 4 \cdot 3 = 4 + 12 = 16$

$$6 + 8 + 4 = 16 + 4 = 20$$

Height of 5 cubes:  $5 + 4 \cdot 4 = 4 \cdot 3 + 4 \cdot 2 + 4 \cdot 1$

$$16 + 12 + 8 + 4 = 40$$

Height of 6 cubes:  $6 + 4 \cdot 5 = 4 \cdot 4 + 4 \cdot 3 + 4 \cdot 2 + 4 \cdot 1$

$$20 + 16 + 12 + 8 + 4$$

$$n = h + 4(6 + 5 + 4 + 3 + 2 + 1)$$

$$h + 4(21)$$

5 cubes:  $5 + 4(4 + 3 + 2 + 1)$     4 cubes:  $4 + 4(3 + 2 + 1)$

$$10$$

$$5$$

$$h + 4 \cdot n - 1$$

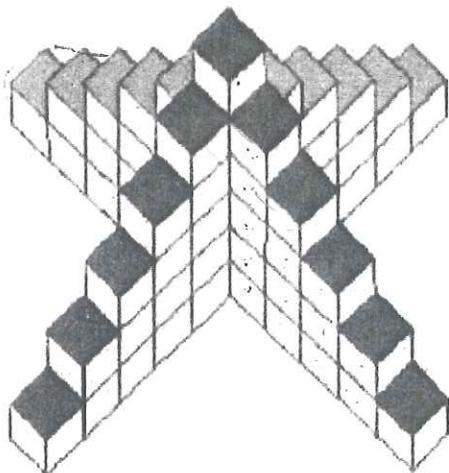
$\sqrt{7} \approx 2.64575$

Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: Brianne Muth  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

- The tower has rotational symmetry at intervals of  $90^\circ$  ← Didn't see this until grading.
  - has a height of six cubes
  - each leg has a height and width of 5 and built like a stairs
- Nice Observation!

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$$15(4) + 6 \quad \begin{array}{l} \text{First I counted the area of 1 leg. Then} \\ \text{since each of the 4 legs are the same, I multiplied the #} \\ \text{of cubes in 1 leg by 4 then added the 6 cubes in the middle} \\ \text{of the tower.} \end{array}$$

66 cubes

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

My strategy would not easily work for a tower with a height of 300 because I counted the # of cubes in 1 leg. This would take a long time for a leg that has a height of 299 cubes.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{array}{ll} 1:1 & \text{unit} \\ 2:6 & h=3 \\ 3:15 & h=5 \\ 4:28 & h=7 \\ 5:45 & h=9 \end{array}$$

$$A = h(2h-1) \quad \begin{array}{l} A = \# \text{ of cubes} \\ h = \text{height} \end{array}$$

$A = 2h^2 - h$

$$A = 2(6)^2 - 6$$

$$\begin{array}{l} A = 2(36) - 6 \\ A = 66 \end{array}$$

+1.5

Provide more work to explain how you got  
 $A = 2h^2 - h$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2h^2 - h$$

$$2h^2 - h - 19,900 = 0$$

$$(2h+199)(h-100) = 0$$

$$h = \frac{-199}{2}, 100$$

$$h = 100$$

Great  
Job stating  
this.

+5

Using my equation, I

could solve for the height.

I got a quadratic equation and factored. Of the two answers I found, one was negative. The height cannot be negative so therefore the height was 100.

Good job!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

b7

In painting the tower I will not paint the bottom. this.

The total surface area is 165 units<sup>2</sup>.

To find the surface area of the tower I did almost

the same thing as I did for the # of cubes. I multiplied the surface area of 1 leg (40) by 4. So I got 160. Then the surface area of the middle spine is 5 at each height so I added 5 to get a total of 165

Good job using  
a pre-existing  
strategy.

$$4((n^2 - (2-n)) + 5)$$

$$\begin{array}{r} 1: 5 \\ 2: 16 \\ 3: 25 \\ 4: 36 \\ 5: 45 \end{array}$$

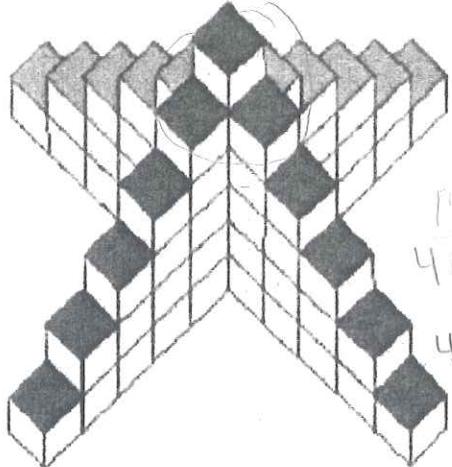
Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: Josh Oetters  
Date: \_\_\_\_\_

Per. 3

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$n=2$$

$$5+9+3 \cdot 4 + 4 \cdot 1 = 26$$

~~$$5+5+11+1$$~~

$$\begin{array}{l} \text{Base } 4 \cdot 3 \\ 4 \cdot 4 \cdot 3 + 1 \end{array}$$

$$\begin{aligned} &4(n(n+1)^2) + (n-1)^2 n^3 \\ &4(n^2 + n - 2) + n - 1 + n^3 \\ &4n^2 + 4n - 8 + n - 1 + n^3 \\ &4n^2 + 9n - 12 \end{aligned}$$

SA

$$5(4 \cdot 12 + 4 \cdot 10 + 4 \cdot 8 + 4 \cdot 6 + 4 \cdot 4)$$

$$(4 \cdot 5 \cdot 3 + 1)$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1

The tower is like a pyramid but not completely.  
It is like 4 sets of steps that lead up to a block.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

$$1 \cdot 6 + 4 \cdot 5 + 4 \cdot 4 + 4 \cdot 3 + 4 \cdot 2 + 4 \cdot 1 = 66 \text{ cubes are needed}$$

I knew there was 1 length of 6 and 4 lengths of everything else, so I just added them up after that.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

Yes, the strategy would work but it would take a lot of button pushing on a calculator.

True, it is "easy" but highly inefficient.

+2

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

I got this because there is 1 block of height  $N$  and 2 rectangles of

block  $N \times (N-1)$ . The 2 sets of steps make a rectangle of  $6 \times 5$ ,  This also works if the height  $n$  is 7. It would make 1 (block) 7 + 2 (rectangle) of  $7 \times 6$  which gives you 91 blocks, just like it would in my equation.

$$2n(n-1) + n = \# \text{ of cubes}$$

Great Illustration

Awesome job expanding but gives you 84 not 91.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\begin{aligned}19,900 &= 2n(n-1) + n \\19,900 &= 2n^2 - 2n + n \\19,900 &= 2n^2 - n \\0 &= 2n^2 - n - 19,900 \\n &= 100\end{aligned}$$

+4.5

Provide the steps  
of the quadratic formula,

Calculators are helpful for  
answering questions, but usually  
other work/processes are better ways to show/explain work.

The height would be  
100 cubes and I  
found this by using  
my equation from  
the number before.  
I multiplied everything out,  
made it simpler and used  
the quadratic equation  
on my calculator to  
find the height to  
be 100 cubes,

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$4+1+(4+3 \cdot 4)+(4+5 \cdot 4)+(4+7 \cdot 4)+(4+9 \cdot 4)+(4+11 \cdot 4)+4 \cdot 5+1 =$$

$$5+16+24+32+40+48+21 = 186 \text{ squares}$$

There would be 186 squares painted because I took one row at a time  
to figure out how many squares were sharing and I added them together  
and then added the bottom of the tower.

6. Find a formula to find the surface area of a tower of height  $n$ . (Bonus added)

$$4(n \cdot 0 + 6 \cdot 1 + 6 \cdot 2 + 4 \cdot 6 \cdot 3 + 6 \cdot 4 + 8)$$

$$5+4 \cdot 12+4 \cdot 10+4 \cdot 8+4 \cdot 6+4 \cdot 4+(5 \cdot 4 \cdot 11)$$

$$5+48+40+32+24+16+64$$

$$\times(2n-2r)$$

$$SA = 4n^2 + 9n - 12$$

Look at my work next  
to the diagram on the front.

Building Towers (Yellow Worksheet)

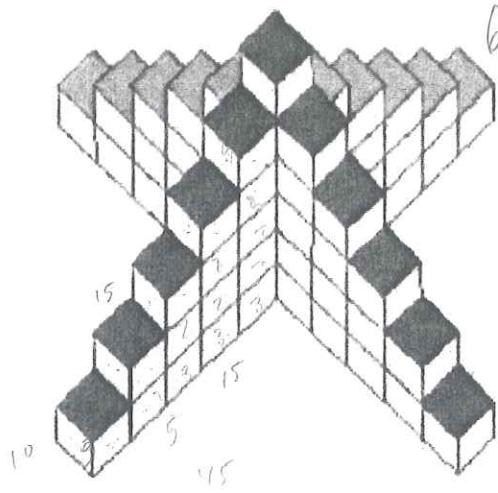
10 points possible

10  
10

Name: Christian Lytle

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



5 3  
6 6  
7 10  
8 15

- 1) (1 point) Describe what you notice about the structure of the tower above.

- 4 steps of blocks all leading to one highest point
- Made of blocks
- each set of steps is identical

+1

Good Observation

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1

66 blocks. Each set of stairs (not including the middle) has 15 blocks. So I did  $4 \times 15$  and then added the remaining 6 blocks in the middle. Used your observation here. Nice!

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1

No, you would have to find out how many blocks are in each side which would take a very long time.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\text{Total blocks} = n + (n-1)(n)(2)$$

$$\begin{aligned} & n + 2n^2 - 2n \\ & 2n^2 - n \end{aligned}$$

+1.5

Don't forget to check your formula with  $n=6$ .

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19900$$

$$0 = (2n+199)(n-100)$$

$\rightarrow n = 100 \text{ or } \frac{-199}{2}$

The height is 100 blocks

+4.5

How do you know

$$2n^2 - n - 19900 = (2n+199)(n-100)$$

Show all steps

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

I found the formula  $2n^2 - n$  by turning the tower into 3 rectangles. I made the highest point a rectangle of  $n \times 1$  cubes. I then put together 2 sides with each other to make another rectangle of cubes  $N \times (N-1)$ . There were two of these rectangles. I added all the rectangles up to get a final formula of  $2n^2 - n$ .



+1      6+

Show your work and process.

186

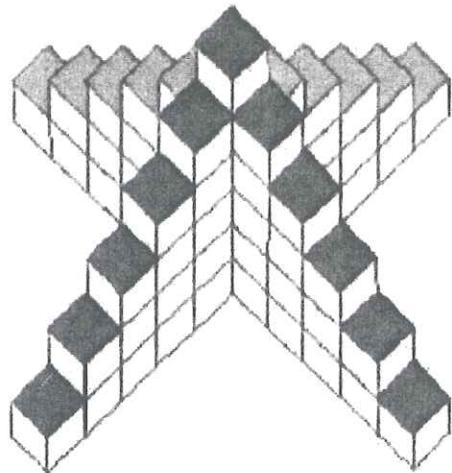
How did you come up with this?

Building Towers (Yellow Worksheet)  
10 points possible

11.5 / 10

Name: JT Hollon  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

(+1) The four sides of the tower are congruent. The sides are symmetrical to each other. ← Good Observation.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

(+1)  $(15 \cdot 4) + 6 = 66$  Each side requires 15 cubes and that number is multiplied by 4 to account for each side. Then the six blocks in the center must be added.  
Great job showing your equation and written explanation.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

(+1) No because it would be more difficult to find the number of cubes required for each side.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

(+1.5) Where does this come from?

$$2n(n-1)+n$$

$2n^2 - n$  total # of cubes

$$2(6^2) - 6$$

$$2(36) - 6$$

$$72 - 6 = 66$$

This formula provides the same answer that I found in question 2 pt a.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = 19900$$

$$2n^2 - n - 19900 = 0$$

$$\frac{1 \pm \sqrt{1 - 4(2)(-19900)}}{2(2)}$$

$$\frac{1 \pm \sqrt{159201}}{4}$$

$$\frac{1 \pm 399}{4}$$

$$\frac{1+399}{4} \quad \frac{400}{4} = 100 = n$$

*Discarded the negative, which is correct but make sure you explain why.*

+5

Good  
Strategy

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$4(15 \cdot 2 + 10) + 5$$

$$4(40) + 5$$

$$165$$

+2

Each side of the tower would have 40 square faces that need to be painted. Only the top cube of the center stack would need to be painted so five faces need to be added. ✓

Building Towers (Yellow Worksheet)

10 points possible

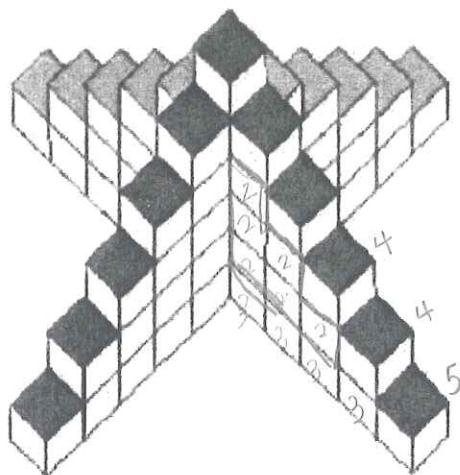
10.5 / 10

Name: Ana Bayer

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

Prob. #3 Work



Column height decreases by 1.

$$\begin{aligned} h_1 &= 7 & n(n+1) &= (n+2)+(n+6) \text{ etc.} \\ 5+2 &= 7 & 3+4 &= 7 \\ 3+4 &= 7 & \end{aligned}$$

Find # of cubes of this pattern  
by equation  $\frac{n}{2}(n+1)$

4 corners = multiply by 4, but the center column only occurs once, so you subtract the result by  $3n$ .

$$\begin{aligned} \text{This results in equation } 4\left[\frac{n}{2}(n+1)\right] - 3n &= 2n(n+1) - 3n = 2n^2 - n \\ \text{Great Work for #3!} & \end{aligned}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 As you move further from the center column, the columns decline in height by one cube.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6 + 5(4) + 4(4) + 3(4) + 2(4) + 1(4) = \text{total}$$

$$6 + 20 + 16 + 12 + 8 + 4 = 66$$

Great job using your explanation.

There are columns moving outwards from the center column in 4 directions. Since each column is one less, I multiply one less of each column after 6 by 4 and then add it to the 6 cubes in the center column to get the total.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 No, because if it was used for a larger cube tower it would be too arduous and time consuming.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n^2 - n = \# \text{ of cubes.}$$

$$2(6)^2 - 6 = 66$$

+2 ~~I see your work at the top.~~

+2 I see your work at the top.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\textcircled{1} \quad 2n^2 - n = 19,900$$

$$\textcircled{2} \quad 2n^2 - n - 19,900 = 0$$

$$n = \frac{1 \pm \sqrt{1-4(2)(-19,900)}}{2(2)}$$

$$n = \frac{1 \pm \sqrt{159,201}}{4}$$

$$n = \frac{1+399}{4}$$

$$n = \frac{400}{4} = 100$$

Answer: The height of the tower is 100 units.

\textcircled{1} Since I have an equation to find the number of cubes based on the height, I can use that same one for the reverse situation.

\textcircled{2} The equation is not factorable, so I must use the Quadratic Formula.

+5

Good job!  
Using your  
tools/resources.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$5(5) + 4 \cdot 4 + 4 \cdot (6-2) + [2(3) + 2(2) + 2(1)] =$$

↑              ↑              ↑              ↑              ↑  
 faces    top    faces    branches    top cubes    painted    small section left to  
 painted    4 end    painted    of each    of each    very    paint.  
 ↑              ↑              ↑              ↑              ↑              ↑  
 faces    top    faces    branches    top cubes    painted    small section left to  
 painted    4 end    painted    of each    of each    very    paint.

+0.5



Good Attempt  
 But I believe  
 you went  
 wrong looking at  
 your relation  
 of cubes with  
 #faces  
 to "paint"

$$25 + 64 + 6 + 4 + 2 = 101 \text{ units}^2$$

Method: Figure out the surface area of each section and add them together.

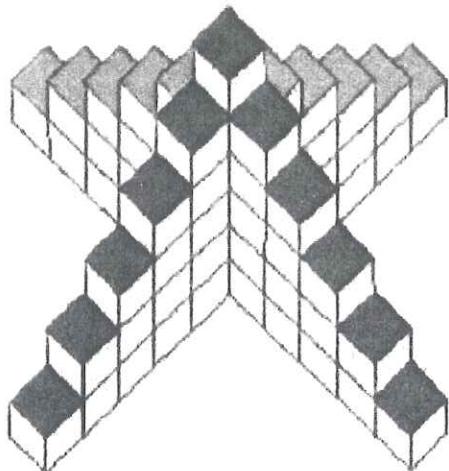
equation:  $28n - 31 + \frac{n-3}{2}(n-2)$

I applied the method I used to solve the question to form an equation for all heights.

3/10

Name: ZACH MEYERS  
Date: 5-5-14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 THE NUMBER OF BLOCKS HAS A PATTERN OF 1, 5, 9, 13, 17, 21. BEGINNING AT THE TOP. THIS IS AN ARITHMETIC SEQUENCE, REPRESENTED BY THE LINEAR EQUATION  $y = 4x - 3$  WHERE  $x$  IS THE BLOCK HEIGHT AND  $y$  IS THE NUMBER OF BLOCKS IN JUST THAT ROW.  
Good Observation!

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

+1 I ADDED THE FORMULA ABOVE 6 TIMES FOR A TOTAL OF 66 CUBES

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 NO, BECAUSE MY STRATEGY ONLY FINDS ONE LAYER AT A TIME  
Good Observation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

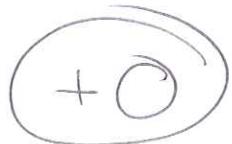
$$4(n^2 - 15) - 3n = \text{NUMBER OF CUBES}$$

+0 Only works for  $n=6$ . If  $n=1$ , this equation wouldn't work. Generalized formulas should work for all  $n$ 's. Check with  $n=1, n=2, n=3, \dots$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$h = 4(n^2 - 15) - 3n = 19900$$

h IS HEIGHT, PLUG INTO CALC AND FIND  
THE ZEROS.



Correct Strategy, but your equation from #3 is wrong. You should use the quadratic equation instead of your calculator.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

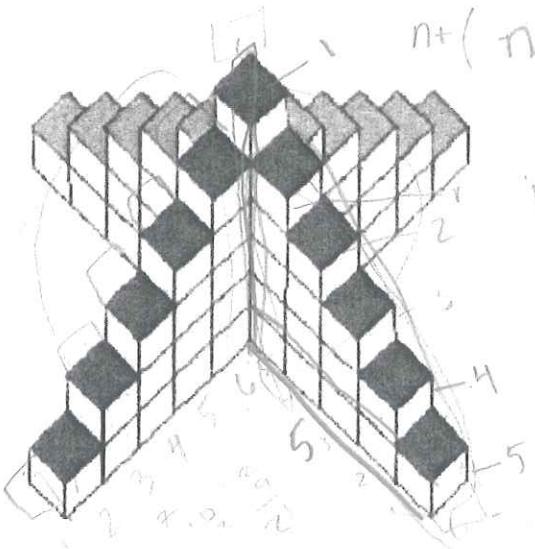
Building Towers (Yellow Worksheet)  
10 points possible

4/10

Name: Virginia Langfield

Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

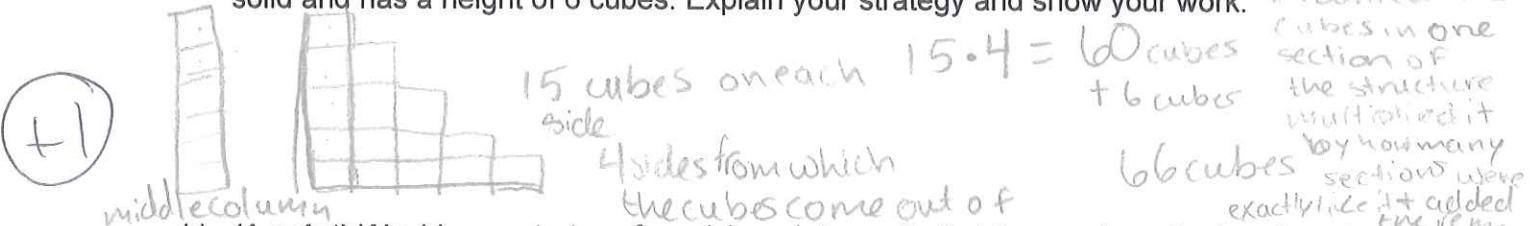


- 1) (1 point) Describe what you notice about the structure of the tower above.

+1 Four equal protrusions of cubes descending from the middle structure at a height of six? height decreased by 1 cube as you move down from that point on all four sides forms an X if seen from above. column of cubes

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.



- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

+1 Probably not because yes it would work but it would take a lot of time to find the number of cubes. Good Observation.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

+0

$$4(5n-15)+n$$

Works only for  $n=6$

Generalized formulas should work for all  $n$ 's. Next time test  $n=1, n=2, n=3$ . That's a good test to see if your formula works.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = n + 4(5n^2 + 1)$$

$$19900 = n + 20n^2 + 4$$

$$19900 = 21n^2$$

$\therefore 950.47$  cubes

Can't have  
47 of  
a cube.

(Formula is wrong)

+1

You recognized here that your formula was incorrect, but you should have gone back to #3 and correct the formula there.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

## **Building Towers (Yellow Worksheet)**

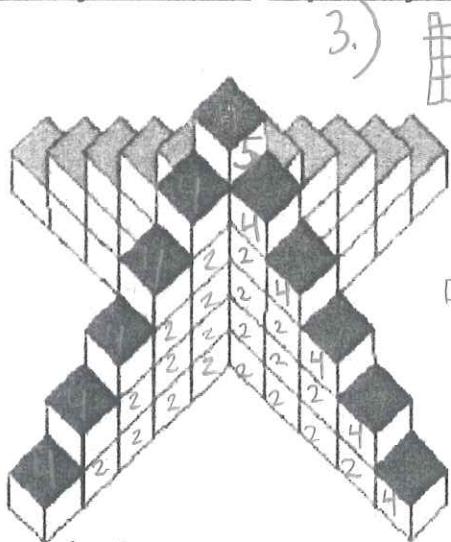
**10 points possible**

5/10

Name: Jay Pullyblank  
Date:

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$f(x) = n(n + (n - 1)) = 2n^2 - n$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

- made of 66 cubes
  - consists of less cubes as it rises vertically ← what consists of less cubes?
  - it's 3 dimensional
  - it's symmetrical across a line that travels completely through the tower →

Reference the diagram shown above:



- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Counting the blocks in the picture, I found the number of cubes to be 66. To solve this problem mentally I found the number of blocks in one of the four sides () multiplied it by four and added it to the number of blocks in the center column (the height) and got 66 cubes.

- b) (1 point) Would your strategy from (a) work to *easily* find the number of cubes for a tower that

- has a height of 300? Why or why not? My strategy from (a) wouldn't work easily to find the number of cubes in a tower with a large height because it involves mostly counting and it would take a long time to add  $4(299+298+297+296+295\dots)+300$

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Since the height was the variable that was changing, I decided to view the structure from a vertical standpoint rather than a horizontal one. Doing this, I found the number of cubes that were added on each step of the height as it increased. I then found the relationship between those numbers and the height in order to derive a formula that can find the total number of cubes in the structure when only given the height.

$$f(n) = 2n^2 - n$$

$$f(6) = 2(6^2) - 6$$

$$f(6) = 72 - 6$$

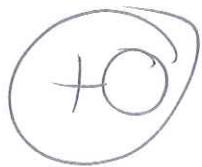
$$f(6) = 66 \checkmark$$

$f(6) = 66$  ✓  
← Great Explanation!

$$f(n) = n(n + (n - 1)) \text{ simplified to } f(n) = 2n^2 - n$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 2n^2 - n$$



You started out the problem correctly. You should still go back and solve this to practice skills of problem solving ~~at~~ a quadratic equation.

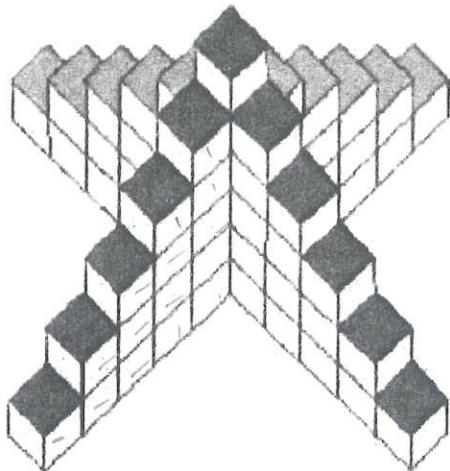
5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

Building Towers (Yellow Worksheet)  
10 points possible

12/10

Name: Ben Paul Indivizzi  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

Tower of 6 cubes with 4 "staircases" descending by one cube each step on each side, down to one cube

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6+5(4)+4(4)+3(4)+2(4)+1(4)=66$$

My strategy was to add the height of the tower, 6, by the height of each stair multiplied by the number of staircases.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

My initial strategy would not work to easily find the number cubes for a tower with a height of 300 as it would take a long time to add up all of these numbers.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\underline{2n^2 - n}$$

How did you come up with this?

$$2(6)^2 - 6$$

$$\begin{aligned} &= 2(36) - 6 && \text{good} \\ &= 72 - 6 \\ &= 66 \checkmark \end{aligned}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = 19,900$$

$$2n^2 - n - 19,900 = 0$$

$$n = \frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{4}$$

$$n = \frac{1 \pm 399}{4}$$

$$n = \frac{400}{4}$$

$$n = 100$$

I used the equation I came up with in problem 3 and put in 19,900 as the solution. I turned that equation and turned it into a quadratic and solved for  $n$  using the quadratic formula. I ignore the  $1-399$  as  $n$  cannot be negative. good!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$15 \times 8 = 120$$

$$10 \times 4 = 40$$

$$5 \times 1 = 5$$

$$SA = 165$$

not including

the bottom  
of the tower

I counted the cubes on the side of one "staircase" and multiplied it by the number of sides of staircases, 8. Then counted the front and tops of each stair and multiplied it by the number of staircases, 4. Then counted the number of sides on the top edge, 5. I then added these together.

Do you think that you find an equation for surface area?

Building Towers (Yellow Worksheet)

10 points possible

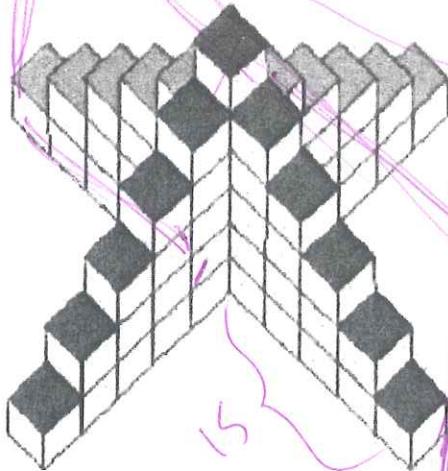
~~MANNED~~

+ 8.75 / 10

Name: UXIDULLU

Date: MARCH 6, 2019

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$[n + (n-1)] n$$

$$\begin{aligned} & 6 \quad 6 \times 1 \\ & + 20 \quad 5 \times 4 \\ & + 16 \quad 4 \times 4 \\ & + 12 \quad 3 \times 4 \\ & + 8 \quad 2 \times 4 \\ & + 4 \quad 1 \times 4 \\ & \hline 60 \end{aligned}$$

AN!

$$\begin{aligned} & 6 \quad (6-1) \text{ mult} \\ & + 20 \quad (6-1) \text{ add} \\ & + 16 \quad (6-1) \text{ add} \\ & + 12 \quad (6-1) \text{ add} \\ & + 8 \quad (6-1) \text{ add} \\ & + 4 \quad (6-1) \text{ add} \\ & \hline 60 - 6 = 60 \end{aligned}$$

add (6-1)

$$\begin{aligned} & 15 \\ & 5+4+3+2+1 \\ & (n-1) \cdot 1 + n = C \\ & \text{addition} \end{aligned}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower begins with a maximum height that decreases into four sections remaining one block each time.



- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

NO - I would have to manually add each height of blocks ( $300 + (300-1) + (300-2) + \dots$  etc.)

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$C = \# \text{ OF } \text{CUBES}$

$n = \text{HEIGHT}$

$$C = [n + (n-1)] n$$

How did you  
find this?  
(not multiplication)  
factorial  
addition

~~4~~ (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

OK



5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

② Follow by attaching to that (left-right) to  
each number you've mark to (left-right factorial)  
and multiplying each of these numbers by  $x$ .

# OXIDATION

- ① The tower begins with a maximum height ( $n$ ) that branches out into sections of four with each height decreasing by 1 each time.
- ② 66 → I added the first  $n$  (height) to each
  - a)  $(n-1), (n-2), (n-3), (n-4), (n-5)$  and multiplying each of those by 4!
- ③  $n=1$  would have to manually add each number of blocks as the height decreases by one. I would have to add 300 numbers.
- ④ if you move the blocks to create a perfect rectangle, you get height  $n$  and width  $n + (n-1)$ . To get the formula you multiply the height and width together.

$$C = [n + (n-1)] n$$

$$\textcircled{1} \quad 19,900 = (n + (n-1)) n$$

$$\textcircled{2S} \quad 19,900 = (2n-1)n$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

good!  
but why did you  
set the equation  
equal to 0?  
explain ☺

→ quadratic  
formula

$$\frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{2(2)} = n$$

$$\hookrightarrow \frac{1 \pm \sqrt{159,201}}{4} = n$$

$$\frac{1 \pm 399}{4} = n$$

$$\boxed{n = 100}$$

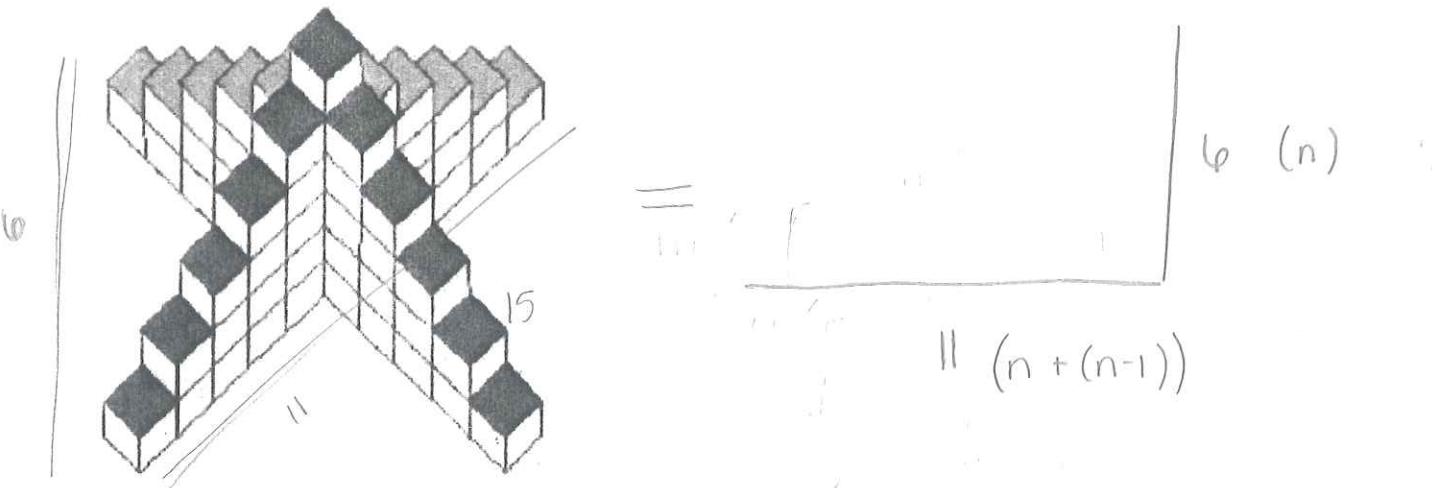
is there  
another  
solution?

the height would be 100 cubes tall.

+ 9.75 / 10

**Building Towers (Yellow Worksheet)****10 points possible****Name:** Aisha Fichtner**Date:** \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

It increases upward by increments of 1. From top to bottom and across it's  $6 \times 6$ . 15 blocks on each side. 6 going down the middle. 66 blocks total.  $1+2+3+4+5+6$ . For one side plus the middle. If you put one side on top the other side on both sides it makes a rectangle.

- 2) Reference the diagram shown above:

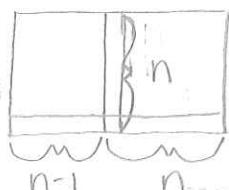
- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

66 blocks. Each of the 4 sides has 15 blocks.  $15 \times 4 = 60$  then I added the 6 blocks in the middle.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, I would have to count all the blocks on one side and then multiply by 4 and add the 300 in the middle.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.



$$[n + (n - 1)]^n$$

$$[6 + (6 - 1)]^6$$

66

Could you explain this further?



4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

*why set it equal?*

$$19,900 = [n + (n-1)]n$$

$$19,900 = n^2 + n(n-1)$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

quadratic formula

*what are the solutions?*

$$\frac{-b \pm \sqrt{(b)^2 - 4AC}}{a^2} = \frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{4} = \frac{1 \pm \sqrt{159201}}{4} = \frac{1 \pm 399}{4} =$$

The height would be 100

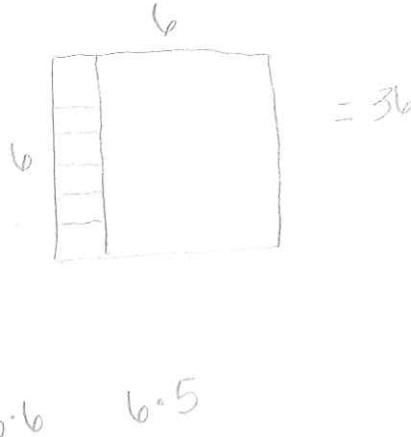
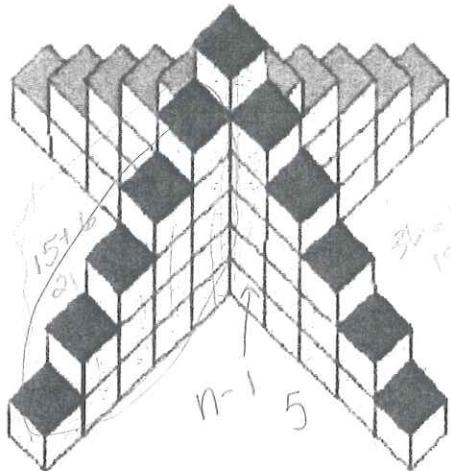
5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

**Building Towers (Yellow Worksheet)**  
10 points possible

+12/10

Name: Lindsey shaffner  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

Each side of the tower is symmetrical.

What does symmetrical mean in this case?

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$(6+5+4+3+2+1) \quad (5+4+3+2+1)$$

21                          15 + 3

$$21 + 15 =$$

36

\*There is only one time  
you have to count the  
height of 6 cubes good!

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No because I added up each section which would take awhile to do with a height of 300.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Where did this come from?

$$n \cdot n + n \cdot (n-1)$$

$n^2 + n(n-1)$

$$n^2 + n^2 - n$$

$$2n^2 - n$$

$$6^2 + 6(6-1)$$

$$36 + 30$$

66

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = n^2 + n(n-1)$$

$$19,900 = n^2 + n^2 - n$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

How did you plug it in? Did you plug it in for  $n$  or for another term?

The height is 100.

I plugged in 19,900 into the answer to the formula. Then used the quadratic formula to find  $n$ .

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

~~5~~

5

6.

$$40+40+40+40$$

160+5

165

$$\frac{1 \pm \sqrt{1^2 - 4(2)(-19900)}}{4}$$

$$1 \pm \frac{\sqrt{1 + 15920}}{4}$$

$$1 \pm \frac{\sqrt{15920}}{4}$$

$$1 \pm \frac{399}{4}$$

why is  
this incorrect?  
~~-398~~  
A

$$\frac{100}{4} = 100$$

I counted up how many would be showing which was 40. There are 4 that are equal.

$$10 \cdot 4 = 40$$

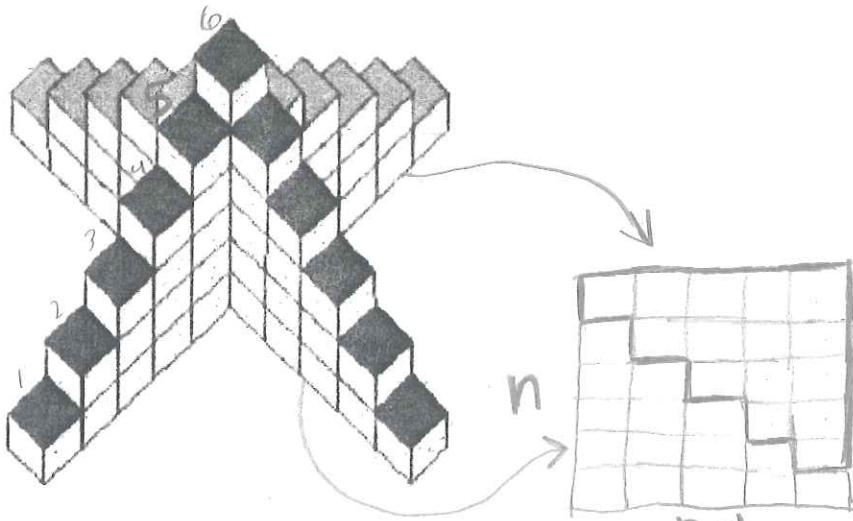
The the only thing showing on the highest is 5.

**Building Towers (Yellow Worksheet)**  
10 points possible

+12/10

Name: Lauren Cannatell,  
Date: March 6, 2014

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$2n(n-1) + n$$

Love this drawing!

- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure of the tower above is symmetrical and seems to be rotated around the center, tallest point.  
interesting!

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Find the number of cubes for one "arm" of the tower and then multiply by 4. Add to the number of cubes needed to make the tallest point (its height).

$$15 \times 4 = 60 + 6 = \boxed{66 \text{ cubes}}$$

needed to build the tower

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because there are many more cubes, it would be difficult to count all of the cubes as I did in part a, so my strategy would not work easily.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n(n-1) + n \quad \text{or} \quad 2n^2 - n$$

$$2(6)(5) + 6 = 66 \checkmark$$

good! but could you explain further? how did you get this formula?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n(n-1) + n$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19,900$$

$$n = \frac{1 \pm \sqrt{1^2 - 4(2)(-19,900)}}{4}$$

$$n = \frac{1 \pm 399}{4}$$

$$n = \frac{400}{4} = 100 \quad \cancel{-399} \cancel{n}$$

good!

The height of a tower with 19,900 cubes is 100 cubes. We set our equation equal to the total number of cubes and used the quadratic formula to solve for  $x$ . This resulted in two solutions, but only the positive solution would work. So, the height is 100 cubes.

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

There are 15 square faces on one side of an "arm". So 30 square faces on the sides of an "arm". We have to add the number of faces going up the "steps" of one of the arms which is 10, so 40 square faces on each arm.  $40 \times 4 = 160$  and now we have to add the tallest block on the top of the tower which has 5 square faces, so

$$SA = 165 \text{ units}^2$$

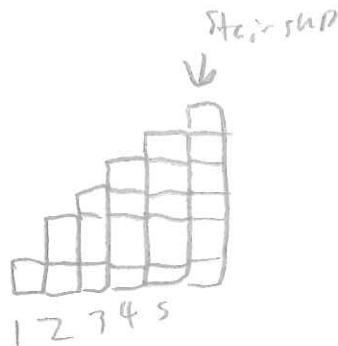
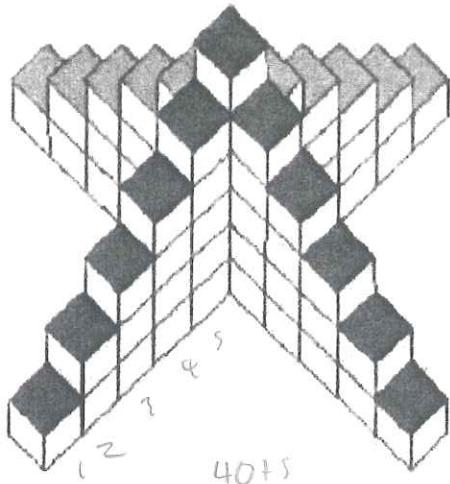
Would it be easy to find an equation for surface area?

**Building Towers (Yellow Worksheet)**  
10 points possible

+12/10

Name: Connor Lomax  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

It is 6 blocks high at its highest point, and its width and length are both 11 blocks long. Each "stair step" increases in height by one block each step.

show this through a drawing?

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{r}
 5 \\
 + 4 \\
 \hline
 9 \\
 + 7 \\
 \hline
 16 \\
 + 2 \\
 \hline
 18 \\
 + 14 \\
 \hline
 32 \\
 + 18 \\
 \hline
 50
 \end{array}
 \times 4 = 60 \text{ blocks} + \boxed{6 \text{ blocks}} = 66 \text{ blocks are needed.}$$

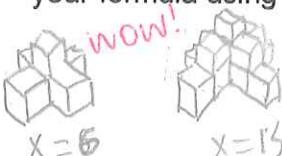
Each stair step, without the top four blocks, has 18 blocks. There are 4 of them so  $18 \times 4 = 72$ . 72 + the highest tower is 66 blocks.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because having to add all the numbers up to 299 would take a long time. It would not be easy at all.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$n$  = greatest height       $x$  = total # of cubes



28  
4 height  
 $6, 15, 28, 45, 66$   
 $4, 9, 16, 25, 36$

$$n(n+1) = x$$

$$2(2+1)$$

$$3(3+2)$$

$$4(4+3)$$

$$n^2 + n^2 - n$$

$$2(36)-6=x$$

$$72-6=x$$

$$66=x$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = X$$

$$2n^2 - n = 19900$$

$$2n^2 - n - 19900 = 0$$

$$(2n+199)(n-100)$$

$$n = \frac{-199}{2}, 100$$

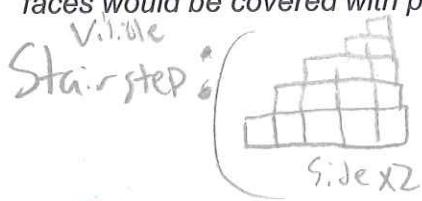
$$n \neq \frac{-199}{2}$$

$$\boxed{n = 100}$$

$X$  would equal 19,900 in this scenario  
So factoring allows us to get  $\frac{-199}{2}$  and 100.  
good! This can't be  $\frac{-199}{2}$  because the height can't  
be negative, the height must be 100.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2



$$(1 \times 2) + (5 \times 3) \times 4 =$$

$$(20 + 15) \times 4 =$$

$$70 \times 4 =$$

$$\boxed{180 \text{ units}^2 \text{ for staircase}}$$

Love your drawings!

The only sides being painted for each stairstep are the Top, front, bottom, and back sides. The back of each are not being painted.

\*each block is 1x1x1  
the units used are "units"

Visible  
Tower:  $(\square + \square + \square)$   
Top Bottom Sides

$$(1 + 1 + 4)$$

$$(6)$$

$$\boxed{6 \text{ units}^2 \text{ for tower}}$$

The only visible parts of the tower is the bottom square, top square, and 4 sides. This means only 6 units<sup>2</sup> are painted.

$$180 \text{ units}^2 + 6 \text{ units}^2 = \boxed{186 \text{ units}^2 \text{ total}}$$

Y  
→  $186 \text{ squares total will be painted.}$

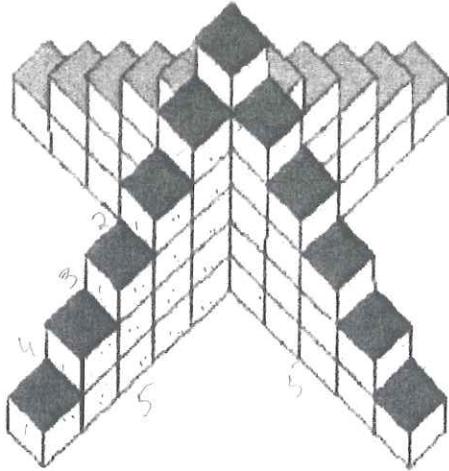
\*I included the bottom in the total

**Building Towers (Yellow Worksheet)**  
10 points possible

+12/10

Name: Luke Johnston  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{aligned} n &= 2 = 21 \\ 9 \cdot 3 &= 27 \end{aligned}$$

$$11 \cdot 4$$

$$4n^2$$

$$40$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

It is in the shape of an X, and has stairs or pyramid steps.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$5+4+3+2+1=15(4)=60+6=66$$

Each set of stairs has 15 cubes so I multiplied that by 4 for the 4 sets of stairs. Then added 6 for the center column.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No because my first strategy involved counting one set then multiplying  
And counting is not easy! ☺

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2(n^2) - n = \text{number cubes}$$

$$2(6^2) - 6 = \text{How did you get this?}$$

$$2(36) - 6 = 66 \checkmark$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2(n^2) - n$$

$$2n^2 - n - 19,900 = 0$$

$$(2n+199)(n-100)$$

$$2n+199=0 \quad n-100=0$$

$$2n=199 \quad n=100$$

$$n=99.5$$

$$n=100$$

The height would be 100 because I plugged ~~99.5~~ into my formula then factored.

What is it?

Be more specific!

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$40 \times 4 = 160 + 5 = 165 + 21 = 186$$

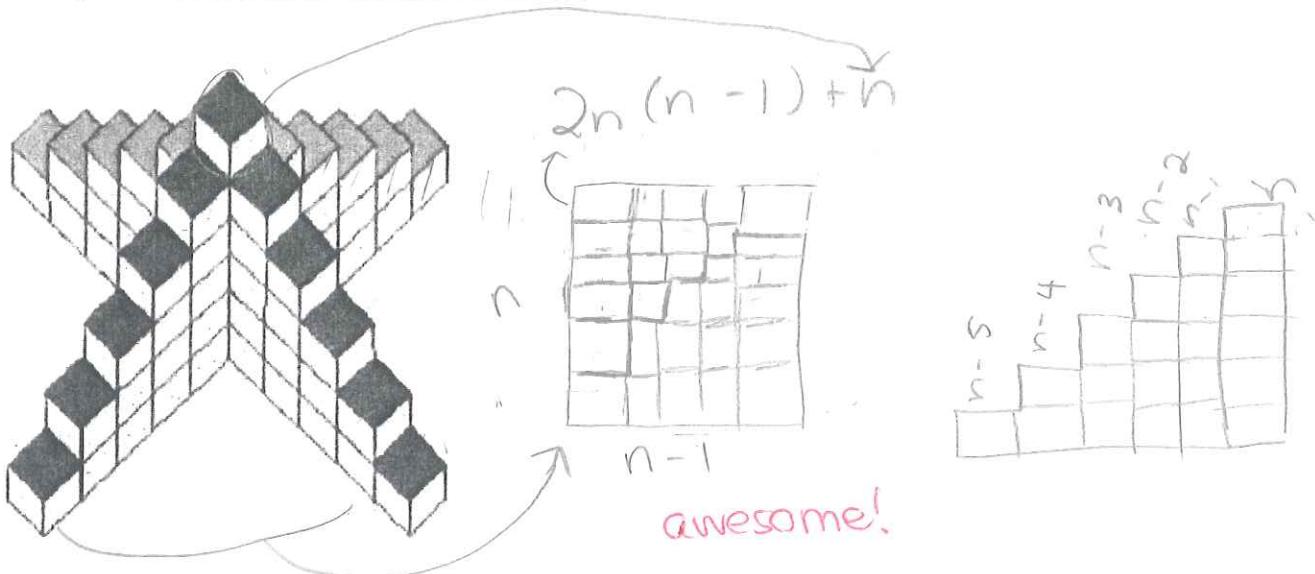
I counted on set of stairs and one more. Then times that by 4 to get all the sets and add 5's for the top one. Then add 21 for the bottom.

## **Building Towers (Yellow Worksheet)**

+11/10

Name: Molly Hofferbam  
Date: 3/16/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure is symmetric on all four sides. The number of blocks increases by one for each "step" of the stair.

- 2) Reference the diagram shown above:

- a) **(1 point)** Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$4(5 + 4 + 3 + 2 + 1)$  All four sides are equal,  
 $(60 + 6 \text{ (height)})$  so you take the number  
 $\boxed{(60 \text{ cubes})}$  of cubes on each side  
 and multiplying it by 4. Then there is only one stroke within.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

no, then would be too many numbers <sup>(to do you)</sup> just add it  
to add, you could make an equation  
to make it easier

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n(n-1) + n$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n(n-1) + n = 19900$$

The height of the tower is 100. In order to solve the equation, you plug in 19900 where? in for  $n$ ? to the equation and use the quadratic formula to solve.

$$0 = 2n^2 - n - 19900$$
$$0 = (2n + )(n - )$$
$$\frac{1 \pm \sqrt{(1)^2 - 4(2)(-19900)}}{4}$$

$$\frac{400}{4}$$

$$\boxed{n = 100}$$

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

165 I reasoned that the highest cube had 5 sides showing. The other sides could not show any more than 5 sides. So, I added the number of cubes with 4 sides and 2 sides showing and added all the surfaces together to get 165.

**Building Towers (Yellow Worksheet)**

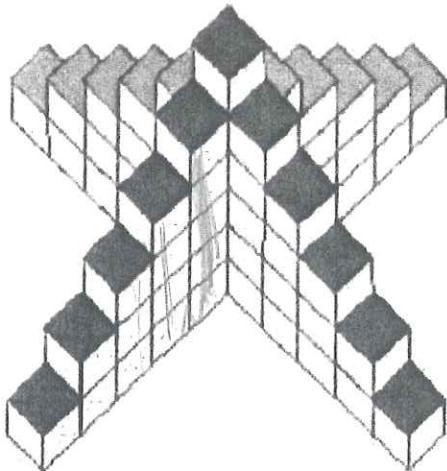
**10 points possible**

+11.75/10

Name: Taylor H

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{aligned} 6 - 15 &= \\ 5 &= 10 \\ 4 &= 6 - 14 \\ 3 &= 3 - 13 \\ 2 &= 1 - 12 \end{aligned}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

Central column

- ~~4 sticking out parts are the same~~
- 4? OK

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{ll} 4(5+4+3+2+1) & +6 \\ (\text{sides}) & (\text{central}) \\ \text{sticking out} & \text{(column)} \\ \text{part}) & \end{array}$$

$$\begin{array}{l} 4(15) + 6 \\ 60 + 6 \\ \hline 66 \text{ cubes} \end{array}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, there are too many numbers to add.

- 3) (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Does it work for  $n = 6$ ?

$$[2(n-1) + 1] n$$

$$(2n-2+1)n$$

$$2n^2 - 2n + n \checkmark$$

one side

bottom length of  
149

center column  
cube

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

-25

center  
column

$$n(2(n-1)+1) = 19900$$

$$n(2n-2+1)$$

$$2n^2 - n - 19900 = 0$$

$$n = \frac{1 \pm \sqrt{1 - (4)(2)(-19900)}}{4}$$

$$n = \frac{1 \pm \sqrt{159201}}{4}$$

$$n = \frac{1 + 399}{4}$$

$$n = 100$$

Explain  
your method  
more!!

why did you do  
what you do?

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+1

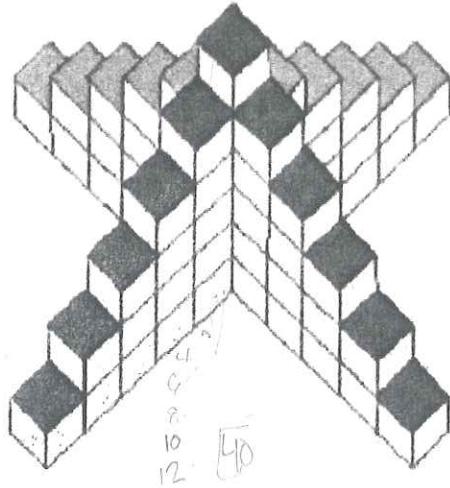
$$(15)(2)(4) = 120 + 4 + 71 + 20 + 21 = 186$$

Explain!

5 faces  $\rightarrow$  5

4 faces  $\rightarrow$

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



5x2

8+5+3



3.

$$15 \cdot 6 = 30$$

$$n \cdot n - 1 \cdot 2 + n$$

$$6 \cdot 5 - 2 + 6$$

15

3.

$$6 \cdot 4 = 16$$

21

- 1) (1 point) Describe what you notice about the structure of the tower above.

It's 3D, there are six blocks on the bottom, and the highest is six tall, and then it goes down by one, there are four sections.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$(15 \times 4) + 6$$

$$60 + 6 = \boxed{66} \text{ blocks}$$

each section has 15 blocks, there are 4 sections  $(15 \times 4)$  and then there are the 6 in the middle that all the sections share/ connect to, so you have to add 6.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because you had to count the blocks, and counting 300 blocks would take awhile. It would not be efficient.

nice word !

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.



$$(b-1) \cdot (h-1) \cdot 2 + h$$

$$\begin{aligned} b &= \text{base} \\ h &= \text{height} \end{aligned}$$

$$2n^2 - 2n + n$$

$$6 \cdot (6-1) \cdot 2 + 6$$

$$6 \cdot 5 \cdot 2 + 6$$

$$30 \cdot 2 + 6$$

$$60 + 6 = 66$$

$$n \cdot (n-1) \cdot 2 + n$$

$$6 \cdot (6-1) \cdot 2 + 6$$

$$6 \cdot 5 \cdot 2 + 6$$

$$30 \cdot 2 + 6$$

$$60 + 6 = \boxed{66 \text{ blocks}}$$

$$60 + 6 = 66$$

$$66 + 6 = 72$$

$$72 + 6 = 78$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work. *plug the number of cubes into the*

25

$$h=100$$

$$n \cdot (n-1) \cdot 2 + n = 19980$$

$$h^2 - n \cdot 2 + n$$

$$2h^2 - 2h + h = 19900$$

$$n(2n^2-n+1) \text{ terms}$$

$$V_1(2n-1) = 100.00$$

$$2n^2 - n - 19900 = 0$$

14-4159201

H 399

100

↓ explain more  
plug it in where?  
why?

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

165

$$40 \times 4 + 5$$

160+5

165

40 Surface area of  
1 section of block + 5

So multiply by 4

to get all sections

add 5 because it  
is the top block

412

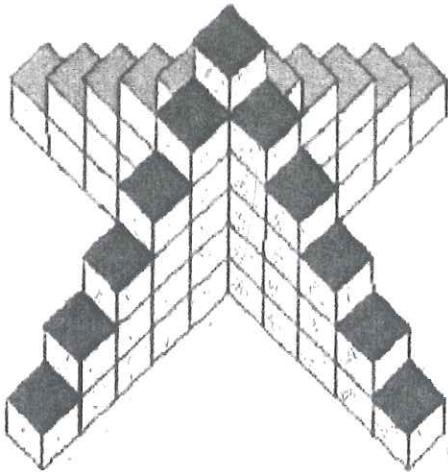
Building Towers (Yellow Worksheet)  
10 points possible

+12/10

Name: Nick Engle

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$2S = 5 \times 5$$

$$\frac{n^2}{2} + \frac{1}{2}$$

$$h = h$$

$$A = \frac{h(h-1)}{2}$$

$$A = A,$$

Diagram showing a 5x5 grid of cubes. The center column is shaded grey. The sides are labeled with 'A' and 'h'. The bottom row is labeled 'h=1'.

- 1) (1 point) Describe what you notice about the structure of the tower above.

It is 6 blocks high in the center, and it climbs down 1 per block in each of the four directions from 6, 5, 4, 3, 2, 1.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$6 + 4(5) + 4(4) + 4(3) + 4(2) + 4 = 66$  cubes I added the center 6 cubes to all the cubes on 5 tall stacks, then 4, etc.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because it would be a lot of work to total up all the rows.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n^2 - n$$

$$2(6^2) - 6$$

$$2(36) - 6$$

$$72 - 6 \quad (6) \checkmark$$

way to  
show your  
work for your  
formula!

$$4\left(\frac{n(n-1)}{2}\right) + n$$

$$2(n^2 - n) + n$$

$$2n^2 - 2n + n$$

$$2n^2 - n$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$2n^2 - n = 19,900 \quad \text{I set up an equation}$$

$$4\left(n^2 - \frac{1}{2}n + \left(\frac{1}{4}\right)^2\right) = 9950 + \frac{1}{16}$$

$$\left(n - \frac{1}{4}\right)^2 = \frac{159201}{16}$$

$$n - \frac{1}{4} = \pm \sqrt{\frac{159201}{16}}$$

$$n = \pm \frac{399}{4} + \frac{1}{4}$$

$$n = 100, -99.5$$

Solve for n

100 blocks tall

why not?

Using  $n$  as the height of the tower, I solved for the area of the side parts with  $4\left(\frac{n(n+1)}{2}\right)$  and add that to the middle section of the tower with value  $n$ . Simplifying this gives  $2n^2 - n = T$   
 $T = \text{total blocks}, I can then plug } 19,900 \text{ for } T \text{ to get the answer.}$

- +2  
5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$n=6$$

$$1S(8) + S(2)(4) \quad S(4) + 5 + 1 = 186$$

↑                      ↑                      ↑                      ↑  
 Area of Side   Stair sides   bottom faces   middle top   bottom

$$\text{Bonus #2: } 8\left(\frac{h^2 - h}{2}\right) + 8(h-1) + 4(h-1) + 5 + 1 = SA$$

$$4h^2 - 4h + 8h - 8 + 4h - 4 + 5 + 1$$

$$4h^2 + 8h - 6 = SA$$

$$SA = 4n^2 + 8n - 6$$

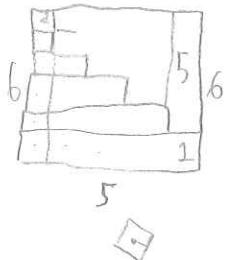
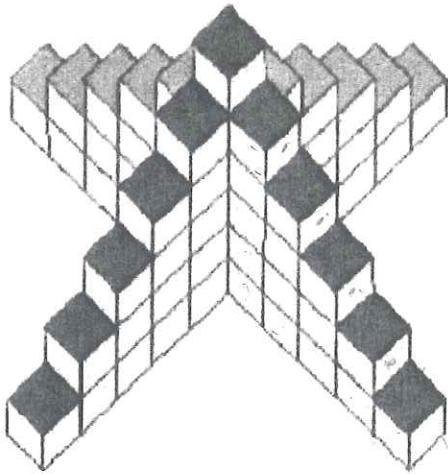
I counted the bottom of the tower as part of the Surface Area.

**Building Towers (Yellow Worksheet)**  
10 points possible

10/10

Name: Michael S.  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



what?

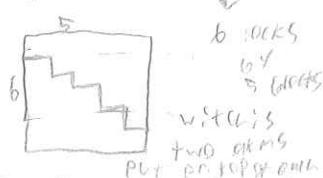
- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure is made of congruent cubes, if you rotate the structure to form a shape with a regular base with a radius of 5 blocks, the structure has a tower of 6 blocks in the center with a 5 row of blocks coming out from the base block, then a layer of 10 at top, then 3 then 2 then 1

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

My strategy would be to find area of blocks in a rectangle  
 $6 \times 5 = 30$  this would be the blocks for the two ends so  
 $30 \times 2 = 60$  Then add the 6 from the tower in the middle  
 $60 + 6 = 66$  blocks



- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? Yes, because the rectangle length would be equal to the tower's height ( $h$ ) and the width would be less than the height of the tower ( $h-1$ ), then add the next after

$$A = 2h(h-1) + h \quad A = 2h^2 - 2h + h \quad A = 2h^2 - h \quad A = 300(2(300)-1) \quad A = 300(599) \quad A = 179700 \text{ Wow!}$$

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units. The number of cubes need would

$$6e \quad A = 0(20-1)$$

where  $A = \# \text{ of blocks}$

$$A = 6(2(6)-1)$$

$$A = 6(11)$$

$$A = 66$$

Yes this works

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work. The height of the tower would be how many cubes high the center tower would have.

Plugging the numbers into the formula I found gives,

$A = n(2n-1)$  where  $A$  is # of cubes, and  $n$  is the height of the tower.  
OK

$$14,900 = n(2n-1)$$

$$19,400 = 2n^2 - n$$

$$-2n^2 - n - 14400 = 0$$

found this polynomial

Plugged into calculator formula

$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-19900)(2)}}{2(2)}$$

$$n = \frac{1 \pm \sqrt{1 + 159200}}{4}$$

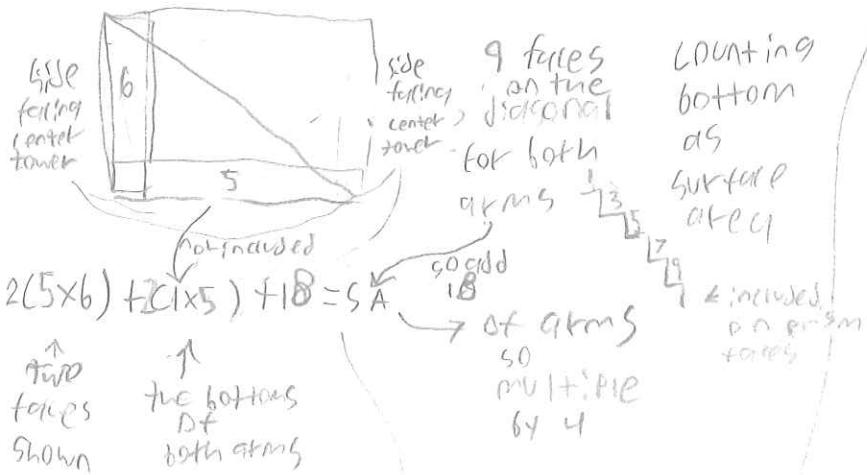
$$n = \frac{1+39}{4}$$

$$n = 100 \text{ or } -\frac{399}{4} \quad \text{and be respective}$$

The height of  
the tower would  
be 110 meters

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

with the same rectangle in 2D



$$2(5 \times 6) + 2(1 \times 5) + 18 = 54$$

<sup>↑</sup>  
two  
faces  
shown

Then add the  
one remaining side of  
the central flower which  
has 5 faces showing  
 $176 + 5 = 181$  sides of  
surface area

$$2(2(n-1 \cdot n) + 2(n-1) + (n-1+n-2) + 5)$$

$$2(2(n^2-n) + n + (2n-3)) + 5 = 5k$$

$$2(\frac{1}{2}n^2 - 2n + n + 2n - 3) + 5 = SA$$

$$2(2n^2+n-3) + 5 = 5A$$

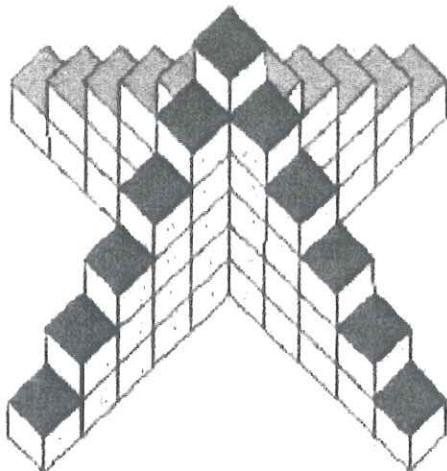
$$4n^2 + 2n - 6 + 5 = 5A$$

Building Towers (Yellow Worksheet)  
10 points possible

+10  
10

Name: Amanda Drago  
Date: 3/6/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower's highest point is 6 stories. There are 4 sections like stairs of 6 blocks each side on a central stack of 6 blocks. The top blocks have 4 sides showing except the center block which shows 3 sides. The blocks in between show 2 sides except the center ones which show 3 sides.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{aligned} & 6 \times 5 + 4 + 4 + 3 + 4 + 2 + 1 + 1 \\ & 6 \times 20 + 16 + 12 + 8 + 4 = 66 \text{ cubes} \end{aligned}$$

There are 6 per layer of the 3 per layer cubes in each level for 1 staircase. There are 2 staircases going to 6 levels, so that would be 2 times 6 per layer by 4 per level added.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

It would not work easily because you would need to add 300 times versus using 6 layers for each 6 units = 360.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$702,570+874$$

$$4n^2 - 13n$$

$$11n^2 - 3$$

$$9n^2 - 5$$

$$5n^2 - 1$$

$$2n^2 - 3$$

$$(2+6n-1)n$$

$$30+30=60$$

how did you get this?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$n^2 \times (n+1)n = 19900$$

$$n^2 + n^2 + n = 19900$$

$$2n^2 + n = 19900$$

$$n(2n+1) = 19900$$

$$2n^2 + n - 19900 = 0$$

$$(2n+1)(n-100) = 0$$

$$n = 100$$

$$2n = 199$$

$$n = 199$$

why  
not?

$$n^2 + (n-1)n = 19900 \text{ cubes}$$

The height is 100.

I used factoring of  
my equation after factoring  
in 19900 to find it.

↙ where did you  
plug 19900?

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

**Building Towers (Yellow Worksheet)**

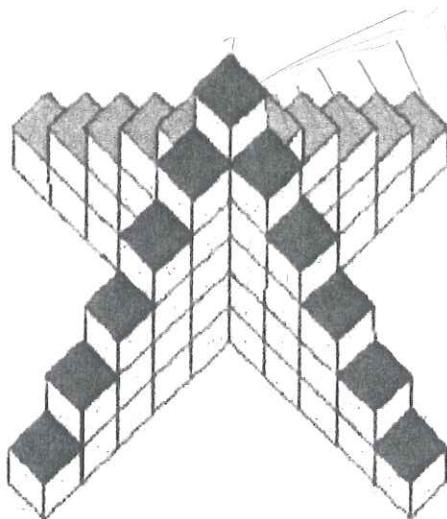
10 points possible

~~10~~  
10

Name: Niki Moffatkar

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$b \cdot h$$

$$l \cdot w \cdot h$$

$$\cdot b \cdot l$$

$$h \cdot (2(n) + 1)$$

$$E$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The base has the most cubes and as you go up the number of cubes on each level gets smaller and smaller. Similar to a cube tower but is missing parts.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6 + 4(5 + 4 + 3 + 2 + 1)$$

$$6 + 4(15) = 6 + 60$$

66 cubes

I found the sum of each edge and then multiplied that by the 4 edges of the cube and added it to the 6 top makes 66 cubes.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

I believe that it would work, maybe not as easily since the number is bigger and my method included counting down numbers from 6 and 300 is much larger.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$1. \text{ height} = 2(n-1) + (n-1)^2$$

↑ check with  $n=6$

$$2(6-1) + 1 = 11 \times$$

6 = 66

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2(n+1)^2 + 1$$

$$19,900 = n(2(n-1)) + 1$$

$$19,900 = n(2n-1) + 1$$

$$19,900 = n(2(n-1) + 1) \quad \begin{array}{l} \text{center column} \\ \text{one side bottom length of red} \\ \text{bottom} \end{array}$$

$n(2n^2 - 2 + 1)$

$2n^2 - n - 19900$

When you flip the sides of the cube it just makes it a rectangular prism and that way you can do l.w.h.

$$n = 100$$

Where did you get this formula?

$$n = \frac{1 \pm \sqrt{1 - 4(2)(-19900)}}{4}$$

$$\frac{1 \pm \sqrt{399}}{4} = \frac{400}{4} = 100$$

height is 100 units  
Is there another answer?

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

$$(15)(2)(4) = 120 + \text{top edge faces} + \text{bottom edges} + \text{side edges}$$

$\frac{1}{2} \times 15 \times 2 \times 4 = 120$

$120 + 21 + 21 + 21 = 186$

186 units<sup>2</sup>

wavy line

$n = \text{number of cubes}$

$$n(2)(4) +$$

**Building Towers (Yellow Worksheet)**  
10 points possible

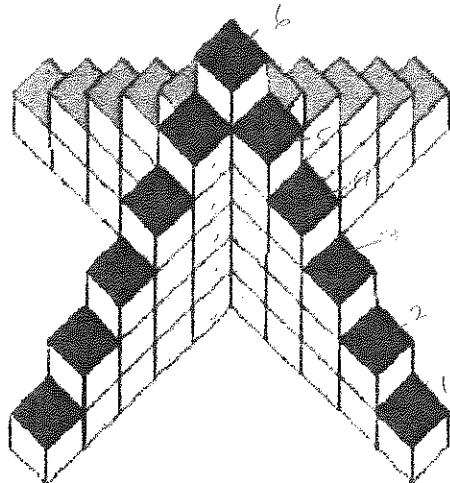
6.5

10

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$1 + 2 + 3 + 4 + 5 + 6 = 21$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

- o The structure of the tower above has cubes in a triangle.
- o It is made of right triangles.
- o

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

The total number of cubes needed to build the tower above is 66,

If you find the height of the center tower and add  
the height of each descending tower and multiply each by 4 (because the  
center tower) then you get the total of 66 cubes.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

*Model that gives you the answer. It is the same concept, due to its symmetrical  
shape, one can find the height of each surrounding tower on  
one side and multiply it by four and add the center tower.  
You will get the right answer. Would that be "easy"?*

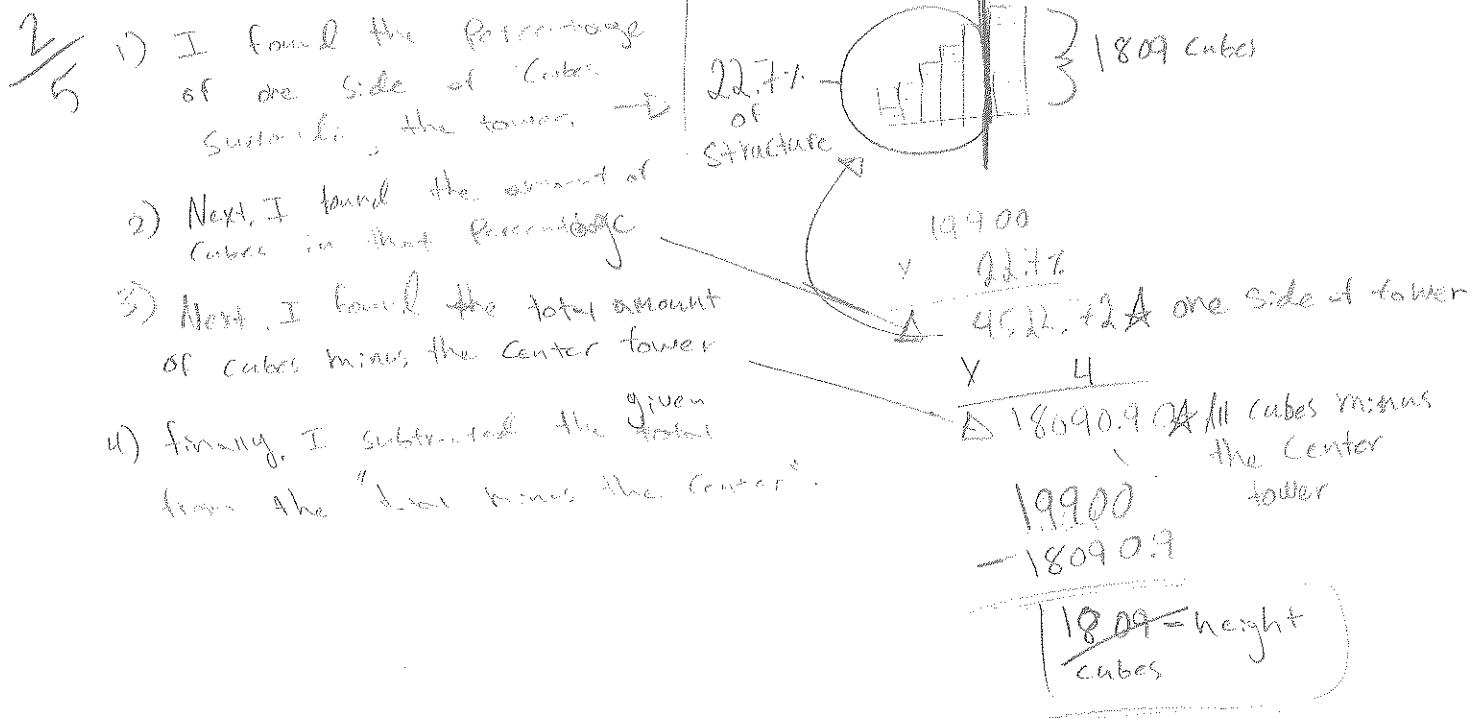
3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Would this formula  
work for a tower  
of any height?

$$\begin{aligned} n + 4(n-1) + 4(n-2) + 4(n-3) + 4(n-4) + 4(n-5) \\ + 4(n-6) + 4(n-7) + 4(n-8) + 4(n-9) \end{aligned}$$

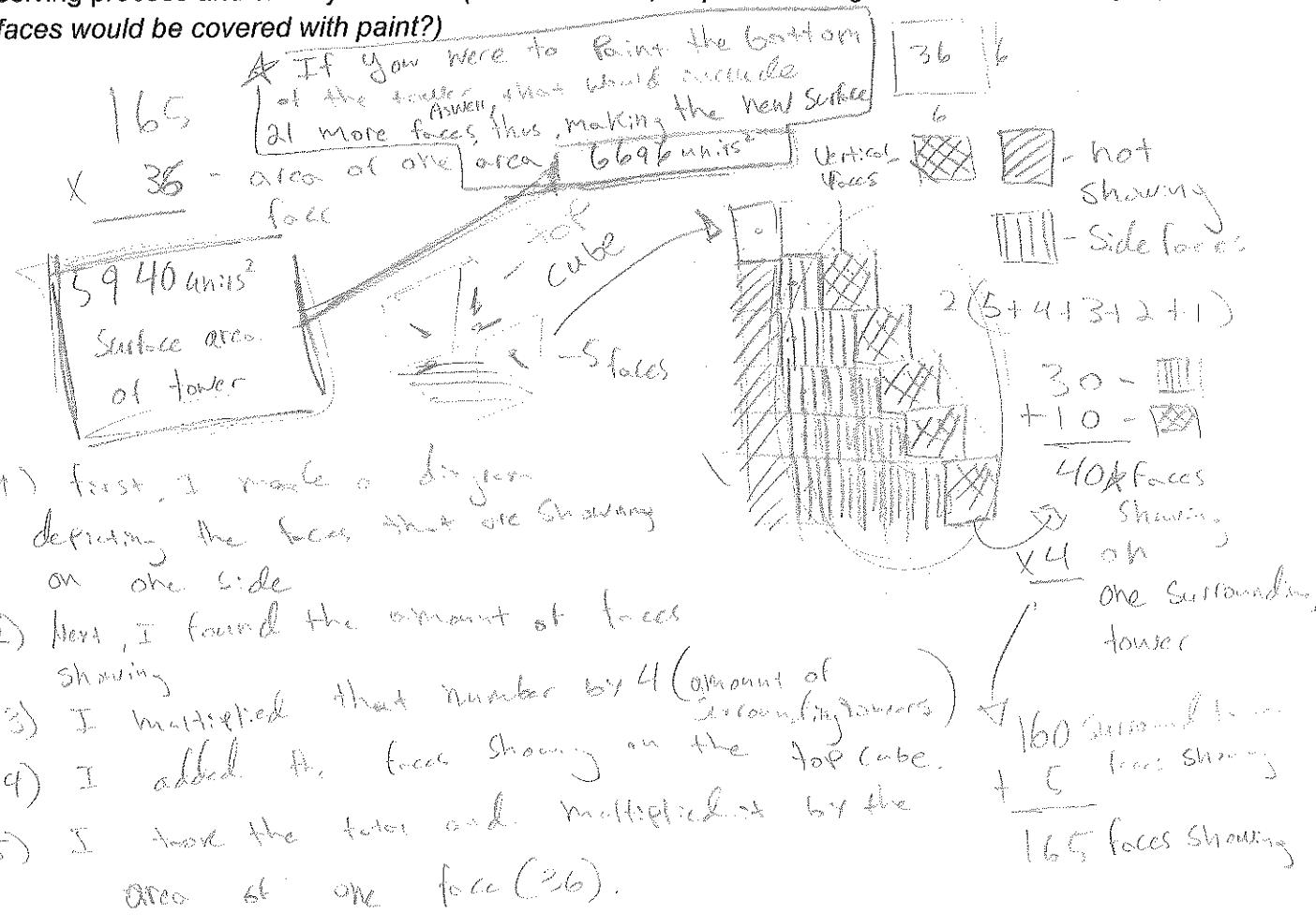
66 cubes

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.



5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

25

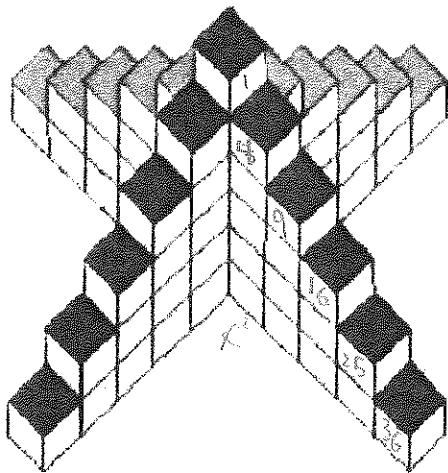


Building Towers (Yellow Worksheet)  
10 points possible

25  
10

Name: Sean Kelley  
Date: 3/5/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



Top section

$$f(x) = (x+2)(x+3)+10(2x)$$

$$f(x) = 15x + 61$$

Bottom

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower has 1 block at the top, the second layer has 2 blocks, the third layer has 3 blocks, the fourth layer has 4 blocks, the fifth layer has 5 blocks, and the bottom layer has 6 blocks. The tower is solid and has a height of 6 cubes.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

I noticed the cubes form a parabolic function, where the height of the blocks follows a defined parabolic function. I used the formula  $f(x) = \frac{1}{2}x^2 + bx + c$  and found  $f(6) = 36$ .

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, it would not work because the formula is not linear.

Agreed!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$(C_6) = 2(2+3n)2 (= 15n + 6)$$

$$(C_6) = n^2 + n + 1 - 1$$

$$(C_6) = 2(2+3(6))+1$$

$$n^2 + n + 1 - 1$$

$$(C_6) = 1 + 0 - 0 \quad (C_6) = 1 + 4 + 1$$

would this work  $72 + 18 + 1 = 56$  vs  $2(2+3(6)) + 1 = 56$ . So  $(C_6) = 4 + 1 + 1$  for all cases? What's the formula you are using?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

5  
5

I take my equation from #3 and  
solve for n

$$19,900 = 2n^3 - 3n^2$$
$$\rightarrow -2$$

$$19,898 \quad 2$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

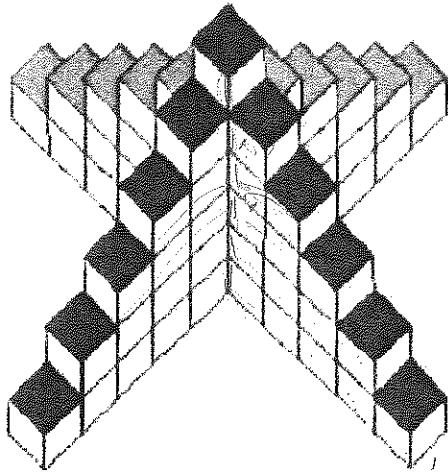
**Building Towers (Yellow Worksheet)**

**10 points possible**

3.5  
10

Name: Jessica A.M.  
Date: 10/10/17

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$6 + 4(5 + 4 + 3 + 2 + 1)$$

$$5 + 4 + 3 + 2 + 1$$

$$5 - 1$$

$$5 - 2 \quad 20 - 10$$

$$5 - 3$$

$$5 - 4$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

It is 6 blocks tall the base contains 24  
most blocks has 4 sides each making a staircase shape

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$6 + 4(5 + 4 + 3 + 2 + 1) = 66$  I took the width of the center of the tower and added the residing rows until I got to 1 then multiplied by 4.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

If you would do  $300 + 4(300 + 299 + \dots)$  however

there could be an easier way ↑ how would you easily compute this?

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

2  
 $n + 4($

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

~~5~~  
5

$$19,900 = n + 4(n-1)$$

would this formula

work for any height?

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

x 10

5 = top block

$$((5(2)+5) \cdot 4) + 5 = 145 \text{ blocks}$$

$$(30+5) \cdot 4$$

$$\begin{array}{r} 35 \\ \times 4 \\ \hline \end{array}$$

$$\underline{140+5}$$

$$6 + 15(n)$$

$$66$$

if  $n=6$

$$n + 4((n-1)($$



$$6 + 4(24) = 14$$

$$6 + 4((6-1)$$

$$6 + 4(5 + 0) = 6 + 20 = 26$$

$$7 + 4(7 + 6) = 7 + 4(13 + 2 + 1) = 7 + 26$$

$$8 + 4(8 + 6 + 5 + 4 + 3 + 2 + 1) = 8 + 36 = 44$$

$$9 + 4(9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) = 9 + 36 = 45$$

5



$$n + 4(10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) = 55$$

12

$$8 + 9$$

8

9

10

$$\frac{66}{4}$$

$$16.5$$

$$66 = 4 \cdot ?$$

$$66 = n^2 + 4(n-1)(n+1) = n(n+1)(4n+3)$$



**Building Towers (Yellow Worksheet)**

**10 points possible**

5  
10

Name: Griffen, Jordan

Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>

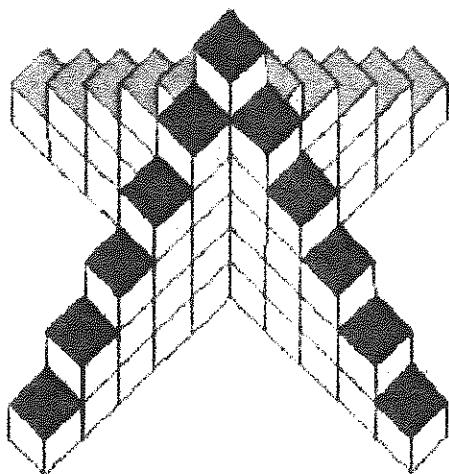


Diagram below shows

5 + 5

5 + 5

- 1) (1 point) Describe what you notice about the structure of the tower above.

The diagram shows a central vertical column of 6 cubes, with 5 additional cubes branching off to the left and 5 branching off to the right at each level above the base.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6 + 4(5) + 4(5) + 4(5) + 4(5)$$

$$6 + 4(5) + 4(5) + 4(5) + 4(5)$$

66 cubes

66 cubes  
The diagram shows a central vertical column of 6 cubes, with 5 additional cubes branching off to the left and 5 branching off to the right at each level above the base.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

My strategy could be adjusted to calculate exactly!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$C = n + 4($$

Try analyzing how you found the # of cubes in the provided figure

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

5

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

x2

$$5 + 4(20) + 2(40)$$

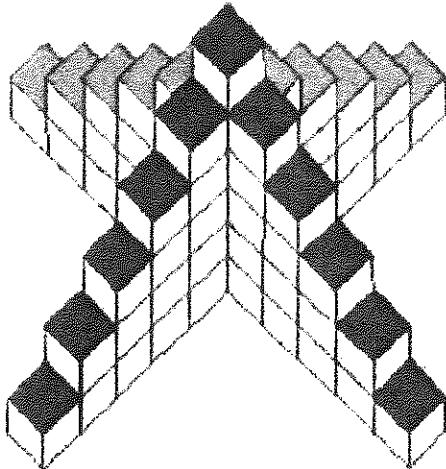
$$5 + 80 + 80$$

$$165 \text{ sf}$$

The solution for the 400x1 is:  
The solution for the 80x80 is:  
the 20x20 is:  $2(20) = 40$  is all of it  
the 5x5 is:  $5 \times 5 = 25$  is all of it

Great!

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

the tower has 4 sides all with equal number of cubes. The center has the largest height but in this case also the fewest cubes.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$1+2+3+4+5 = 15$$

$$15 \cdot 4 = 60$$

$$60 + 6 = 66 \text{ cubes}$$

the tower has 66 cubes. Each side has 15 cubes in it and there are four sides. Added with the middle six there are 66.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

NO, I would have to add numbers 1 through 299, then multiplying that by 4, this would just require a lot of time.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$4\left(\frac{n}{2}(n)\right) - n$$

I noticed a relationship

Great!

$$4\left(\frac{6}{2}(6)\right) - 6$$

$$4(18) - 6$$

66 cubes

between the side blocks and the height, then multiplied by 4 for the sides. However I was always 6 off so I just subtracted it.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\times 5 \quad 19900 = 4(\frac{1}{3}(n)) + n$$

$$2(4975) = (\frac{n^2}{3} + n) 2$$

$$9950 = n^2 + 3n$$

$$0 = n^2 + 3n - 9950$$

$$a=1 \quad b=3 \quad c=-9950$$

$$-\frac{b}{2} \pm \sqrt{\frac{b^2 - 4ac}{4}}$$

$\Delta > 0$

$$\frac{2 \pm \sqrt{4 - 4(1)(-9950)}}{2}$$

$$x = 100 \text{ or } -98.75$$

the tower is 100 cubes high. I took the formula I got in 3, then used the answer as the x. I knew if I put the equation into a quadratic I could get the answer using the quadratic formula. I threw out the negative answer and got 100.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

x2

$$5+5+1$$

$$4+4+1$$

$$3+3+1$$

$$2+2+1$$

$$1+1+1$$

$$\underline{3S+S}$$

$$40+4$$

$$160+S$$

$$165$$

165 units.

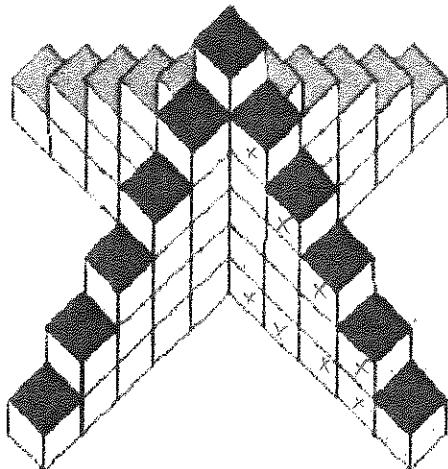
I found how much was on one side multiplied it by 4 and added 5 for the top.

**Building Towers (Yellow Worksheet)**  
10 points possible

9.5  
10

Name: Robert O'Keefe  
Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

a central stack of cubes with four stacks coming off each side of the central stack. each stack coming off the previous stack decreases by one, after the first stack only one stack comes off that one.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{cccccc} 6 & + & (3 \times 4) & + & (4 \times 4) & + (3 \times 4) + (2 \times 4) + (1 \times 4) = 66 \text{ cubes} \\ & 6 & 20 & 16 & 12 & 8 & 4 \end{array}$$

Finding the amount each individual step down and add them up

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No because you would need to write out  $n \times n$  for all 300 steps of the tower except center and add them up

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n^2 - n$$

How did you get this?  
Does it work for a height of 6 units?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$\times 5$

Using the formula from  
the last problem 19,900  
was plugged in for the  
answer and solved for  
n or the height of the  
tower.

$$\begin{aligned}2n^2 - n &= 19,900 \\2n^2 - n - 19,900 &= 0 \\(2n + 199)(n - 100) &= 0 \\2n + 199 &= 0 \quad n - 100 = 0 \\n &= \cancel{-199} \quad n = 100\end{aligned}$$

*Good!*  
the height of the tower  
is 100 cubes

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$+ .5$

$$1 + (12 \times 2) + (12 \times 3) + (12 \times 4) + (5 \times 5)$$

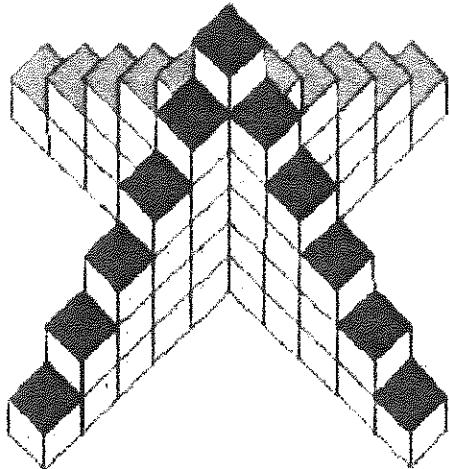
$$\boxed{150 \text{ units}^2}$$

**Building Towers (Yellow Worksheet)**  
**10 points possible**

10.5  
10

Name: Greg Lefay  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

4 legs, the steps go up by 1 block, the maximum number of blocks in the middle is 6, there are 6 levels. one leg is 15 blocks excluding the middle

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{aligned} & 4(5+4+3+2+1) + 6 \\ & 4(15) + 6 \\ & 60 + 6 \\ & 66 \end{aligned}$$

You multiply one leg by 4 to get the 4 legs, then you add the middle to find the total # of blocks it takes.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because it would be very tedious to find the final answer,  $4(299+298+297\dots+1) + 300$ . This would take a very long time to complete, a very inefficient way to solve the problem.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n^3 + (n^2 - n)$$

The total number of cubes  
is equal to  $n^3 + (n^2 - n)$

Does it work?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$x 5 \quad n^2 + n^2 - n = 19900$$

$$2n^2 - n = 19900$$

$$2n^2 - n - 19900 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nice!  $n = \frac{1 \pm \sqrt{1 + 4(2)(19900)}}{2(2)}$

$$n = \frac{1 \pm \sqrt{159200}}{4}$$

$$n = 100$$

The height of the tower is 100 cubes. I took the formula I found in part 3 and I set it equal to the max number of cubes, then I set the equation equal to 0 by getting everything on the left side of the equation and you get a quadratic. At first I attempted to factor then I realized I could just use the quadratic formula and I plugged the numbers in and got  $n=100$ . I then plugged 100 into my original equation and got 19900 blocks.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+1.5

Total Number of  
cubes: 66

$$4(21) + 2(40) + 0(5)$$

$$84 + 80 + 0$$

$$164 \text{ blocks}$$

$$4(n-1) + 1 + 2(2(n-1)) + 0(n-1)$$

$$4(n-1) + 1 + 2(8(n-1)) + 0(n-1)$$

$$4(n-1) + 1 + 16(n-1) + 0(n-1)$$

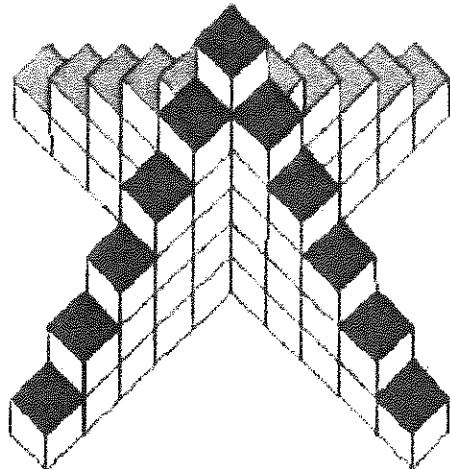
I took down for the tower from a birds eye view and found there are 21 blocks on the top, and now including the tip, there are 21 blocks. All the top blocks have 4 sides exposed, so  $4(21)$  then I found the amount of blocks in the middle that would have no paint on them so  $0(5)$ . The rest of the blocks only have 2 sides showing, so  $2(40)$  all the numbers in the parentheses add up to the total number of blocks in the tower.

**Building Towers (Yellow Worksheet)**  
10 points possible

9.5  
10

Name: Jenna Eagle  
Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

\* it looks like 2 planes, each with an absolute value graph ← interesting!  
\* to be more basic, it looks like 4 identical staircases leading to one point.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$4(1+2+3+4+5)+6 = 60 \text{ cubes}$$

each of 4 sides has a column consisting of 1, 2, 3, 4, and 5 cubes  
the center consists of 6 cubes

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? no, it would be easier to write  $n$  as the total height and use factorial numbers, with a calculator, to solve.  
 $4((n-1)!) + n = n$  of cubes needed  
because writing out each number 1-299 would be tedious.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

Yeah, I just did that in 2b. I'm always jumping the gun :/ that's 6k!

$$n: 6 = 4((6-1)!) + 6 = 60 \text{ yay it works.}$$

I later found out I did that wrong. see \*4 :/

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

2.5

if # of cubes =  $4((n-1) + n)$  and  $n = 19,900$ , then use a calculator to find that # of cubes = a number that's too big for my calculator to handle maybe  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ , not  $5+4+3+2+1$ . oops. time for a new solution.

$$\begin{array}{c}
 \text{6 tower} \\
 \left( \begin{array}{l} 6 \cdot 6 \\ 1+5=6 \end{array} \right) \\
 2+4=6 \\
 3=3 \\
 4(2.5)(6+6) = 4(3) \cdot 13
 \end{array}
 \quad
 \begin{array}{c}
 \text{7 tower} \\
 \left( \begin{array}{l} 7 \cdot 7 \\ 1+6=7 \end{array} \right) \\
 2+5=7 \\
 3+4=7 \\
 4(3.5)(7+7) = 4(3.5) \cdot 14
 \end{array}
 \quad
 \begin{array}{c}
 \text{8 tower} \\
 \left( \begin{array}{l} 8 \cdot 8 \\ 1+7=8 \end{array} \right) \\
 2+6=8 \\
 3+5=8 \\
 4+4=8 \\
 4(4)(8+8) = 4(4) \cdot 16
 \end{array}
 \quad
 \begin{array}{c}
 \text{9 tower} \\
 \left( \begin{array}{l} 9 \cdot 9 \\ 1+8=9 \end{array} \right) \\
 2+7=9 \\
 3+6=9 \\
 4+5=9 \\
 5+4=9 \\
 4(4)(9+9) = 4(4) \cdot 18
 \end{array}
 \end{array}$$

$$\# \text{ of cubes} = 4n(\frac{1}{2}n - \frac{1}{2}) + n \checkmark$$

NOW I will solve that problem.

$$n = 4(19,900)(\frac{1}{2}(19,900) - \frac{1}{2}) + 19,900$$

$$n = 79,600 \cdot 9,949.5 + 19,900$$

$$n = 781,980,200 + 19,900$$

$$n = 781,980,200 + 19,900 \text{ cubes}$$

$$19,900 = 4n(\frac{1}{2}n - \frac{1}{2}) + n$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) I'm not including the bottom of the tower.

x2

most of the side faces would be  $2 \cdot 4n(\frac{1}{2}n - \frac{1}{2})$    $\times 8$

all of the top faces would be  $4(n-1) + 1$  

the rest of the side faces would be  $4n$  

$$8n(\frac{1}{2}n - \frac{1}{2}) + 4(n-1) + 1 + 4n$$

$$8(6)(\frac{1}{2}(6) - \frac{1}{2}) + 4(6-1) + 1 + 4 \cdot 6$$

$$48(2.5) + 4(5) + 1 + 24$$

$$120 + 20 + 25$$

165 faces

**Building Towers (Yellow Worksheet)**

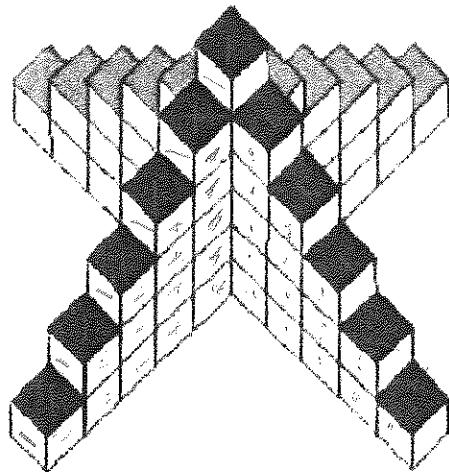
10 points possible

12  
10

Name: Jenna Singer

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure is symmetric, increasing by 3 blocks each time as you work from the outside in.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{l} \text{Front view: } 6+5(1)+4(1)+3(1)+2(1)+1(1) \\ = 6 + 10 + 16 + 12 + 8 + 4 \\ = 66 \text{ cubes} \end{array}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

: NO, because it requires drawing a diagram and writing out each row. A formula would be easier - yes!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{array}{l} \text{# of cubes} \\ y = n + 2(n)(n-1) \end{array}$$

$$\begin{aligned} y &= 6 + 2(6)(5) \\ &= 6 + 2(30) \\ &= 66 \end{aligned}$$

\* Two legs together make a height of  $n$ , and length of  $n-1$ .

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

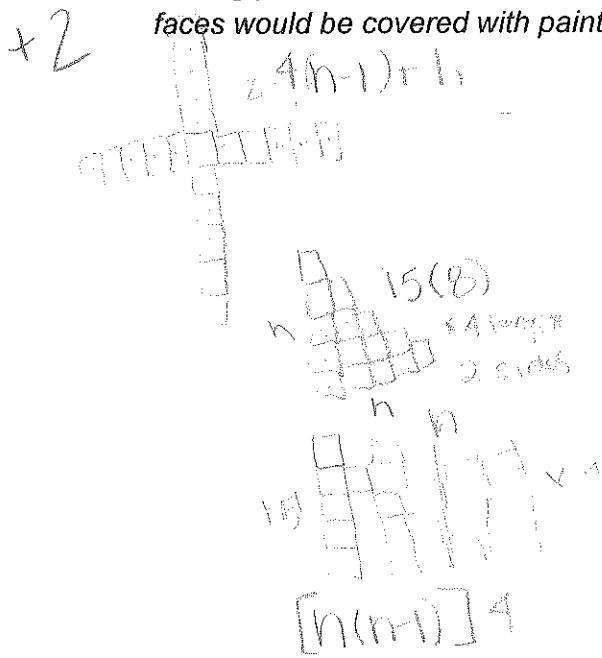
$$\begin{aligned} \text{a)} \quad & y = n + 2(n)(n-1) \\ \times 5 \quad & 19,900 = n + 2(n)(n-1) \\ & = n + 2(n^2 - n) \\ & = n + 2n^2 - 2n \\ & 19,900 = 2n^2 - n \end{aligned}$$

$$\begin{aligned} 0 &= 2n^2 - n - 19,900 \\ (2n+199)(n-100) &= 0 \\ 2n+199 &= 0 \quad n-100 = 0 \\ 2n &= -199 \quad |n=100| \\ n &= -99.5 \quad |n=100| \end{aligned}$$

Great!

I plugged 19,900 into the formula for  $y$ , then set the equation equal to zero. Since I received two answers, I knew the height couldn't be a negative number, but 100 was not in the equation.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)



 6(4)  $\boxed{\text{TSA} = 165}$

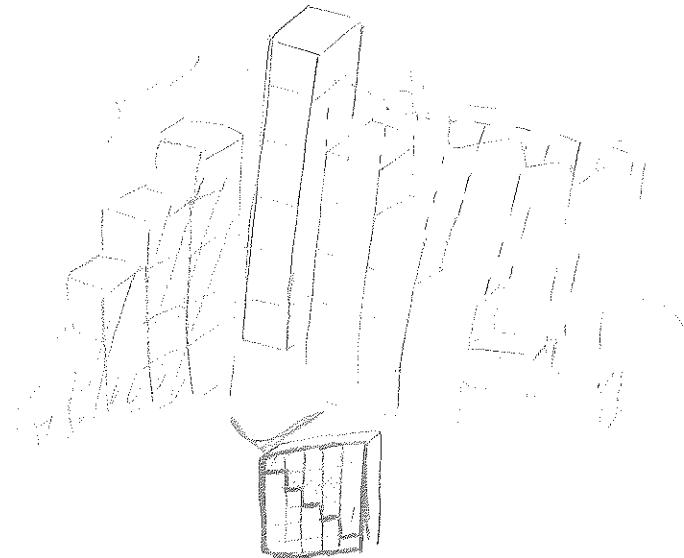
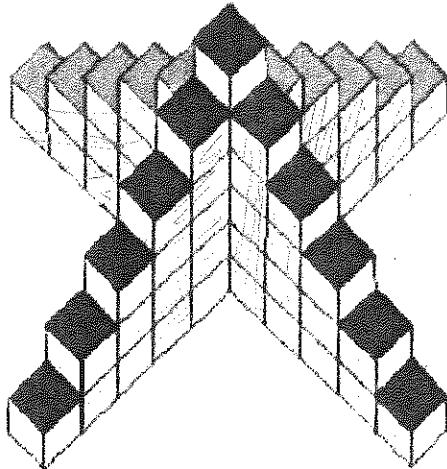
4n To solve this, I looked at the tower from all sides, drew a 2-D diagram, and created an equation/formula for each, then put in the height of 6.

**Building Towers (Yellow Worksheet)**  
10 points possible

85  
10

Name: Jennifer Sung  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

There is a stack of 7 cubes in the center of the tower with 16 cubes on four sides, forming a staircase pattern with descending steps on each side.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Height of central stack +  $4(5+4+3+2+1)$

$$6 + 4(15) = 6 + 60 = 66$$

cubes necessary for each of the descending structures on each side

66 cubes are needed

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, the equation used would be  $300 + 4(299 + 298 + 297 + 296 \dots)$   
it would be tedious & take too long to put set it up.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$x = n + 2(n(n-1))$$

$$x = n + 2(n^2 - n)$$

$$x = n + 2(n^2 - n)$$

$$6 + 2(36 - 6)$$

$$6 + 2(30)$$

$$6 + 60 = 66 \checkmark$$

Great!

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

3/6

You can use the equation from question 3 and solve for n with 19,900 as x, since the height(n) is needed, and the number of cubes(x) is provided.

$$x = n + 2(n^2 - n)$$

$$19,900 = n + 2n^2 - 2n$$

$$19,900 = 2n^2 - n$$

quadratic formula?

$$n_{\text{approx}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2n^2 - 1 = 19,900$$

$$9,950.5$$

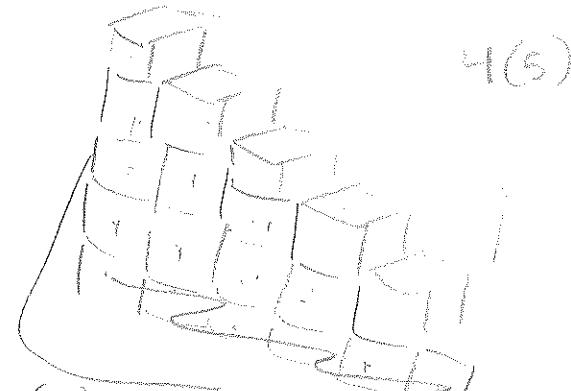
$$\underline{\underline{9,950.5 = n}}$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+ .5

5/15

$$\text{SA} = 144 \text{ ft}^2$$



~~2(3)~~ 2(3)

4(6)

~~236~~  
236  
144

$$5 + 4(2 \cdot 8 + 4 \cdot 3)$$

$$5 + 4(16 + 12)$$

$$5 + 4(30) = 125$$

$$5 + 144$$

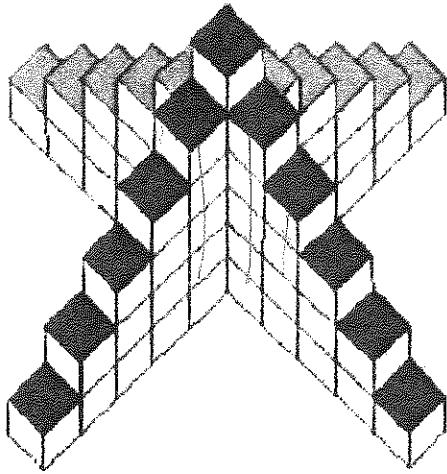
$$= 149$$

Building Towers (Yellow Worksheet)  
10 points possible

2.5  
10

Name: Sydney Richardson  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$6 + (n-1)4 + (n-2)4 + (n-3)4$$

$$6 + (n-5)4$$

$$6 + 5(4) + 4(4) + 3(4) + 2(4) + 1(4)$$

$$n + (n-1) + (n-2) + (n-3)$$

$$n + 4(n-4)$$

$$n + 4(n-1)$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure resembles four staircases banded together by a tower in the middle that is one cube higher than the next highest height of the first staircase.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

are four staircases,  $6 + 5(4) + 4(4) + 3(4) + 2(4) + 1(4) = 66$  cubes and added together since the height of the tower is six cubes, that height to get the height has to be added on. Since the staircases descend by of the staircase are each time, that number multiplied by four. See there

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

-5

I wouldn't easily

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

~~$n + (n-1)4 + (n-2)4 + (n-3)4$~~

Would this work for any height?

-2

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

-5

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (*In other words, if I paint the original tower, how many square faces would be covered with paint?*)

**Building Towers (Yellow Worksheet)**

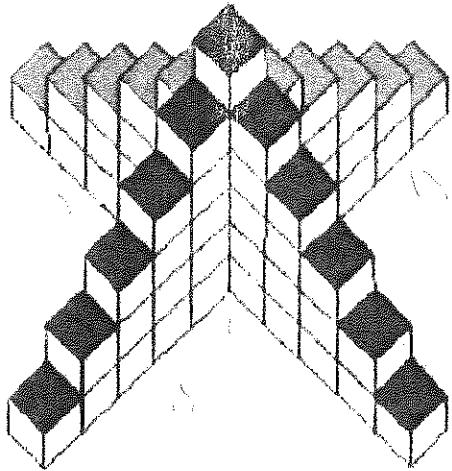
10 points possible

11.5  
10

Name: Jordan Fine

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

rotational symmetry around its side edges  
 - height of 6 units volume of 66 cubes

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

66. Legs = 3 in height (one on the left)  
 so legs having 5, 4, 3, 2, 1, add together give 15. 15 x 4 = 60  
 Four legs total so  $15 \times 4 = 60$ , plus bottom center column  
 $6 + 6 = 12$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

at especially easy, but easier  
 than counting by hand. This is  
 because my method only deals with one leg

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{aligned} V &= n + 2(n-1)(n) & V &= 2(36) + 6 \\ V &= n + 2n^2 - 2n & V &= 72 + 6 \\ V &= 2n^2 - n & V &= 78 \end{aligned}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\frac{4.5}{5}$$

$$19900 = 2n^2 - n$$

$$2n^2 - n - 19900 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{1 \pm \sqrt{1 + 4(2)(19900)}}{2(2)}$$

$$n = \frac{1 \pm \sqrt{1 + 16(2)(19900)}}{4}$$

$$n = \frac{1 \pm \sqrt{1 + 16(39800)}}{4}$$

$$n = \frac{1 \pm \sqrt{1 + 636800}}{4}$$

$$n = \frac{1 \pm \sqrt{636801}}{4}$$

$$n = \frac{1 \pm 791}{4}$$

$$n = 200$$

Simplifies into an  
eliminator, then  
uses quadratic  
formula.  
Height is 200  
cm.

further  
explanation  
of process?

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$+2$$

$$2 \text{ sides of 6x6 base} = 2(6)(6) = 72$$

$$6 \text{ front edges on base} = 6(6) = 36$$

$$(3) \quad (1)$$

Great!

$$\text{bottom} = 6(6) = 36$$

$$4 \text{ top edges} = 4(6) = 24$$

$$= 36 + 24 = 60$$

$$\text{total surface area} = 60$$

$$196 \text{ units}^2$$

**Building Towers (Yellow Worksheet)**

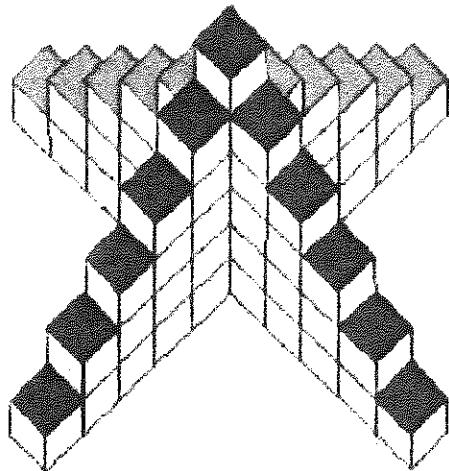
**10 points possible**

10  
10

Name: Bayley Costner

Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower has columns of blocks that branch out from the center column, and decrease in height by 1 block each column

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\rightarrow 4(5+4+3+2+1) + 6 = 66$$

multiply by 4 for the number of the number  
each "branch" of cubes in each of blocks  
the tower "branch" in the center column

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

no, because finding the number of cubes in each "branch" would take too long

exactly!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\checkmark n + 4\left(\frac{n}{2} \cdot (n-1)\right) = c$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$n + 4 \left( \frac{1}{2} (n-1) \right) = 19,900$$

$$\cancel{n} + 4 \left( \frac{n^2}{2} - \frac{n}{2} \right) = 19,900$$

$$n + 2n^2 - 2n = 19,900$$

$$2n^2 - n = 19,900$$

$$2n^2 - n - 19900 = 0$$

$$(2n+199)(n-100) = 0$$

$$2n = -199 \quad n = 100$$

$$n \geq 99.5$$

$$(n = 100)$$

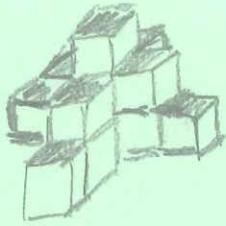
Explain your problem-solving  
method

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

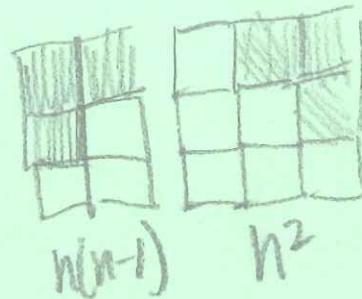
$\times 2$

$$(5 \cdot 5) + (4 \cdot 16) + (3 \cdot 16) + (2 \cdot 24) + (1 \cdot 1) + (0 \cdot 4) = 186 \text{ units}^2$$

$\uparrow$ number of blocks with 5 sides showing	$\uparrow$ # of blocks with 4 sides Showing	$\uparrow$ # of blocks with 3 sides showing	$\uparrow$ # of blocks with 2 sides Showing	$\uparrow$ # of blocks with 1 side showing	$\uparrow$ # of blocks with 0 sides showing
-----------------------------------------------------------	------------------------------------------------------	------------------------------------------------------	------------------------------------------------------	-----------------------------------------------------	------------------------------------------------------



4 parts w/ a middle



$$1+2+3+4+5+6 = 3(7)$$

$$4[3(7)-3(6)] = 18 \quad 84-18 \\ 66 \checkmark$$

$$4(1)+4(2)$$

$$4(1+2+3+4+5)+6$$

$$4(2.5)(6)+6 = 66 \checkmark$$

36x25

yes!

$$n^2 + \cancel{6n}$$

$$\cancel{n^2} + 2n^2 + 1$$

$$2n^2 + 2n + 1$$

$$n^2 + n(n-1)$$

$$2n^2 + 2n - 3n = 2n^2 - n \quad \checkmark$$

$$n^2 + n^2 - n$$

$$\underline{n + (2n-2)(n)}$$

$$2n^2 - n$$

$$n + 2n^2 - 2n \\ 2n^2 - n \quad \checkmark$$



**Building Towers (Yellow Worksheet)**

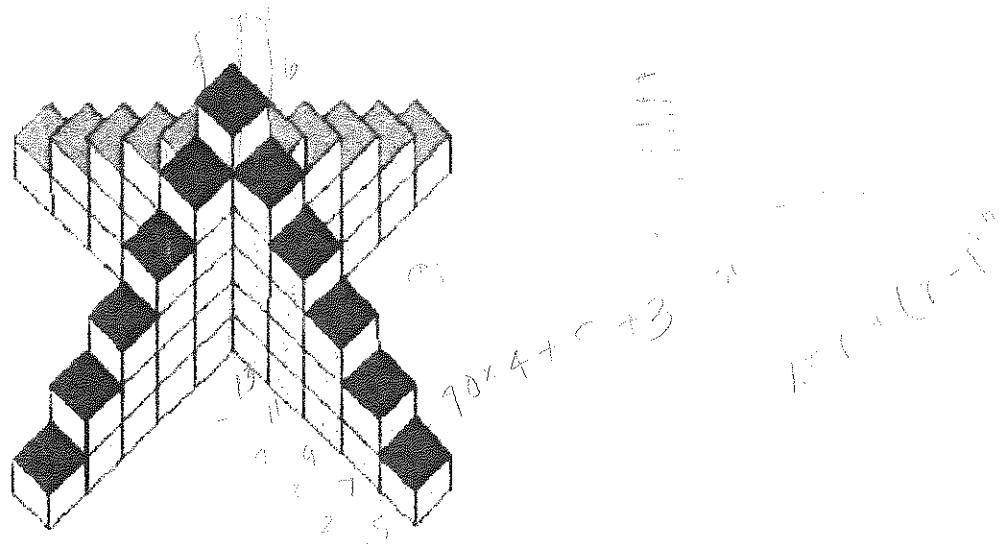
**10 points possible**

10.5  
10

Name: KATHYANNE WILSON

Date: 3/5/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

AS EACH COLUMN GETS CLOSER TO THE CENTER,  
 ONE BLOCK IS ADDED TO THE TOP. THERE ARE 6  
 EQUAL SPOND LEGS EACH WITH THE SAME  
 NUMBER OF BLOCKS.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$(15 \times 4) + 6 = 66$  CUBES. EACH "LEG" HAS 15 BLOCKS  
 AND THE CENTER COLUMN HAS 6. I MULTIPLIED  
 THE LEGS BY THE NUMBER OF BLOCKS AND ADDED  
 THE CENTER COLUMN.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

NO, BECAUSE IT WOULD TAKE TOO LONG AND WOULD BE  
 INEFFICIENT. A SIMPLIFIED VERSION OF THE FORMULA IS GOOD!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

NUMBER  
OF CUBES

1 unit: 6

2 units: 16

3 units: 26

$n^2 + 4(n-1)^2$

$n^2 + 4(n-1)^2$

$n^2 + 4(n-1)^2$

$n^2 + 4(n-1)^2$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

4.5

$$19,900 = 2n^2(n+1) + n$$

$$= 2n^3 + 2n^2 + n$$

$$= 2n^2 + n + 19,900$$

$$1 \pm \sqrt{1 + 4(2)(19,900)}$$

$$\pm$$

$$\frac{1}{4} + \frac{397}{4}$$

$$\boxed{n = 100}$$

$$19,900 = 2(100)(100) + 100$$

$$\checkmark = 19,900$$

The height of  
this tower is  
100 cubes.

My method was  
using the formula &  
solve for n  
further  
explanation?

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

X	Y	SP = 185 186?
1	2	I counted the open faces,
2	7	multiplied by if you added
3	9	the top.
4	11	
5	13	

- can you also find  
a formula for the  
surface area of  
a tower of height n?

$$\text{new } S = 45 \\ 6(1, 5, 6) = 45$$

$$2n+3$$

Building Towers (Yellow Worksheet)

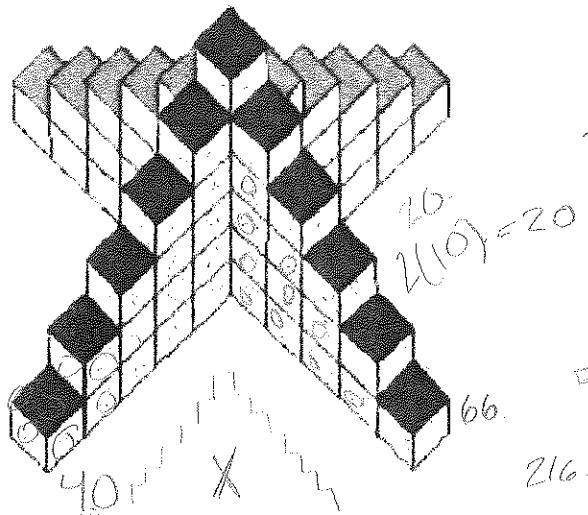
10 points possible

12  
10

Name: Sumeet Sidhu

Date: 3/6/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$7 \cdot 21 \cdot 4 = 84 + 7 = 91 \\ 2(7^2) - 7 = 91$$

$$\begin{array}{r} 5 \cdot 4 = 20 \\ \times 4 \\ \hline 80 \\ 80 \end{array}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure looks like a X if you look from the top view. Looking from one of the sides it looks like a triangle with steps. It's like a pyramid but its missing sides. Going up the cubes increase by one.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

For this tower you need 66 cubes. I added up the sides first leaving the middle for later. One side had 15 and  $15 \times 4 = 60$  and adding that made 66.  $1+2+3+4+5+6 = 21$  and  $21 \times 4 = 84$ . Then  $84 + 6 = 90$ .

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? No it wouldn't work easily because counting the amount of cubes on one side would become a very tedious job, yes!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

To find this formula I played around trying to find out what 6 and 66 had in common. I noticed that  $6^2 + 6^2 = 72$  is close to 66. The difference is of only 6. So I derived the formula  $2(n^2) - n$ .

$$\begin{aligned} & 2(6^2) - 6 \\ & 2(36) - 6 \\ & 72 - 6 = 66 \text{ ✓} \end{aligned}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2(n^2) - n$$

$$2n^2 - n - 19,900 = 0$$

$$(2n+199)(n-100) = 0$$

$$2n+199=0 \quad n-100=0$$

$$n=-99.5 \quad n=100$$

Bonus!  
The height would  
be 100 cubes.

I took the formula I had and made it equal to 19,900, then I solved for n. I got two different answers but I knew the height couldn't be negative so it had to be the positive answer 100.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

12



40 squares shown  
on one wing of  
the shape.

$$\begin{aligned} 20 &= \frac{\text{top 5 have 4}}{\text{sides showing}} \\ + 20 &= \frac{\text{bottom 10 have}}{\text{2 showing}} \\ \hline 40 &= \end{aligned}$$

The surface area without the bottom would  
be 165 sq. cubes.

$$\begin{array}{r} 40 \times 4 = 160 \text{ blocks shown of } \\ + 5 \leftarrow \text{ shown of the very top cube.} \\ \hline 165. \end{array}$$

a) Surface area formula:

$$4(4(n-1)) + 5 + 2$$

**Building Towers (Yellow Worksheet)**

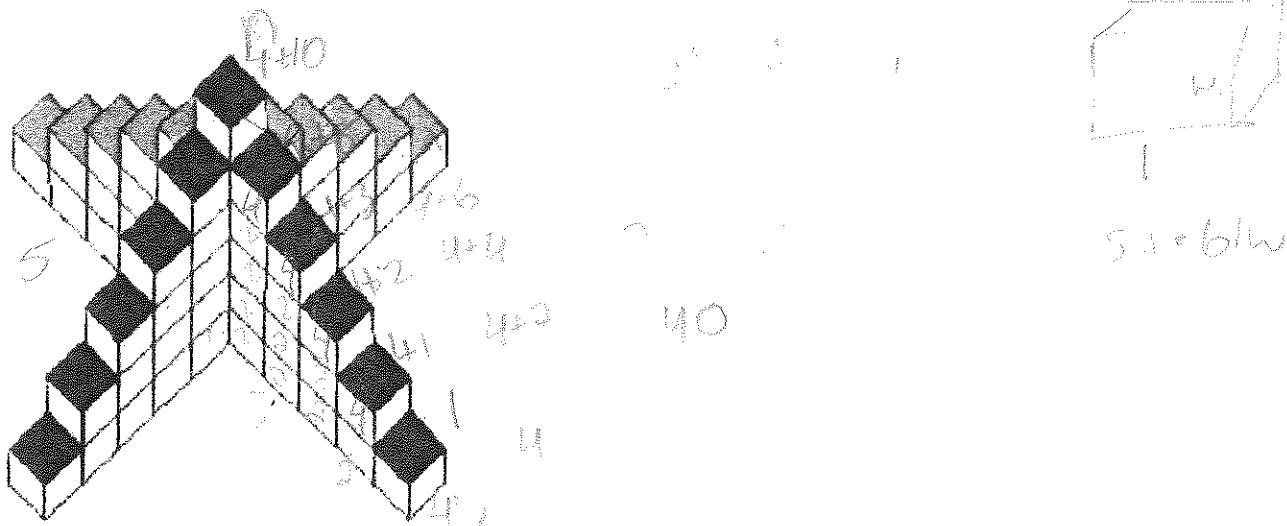
10 points possible

11.5  
10

Name: Valeria M.

Date: 2/6/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

It's three-dimensional, there are 11 rows, 25 blocks all connected in the center, the tower is 6 blocks high and 5 blocks long with a width of 5 blocks. And the depth is 5 blocks.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

The length one side is 5 blocks and to find how many blocks are on two sides we would multiply the height times one of the sides. ( $6 \times 5 = 30$ ) blocks. And that is only half of 30(2) = 60 blocks. Also the center must be accounted for, so add the center ( $6 - 6 = 36$  blocks).

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Yes because to find the height of the tower you would multiply the height times one of the sides. If you do that you get the amount of blocks for just the center ( $300 \times 5 = 1500$  blocks) and then  $\times 2 = 3000$  blocks. Then multiply by 3 and add the height once more to get 3000 blocks + 3000 blocks = 6000 blocks.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$2n \times (n-1) + n$$

$$2n^2 - 2n + n$$

$$2n^2 - n$$

$$2(6)^2 - 6$$

$$66$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

4.5

$$2n^2 - n = 19900$$

- used the formula

$$2n^2 - n - 19900 = 0$$

- all on one side

$$-b \pm \sqrt{b^2 - 4ac}$$

- quadratic formula

$$1 \pm \sqrt{1 + 4(2)(-19900)}$$

- Solve

$$\frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{4}$$

Further explanation  
of problem-solving  
method?

$$\frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{4} = \frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{4}$$

$$\frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{4} = \frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{4}$$

Answer: [100 blocks]

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

x2

$10 \times 4 = 40$  blocks  $\rightarrow$  one side of the tower  
is 40 blocks wide  
+ 4 sides  
+ 1 top

the 40 blocks covering  
the 4 sides  
+ the other 3 sides  
(not covering the bottom)

$5 \times 4$  blocks + 4 sides  
are showing

The top blocks on one  
row have 3 sides  
that are not painted

1 block at top with 3  
sides showing

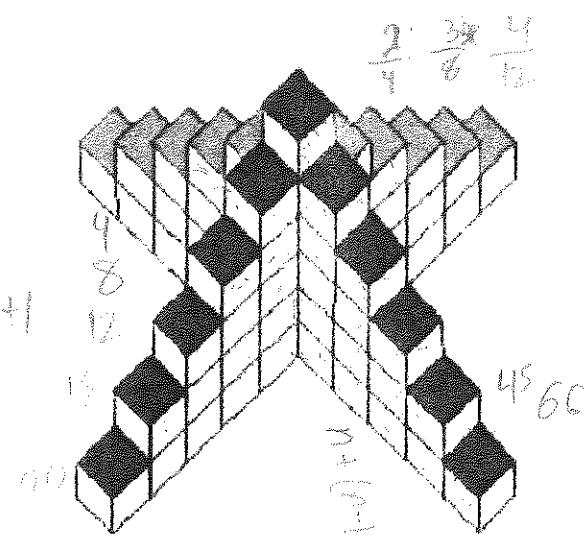
the 3 sides are not  
painted

$$(40 \times 2) + (5 \times 4)(4) + 3 =$$

160 + 80 + 3 = 243

Surface Area [163 square faces]

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{array}{c} 2 \\ 3 \\ 4 \\ \hline 4 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array}$$

$$\begin{array}{c} 3 \\ 6 \\ 9 \\ \hline 3 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array}$$

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{array}$$

$$\begin{array}{c} 4(n-1) \\ n(4n-9) \\ n-1 \end{array}$$

$$\begin{array}{c} 6 \\ 3 \\ 2 \\ 1 \end{array}$$

$$\begin{array}{c} 4(n-3) \\ n+4(n-1)+4(n-2)+4(n-3) \\ n+4n-4+4n-8+4n-12 \\ n+12n-24 \\ n+16n-24 \\ n^2-4n^2 \end{array}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

one block down the middle with 3 more  
cubes coming out of it, each layer has one  
more block than the one above it. the top  
of each block is shaded.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{c} \text{Sum all of the cubes in our big matrix } 6 \times 6 \\ n+4(\sum(n-1)) \text{ and add the center column. } 6 \times 6 = 36 \end{array}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

yes, because if we use the formula which  
the height  $n$  can be plugged in to find the  
how "easy" would summing  $(n-1)$  from 1 to 300?

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n+4(\sum(x-1))$$

How would you compute this?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

The Height is 100.

$$19900 = 10 + 4 \left( \sum_{x=1}^{100} (x-1) \right)$$

\*2.5

$$19900 = 10 + 4 \left( \sum_{x=1}^{100} (x-1) \right)$$

After I got my first formula, I didn't know how to solve it for  $N$ . I guess and checked and found that when I plugged 100 in, it came out as 19900 how?

5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

# Building Towers (Yellow Worksheet)

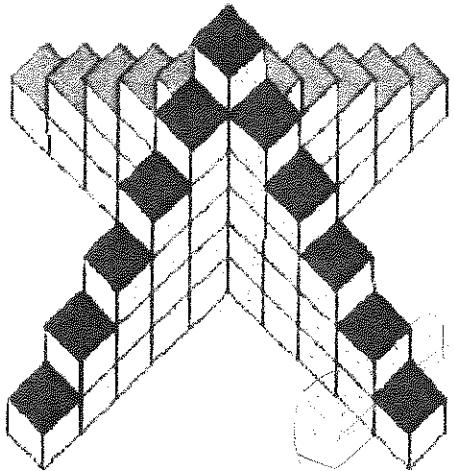
10 points possible

12  
10

Name: Trent Hock

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$(2n)^3 + 4(n-1)$$

$$2 \cdot 6^3 + 4(6-1)$$

$$144 + 4(5) = 176$$

$$176 + 4$$

$$= 180$$

$$6 \cdot 6 \cdot 6 = 216$$

$$5 \cdot 6 \cdot 5 = 150$$

$$4 \cdot 6 \cdot 4 = 96$$

$$3 \cdot 6 \cdot 3 = 54$$

$$2 \cdot 6 \cdot 2 = 24$$

$$1 \cdot 6 \cdot 1 = 6$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower is shaped like a cross, where each leg of the cross is 1 block wide. At the end of each leg, the tower is 1 block high. At the second block in, the tower is 2 blocks high. At the third, it is 3, and so on until the legs meet at the center, which is 6 blocks high.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

6 + 4(5, 4, 3, 2, 1) Each leg consists of columns of 5, 4, 3, 2, and 1.  
 6 + 4(15) So, I can add up how many cubes were in each leg.  
 6 + 60 And since there are 4 legs, multiply by 4.  
 66 cubes Just had to add 6 for the middle column.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? No, it would not, because to have a height of 300, each leg would have 699 columns, so 1,800. You'd have to add 1,99+198+197... down until 1.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{aligned} 1 \cdot 6 &= 6 \\ 2 \cdot 6 &= 12 \\ 3 \cdot 6 &= 18 \\ 4 \cdot 6 &= 24 \\ 5 \cdot 6 &= 30 \\ 6 \cdot 6 &= 36 \\ 7 \cdot 6 &= 42 \end{aligned}$$

$n$  height

$$2 \cdot (6)^2 - 6$$

$$2 \cdot 36 - 6$$

$$72 - 6$$

$$66$$

$$\text{cubes} = 2n^2 - 6$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2n^2 - n$$

$\times 5$

$$2n^2 - n - 19,900 = 0$$

$$(2n + 199)(n - 100) = 0$$

$$2n + 199 = 0 \quad n = 100$$

~~$n = -\frac{199}{2}$~~

*Bright.*

The height is 100 cubes

I used the formula I found in part 3 and set it equal to 19,900 since that is how many cubes are in the tower. Then, I solved for  $n$  to find the height. I got two answers, but since height must be positive,  $-\frac{199}{2}$  isn't valid, so the answer is 100 cubes.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) Includes Bottom!

$\rightarrow$   $SA = (2n)^2 + 2(4n - 3)$

$$= (2 \cdot 6)^2 + 2(4 \cdot 6 - 3)$$

$$= (12)^2 + 2(24 - 3)$$

$$= 144 + 2 \cdot 21$$

$$= 144 + 42$$

$$\checkmark SA = 186.$$

I found a formula that represents the surface area, not just the top, covering up the surface area of the bottom. But the surface area of the bottom, excluding the top of the cubes in the bottom, was 144, or  $12^2$ . In the bottom, there were 6 rows of  $(2 \cdot 6)^2$ , which totals 144. So, the surface area of the bottom, including the top, is 186.

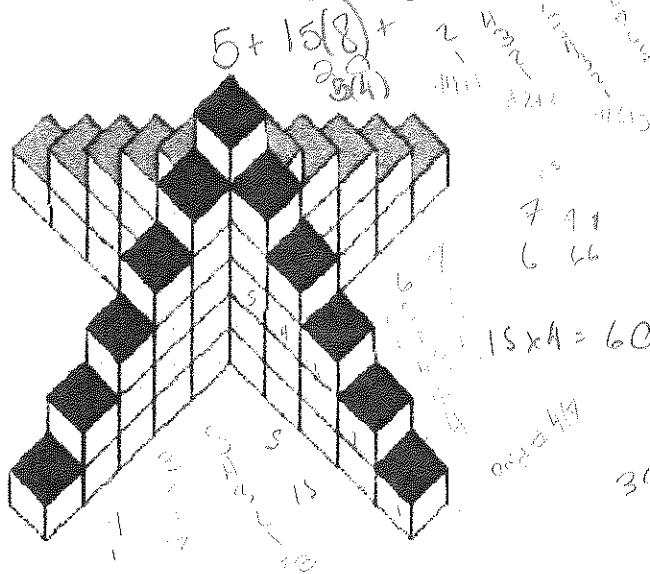
When each layer is 6 long including the center, so, since there is only 1 center once, the top (and bottom), we know the same) can be found with  $4n - 3$  where  $n$  is the number of layers. The center was accounted for, so adding that to our previous equation, we get  $SA = (2n)^2 + 2(4n - 3)$ , which is then plug in  $n = 6$  to get 186.

## **Building Towers (Yellow Worksheet)**

10.25

Name: Caroline Carroll  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

Three Dimensional figure Made with stacking cubes  
in a stair like fashion from each edge of the center  
cube. Height is 6 cubes

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

The Cubes stack in an order of increasing by one cube. Therefore the side contains a row of 3, 4, 3, 2, 1 cubes which can be multiplied by 4 to get each side. Then an additional 6 cubes must be added for the central stack of the tower, to get 66.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Although possible, my strategy would not be an efficient tool to find the height of the tower, since that would require adding up the digits from 1-899, multiplying by 4 and then adding 300. It would take a long time.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\text{Volume} = 2n^2 - n$$

$$V = n^3 + (n-1)^3 + (n-2)^3 + \dots + (n-3)^3 + (n-3)^3 + \dots$$

$$h = 2(6)^2 - 6 = 66 \checkmark$$

↑ Weeks 6-11 not  
feasible

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$V = 2n^2 - n$$

$$19,900 = 2n^2 - n$$

$$2n^2 - n - 19,900 = 0$$

$$2(h-100)(h+99.5) = 0$$

$$h = 100, -99.5$$

The height is 100 cubes

Plugged into Volume equation

Quadratic formula

{ more detail about process}

$$V = 2(100)^2 - 100$$

$$V = 19,900$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$n = \# - 1$$

+ .5

$$\begin{aligned} SA &= n + 3n(8) + n(4) + n(4) + 1 \\ &= 5 + 15(6) + 20 + 20 + 1 \\ &= 166 \text{ units}^2 \end{aligned}$$

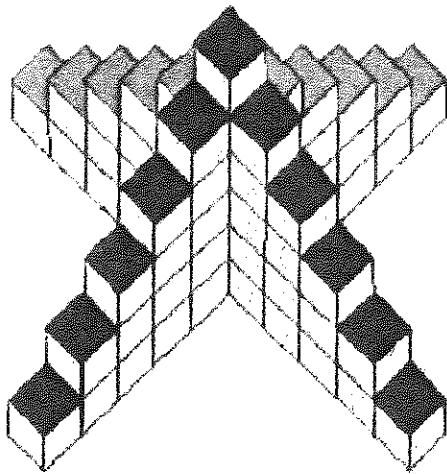
## **Building Towers (Yellow Worksheet)**

**10 points possible**

Name: Beth Palmer

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

at the 4 <sup>ed</sup> day, 6:30 a.m. to 10:30 a.m.  
blocks high rising into a broad ridge of  
at night a short sun two other ridges.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$(1+2+3+4+5)(4) + 6 = \text{Total students} \rightarrow 41$$

$(5)(4) + 6 = \text{Total}$

$66 = \text{Total students in class}$

I took the first 7 cubes  
I built a model by  
I can make  
by  $\frac{1}{2}$  to get 41 cubes  
easily find the number of cubes for a tower that

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Now my strategy would be to make the city think the number of  
bus drivers up to a certain point will result in very little  
loss, all the way up to a point where it becomes inefficient.  
int) How many bus drivers should be used?

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

The effect of  $\Delta$  on the value of  $\alpha$

units.

and the  $\beta$ -series of  $\text{Li}^{+}$  ions.

- 6 -

T-366

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 100 \cdot n^3$$

$$\sqrt[3]{19900} = \sqrt[3]{100n^3}$$

$$(10n)^3 = 19900$$

$$100\sqrt[3]{19.9} = (10)^3 \sqrt[3]{(19.9)(n^3)} = 10(1 + \sqrt{19.9})$$

$n = 20$

Great!

$$\frac{14399}{11} \quad \cancel{\frac{1398}{11}}$$

can't have negative numbers  
 $1300 - 100 = 0$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

10x10 blocks with 3 sides showing

5x9 blocks with 4 sides showing

1 block with 5 sides showing

$$(10 \cdot 3 \cdot 10) + (20 \cdot 4 \cdot 10) + (1 \cdot 5 \cdot 10)$$

$$80 + 80 + 5 = 165$$

$165 \text{ square units of area}$

After setting 19900 equal to the volume of the quadratic formula was needed to determine the height. Once the number of sides was found, it was found to be 165. This was either  $\frac{14399}{11}$  or  $1398$ .

Since  $1398$  could not be a negative number,  $14399$  was the only solution, implying a height of 100 units.

The block has 11 faces. The top of a stack adds one more face. The bottom adds two more faces. The sides add three more faces. The very top block has no faces showing, and there are 6 blocks with 3 faces showing each. The number of blocks by the width ( $10 \cdot 10$ ) is 100. The height is 6. The total is  $(10 \cdot 3) + (20 \cdot 4) + 100 \cdot 6 = 760$ . The answer is 760.

## Building Towers (Yellow Worksheet)

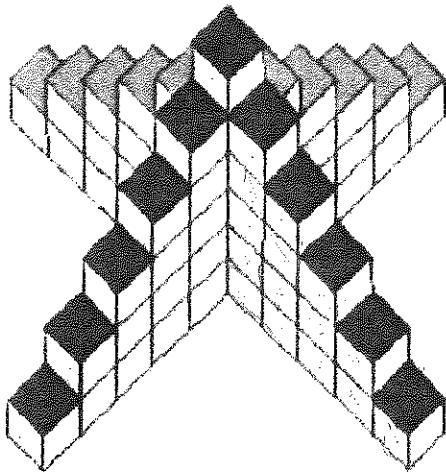
10 points possible

11  
10

Name: Alyssa Troxel

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

It looks like a pyramid with the corners taken out also the tower gets 1 cube shorter every time it goes another row away from the center

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{aligned} & 6+4(5)+4(4)+4(3)+4(2)+4(1) \\ & 6+20+16+12+8+4 \\ & \boxed{66 \text{ cubes}} \end{aligned}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

not easily because  
My strategy would work but the numbers would get very large and maybe hard to work with without a calculator.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{array}{llll} n = \text{height} & n=1, & n=2, & n=3, \\ \# = \text{total} & \# = 1, & \# = 5, & \# = 15 \\ \text{number} & \# = 1 & \# = 6 & \# = 15 \\ \text{of cubes} & \# = 1 & \# = 6 & \# = 15 \end{array}$$

$$\begin{aligned} & n(2n-1) \rightarrow n=6 \\ & 6(2(6)-1) \\ & 6(12-1) \\ & 6(11) \\ & \boxed{66 \text{ cubes}} \end{aligned}$$

$$\begin{array}{l} \boxed{n(2n-1)} \\ C = \text{total cubes} \end{array}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.  $C=19900$

$$\begin{aligned} & \times 5 \quad n(2n-1) = 19900 \\ & 2n^2 - n - 19900 = 0 \\ & \frac{-b \pm \sqrt{b^2 - 4AC}}{2A} = n \\ & \frac{1 \pm \sqrt{1 + 4(2)(-19900)}}{2(2)} = n \\ & \frac{1 \pm \sqrt{1 + 159200}}{4} = n \\ & \frac{1 \pm \sqrt{159201}}{4} = n \end{aligned}$$

Great!

$$\frac{1 + \sqrt{159201}}{4} = n$$

$$\frac{400}{4} = n$$

$$100 = n$$

I plugged in 19900 into my equation and solved to get a height of 100 cubes

- +1.5 5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$\begin{array}{cccc} n=1 & n=2 & n=3 & n=4 \\ C=1 & C=6 & C=15 & C=28 \\ S=6 & S=28 & S= & S= \end{array}$$

$15 \cdot 2 = 30$  - each side of inner corners

$30 \cdot 4 = 120$  - all 4 inner corners

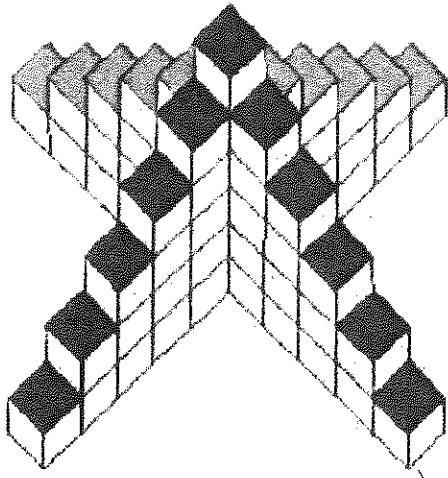
$11 \cdot 4 = 44$  - all 4 sides going up

$$120 + 44 + 1 + 2 \cdot 16 = 186$$

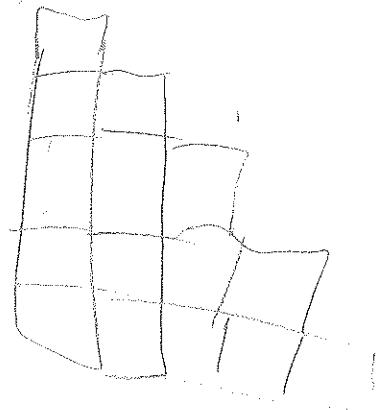
top cube all bottoms

186 cubes<sup>2</sup>  
to paint the  
tower including  
the bottom

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{aligned}
 0 & n=1 \rightarrow 0 \rightarrow 1 \\
 4 & n=2 \rightarrow 1 \rightarrow 6 \\
 12 & n=3 \rightarrow 3 \rightarrow 15 \\
 20 & n=4 \rightarrow 6 \rightarrow 28 \\
 30 & n=5 \rightarrow 10 \rightarrow 45 \\
 40 & n=6 \rightarrow 15 \rightarrow 66
 \end{aligned}$$



- 1) (1 point) Describe what you notice about the structure of the tower above.

It is 6 blocks high. There are 4 branches that come from the single tower. As you get further from the base, the branches get smaller at a 1:1 ratio.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6 + 4(5+4+3+2+1)$$

$$6 + 4(15)$$

$$6 + 60$$

total cubes

I found the number of cubes in each layer then I added it to the number of cubes in each branch. I multiplied that number by 4 because the tower is solid.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, You would have to add up the numbers 1-300 and that would not be easy.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\text{cubes} = \frac{n}{2}(n+1) + n$$

$$\text{cubes} = 2n^2 - 2n + n$$

$$\text{cubes} = 2n^2 - n$$

$$n(n-1)$$

$$\text{cubes} = 2n^2 - n$$

does your formula work?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$x 5 \quad \text{Cubes} = 2n^2 - n$$

$$19900 = 2n^2 - n$$

$$2n^2 - n - 19900 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2A} = n$$

$$\frac{1 \pm \sqrt{1 + 159200}}{4}$$

$$\frac{1 \pm \sqrt{159201}}{4} = n$$

$$\frac{400}{4} = n$$

$$n = 100$$

$$\text{height} = 100$$

I plugged in 19900 into the equation and used the quadratic equation to find n. Since n is height.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$x 1.5$  sides bottom top middle column

$$15(2)(4) + 5(3)(4) + 6$$

$$120 + 60 + 6$$

$$[186 \text{ squares}]$$

Includes bottom.

$$\left(\frac{8n(n+1)}{2}\right)(8) + (n-1)(3)(4) + 6$$

$$16n^2 + 16n + 6n - 12 + 6$$

$$4n^2 + 4n + 12n + 12 + 6$$

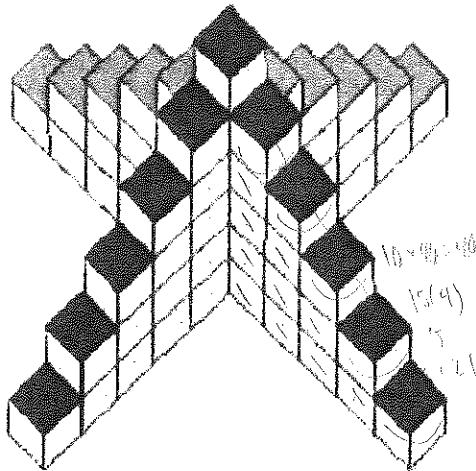
$$4n^2 + 8n - 6$$

Building Towers (Yellow Worksheet)  
10 points possible

12  
10

Name: Avery Hoyt  
Date: 3/6/14

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower has four stacks jutting from the middle, and each side is 5 blocks long. They also add one block when they move over, making the heights 1, 2, 3, 4, and 5 until reaching the middle of a height of 6.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\text{Side } 1+2+3+4+5=15$$

$$\text{all sides } 15(4)=60$$

$$60 \times 6 = 360$$

It will take 60 cubes. I found this by calculating that 15 blocks make a side, so all all 4 would make 60. They added the height to get 60 blocks.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, it would not, because my method takes adding the number of blocks on one side, and so adding from 1 until 299 would be very tedious, and time consuming.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$4\left(\frac{n(n-1)}{2}\right) + n = \# \text{ of cubes}$$

$$2n(n-1) + n = \# \text{ of cubes}$$

$$2n(n-1) + n \\ 2(6)(5) + 6 \\ 60 + 6$$

66 cubes.

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$\# \text{ cubes} = 2n(n-1) + n$$

$$19,900 = 2n(n-1) + n$$

$$19,900 = 2n^2 - 2n + n$$

$$19,900 = 2n^2 - n$$

$$0 = 2n^2 - n - 19900$$

Great.  $0 = (2n+199)(n-100)$

$$0 = 2n+199$$

$$0 = 100 - n$$

$n = 100$

$n = -99.5$  (cannot be positive)

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

1 with 3 sides

20 with 4 sides

40 with 2 showing

21 on bottom

$$5 + 20(4) + 40(2)$$

$$5 + 80 + 80$$

$$165 \text{ faces} + 21$$

186

additional bonus:

$$5(n - (n-1)) + 4(n-1)(4) + 2(n+4)(4)$$

$$5(1) + 4(4n-4) + 2(4n+16) + 4(n-1) + 1$$

$$5 + 16n - 16 + 9n + 32 + 16 + 4n - 4 + 1$$

$$5 + 28n + 18$$

$SA = 18 + 28n$

The height is 100 blocks. I found this by first plugging in the 19,900 as the answer for the equation, and after factoring, the answers were 100 and -99.5, and since height cannot be a negative height, the height must be 100 blocks.

186 faces. I found this because the top block shows 5 faces, and all the blocks on the top of the tower show 4, and there are 20. And the rest (40) only show 2. So,  $5 + 80 + 80 = 165$  faces, but you must add the 21 on the bottom to get 186.

~~$21 + 165 = SA$~~

~~$186 = SA$~~

$18 + 28(6)$

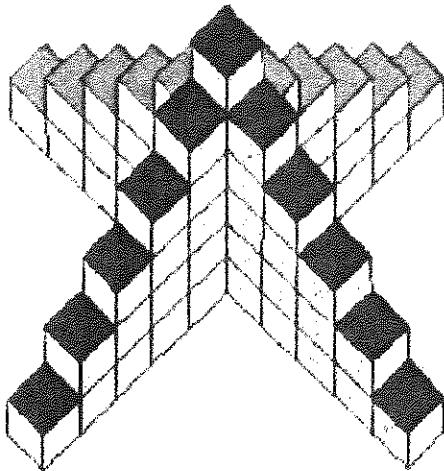
$186 = SA$

**Building Towers (Yellow Worksheet)**  
10 points possible

12  
10

Name: Rishabh K. Budhraju  
Date: 3-6-14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The cubes form part of a structure of a square pyramid.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Number of cubes =  $6 + 4(5 + 4 + 3 + 2 + 1)$  / Each column has a descending order of cubes than the previous when going out from the middle. I added starting from the 5 down to 1 and multiplied by 4 since there were 4 faces. Then I added 6 because there was only one six level column.  
- 66 cubes

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

If it would not work easily because the number of cubes in each column would have to be added starting from 294, Yes!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\begin{aligned} \text{Number of cubes} &= n(n-1) \times 2 + n \\ &= (n^2 - n) \times 2 + n \\ &= 2n^2 - 2n + n \end{aligned}$$

Number of cubes =  $n^2 - n$

X = # of cubes

$$X = 2(6)^2 - 6$$

$$X = 66$$

- $\times 5$  4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19900 = 2n^2 - n$$

$$2n^2 - n - 19900 = 0$$

$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-19900)}}{2(2)}$$

$$n = \frac{1 \pm \sqrt{159201}}{4}$$

$$n = \frac{1 + 399}{4} \quad \text{sqrt!}$$

$$n = \frac{400}{4} \quad n = \frac{400}{4}$$

$$n = 100$$

The height of this tower is 100 cubes. I used 19900 in the formula: number of cubes =  $2n^2 - n$  to get  $19900 = 2n^2 - n$ . Then I used the quadratic formula and got the height to be either  $\frac{-398}{4}$  cubes or 100 cubes. Since the height would have to be positive, the height is 100 cubes.

- $\times 2$  5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$\begin{aligned} \text{Surface area} &= 5(8)(4) + 10(4)(2) + 5 \\ &= 165 \text{ cubes}^2 \end{aligned}$$

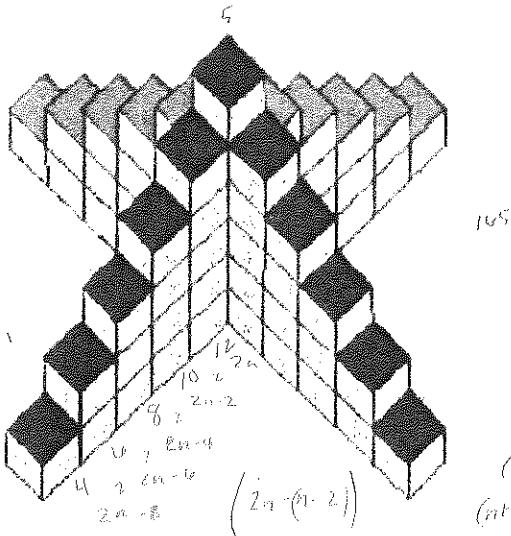
I noticed the top layer for each wall had 5 cubes that showed 4 sides, so I multiplied those and then multiplied that by 4 because there are 4 walls. Next, I had 10 cubes that showed 2 sides on 4 walls. Finally, there were two more added to 5 for the number of sides that the top cube showed.

**Building Towers (Yellow Worksheet)**  
10 points possible

Name: Mehak Chawla

Date: 3/16/2014

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$4(2(2n+3)P_{k-1} - 2)$$

$$4(4n+6+2n-2)$$

$$4(6n+4)$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

It consists of four right triangles with the same height and they all have the same area and base length. The base length is equal to the height for each triangle.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\text{Total Number of Cubes} = 4 + 4(\sum_{k=1}^6 k)$$

$$\begin{aligned} T &= 4 + 4\left(\frac{1}{2}\right)(5)(6) \\ &= 4 + 60 \\ &= 64 \end{aligned}$$

I found the area of one triangle multiplied it by four and then added six for the center.

60 cubes

$$\begin{aligned} T &= h + (2)(k-1)(h) \\ &= h(1+2k-2) = h(2k-1) \end{aligned}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Yes, because the values plugged in for the base and height can be changed easily.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$T = 4(2n-1)$$

$$T = 6(2(6)-1)$$

$$T = 6(11)$$

$$T = 66$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

*x5*

*Great!*

$$\begin{aligned} \text{Total number of cubes} &= h(2h-1) \\ 19,900 &= h(2h-1) \\ 19,900 &= 2h^2 - h \\ 2h^2 - h - 19,900 &= 0 \\ h &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ h &= \frac{1 \pm \sqrt{19900}}{4} \\ h &= 153.99 \end{aligned}$$

$n = 100 \quad h = 153.99$

The height is 153.99 units.

height

Using the formula from question 3,

I substituted the 19,900 cubes in for the total amount of cubes.

Then, I solved for  $h$ , the height, by using the quadratic formula.

The final answer has to be positive, since we're finding the height of the tower, so 100 units is the height of the tower.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) - find a formula for the surface area of a tower of height  $n$

*x2 Top*

$$\begin{aligned} 5 &\quad 4 \text{ triangles} \\ (n-1)(n+2)4 & \\ \boxed{S.A = 5 + 4(n-1)(n+2)} \\ &= 5 + 4(6-1)(6+2) \\ &= 5 + 4(5)(8) \\ &= 5 + 160 \\ \boxed{S.A = 165n^2} \end{aligned}$$

This surface area does not include the bottom of the tower.

There are  $n-1$  steps for each triangle and the amount of cube faces is  $2n$  and decreases by  $n-2$  each step down.

$$5 + 2n - (n-2)$$

$$= 2n + n + 2$$

$n+2$  is the number of cube faces for each step. Since

there are  $n-1$  steps, I multiplied  $n+2$  and  $n-1$  together to get the amount of cube faces for one triangle. I multiplied this quantity by 4 because there are 4 similar triangles. Then,

I added 5 because the top of the tower will always have 5 faces.

$$\begin{aligned} \text{Surface Area} &= \frac{1}{2}(5 + 4(n+2)(n-1)) \\ &= 5 + 4(n^2 + n - 2) \end{aligned}$$

$$= 5 + 4(n^2 + n - 2)$$

$$= 4n^2 + 4n - 3$$

# Building Towers (Yellow Worksheet)

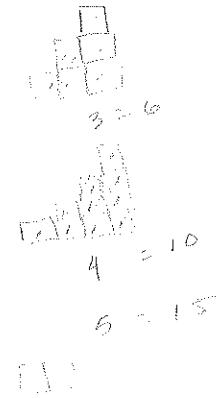
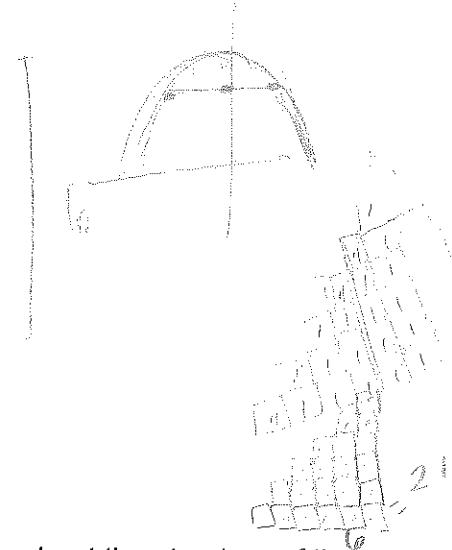
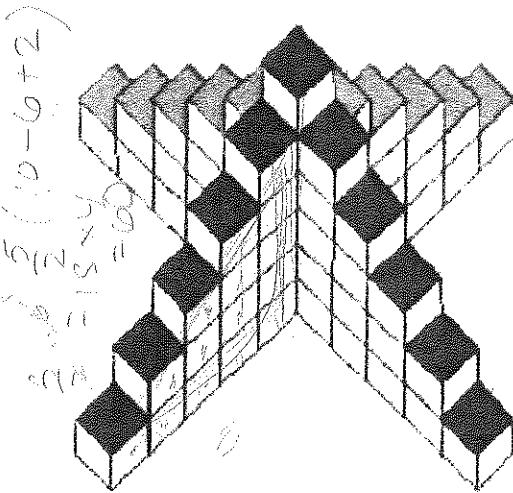
10 points possible

5:25  
10

Name: Romela S. 10/10/15

Date: 10/10/15

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The height is six cubes. It has a width of 10 cubes.  
The front face is a pyramid, or a triangle.  
The top layer of the tower is the top layer of the pyramid.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\text{height} = 6$$

$$(n+1)(n+2)(n+3) = (6+1)(6+2)(6+3)(6+4)(6+5)$$

$$= 6 + ((6+1) + (6+2) + (6+3) + (6+4) + (6+5) + (6+6)) \times 4$$

$$= 6 + (5+4+3+2+1) \times 4 = 6 + (5 \times 4) = \text{Total}$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

My strategy would not work for 300 because it would take too long.

It would take a long time to add up 300 times.

- 3) (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n + ((n+1) + (n+2) + (n+3) + (n+4) + \dots) \times 4$$

$$= n + (n+1) + (n+2) + (n+3) + (n+4) + \dots + (n+6-1) \times 4$$

$$= n + \frac{n+6-1}{2} \times (2(n+1) + (n+2) + (n+3)) \times 4$$

$$= n + \frac{n+5}{2} \times (2n+6) \times 4$$

$$= n + \frac{n+5}{2} \times (4n+12)$$

$$= n + \frac{4n^2+20n+60}{2}$$

$$= \frac{4n^2+20n+60}{2}$$

$$= 2n^2+10n+30$$

Would this work for any height?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

2.25  
5

$$\begin{aligned}
 & h + \left\{ \frac{h}{2} [2(n-1) + (h-2)(n-1)] \right\} \times 4 = n \left( \frac{n-1}{2} [2n-2 + n^2 - 2n] \right) \\
 & = \frac{19900 + (19900 - 1 - 19900)}{2} \\
 & = 492,000,100 \text{ cubes}
 \end{aligned}$$

You found the number  
of cubes in a tower  
of height 19,900.

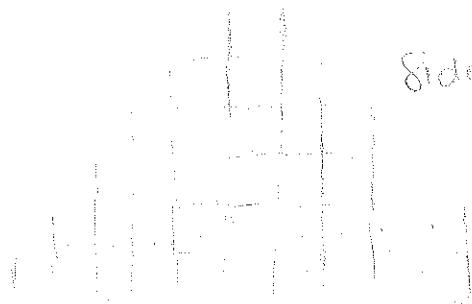
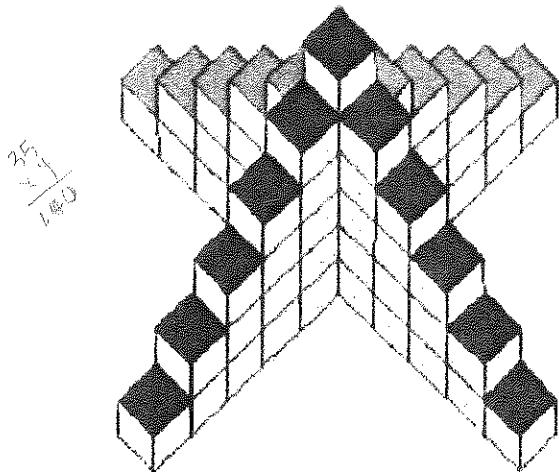
5. (Bonus - up to 2 points). Find the **surface area** of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

**Building Towers (Yellow Worksheet)**  
**10 points possible**

16  
10

Name: Ashley TAYLOR  
 Date: 3/1/16

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



Side Views

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower is built up of cubes and has a solid front shape. The tower has a large hollow center and many sides of the center part of the tower.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{array}{r}
 1 \ 2 \ 3 \ 4 \\
 \times 4 \quad 160 \\
 \hline
 4 \ 8 \ 12 \ 16 \\
 + 4 \ 8 \ 12 \ 16 \\
 \hline
 60 \quad 160
 \end{array}$$

I found how many cubes in one row of the tower, multiplied by 6 to include all rows and added them together. I also multiplied 6 times 6 to find the total number of cubes.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because we would have to multiply the height times the width many times up to 300 and it would take a long time to do that.

(15!)

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$\# = n(2n - 1)$$

length + width

$$2n + 1$$

$$160 = (6)(2)(16) - 10$$

$$160 = 120 + 40 - 10$$

$$160 = 160 - 10$$

$$160 = 160 - 10$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

x 5

$$19900 = n(2n-1)$$

$$19900 = 2n^2 - n$$

$$2n^2 - n - 19900 = 0$$

$$n(2n-1)$$

Great!

$$n = \sqrt{19900/2 + 1/4}$$

4

$$n = \sqrt{19900/2 + 1/4}$$

4

$$n = \sqrt{19900/2 + 1/4}$$

4

$$n = \frac{399}{4} + \frac{1}{4}$$

$$n = 99.5 + 0.25$$

I plugged in 19900 as the number of cubes I created a quadratic equation. It was unsolvable, so I used the quadratic formula & got two values for n and eliminated the one that didn't check out.

$$99.5(2(99.5)-1) = 100(2(100)-1)$$

$$99.5(199-1)$$

$$100(200-1)$$

$$100(199)$$

$\cancel{+19900}$

19900

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

top view  
bottom view



$$\begin{array}{r} 15 \\ \times 8 \\ \hline 120 \\ + 21 \\ \hline 141 \\ + 4 \\ \hline 145 \end{array}$$

21  
square  
faces

141 square faces  
bottom face counted  
in top part.

I multiplied  
15 by 8 because  
the legs have two  
sides each with 15  
square faces. Then  
added 21 because  
the base is 15.

Since there can  
be seen 4 faces at  
a time. Then I added  
it to include the  
remaining 14 faces.  
The top part after  
that I counted 4  
to get the remain-  
ing side of 14 faces  
there is the bottom of the end

**Building Towers (Yellow Worksheet)**

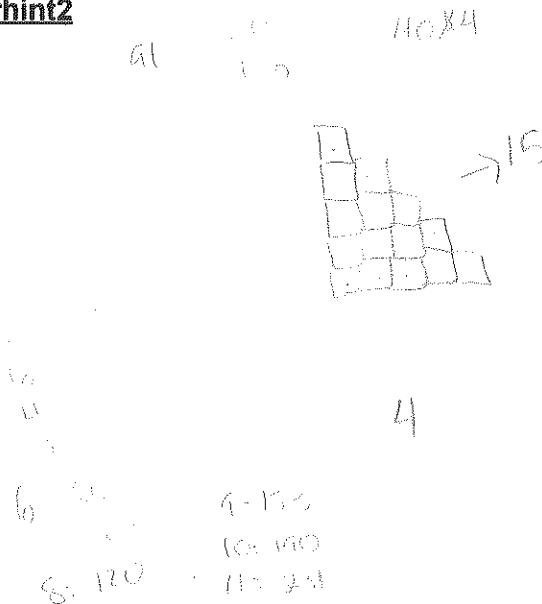
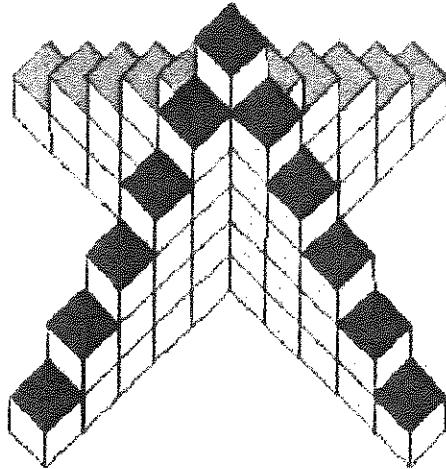
**10 points possible**

12  
10

Name: Andrew Husev

Date: 4/16/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

There are 4 congruent pieces that are all perpendicular to each other. The middle piece is a lone column that is one block higher than the perpendicular pieces' highest column.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$\begin{aligned} & 15(4) + 6 \\ & \text{Total } H = 66 \end{aligned}$$

I counted the number of cubes in one of the four pieces. I multiplied it by 4 and then added 6 to account for the middle column.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No. Since you would have to do a lot of counting to find the number of cubes.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$(2n-1)n$$

$$\begin{aligned} & (15)(4) = 6 \\ & (15)(4) = 6 \\ & 15(4) = 6 \end{aligned}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = (2n+1)n$$

$$19,900 = 2n^2 + n$$

$$2n^2 + n - 19900 = 0$$

$$(2n+199)(n-100) = 0$$

Great!

$$2n = -199 \quad n = 100$$

$$n = -99.5$$

Height = 100 cubes

To solve this, I plugged in 19900 for the number of total cubes in the equation I found and solved for  $n$ .

I had to factor in order to find  $n$  and one of the numbers was negative so I knew it could either be that the other will positive and it makes taller or longer than

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$\times^2 \quad 40(4) + 5$$

$$160 + 5$$

165 square faces

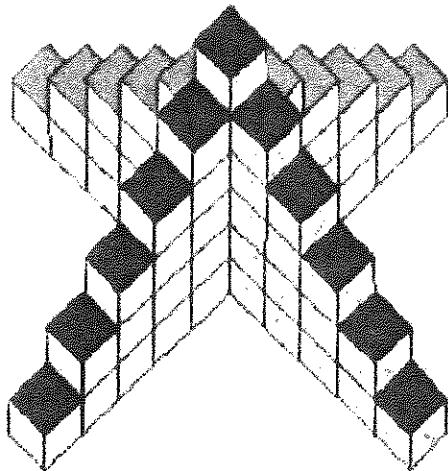
Since I knew there were 16 blocks in each place, I multiplied that by 10 to get 160 square side of paint. I also figured out I added 10 to account for the faces that ~~were~~ showing but not included in the 160. I multiplied 10 by 10 which is equal adding 10 because we can't add 10 to each face.

Building Towers (Yellow Worksheet)  
10 points possible

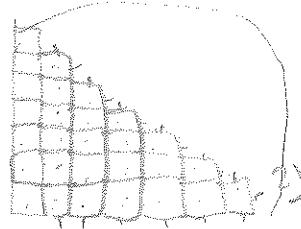
12  
10

Name: Chase Kaisler  
Date: \_\_\_\_\_

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$5 \times 2$$



$$\frac{7+1}{2} = 7$$

$$6 \times 3$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

One center tower, 6 blocks high.

4 branches that are decreasing one block height every block they go away.  
From the center 4 branches are the same.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

1 side  
 $15 + 12 + 8 + 4$   
 $15 + 12 + 8 + 4 + 15$   
 $66$

found how many cubes is needed for one branch  
multiplied that by 4 because there's 4 of them. Then added 6 for the middle piece.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Would not work well. Having to add  $299 + 298 + 297 + \dots$  etc. would take a very long time. Yes!

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$\frac{n(n+1)}{2}$   
 $\frac{6(6+1)}{2} = 6 \times 7 / 2 = 21$

$6 + 4\left(\frac{6^2 - 6}{2}\right)$   
 $6 + 4(15)$   
 $[66] \checkmark$

$n + 4\left(\frac{n^2 - n}{2}\right)$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

x5

$$19,900 = n + 4\left(\frac{n^2-n}{2}\right)$$

$$100 + 4\left(\frac{100^2 - 100}{2}\right) = 19900$$

$$19,900 = n + 2n^2 - 2n$$

$$2n^2 - n - 19,900 = 0$$

$$1 \pm \sqrt{1^2 - 4(2)(-19900)}$$

$$\frac{1 \pm \sqrt{398}}{2}$$

$$\frac{1 \pm 398}{4} \quad \frac{400}{4} = 100$$

$$\frac{398}{4} = 99.5$$

Made 19,900 equal to my equation.

Simplified to  $2n^2 - n - 19,900 = 0$ .

Used quadratic formula and got

2 possible solutions 100 and 99.5.

Plugged those into my equation.

Found 100 to be the

correct answer.

$$99.5 + 4\left(\frac{99.5^2 - 99.5}{2}\right) = \cancel{19900}$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?) ~~bottom included~~

x2

$$(15+2+10+5) \cdot 4 + 6$$

15 on one side of a branch \* 2 for both sides  
plus 10 on the top and sides facing out, + 5  
for the bottom, add all those to get 45 surface  
area for one branch multiply by 4 for all of the  
branches (base on top has 5 sides showing and  
left on the bottom so add 6 to get

$$\boxed{186}$$

$$186$$

$$\left(2\left(\frac{n^2-n}{2}\right) + 2(n-1) + (n-1)4 + 6\right) \leftarrow \text{Bonus #2}$$

$(n-1)$  finds the bottom,

$2\left(\frac{n^2-n}{2}\right)$  finds the 2 sides

$\times 4$  for the 4 branches.

$2(n-1)$  finds the top and  
sides facing out,

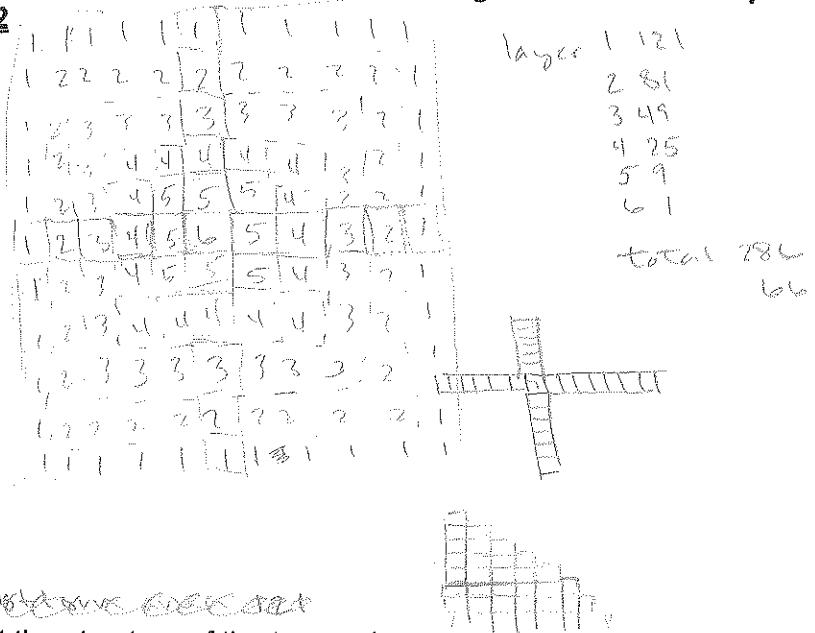
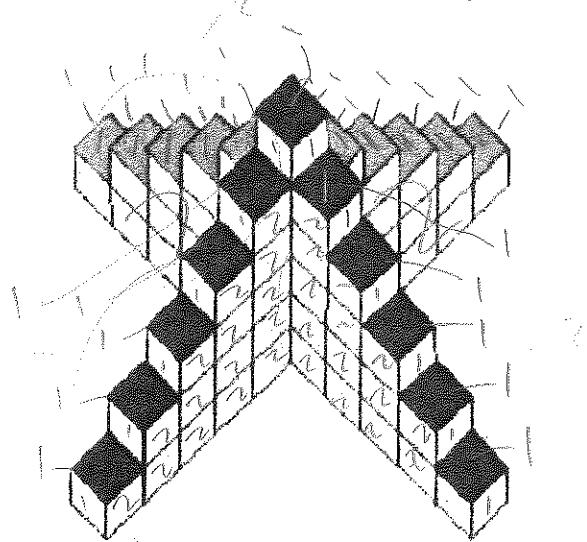
$+ n$  for the 5 on top  
and 1 on bottom remaining

## **Building Towers (Yellow Worksheet)**

12  
10

Name: Tom Wagner  
Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

Tower, but only with non-adhesive legs.  
interesting!

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

60+6 blotters Count the lobes of the legs + 1  
for center wings

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No, because you cannot work in the prison system.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

largest Height = n  
Area  
HDL values = C

$$\left( -2n(n-1) + n \right)$$

$$\begin{aligned}66 &= 2(6)(6-1) + 6 \\66 &= 12(6-1) + 6 \\66 &= 72 - 12 + 6 \\66 &= 66\end{aligned}$$

$$C = 2n(n-1)$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$C = 2n(n-1)$

solving

$19900 = 2n(n-1) + n$   
 $19900 = 2n^2 - 2n + n$   
 $19900 = 2n^2 - n$   
 $2n^2 - n - 19900 = 0$

$\frac{1 \pm \sqrt{1 - 4(2)(-19900)}}{2(2)} = n$   
 $n = \frac{\sqrt{158201}}{4}$   
 $n = 100, n = -99.5$

Use Quadratic  
equation

*Great!*

First, you plug in 19900 into the equation ( $C = 2n(n-1) + n$ )  
 Then you combining to get  
 Then you can estimate  
~~negative~~ factor of the  
 quadratic equation.  
 You will get two answers.  
 But one is negative.  
 So you may just use the  
 positive value because  
 negative

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$\checkmark 4(2n-1)$  finds surface area  
 or all block faces (added at front)  
 $4((n-1)^2 + n-1)$  finds area for sides  
 or three legs (added 2)

$4(2n-1) + 4((n-1)^2 + n-1) = 4(6-1) + 1 + 1$

$8n-4 + 4(n^2 - 2n + 1 + n - 1) + 4n - 4 + 1 + 1$

$8n - 4 + 4n^2 - 4n + 4n - 4 + 2$

$8n - 6 + 4n^2$

$$4(n-1)^2 + 4(n-1) = 4((n-1)^2 + n-1)$$

$4(n-1)^2 + 4(n-1)$  bottom faces  
of blocks

$$4(6-1) + 4((6-1)^2 + 6-1) = 4(6-1) + 1 + 1$$

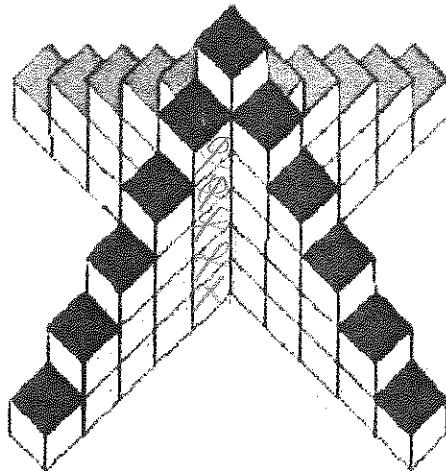
$$8n-4 + 4(n^2 - 2n + 1 + n - 1) + 4n - 4 + 1 + 1$$

$$8n - 4 + 4n^2 - 4n + 4n - 4 + 2$$

$$8n - 6 + 4n^2$$

$$4(6)^2 - 6 + 8(6) = 186$$

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

The structure shown is made up of 4 sections with each part leading to the center. There are 6 cubes in the center and from each face of the cube in the center, a column of cubes is built, each one less than the one before.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

There are 60 cubes needed to build the tower above. The strategy I used to find this number is I added one side of columns by adding  $(5+4+3+2+1)$  and the answer was 15 which I then multiplied by 4 = 60 and added the height of 6 to get 66.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

This strategy would not easily find the number of cubes for a tower that has a height of 300 in that it would be difficult to count the number of cubes on one side ( $300 + 299 + 298 \dots$ ) and also multiplying and then further adding.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$t = 2n^2 - n$$

$$t = 60 \checkmark$$

$$t = 2(6)^2 - 6$$

$$t = 2(36) - 6$$

$$t = 72 - 6$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$\times 5$

$$2n^2 - n = 19,900$$

$$2n^2 - n - 19,900 = 0$$

Quadratic  
Formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Great!

$$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(19,900)}}{2(2)}$$

$$\frac{1 \pm \sqrt{1 + 159200}}{4}$$

$$\frac{1 \pm \sqrt{159201}}{4}$$

$$\frac{1 \pm 399}{4} = \frac{-398}{4}, \frac{400}{4} = 100$$

The height of the tower that is made up of 19,900 cubes is 100 cubes.

I used my formula and set it equal to the number of cubes in the tower. I then solved for n using the Quadratic Formula and got 2 answers. Because the height of the cube cannot be negative, only one answer remained of 100 cubes.

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

+2

There are 40 squares shown on one side of the tower. Therefore the surface area of the tower is 165 sq. units.

165

$$(5)(4)(4) + 10(4)(2) + 5 = 165 \text{ cubes.}$$

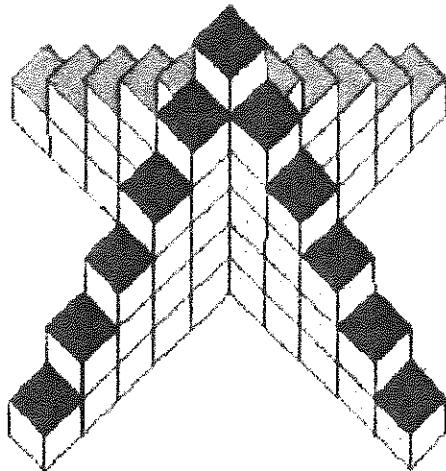
↑  
Blocks shown

## **Building Towers (Yellow Worksheet)**

5.5  
10

Name: Morgan  
Date: 10/10/18

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



- 1) (1 point) Describe what you notice about the structure of the tower above.

4 sides opening out to blocks from the Center Block  
the Center is 6 blocks long

- 2) Reference the diagram shown above:

- a) **(1 point)** Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$b+4(15) = 66$  Because there are 6 boxes of 10 pencils plus 4 more.  
 $b+4(5)+4(2)+11=66$  This is good for 1. Because there are 5 boxes of 10 pencils plus 2 more plus 11 more.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No. Decisions or Grants by different Companies in the block on Way  
or right of way for a New Rail or to Merging of Rail or a junction (200-1).

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n + 4(n-1) + 4(n-2) + \dots + 4(1) + 1 = 4\left(\frac{n(n+1)}{2}\right) + 1 = 2n^2 + 2n + 1$$

$$2^m \cdot 4^{n-m} = 2^{m+3} \cdot (2^{n-m-3})$$

$\alpha = 4(16 + 15)\delta$  guarantees that  $\hat{\alpha}_n \in \mathcal{A}_n$ .

20. 4/10/80 (2)

$$(x^2 + 4)(x^2 - 4) = (x^2 + 4)(2x + 1)(2x - 1)$$

$$\sum_{k=1}^{n-1} (2(n-k), (k+1-n))$$

Would this work for any weight?

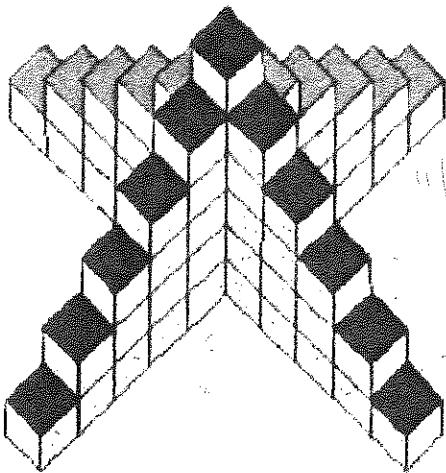
4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$\begin{aligned} 19,900 &= n + 4(n^2 - n - 4(\frac{n+1}{2}(2n+1) + (n-1)(n))) \\ &= n + 4(n^2 - n - 4(\frac{n+1}{2}(2n^2 + 2n - n + 1))) \\ &= n + 4(n^2 - n - 4(\frac{n+1}{2}(2n^2 + n))) \\ &= n + 4(n^2 - n - 2n^2 - n) \\ &= n + 4(n^2 - n - 2n^2 - 2n) \\ &= n + 4(-n^2 - 3n) \end{aligned}$$

$\frac{1}{2}/6$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

Directions: Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\text{number of cubes} = 3n$$

$$n(n+1) \cdot 4$$

$$4(1+2+3+4)$$

$$n + 4((n-1)^2 - (n-4))$$

$$(n-1)^2$$

$$\begin{array}{r} 6 \\ \times 11 \\ \hline 66 \end{array}$$

4

- 1) (1 point) Describe what you notice about the structure of the tower above.

The highest point is in the middle and there are 4 sections coming off the highest point. These sections decrease in height by one block until reaching zero. Made of cubes.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$6 + 4(5) + 4(4) + 4(3) + 4(2) + 4(1) = 66 \text{ cubes}$$

Since you know there are 4 towers with 6 cubes, you multiply 4 by 6 to get the amount of blocks used in that tower. Continue decreasing 5 and multiplying it by 4, then add the amount of blocks used in the middle (the height).

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No my strategy wouldn't easily work because you would have to do 299 times 4 all the way until the 299 gets to zero and then you would have to add all those numbers together.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$n + 4 \left( \sum_{k=1}^n k \right)$$

how would you compute this?

2

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = \text{total # of cubes including center column}$$

3.25    5     $\frac{19900}{4} = 4,975$  cubes in each section

My method would be to set the formula for finding the amount of cubes equal to 19,900 then solving for n.

The tower has a height of 100 cubes. Because if you plug n into my equation, you get the answer of 19,900

$$100 + 4 \left( \frac{n(n+1)}{2} \right) = 19,900$$

How did you compute this?

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

$$6 + 4(5) + 4(7) + 4(9) + 4(11) + 4(13) = 185 \text{ square faces.}$$

+1.5    2    To find the surface area of the tower, I counted how many sides were visible <sup>on the</sup> of each tower, then I multiplied that number by 4 since there were 4 sections (except the height) of each tower. I added those together then added how many sides were on the surface of the height to get my answer. (Bottom included)

**Building Towers (Yellow Worksheet)**

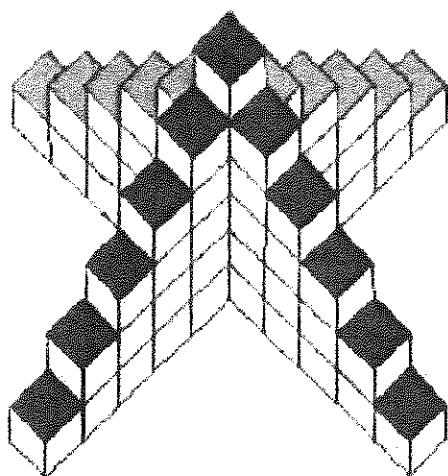
**10 points possible**

10

Name: Celia Goss

Date: \_\_\_\_\_

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$\begin{array}{l} \text{6 cubes} \\ \text{6 cubes + 2} \\ \text{6 cubes + 2 + 2} \\ \text{6 cubes + 2 + 2 + 2} \\ \text{6 cubes + 2 + 2 + 2 + 2} \end{array}$$

$$17 \cdot 6 = 6 \cdot 45$$

$$\begin{array}{l} \text{4 cubes} \\ \text{4 cubes + } n \\ \text{4 cubes + } n + 1 \\ (4n - 3) \cdot \frac{n}{2} + \frac{n}{2} \end{array}$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

It has a pyramid shape.

Each row has 4 more cubes than the last one.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Assuming there is 6 cubes vertically in the center,

I would take 66 cubes to build the tower.

I found the cubes per level on the first level!

(6 cubes \* 6) + 1, 2, 3, ... then added (n-1) until I reached 1.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

No because it would be a lot of adding.

But you could use an arithmetic sequence if it was that long.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$(4n - 3) \cdot \frac{n}{2} + \frac{n}{2}$$

$$2n^2 - \frac{3n}{2} + \frac{n}{2}$$

$$2n^2 - \frac{2n}{2}$$

Does your formula work?

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

+5

$$2n^2 - n = A$$

$$2n^2 - n = 19,900$$

$$2n^2 - n - 19,900 = 0$$

I took the formula I had for area vs. height and plugged 19,900 in for area. I then used the quadratic formula to solve for the height.

$$\frac{1 \pm \sqrt{1^2 - 4(2)(-19,900)}}{2(2)}$$

$$1 + 399$$

(00 cubes)

$$\frac{400}{4} = 100$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

Solving not including

any bottoms only

sides

front

$$\text{Each side} = 4 + (2 \cdot 4)$$

$$= 12$$

height of 6

255 square faces for 6

I used  $n^2$   
because each row has  $n^2$   
and there are  $n$  rows  
 $n-1$  4(6) bottom sides  
because

the top is  $n-1$  rows  $\Rightarrow$   $n-1$  rows

a different equation  
which is factored  
by the 35.

$$4n^2 + 4n - 8 = n - 35$$

$$4n^2 + 4n + 8$$

each row higher  $\rightarrow n-2$

until last

$(12) \cdot (4) + 5$  top square always 5

Equation  
if he tower has  $n$  rows  
is always  $2n^2 + 4n + 5$

Top square is always 5.

$$50 + 10 + 5$$

$$50 + 10 + 5$$

$$4n^2 + 4n - 8 + 4(n-2)(n-1) + 2(n-2)$$

$$4((n^2 + n - 2))$$

4

**Building Towers (Yellow Worksheet)**

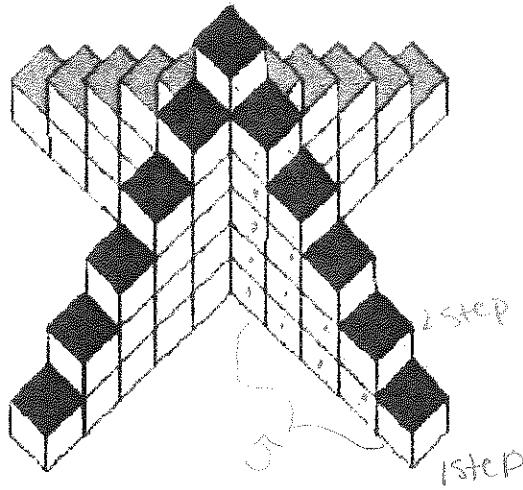
**10 points possible**

Name: Taylor Vohland

Date: March 5th

12  
10

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



for a tower with  $n$  steps

$(n-1)^2 \rightarrow$  for a full square

height  $\downarrow$       width  $\downarrow$

$$\frac{15}{25} \quad \frac{10}{10}$$

$$\frac{3(5)}{5(5)} \quad \frac{(n-1)^2}{(n-1)^2}$$

$$(n-1)(n-2) \quad n^2 - 3(n-2)$$

$$(n-1) \quad (2)(5) = 10$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

I notice that when looking at the tower from a birds-eye view, the tower is in the shape of an X. Also, the tower is built like a pyramid, in the fact that towards the bottom, there are more cubes, and the number of cubes decreases

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

$$(1)4 + 2(4) + 3(4) + 4(4) + 5(4) + 6 \cdot \text{The total number of cubes needed is } 66. \text{ Since there are four sections to the X, I multiplied each step of cubes by 4 and then added it to the middle line because the middle line is part of all four sections.}$$

$$4 + 8 + 12 + 16 + 20 + 6$$

66

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

My strategy for finding the number of cubes would not have easily to find if the height is 300 because I would have to multiply many cubes over and over again and then add them all together to find the total.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$t = 2h^2 - h \quad \leftarrow \text{above} \quad h = \text{height}$$

$$t = (6+4(5)) + 4(4+3(3) + 4(2) + 4(1)) + 1(0) = 66 \checkmark$$

$$t = 2(6)^2 - 6 = 66 \checkmark$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$x^5 + = 19,900 \text{ cubes}$$

$$\text{Greatest } 19,900 = 2h^2 - h$$

$$0 = 2h^2 - h - 19,900$$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{4}$$

$$h = \frac{1 \pm \sqrt{15920}}{4}$$

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

First, I am going to find out how many cubes have four faces visible (the top cube of each step).  $5 \times 4 \times 4 = 80$

Next, I am going to add in the number of cubes below the top cube that only have two faces visible.  $(4+3+2+1=10)$  ~ per section of the tower

$$10 \times 4 \times 2 = 80$$

cubes = bottom faces

Finally, I am going to add in the top face that has 5 visible faces.

$$80 + 80 + 5 = 165 \text{ unit}^2$$

$$\text{Formula: } \text{SA} = 5 + (n-1)(16 + 1)8$$

$$h = 100, \cancel{-98.5}$$

The height of the tower is 100 units (cubes). I used the formula that was derived from problem three, and plugged in the value of 19,900 cubes and then used the quadratic formula to find the value of  $h$ .

**Building Towers (Yellow Worksheet)**

10 points possible

(6x7x7) =

4(4)(4) =

= 64

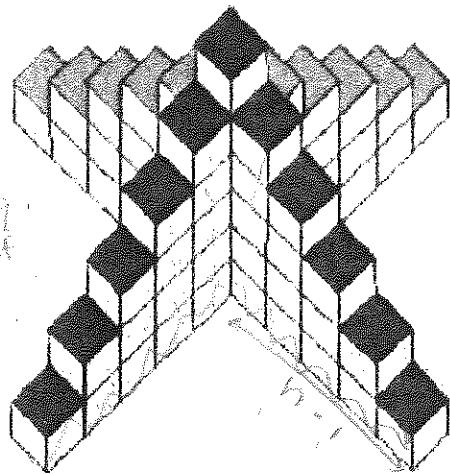
10.5

10

Name: Abby Vito

Date:

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help:  
<http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



I saw the tower at a pub with 4 wings, each of the wings is  $\frac{1}{2} \times 25$  being a part of a possible square  $(n-1)(n)$

the ratio of the square filled to the total square units

$$\frac{(n-1)(n)}{(n+1)^2}$$

if the "wings" were full by wires how many is missing from them

$$6 \times 5 \times 4$$

$$10 \times 16 \times 9$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

As the tower gets taller, it adds 4 blocks per layer.

There are 4 starting extensions at the base, with a new layer of 4x4 blocks.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Since each level is 4 times as tall as the last by 4,

number of blocks in each level, you add 4 more blocks than the previous level.

$$21 + 17 + 13 + 9 + 5 + 1 = 66$$

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not?

Not to work after working all day up to 300 blocks

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$4\left(\frac{n(n+1)}{2}\right)(n-1)^2$$

$$4\left(\frac{n(n+1)}{2}\right)(n-1)^2$$

percentage of  $n(n+1)$  is a square that has a

$$(dn^2 - n)$$

$$\frac{(2n-1)(n-1)^2+n}{n+1}$$

$$4 \left( \frac{(n-1)(n)}{2} \right)$$

$$\boxed{2n^2 - n + 4}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

~~4.5~~  
5

$$2n^2 - n + 19,900$$

$$2n^2 - n - 19,900 = 0$$

$$-b \pm \sqrt{b^2 - 4ac}$$

2. a

$$\frac{1 \pm \sqrt{1 - 4(2)(-19,900)}}{2(2)}$$

plug into quadratic formula.

more explanation of process?

$$\frac{1 + 399}{4}$$

$$\frac{400}{4} = 100$$

100 cubes ✓

5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

~~4.2~~

11 square on top with 5 visible sides

20 sq w/ 4 sides visible (5 blocks w/ 4 sides visible) 4 wings

40 sq. w/ 2 sides visible . 10 blocks w/ 2 sides of avg

10 x 10^2

$$\boxed{165 \text{ units}^2}$$

$$(6(n-1)) + (n-1)(6+5)$$

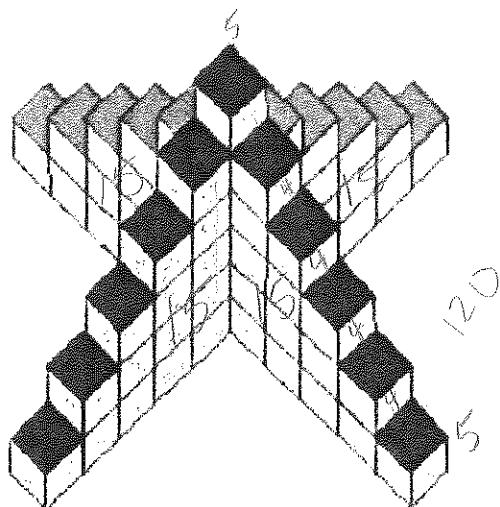
$$32(n-1) + 5$$

**Building Towers (Yellow Worksheet)** 12  
10

10 points possible

Name: Nanditha Ravikiran  
Date: 3/6/14

**Directions:** Consider the cube tower illustrated below, then answer each of the 5 tasks that follow. For each task explain your thinking in words and/or with mathematics work. See the following links for extra help: <http://bit.ly/towerhint1> <http://bit.ly/towerhint2>



$$4(2n-1)+1$$

$$\frac{8((n-1)+h)}{2}$$

$$(n-1)4+1$$

$$15 \times 8$$

$$160.$$

- 1) (1 point) Describe what you notice about the structure of the tower above.

The tower is essentially a square pyramid with a diagonal base length of 11 units. It is symmetric. It has four equal protruding sides with a central column that is one cube taller than the rest of the structure.

- 2) Reference the diagram shown above:

- a) (1 point) Find the total number of cubes needed to build the tower above. Note that the tower is solid and has a height of 6 cubes. Explain your strategy and show your work.

Each side is composed of  $(5+4+3+2+1 = 15)$  cubes. If this number is multiplied by 4 (for each side),  $15 \times 4 = 60$  cubes is the total so far. Then 6 cubes (height of the sides in the middle plus the cube on top) can be added to 60 for a total of 66 cubes needed to build the tower.

- b) (1 point) Would your strategy from (a) work to easily find the number of cubes for a tower that has a height of 300? Why or why not? It would not work to find the number of cubes easily for a height of 300 because my strategy required that I knew the base length of the individual sides from the diagram. For a tower with a height of 300, the strategy would only be easy if one could see the length. Otherwise, it would have to be calculated.

3. (2 points) How many cubes would be needed to construct a tower of height  $n$ ? (In other words, provide a formula that allows you predict the number of cubes used for a tower of any height). Check your formula using a tower of height 6 units.

$$((n-1)2)n$$

$$2n(n-1) + n$$

$$if n=6 \rightarrow 2(6)(6-1) + 6$$

$$12(5) + 6$$

$$60 + 6$$

$$66 \checkmark$$

$$\boxed{2n(n-1) + n}$$

4. (5 points - see rubric) Consider a tower that is made up of 19,900 cubes. What is the height of this tower? Explain your method and show your work.

$$19,900 = 2n(n-1) + n, \text{ where } n = \text{height of tower}$$

$$19,900 = 2n^2 - 2n + n$$

$$19,900 = 2n^2 - n$$

$$2n^2 - n - 19,900 = 0$$

$$n = \frac{1 \pm \sqrt{(1)^2 - 4(2)(-19,900)}}{2(2)}$$

$$n = \frac{1 \pm \sqrt{159201}}{4}$$

$$n = \frac{1 \pm 399}{4}$$

$$n = \frac{400}{4} \text{ or } \frac{-398}{4}$$

$$n = 100 \text{ or } -99\cancel{5}$$

(can't be a negative height)

Height = 100 cubes

The formula from part 3 can be used because it determines the number of cubes needed to construct a tower of height  $n$ . Given the number of blocks needed,  $n$  can be solved for, using the quadratic formula. This yields two answers, of which only one (100) is logical because the other is a negative value.

12. 5. (Bonus - up to 2 points). Find the surface area of a tower with a height of 6. Explain your problem solving process and show your work. (In other words, if I paint the original tower, how many square faces would be covered with paint?)

Edges (excluding top block)  $\rightarrow 4(2n-1)$

top side of top block  $\rightarrow +1$

$$\text{total area of 4 sides} \rightarrow 8((n-1)^2 + (n-1)) = 4((n-1)^2 + n)$$

bottom  $\rightarrow 4(n-1) + 1$

Combined Surface area  $\rightarrow$

$$4(2n-1) + 4((n-1)^2 + (n-1)) + 4(n-1) + 1 + 1$$

$$8n - 4 + 4(n^2 - 2n + 1 + n - 1) + 4n - 4 + 2$$

$$8n - 4 + 4n^2 - 8n + 4 + 4n - 4 + 2$$

$$(4n^2 + 8n - 6)$$

$$4(b^2) + 8(b) - 6$$

$$144 + 48 - 6$$

$$= 186 \text{ units}^2$$

(for a tower of 6)

$$\text{Formula: } SA = 4n^2 + 8n - 6$$

(where  $n = \text{height}$ )

I found an expression for individual components of the structures and then added them together for a final expression. I plugged in 6 for  $n$  in the formula to find surface area of 186 units<sup>2</sup> for a tower of height 6 (including the surface area of the bottom of the structure).