# AI-Enhanced Learning in Calculus Education

A Study of GenAI-Mediated Taylor Series Instruction

AMS Contemporary Mathematics (CONM) Chapter Proposal

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**Abstract**

This chapter examines how genAI scaffolds recursive mathematical understanding in undergraduate calculus through analysis of 127 complete student-AI transcripts from a Taylor series learning module. Using the Pirie-Kieren framework, we developed the Pirie-Kieren Work Analysis Protocol (PK-WAP) to operationalize recursive understanding as observable dialogue patterns. Analysis of 30 anchor cases revealed that most students reached Inventising (the highest applied level), demonstrating agentic moves such as proposing alternative strategies, questioning approximation validity, and critically evaluating AI suggestions. Students engaged substantively in dialogue: while genAI contributed approximately 55% of words overall, individual student participation varied widely, with some students dominating the conversation and others relying more heavily on AI scaffolding. Survey findings indicated most students reported the assignment helped them learn calculus concepts better and developed more positive attitudes toward AI for learning. However, the graduate assistant interview revealed a significant ceiling effect: approximately 40% of students already knew Taylor series from high school, and the TA suggested survey responses may have been inflated by social desirability bias. The study identifies five design principles for AI-mediated mathematics learning and demonstrates how AI tools (VS Code with GitHub Copilot) can support rigorous qualitative analysis of large transcript datasets while maintaining interpretive depth.

## 1 Introduction: AI as Learning Partner in Mathematics

The rapid proliferation of large language models (LLMs) in educational settings presents both opportunities and challenges for mathematics instruction. While intelligent tutoring systems (ITS) have supported mathematics learning for decades, conversational AI agents like ChatGPT represent a qualitative shift: they offer natural language interaction, adapt responsively to student queries, and generate explanations in real time. Yet fundamental questions remain about how students actually *engage* with these tools, whether

AI scaffolding supports genuine conceptual development, and how we can design AI-mediated learning experiences that cultivate mathematical reasoning rather than passive answer-seeking.

This chapter addresses these questions through a design-based research study examining how 127 undergraduate calculus students at Sichuan University explored Taylor series with a custom-designed genAI learning partner. Rather than treating AI as a black-box tutoring system, we designed an intentional pedagogical framework grounded in the Pirie-Kieren theory of recursive mathematical understanding—a model emphasizing that learning proceeds through iterative cycles of conceptual growth, with students "folding back" to earlier understanding levels when encountering obstacles or seeking deeper insight.

Our central research question asks: **How does working with genAI as a learning partner shape the recursive development of students' understanding of calculus?** Addressing this question required not only measuring learning outcomes but analyzing the *process* of student-AI dialogue—examining how students reason, question, struggle, and ultimately construct mathematical meaning in collaboration with an AI agent.

Methodologically, the study demonstrates a dual role for AI: as the *object* of investigation (students learning with AI) and as a *methodological partner* (researchers using AI tools like GitHub Copilot to analyze qualitative data). This chapter models how AI can support rigorous, transparent, reproducible qualitative research while maintaining the interpretive depth essential to understanding complex learning processes.

## 2 Theoretical Framework: Pirie-Kieren Recursive Understanding

The Pirie-Kieren theory provides a sophisticated model for analyzing mathematical understanding as dynamic, recursive, and multileveled (Pirie & Kieren, 1994). Unlike linear stage theories, the framework conceptualizes learning as movement through nested levels of increasing sophistication, with critical "folding back" processes where learners return to earlier understandings to rebuild, refine, or extend their knowledge. The framework identifies eight nested levels: (1) **Primitive Knowing**—intuitive, pre-formal knowledge; (2) **Image Making**—creating mental representations through examples; (3) **Image Having**—possessing manipulable mental images; (4) **Property Noticing**—recognizing patterns and relationships; (5) **Formalising**—articulating generalizations and algorithms; (6) **Observing**—reflecting on formal properties; (7) **Structuring**—constructing theoretical frameworks; and (8) **Inventising**—creating new mathematics by applying understanding to novel contexts. Critically, these levels are not hierarchical stages but *nested contexts*—each level contains and builds upon previous levels. Students move fluidly between levels, and effective learning often involves "folding back": returning to earlier levels (e.g., creating new examples) when confronting challenges at higher levels (e.g., struggling to formalize a pattern).

The framework offers distinct advantages for studying AI-mediated instruction. Rather than assessing only final outcomes (test scores), Pirie-Kieren enables analysis of *how* understanding develops through dialogue, identifying level transitions, folding-back moments, and the quality of reasoning at each level. Effective AI tutoring should facilitate recursive engagement—prompting students to revisit concepts, test understanding through new examples, and connect formal abstractions to concrete instances. The framework guided our AI prompt design: we explicitly incorporated features to encourage folding back, support multiple levels (concrete examples *and* formal definitions), and prompt property-noticing and reflection.

## 3 Study Design and Methodology

Participants were 127 Chinese undergraduate STEM students enrolled in three sections of second-semester calculus at Sichuan University. Approximately 40% reported prior exposure to Taylor series from high school, creating diverse entering knowledge levels. Building on prior proof-of-concept work (Edwards et al., 2024), we designed a three-phase AI-mediated learning experience: **Phase I: Personality Profiling** adapted the AI's teaching persona based on student preferences; **Phase II: Historical Narrative** presented Taylor series through Brook Taylor's 1715 work, connecting mathematical development to contemporary applications; **Phase III: Interactive Problem-Solving** had students work through guided problems involving real-world applications (satellite orbit prediction, medical device design, environmental modeling). The AI scaffolded problem-solving through Socratic questioning, adaptive hints, and requests for student reflection, operationalizing Pirie-Kieren principles to create opportunities for students to move through multiple understanding levels and fold back when needed. We collected complete transcripts of all 127 student-AI dialogues, post-module surveys, and a graduate assistant interview.

Analyzing 127 complete conversational transcripts required a hybrid approach combining manual calibration, automated processing, and AI-assisted coding. We developed the **Pirie-Kieren Work Analysis Protocol (PK-WAP)**—a systematic coding scheme operationalizing the eight understanding levels as observable indicators in transcript text. For each level, we defined inclusion and exclusion criteria with example excerpts. The research team (three authors) independently coded five transcripts manually, achieving 85% inter-rater reliability. Having established reliable manual coding, we used AI tools (GitHub Copilot within VS Code) to assist with transcript preprocessing, pattern detection, word count analysis, and aggregation. Critically, AI *assisted* but did not replace human judgment—all level classifications were reviewed by researchers, with AI tools functioning as efficiency multipliers enabling comprehensive analysis impossible through purely manual methods.

## 4 Findings: Recursive Understanding in Action

Across 127 transcripts, the dialogue was relatively balanced overall, with genAI producing approximately 55

Analysis of 30 anchor cases—chosen to represent a range of engagement patterns and learning trajectories—revealed that across these cases, student talk percentages ranged from 13.3% to 100.0%, reflecting a broad spectrum of participation patterns. Regardless of talk share, all cases showed evidence of significant mathematical reasoning, with students reaching at least Formalising or Observing layers, and several attaining Structuring (the penultimate PK level). The number of observed recursive movements ranged from two to three per transcript, suggesting that the genAI partner frequently prompted cycles of reflection, error-checking, and conceptual reorganization. Analysis revealed that 77% of students reached Inventising and exhibited folding-back, demonstrating agentic moves such as proposing alternative strategies, questioning approximation validity, and critically evaluating AI-generated suggestions.

One illustrative case (P44-G7-S4) involved a student designing a Taylor series-based breathing alert system for deep-sea divers who encountered the "ghost term problem"—when expanding $\sin(\pi x)$ around $x = 0$, initial terms vanish because derivatives are zero. Rather than accepting AI-provided solutions, the student recognized the pattern of vanishing terms, sketched the sine function to understand behavior near zero, proposed calculating additional terms to capture non-zero contributions, and ultimately independently suggested shifting the expansion center from $x = 0$ to $x = 2$ (the depth of interest), demonstrating creative adaptation. This transcript exemplifies how the AI facilitated recursive thinking: the student moved fluidly through levels, folding back when needed, and ultimately invented a novel solution approach.

Survey findings (n=146) indicated that 62% agreed or strongly agreed the assignment helped them learn calculus concepts better (51% agreed, 12% strongly agreed), while 22% reported neutral impact. Regarding attitudes toward genAI for learning, 77% reported more positive attitudes following the assignment (42% much more positive, 35% somewhat more positive), with 19% reporting no change and only 3% more negative. The graduate assistant interview provided crucial context: approximately 40% of students already knew Taylor series from high school, creating a ceiling effect. For these students, the AI's pedagogical approach—starting from basics—felt redundant. The TA also noted that survey responses may have been inflated by social desirability, as students assumed the instructor could see their responses.

## 5 Implications, Design Principles, and Limitations

Our findings suggest several design principles for structuring AI-mediated mathematics learning supporting recursive understanding. First, adapt content difficulty and pedagogical style: personality profiling adjusted the AI's tone, but future systems should incorporate diagnostic assessments routing students toward mathematically appropriate challenge levels. Novices may need foundational scaffolding, while advanced students—particularly those with prior Taylor series experience—benefit from deeper work with convergence, error bounds, and alternative expansion strategies.

Second, explicitly scaffold recursive thinking. Prompts encouraging students to fold back, generate new examples, compare representations, or revisit earlier reasoning support movement across Pirie–Kieren levels. Designing AI dialogue to facilitate property-noticing, reflection, and error-checking helps students reorganize understanding rather than merely follow procedural steps.

Third, balance efficiency and depth through learner control. While some students valued detailed explanations, others found them tedious. Allowing students to regulate pacing ("Would you like a hint?", "Try on your own first?") supports autonomy while maintaining rigor.

Fourth, use real-world contexts that are mathematically meaningful. Application tasks—such as satellite orbit prediction or deep-sea breathing alarms—created opportunities for Inventising-level reasoning. However, contexts must contribute genuine mathematical insight rather than function as superficial wrappers.

Finally, this study illustrates how AI tools serve as methodological partners in qualitative research. AI-assisted preprocessing and pattern detection enabled comprehensive analysis of 127 transcripts while preserving human interpretive judgment, increasing scalability, transparency, and reliability—offering a transferable model for analyzing large text-based educational data.

The chapter demonstrates that carefully designed genAI learning partners meaningfully support recursive mathematical understanding, with most students reaching advanced Pirie–Kieren layers and exhibiting productive folding-back cycles. Future research should explore sustained interventions across multiple topics and settings to refine these principles.

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