Lesson 05: Fermi Estimation - Constructing Reasonable Models (High-Inquiry Version)

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Lesson Title: Fermi Estimation: How Would You Figure That Out?
Intended Grade Level(s): Grades 6-12 (adaptable)
Content Area: Content-Agnostic Quantitative Reasoning & Modeling

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I. Planning

Lesson Focus / Goals

State the big idea(s) of the lesson. Focus on conceptual understanding. Avoid vague objectives. Specify the knowledge and skills students should demonstrate.

The lesson aims to provide the following for students: - Generate multiple possible approaches to impossible-seeming estimation problems - Create and justify their own assumptions rather than receiving them - Experience modeling as decomposition and reasoning, not formula-application - Defend the reasonableness of their approach, not its accuracy - Understand that different valid models can produce different estimates

Learning Objectives

Write clear, measurable objectives. Include both procedural and conceptual goals. Consider potential misconceptions students might have.

By the end of the lesson, students will be able to: - Decompose a complex estimation problem into manageable sub-questions - Generate their own reasonable assumptions and justify them - Construct at least one coherent estimation model (problem breakdown + assumptions) - Defend the reasonableness of their assumptions and approach - Compare different estimation models and explain why they produce different results - Distinguish between "reasonable" and "accurate" in the context of estimation

Potential Misconceptions: - Students might think there's ONE correct way to break down the problem - Students might seek "the right assumptions" rather than defensible ones - Students might think estimation is about getting close to the "real answer" - Students might equate modeling with following formulas - Students might be uncomfortable defending choices without knowing if they're "correct"

Standards Alignment

Note: This is a content-agnostic lesson that places math and science students on equal footing. However, the reasoning processes naturally align to core practices in both disciplines.

Standards for Mathematical Practice (Common Core): - MP1 – Make sense of problems and persevere in solving them.
Students decompose complex problems and make epistemic choices about how to model them. - **MP2** – Reason abstractly and quantitatively.
Students construct estimation models by generating and justifying assumptions about real-world quantities.

NGSS Science and Engineering Practices: - Using Mathematics and Computational Thinking – Students use mathematical reasoning to model real-world phenomena when direct measurement is impossible. - **Developing and Using Models** – Students create and defend mathematical models that represent complex systems through reasoned assumptions.

II. Implementation

Materials Needed

List all physical and digital resources, manipulatives, and technology needed. For each item listed, provide a brief justification/explanation for its inclusion.

The following materials are used in the lesson: - **Blank estimation worksheets** with space for: problem decomposition, assumptions + justifications, calculations, confidence ratings - **Chart paper** for groups to display their models publicly - **Sticky notes** for peer questions about assumptions - **Calculators** for computation (once models are built) - **Optional: Reference sheets** with general facts (US population, etc.) but NOT problem-specific assumptions - **NO teacher worked example** - students build approaches from scratch - **NO answer key** - reasonableness is judged by logic, not accuracy

Preparation: Prepare blank worksheets that scaffold model-building without prescribing structure. Prepare chart paper for public display. Do NOT prepare assumptions or formulas. Be ready to NOT reveal "the actual answer" (or delay it significantly).

Lesson Flow

(Before-During-After)

Organize your plan using the Before–During–After framework. Include approximate timing, key questions, and anticipated student responses.

Note for instructors: The essential inquiry phases are (1) decomposition, (2) assumption justification, and (3) model comparison. These three phases distinguish modeling from calculation. Some meta-cognitive prompts in the After section are optional or extensible depending on time. Prioritize students building, defending, and comparing their own models over covering all reflection questions.

Before: (Launch – 7 min)

1. Write the **Fermi question** on board: "How many piano tuners work in Chicago?"
2. Pause. Let the impossibility land.
3. Ask: "What's your first reaction? Turn and talk."
4. Collect reactions: "We can't know!" "We'd have to look it up!" "That's random!"
5. Pose the challenge: "What if you couldn't look it up? What if you HAD to figure it out? How would you even start?"
6. Brief historical note: "Physicist Enrico Fermi was famous for making surprisingly good estimates of things that seemed impossible to know. The key? Breaking big questions into smaller, answerable questions."
7. Frame the work: "Today, you're going to build your own estimation model. There's no formula I'm giving you. You'll decide how to break down the problem, what to assume, and how to justify your thinking."
8. Clarify: "Your job isn't to get THE right answer. It's to build a REASONABLE approach and defend it."

During: (Explore – 23 min) Phase 1: Build Your Model (10 min) - Students work in pairs/small groups - Challenge: "Break down the big question into smaller questions you CAN estimate" - Worksheet prompts: - "What do we need to know?" - "How could we figure that out?" - "What assumptions do we need to make?" - "Why is that assumption reasonable?" - Teacher circulates with open questions ONLY: - "How did you decide to break it down that way?" - "Where did that number come from? Is it reasonable?" - "Could you approach this differently?" - NO validation of approaches or assumptions - Students calculate once they have a model

Phase 2: Public Model Display (5 min) - Groups post their models on chart paper showing: - Their decomposition approach (what sub-questions they asked) - Their key assumptions with justifications - Their final estimate - Quick gallery walk: Notice different approaches

Phase 3: Model Defense & Comparison (8 min) - Select 2-3 models with DIFFERENT decomposition structures - Groups present (2 min each): - "Here's how we broke it down…" - "Here's why we assumed…" - "Our estimate is…" - After each: Peers ask clarifying questions or challenge assumptions - "Why did you use that number?" - "What if that assumption is off by 2x?" - "Could you have broken it down differently?" - Teacher facilitates but doesn't judge

After: (Meta-Discourse on Estimation & Modeling – 10 min) Compare Approaches (4 min) - Display estimates from all groups - Note: They likely vary (maybe 200 to 800) - Ask: "Why are our estimates different? Is someone wrong?" - Guide toward: Different decompositions and assumptions produce different estimates - Key question: "Which assumptions matter MOST? Which could be way off without changing the estimate much?"

Surface Big Ideas About Modeling (4 min) - “What makes an assumption ‘reasonable’ even if you don’t know if it’s right?” - Based on experience, logical constraints, general knowledge - Defensible, not perfect - “Is estimation about getting the right answer?” - No—it’s about building a defensible model when you CAN’T know the answer precisely - “Where do scientists and engineers use this kind of thinking?” - Climate models, drug dosing, infrastructure planning, public health, space missions - ANY time you need to reason about systems you can’t measure directly

The Reveal Decision (2 min) - Option A: DON’T reveal the actual number - “In real inquiry, you often don’t get to check your answer” - “The quality of your thinking matters more than matching reality” - Option B: Reveal it, BUT frame carefully with deliberate scripting: - “The ‘actual’ number is about 290 piano tuners” - “Notice: Many of your estimates are close! That’s not because you guessed well—it’s because your MODELS were reasonable” - “Even if you were off, that doesn’t mean your model was bad. It means assumptions differed” - **Critical framing:** “We’re not checking if you were right. We’re seeing how different reasonable assumptions lead to different estimates. Your thinking is what matters, not the match.” - **Important:** Without explicit framing, students may default to accuracy-seeking and retroactive validation. Script this moment to protect epistemic integrity. - Emphasize: “Inquiry isn’t about being right. It’s about being reasonable and defensible.”

III. Assessment

Formative Assessment

Describe how you will check for understanding during and after the lesson.

During Phase 1 (Model-Building): - Observe whether pairs move beyond “I don’t know” to generating sub-questions - Listen for justification language: “We’re assuming X because…” - Note which groups generate assumptions vs. seek teacher’s “right” assumptions - Check if decompositions are coherent (do sub-questions connect logically?)

During Phase 3 (Defense): - Monitor whether groups can defend assumptions when challenged - Listen for epistemic language: “reasonable,” “plausible,” “based on” - Observe whether students distinguish between assumption quality and calculation accuracy - Note whether students revise thinking in response to peer questions

During Meta-Discourse: - Check if students recognize that different valid models produce different estimates - Listen for understanding that “reasonable” “accurate” - Observe whether students connect to broader modeling contexts

Exit Ticket: Students answer: 1. “Describe ONE assumption you made and WHY it’s reasonable (even if you don’t know if it’s exactly right)” 2. “If your assumption was off by 2x (twice as big or half as small), how would that change your estimate? A lot or a little?" 3. "What's one way you could break down this problem DIFFERENTLY than you did?"

Self-Assessment: "How confident are you that your model is reasonable (not accurate, but reasonable)?"

IV. Reflection & Next Steps

After teaching: Note what worked well and what didn't. Identify topics or skills to revisit. Record surprising student thinking. Suggest changes for next time.

I will aim to answer the following questions after the lesson has been taught: - Did students generate their own decompositions, or seek teacher guidance on "the right way"? - How comfortable were students with not knowing if their assumptions were "correct"? - Did students defend assumptions with reasoning, or just state them? - What language did students use: "the answer" vs. "our estimate" vs. "one approach"? - Did peer challenges improve model quality or just create defensiveness? - Which assumptions varied most across groups? Which were similar? - Did students distinguish between accuracy and reasonableness? - If I revealed the actual number, how did students respond? (Validation-seeking? Surprise? Indifference?) - Did students transfer the big idea: modeling as constructing defensible representations? - Which groups struggled most? With decomposition? Assumption generation? Justification?

Note: Please attach student handouts and any other printed materials that students will need to complete the lesson.

Student Estimation Model Worksheet

Group Members: ______________________________ **Date:** ______________________________

Fermi Estimation: Building a Reasonable Model

The Big Question:
How many piano tuners work in Chicago?

Step 1: Break Down the Problem

What smaller questions do we need to answer to estimate this?

1. ______________________________
2. ______________________________
3. ______________________________

4. ______________________________

5. ______________________________

Step 2: Make Assumptions (and Justify Them!)

For each smaller question, what do you need to assume? Why is that reasonable?

What We Need to Know	Our Assumption	Why This Is Reasonable
Example: How many people in Chicago?	~3 million	It's a major US city, probably similar to other big cities

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Step 3: Build Your Model

Show how your sub-questions and assumptions connect. Use arrows, diagrams, or formulas—whatever makes sense to you.

[Large workspace area]

Step 4: Calculate Your Estimate

Show your work:

Our estimate: About ____________________ piano tuners in Chicago

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Step 5: Confidence Rating

Which assumptions are you MOST confident about?

Which assumptions are you LEAST confident about (biggest guesses)?

If your least confident assumption is wrong by 2x, how much does your final estimate change?

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Step 6: Alternative Approach (After Discussion)

After seeing other groups' models, describe ONE different way you could have broken down this problem:

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Would that approach give a very different estimate? Why or why not?

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Exit Ticket

1. Describe ONE assumption you made and WHY it's reasonable (even if you don't know if it's exactly right):

Assumption: ______________________________

Why it's reasonable: ______________________________

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2. If your assumption was off by 2x (twice as big or half as small), how would that change your estimate?

Change a lot (more than 2x) Change by about 2x
Change a little (less than 2x)

Explain: ______________________________

3. What's one way you could break down this problem DIFFERENTLY than you did?

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4. Self-Assessment: How confident are you that your model is reasonable (not accurate, but reasonable)?

Low confidence Medium confidence High confidence

Why? ______________________________

V. Further Revision Ideas

These are additional inquiry-enhancing moves suggested through analysis of the lesson's revision capacity. While not included in this version, they represent growth opportunities for continued development.

Compare Sensitivity to Assumptions (Dimension 5 & 9)

- After initial estimates, ask: "Which assumptions, if they're wrong by 2x, would change your estimate the MOST?"
- Students test: double one assumption, recalculate, see impact
- Surfaces which factors dominate the model
- Connects to scientific modeling: identifying key variables vs. noise
- Deepens causal reasoning about multiplicative relationships

Tolerate Initial Messiness - No Structure at All (Dimension 3)

- Remove even the minimal scaffolding: just pose the question
- Let groups flounder, try dead-ends, start over
- After 5 minutes of struggle, pause for meta-commentary: "What's hard about this?"
- Then provide light structure: "Think about what you need to know"
- Pedagogically challenging for novices but reveals authentic problem-solving
- Makes problem-before-method visceral

Never Reveal the Actual Number (Dimension 8 & 9)

- Keep epistemic authenticity fully intact
- Close with: "In real modeling, you rarely get to check your answer against reality"
- Emphasize: Quality is judged by defensibility, not accuracy
- Harder emotionally for students and teachers
- But philosophically aligns with inquiry as constructing understanding, not finding truth

Connect Across Problem Types (Dimension 10)

- After piano tuners, pose 2-3 more Fermi problems in quick succession:
  - "How many gas stations in your state?"
  - "How many pizzas are ordered in the US on Super Bowl Sunday?"
  - "How much water flows through Niagara Falls in a year?"
- Ask: "What's the SAME about how you approach these? What's DIFFERENT?"
- Surface transferable strategies: decomposition, assumption generation, reasonableness checking

- Build meta-cognitive awareness of modeling as a practice

Add Constraints or Complications (Dimension 2)

- "What if I told you only 1 in 10 pianos gets tuned regularly, not all of them?"
- "What if tuners work part-time, not full-time?"
- "What if some tuners service organizations (schools, concert halls), not just homes?"
- Forces model revision and refinement
- Extends ceiling: sophisticated students can layer complexity
- Shows that models can be iterated and improved

Debate "Best" Approaches (Dimension 7)

- After sharing models, run a structured debate:
  - "Which decomposition approach is most defensible? Why?"
  - "Which assumptions are strongest/weakest across all models?"
- Require evidence-based argumentation
- Shifts from individual model-building to collective evaluation
- Strengthens discourse dimension significantly

Capacity Analysis Summary

Why This Lesson Has Very High Revision Capacity:

This lesson is **mathematically coherent but epistemically over-specified** in its LOW version. Its constraint signature: - **Lesson 01 (WODB):** authority & closure - **Lesson 02 (Mystery Graphs):** interpretation vs. explanation - **Lesson 03 (Headbandz):** efficiency vs. agency - **Lesson 04 (Marble Maze):** procedure vs. uncertainty - **Lesson 05 (Fermi): modeling vs. formula-following**

Key Insight: "Inquiry isn't about getting close to the real number; it's about constructing and defending a way of thinking about the world."

Core Message: Estimation is not calculation. **Modeling is not formula-following.** Inquiry lives in the choices, not the arithmetic.

Very High/High-Capacity Dimensions for Novice Revision:

1. **Openness & Multiple Pathways** (VERY HIGH) - Fermi problems naturally afford multiple decompositions; lesson artificially collapses to single formula; allowing alternative breakdowns is one of strongest inquiry levers; very accessible revision

2. **Integration of Big Ideas** (VERY HIGH) - Estimation, modeling, scale, uncertainty are foundational ideas; lesson already gestures at them; novices can foreground estimation as epistemic practice, connect broadly
3. **Curiosity & Genuine Puzzlement** (HIGH) - Question feels impossible; that's exactly what makes Fermi problems powerful; inquiry suppressed by immediate formula reassurance and premature reveal; revision is largely subtractive
4. **Student Agency in Reasoning** (HIGH) - Agency suppressed ONLY because choices removed; students capable of generating assumptions and decompositions; very teachable shift to let students choose and justify
5. **Low Floor / High Ceiling** (HIGH) - Entry fully supported; ceiling capped by fixed assumptions, steps, validation; novices can loosen assumptions, invite refinement, allow sophistication

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Medium-High/Medium Capacity Dimensions:

6. **Context-Rich / Phenomena-Based** (MEDIUM-HIGH) - Context (piano tuners) is authentic and grounded; missing student sense-making about WHY assumptions reasonable; novices can interrogate assumptions, ground in experience
7. **Connection-Making** (MEDIUM-HIGH) - Exit ticket introduces transfer; method remains fixed; novices can compare Fermi problems across contexts, surface what stays same vs. changes
8. **Causal Explanation** (MEDIUM) - Relationships are multiplicative and logical; causality is procedural not explanatory; novices can ask WHY each factor matters, explore sensitivity to assumptions
9. **Collaboration & Discourse** (MEDIUM) - Collaboration is procedural (check calculations); no space for epistemic disagreement; novices can compare models, debate assumptions, argue about "best" estimates
10. **Problem Before Method** (MEDIUM) - Problem is compelling but method front-loaded and privileged; requires tolerating messiness, delaying structure, allowing failed attempts; pedagogically harder for novices but powerful

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Why This Is a Particularly Important Low Anchor:

Three key teaching virtues:

1. **Exposes "modeling as calculation" thinking** - Candidates often equate modeling with formulas; this lesson makes that assumption visible and challengeable

2. **Foregrounds assumptions as epistemic moves** - Assumptions are present but treated as given, not chosen; HIGH version shifts ownership to students

3. **Inquiry lives in reasonableness, not accuracy** - LOW version rewards matching answer range; HIGH version rewards defensibility; fundamental epistemic shift

Pedagogical Leverage: This lesson teaches candidates that giving students a formula is not teaching them to model. The transformation happens when students must CHOOSE how to think about a problem, not just FOLLOW a procedure. Assumptions become epistemic commitments, not recipe ingredients.