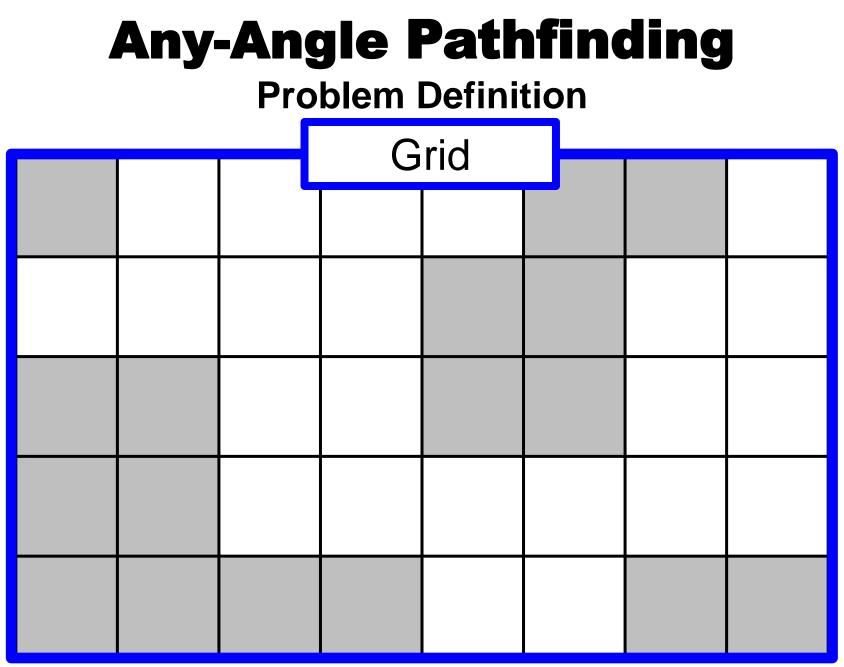


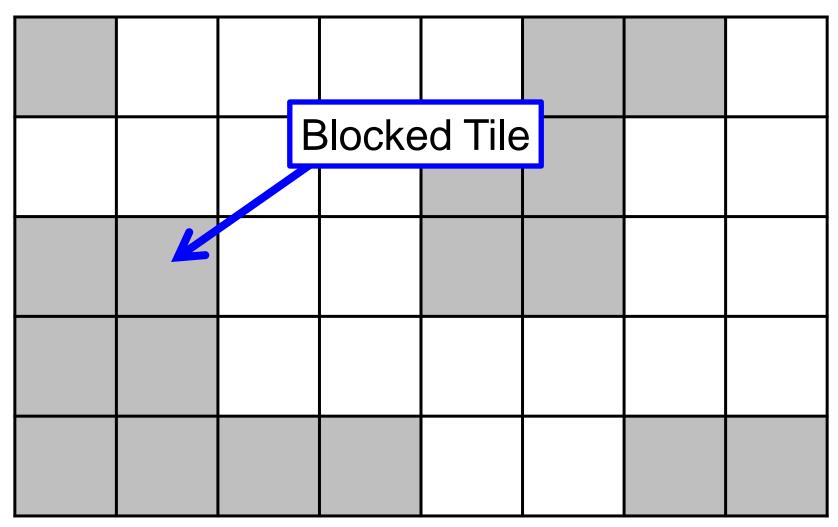
Edge N-Level Sparse Visibility Graphs: Fast Optimal Any-Angle Pathfinding Using Hierarchical Taut Paths

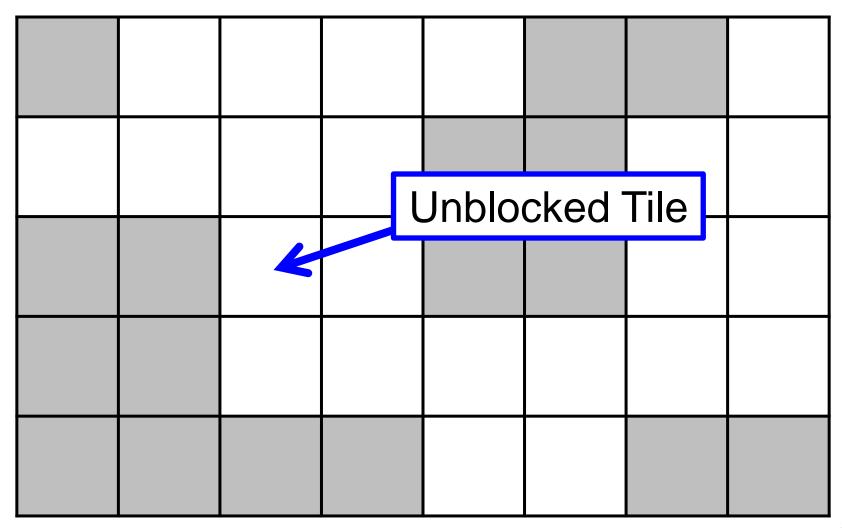
Shunhao Oh ohoh@u.nus.edu

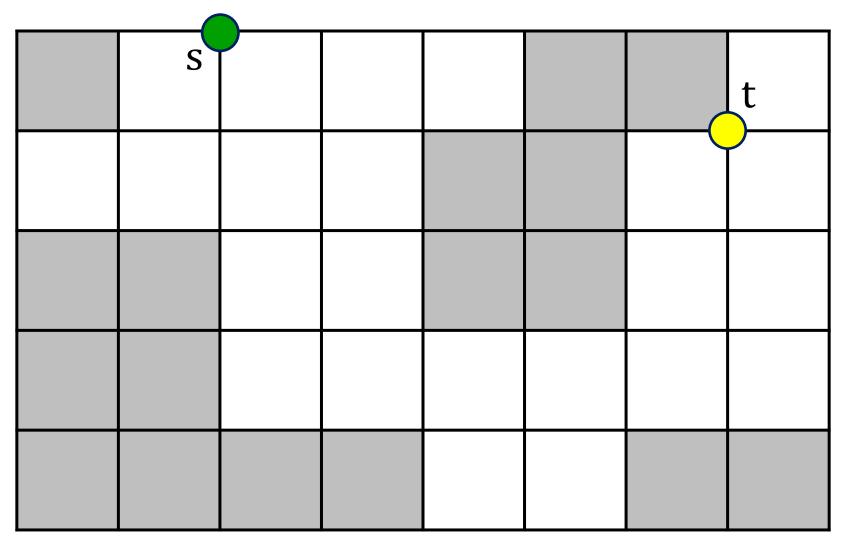
Hon Wai Leong leonghw@comp.nus.edu.sg

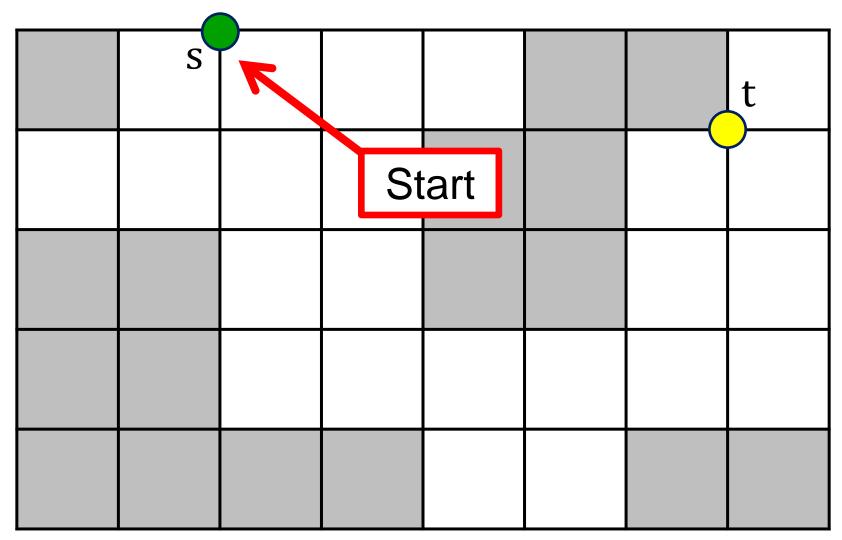
Department of Computer Science National University of Singapore

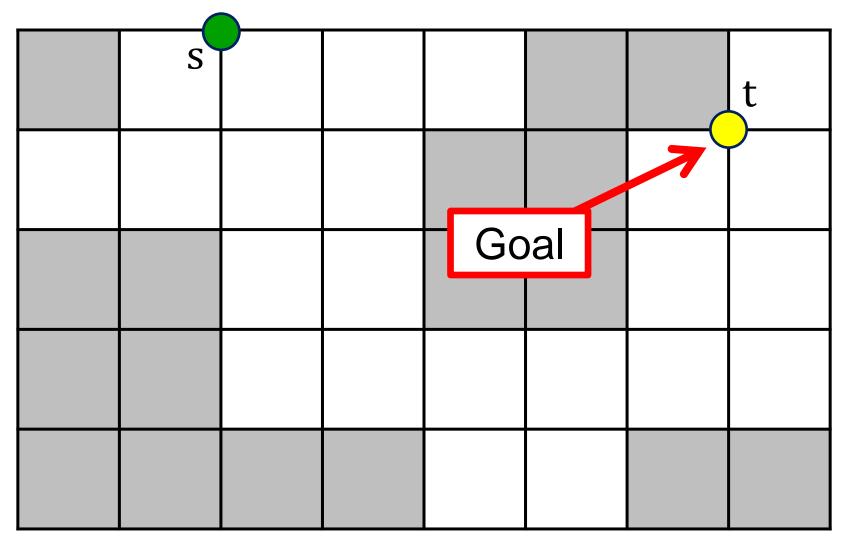




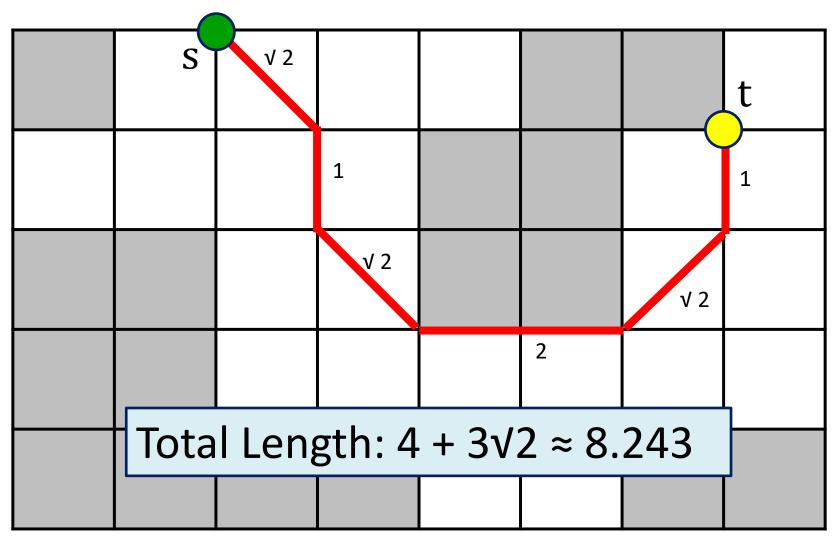




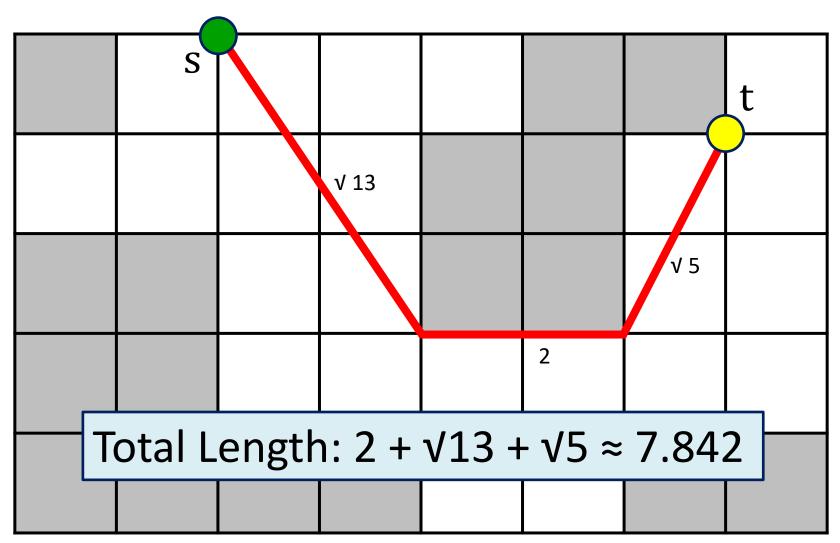




8-Directional Path



Any-Angle Path

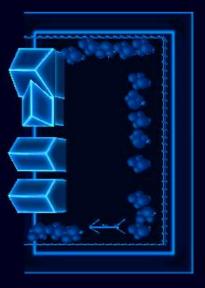


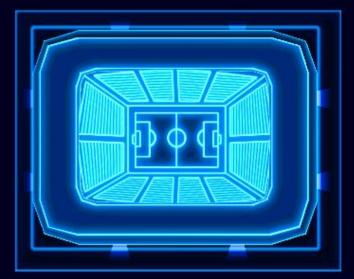




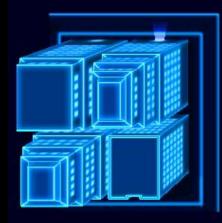
















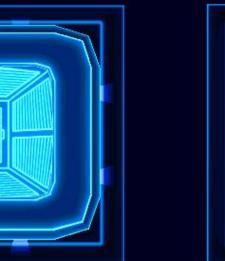


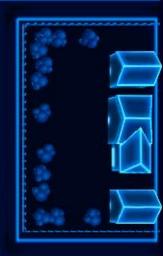




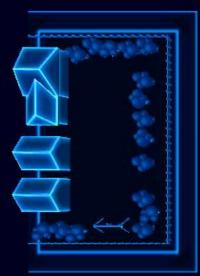


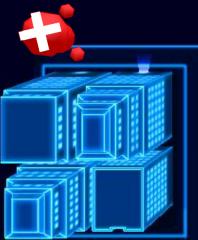


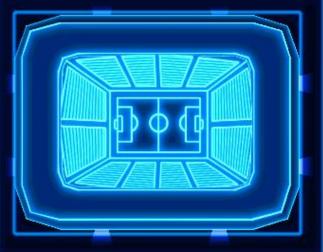






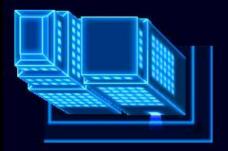


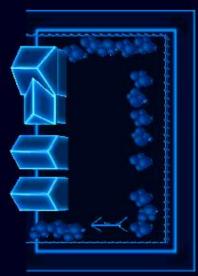


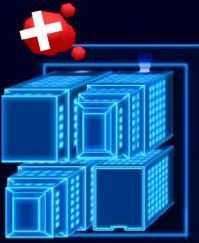






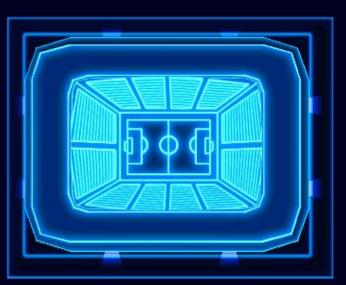










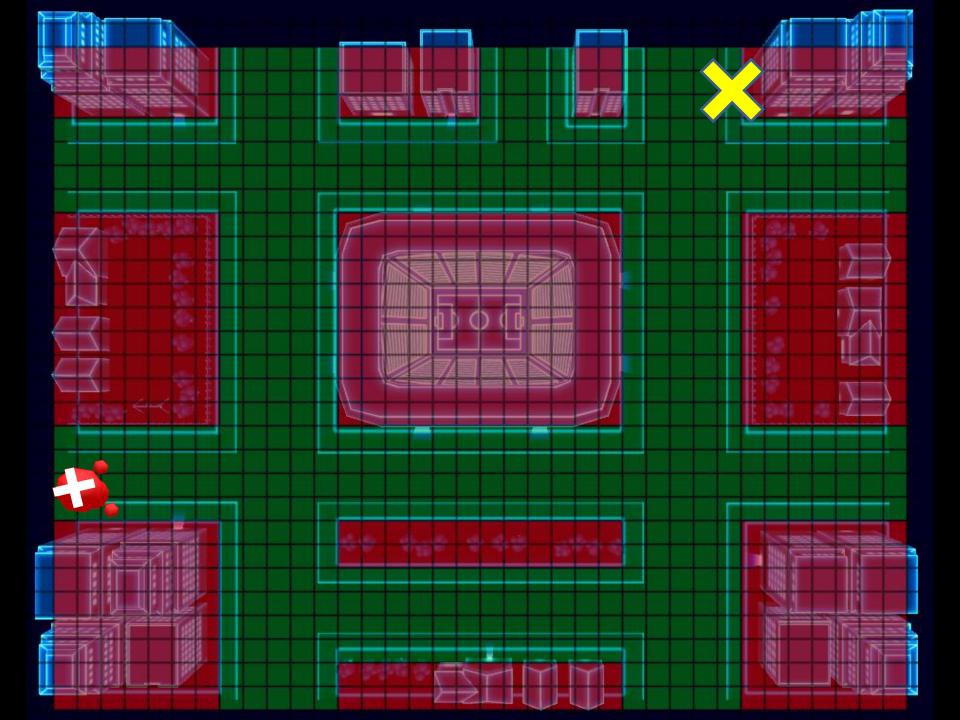


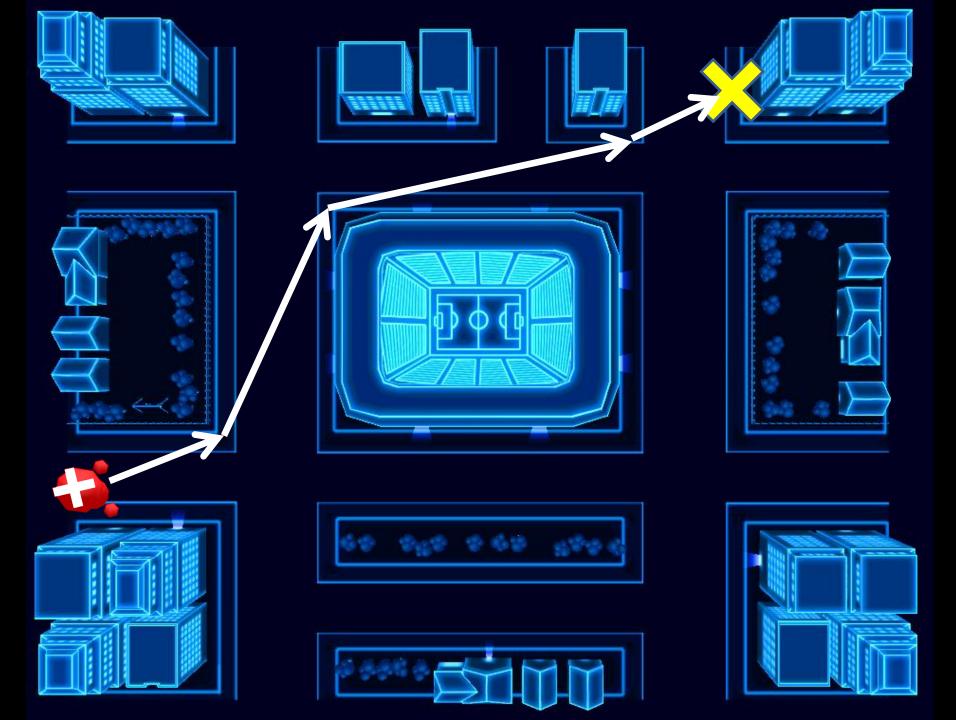


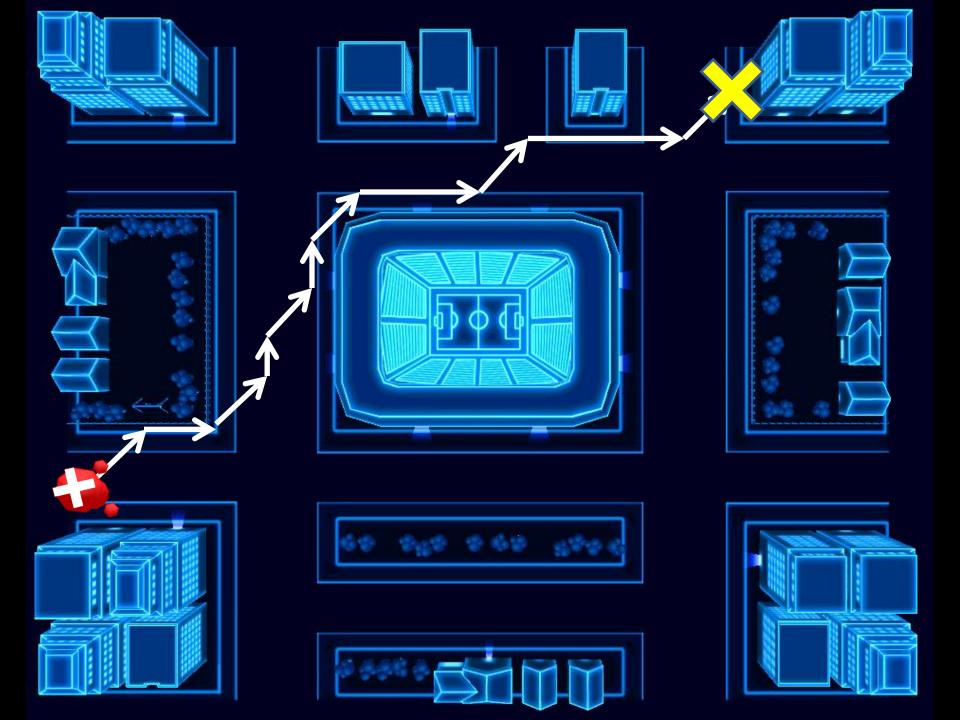


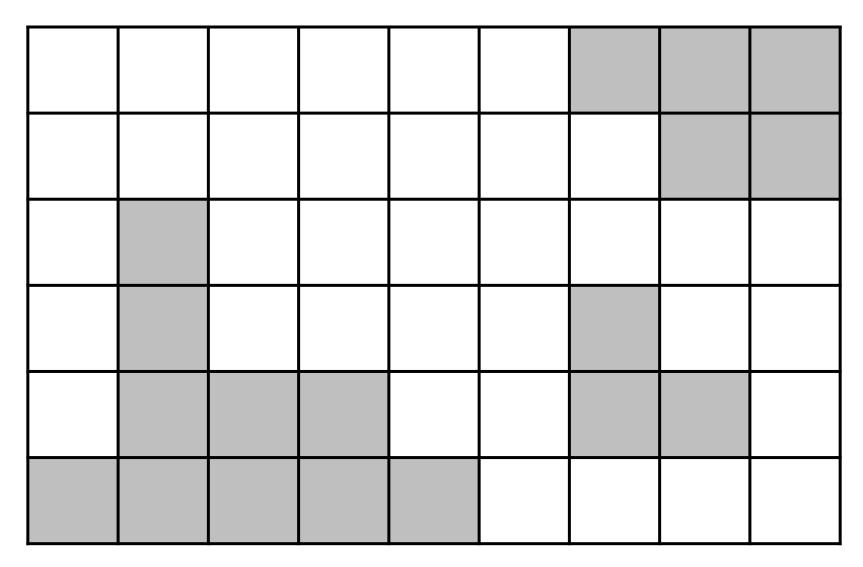


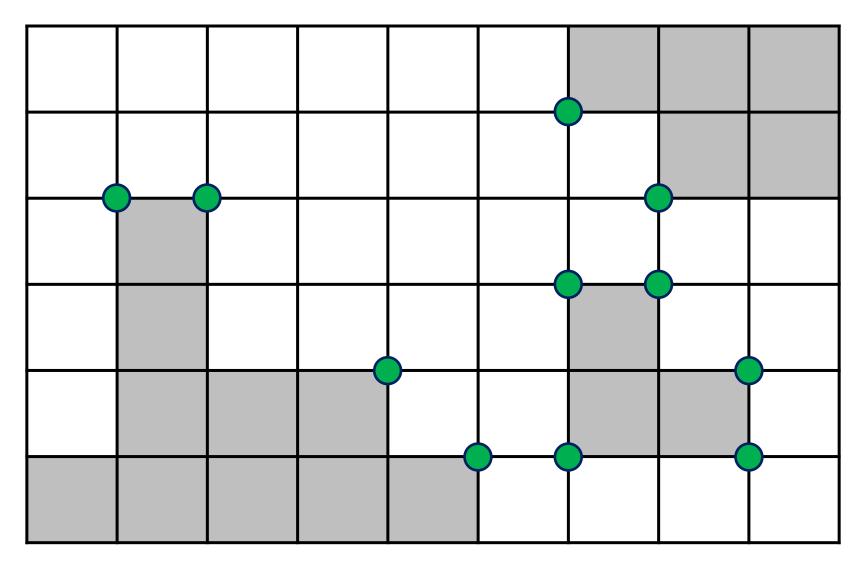


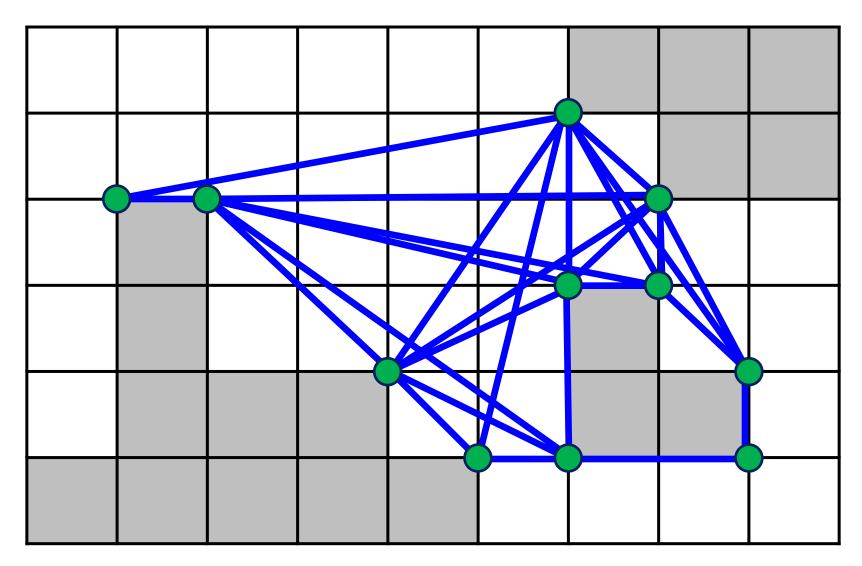


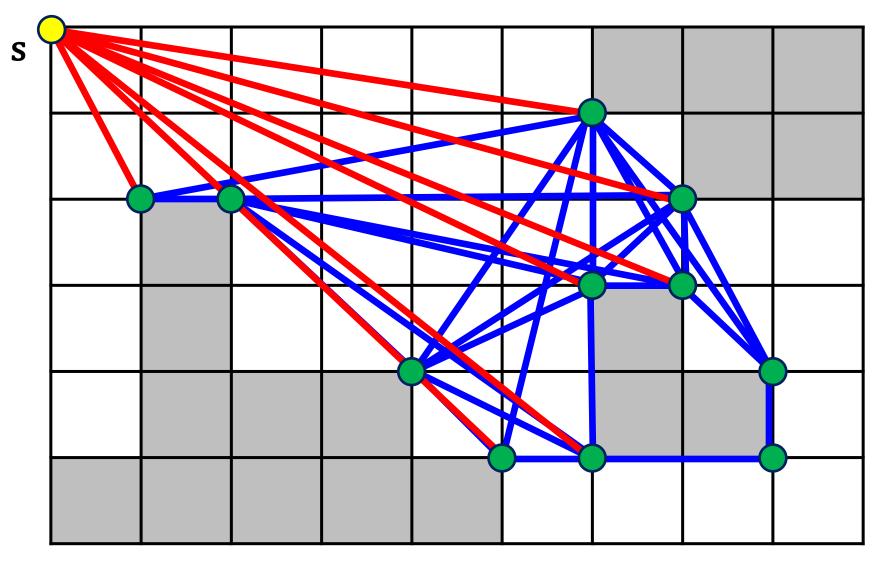


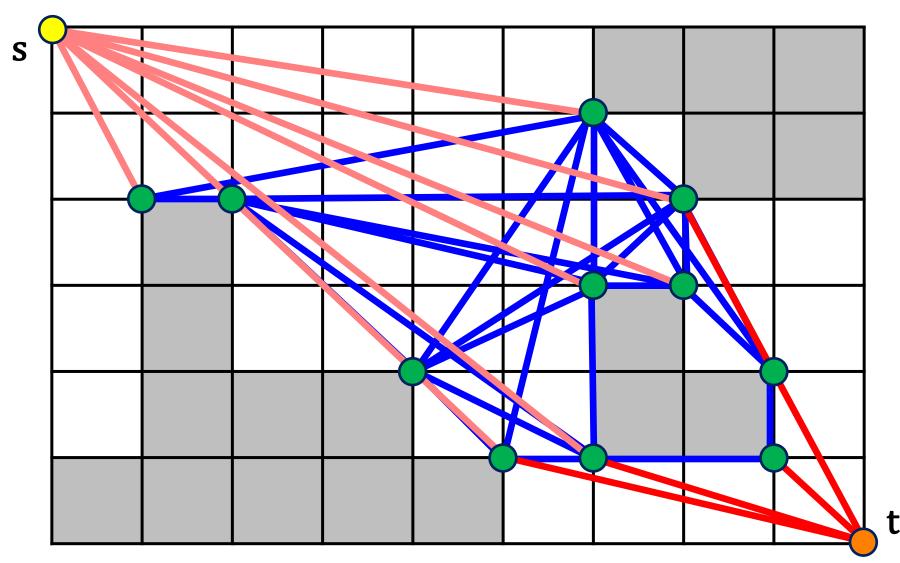


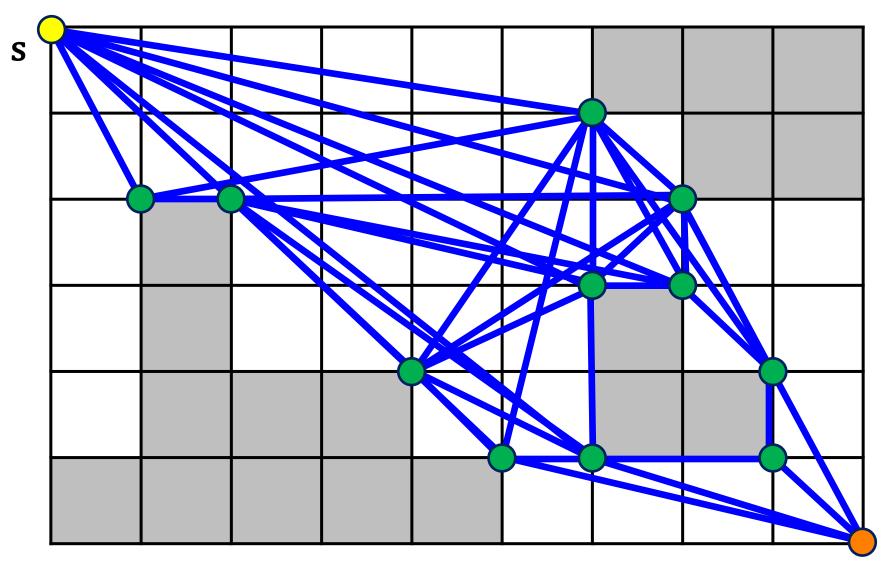


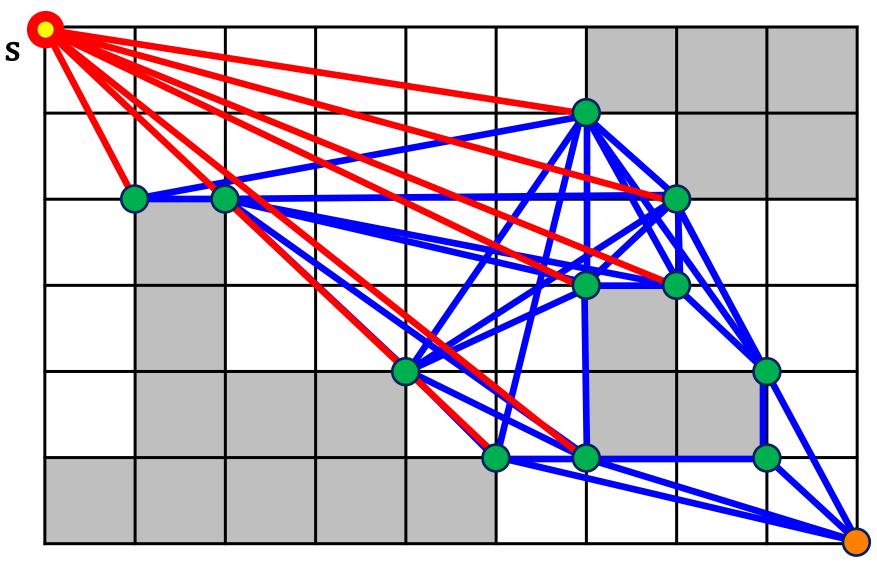


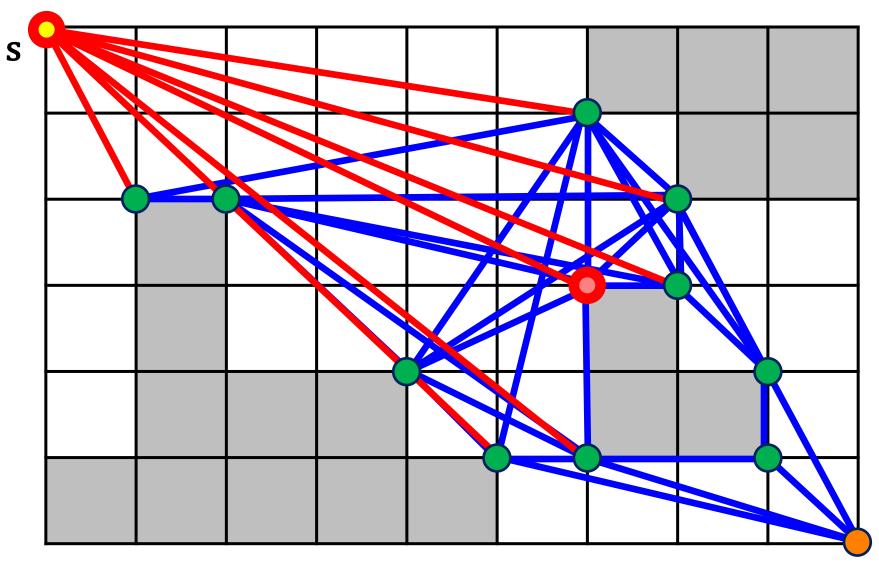


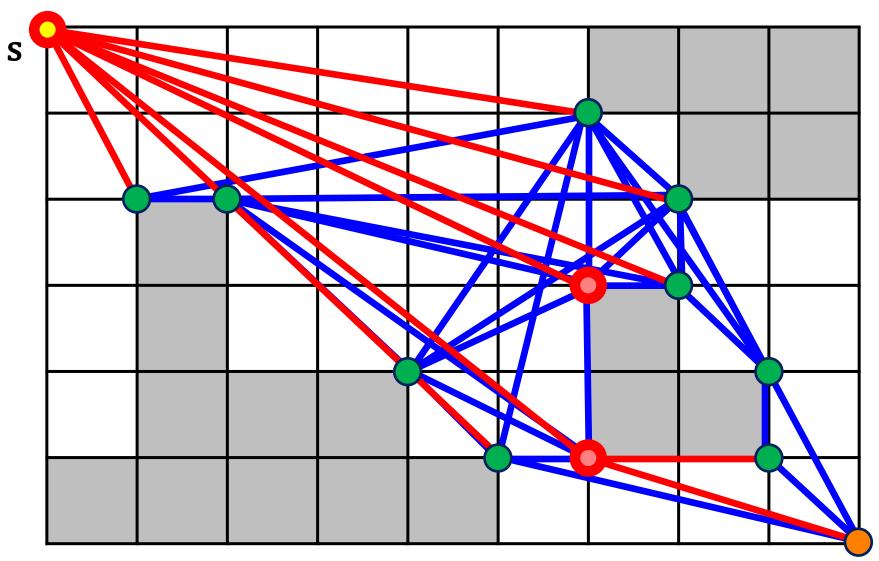






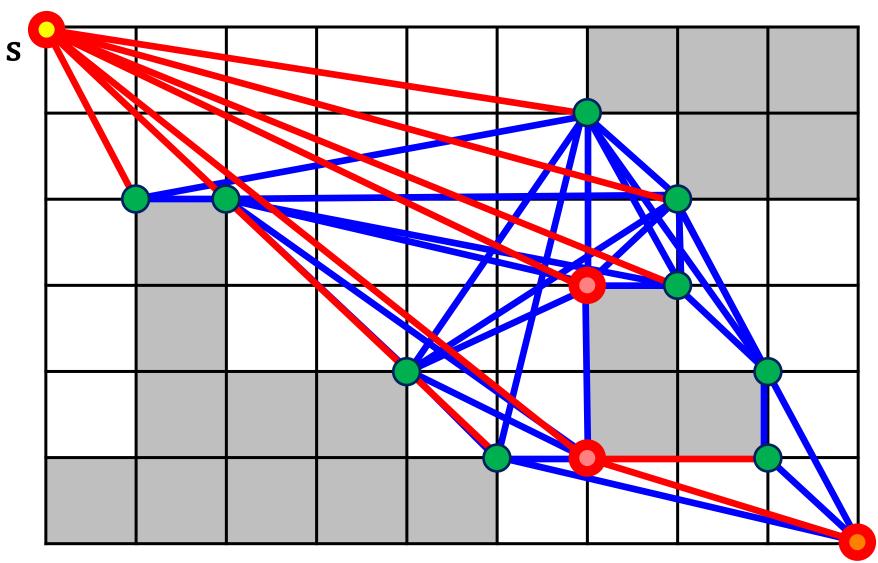






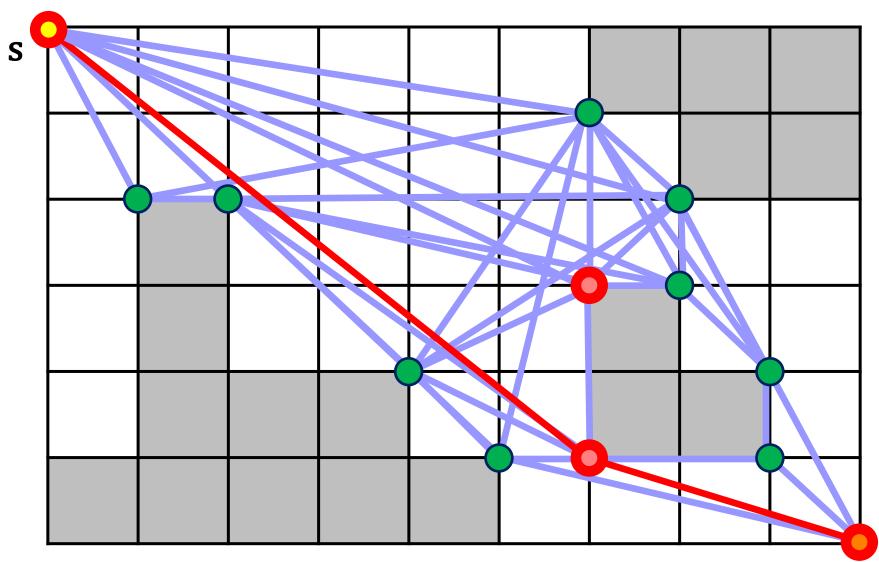
Visibility Graphs

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Visibility Graphs

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Visibility Graphs

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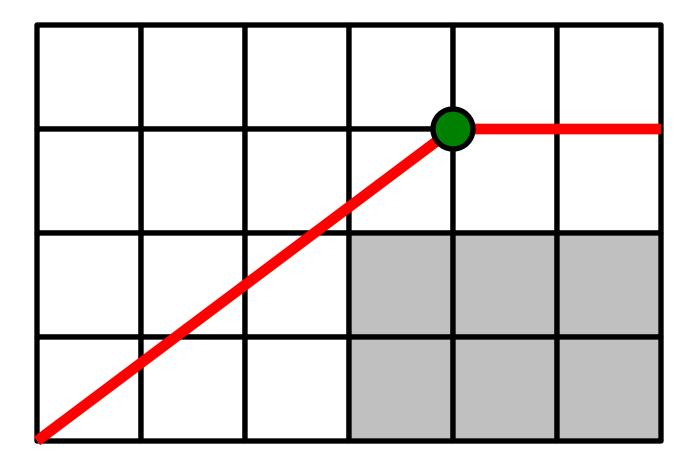
Taut Paths



Taut Paths Optimal Paths must be Taut!

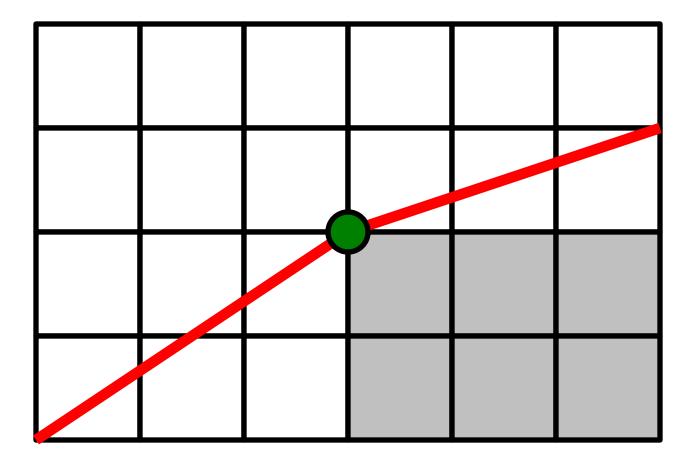


Non-Taut Path



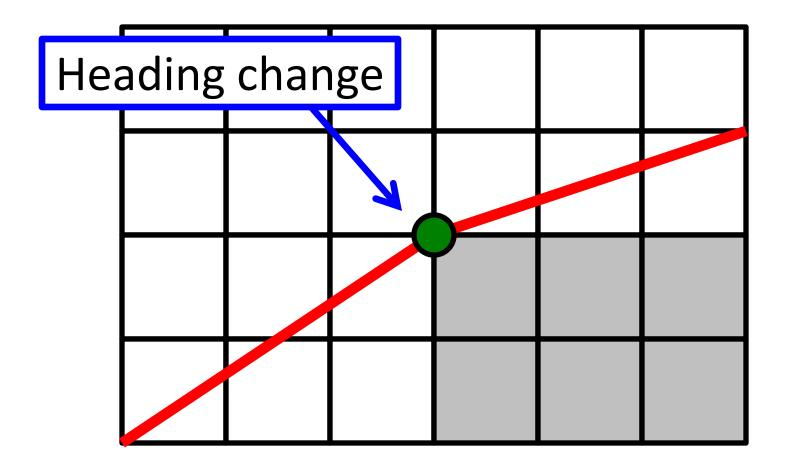
Taut Paths

Taut Path



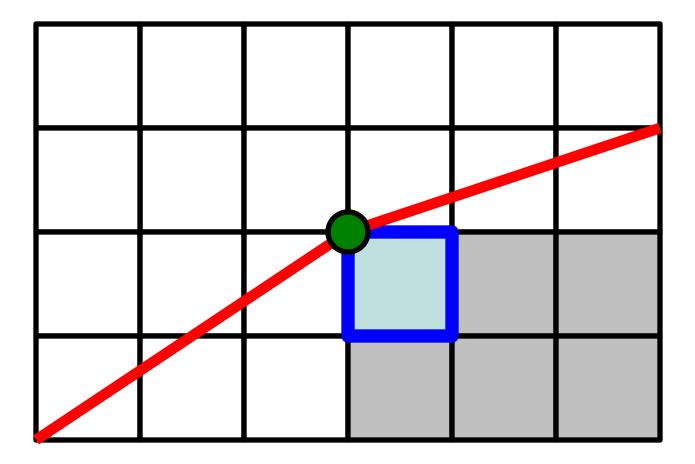


Taut Path





Taut Path



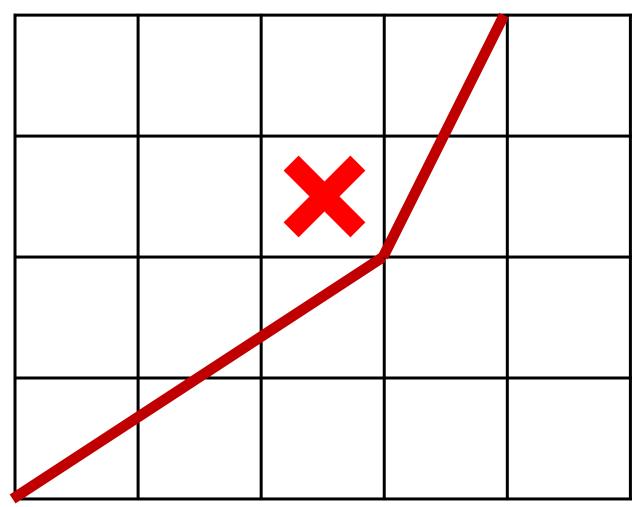


Tautness Checks



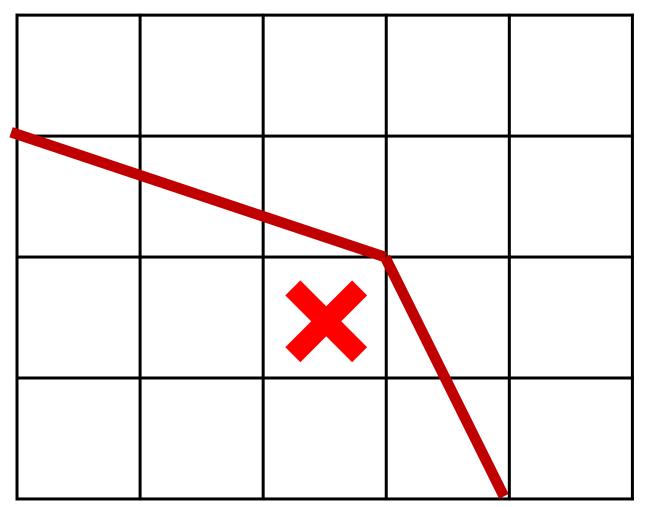
Tautness Checks

We need only check one tile.



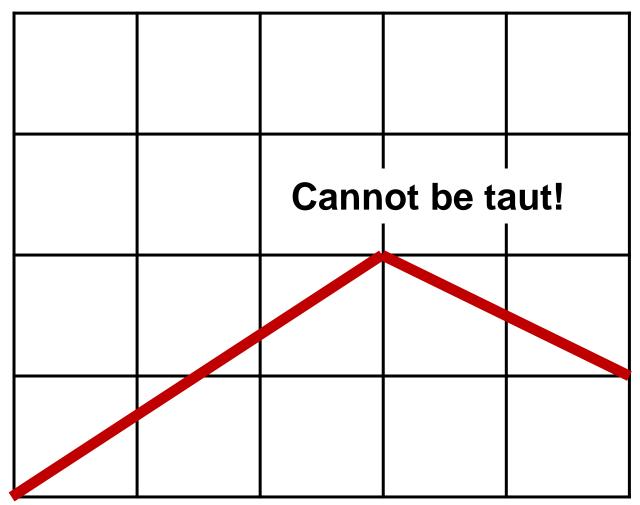
Tautness Checks

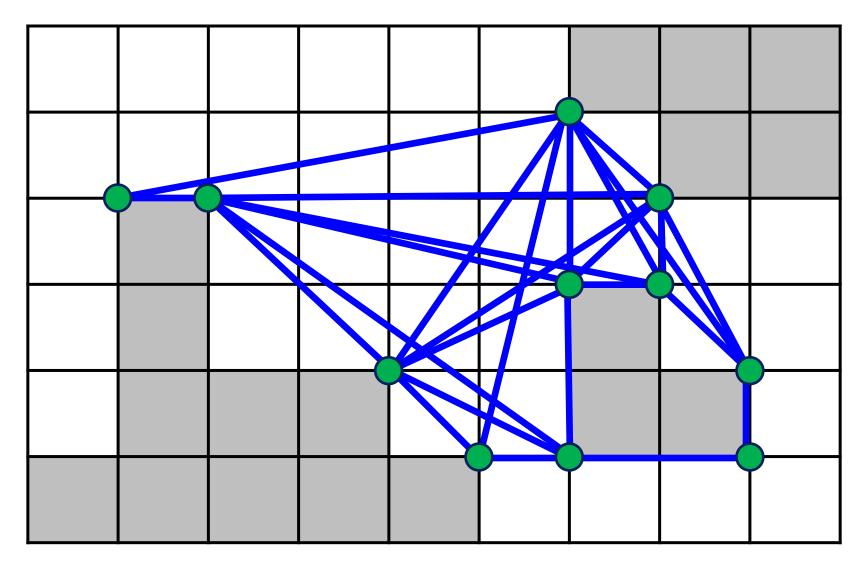
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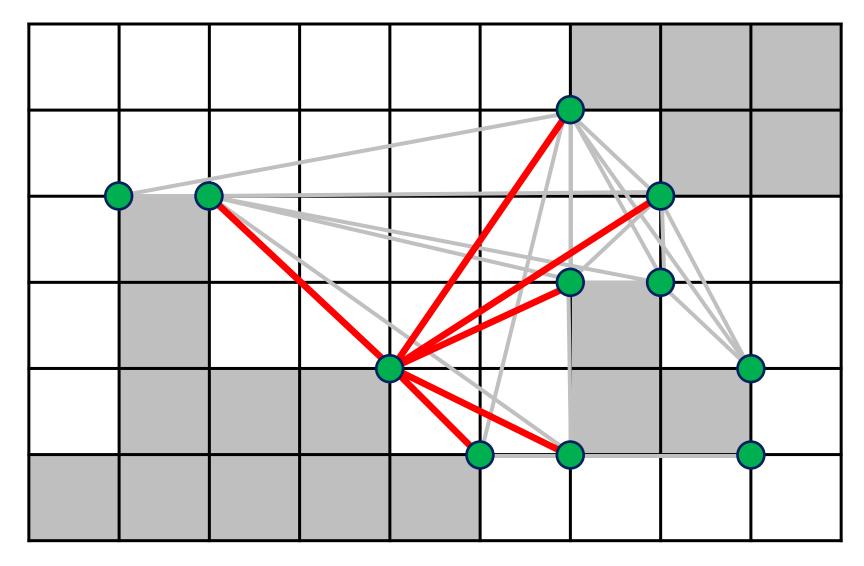


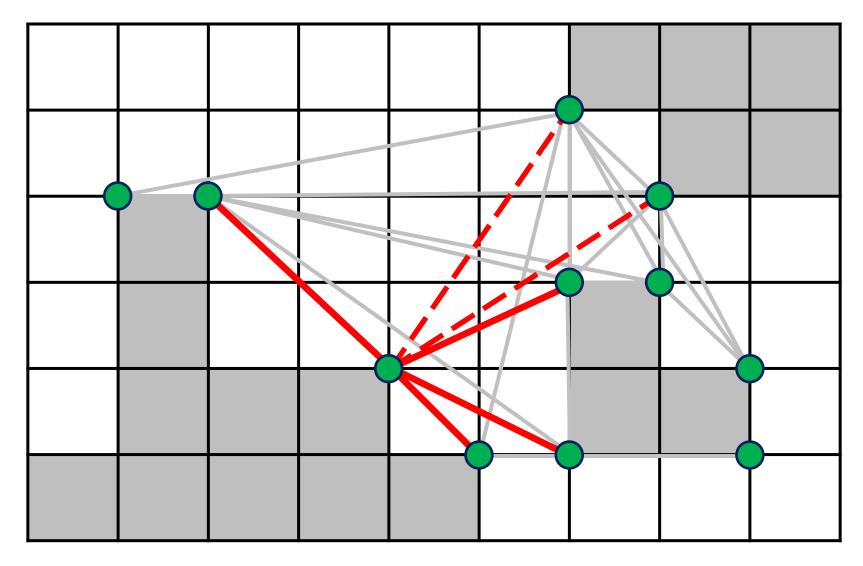
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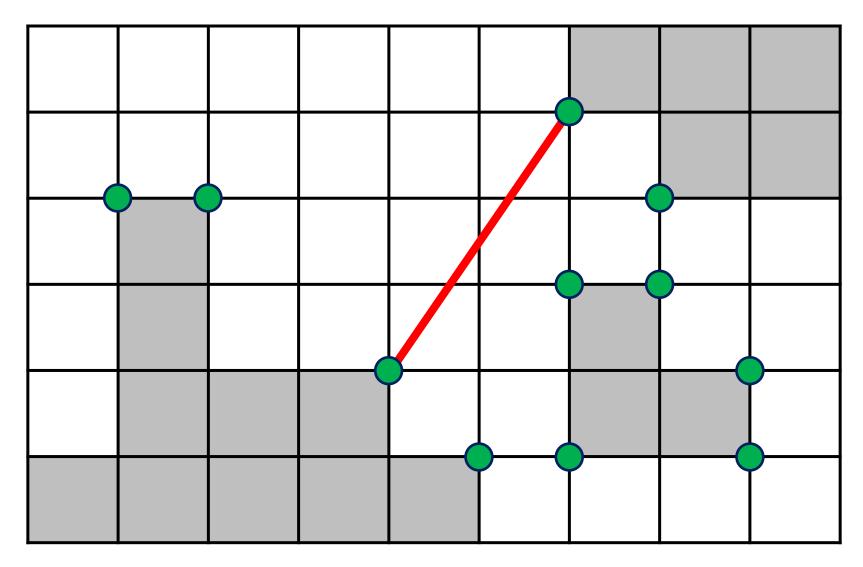
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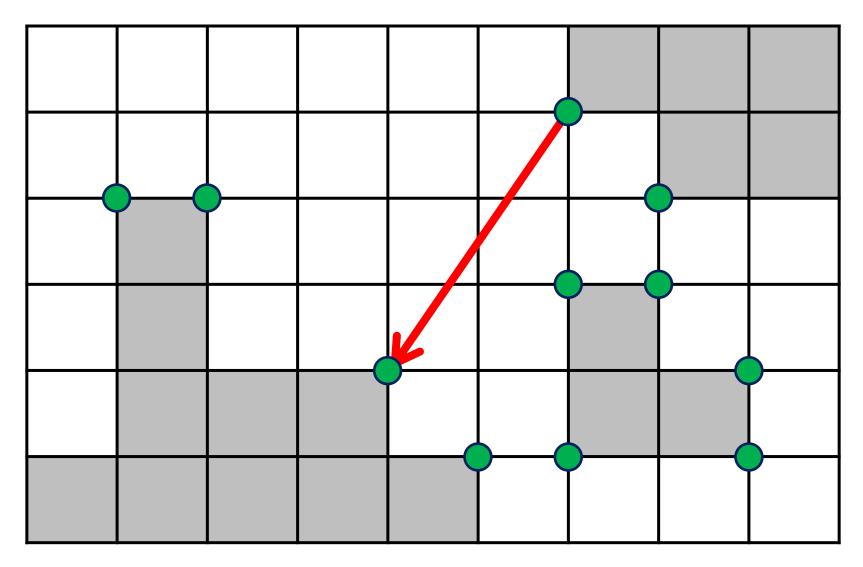


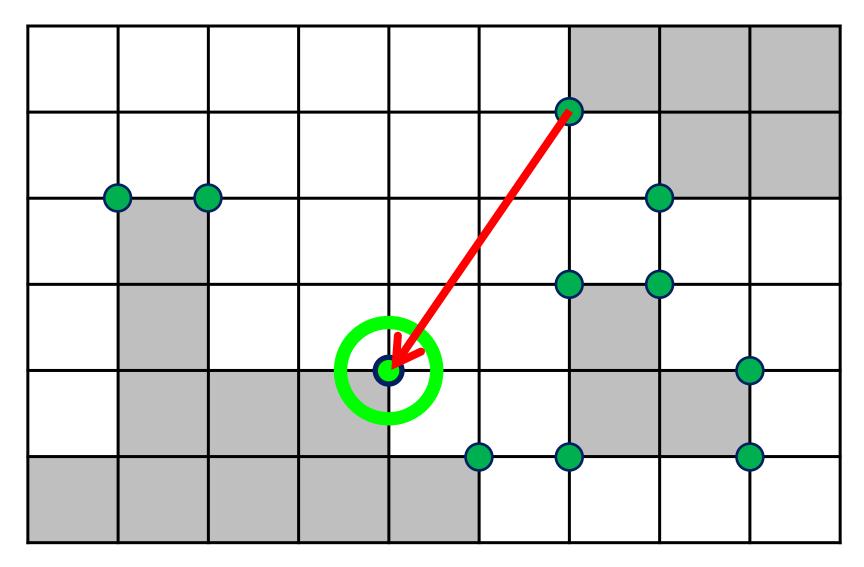


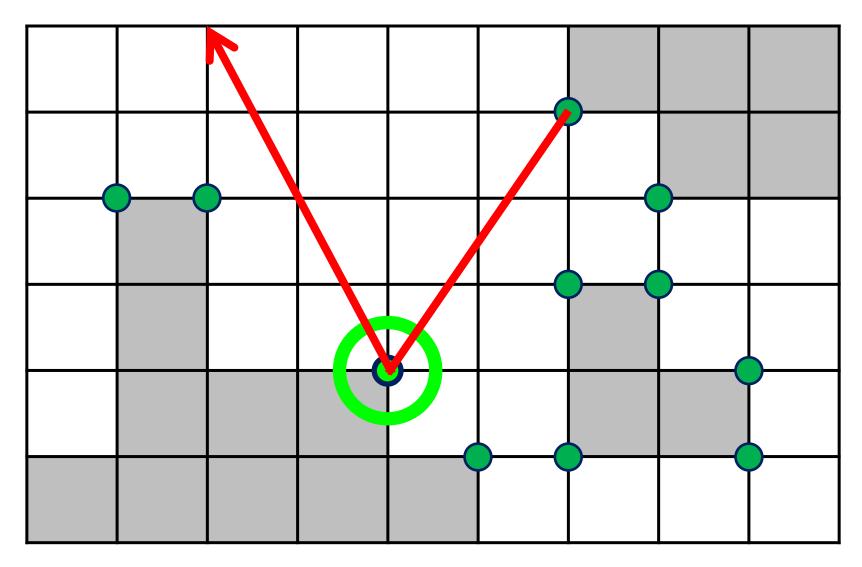


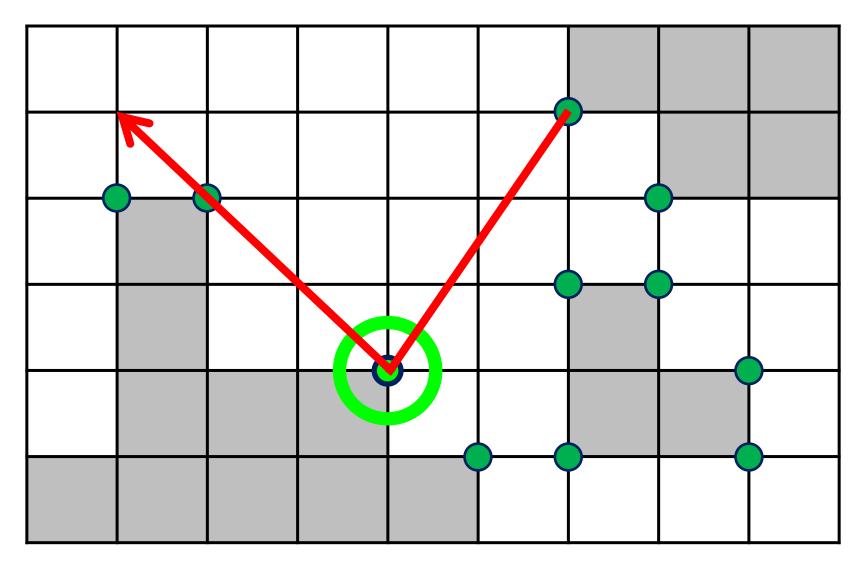


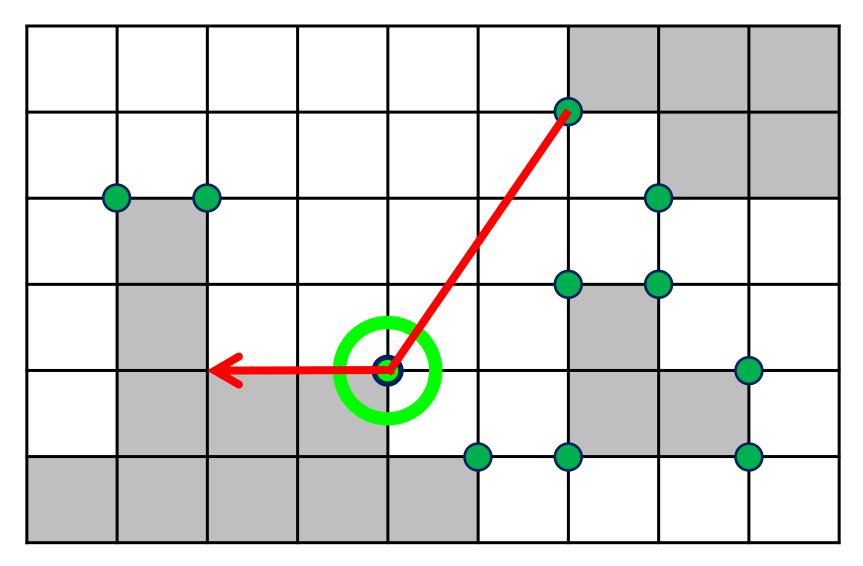


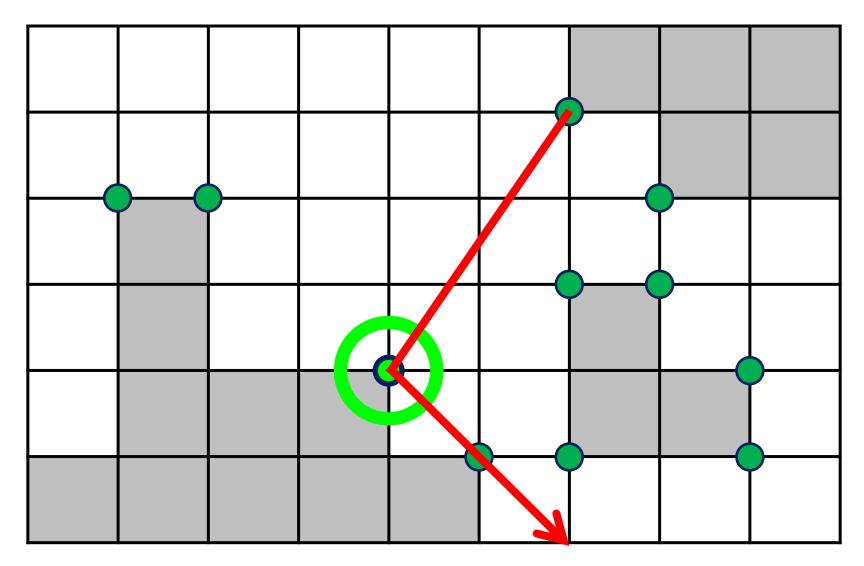


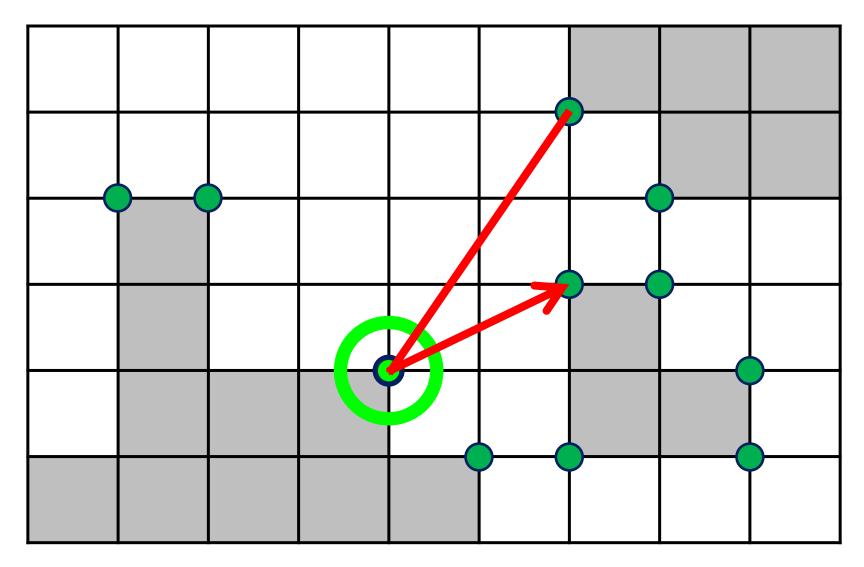


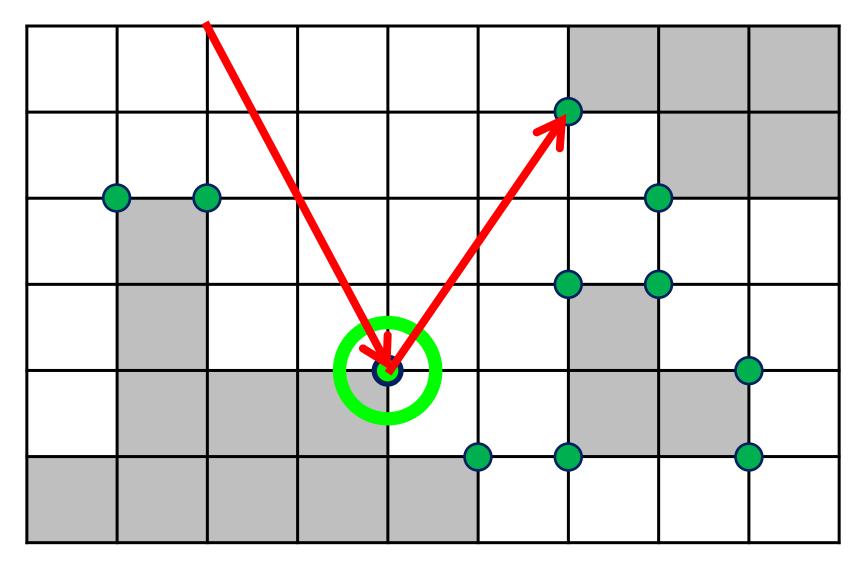


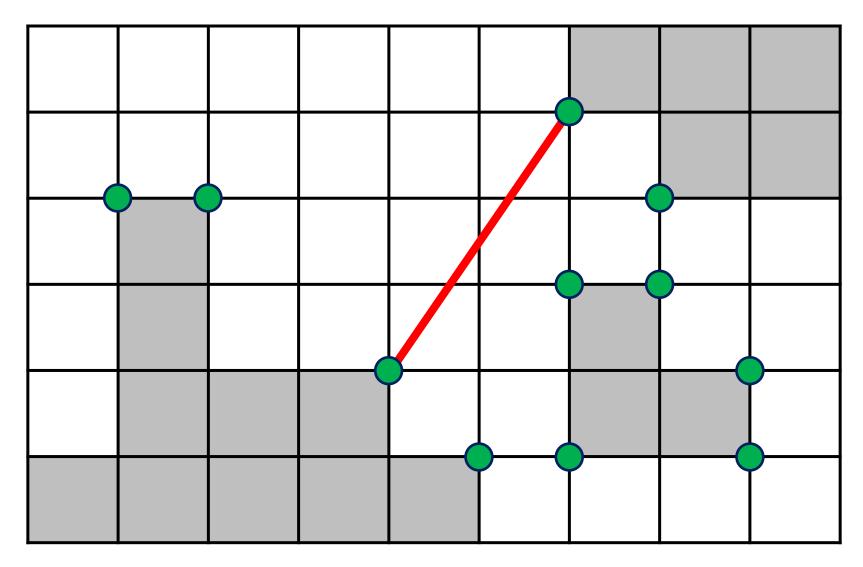


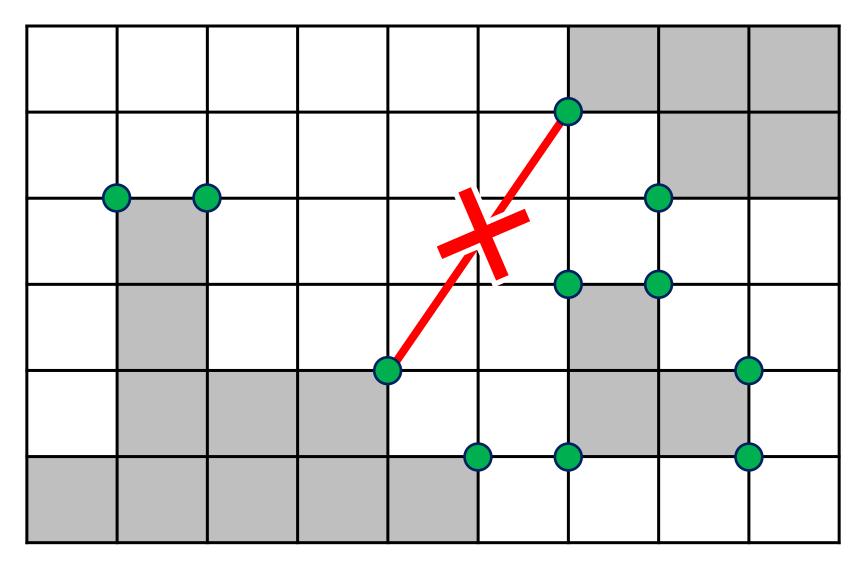


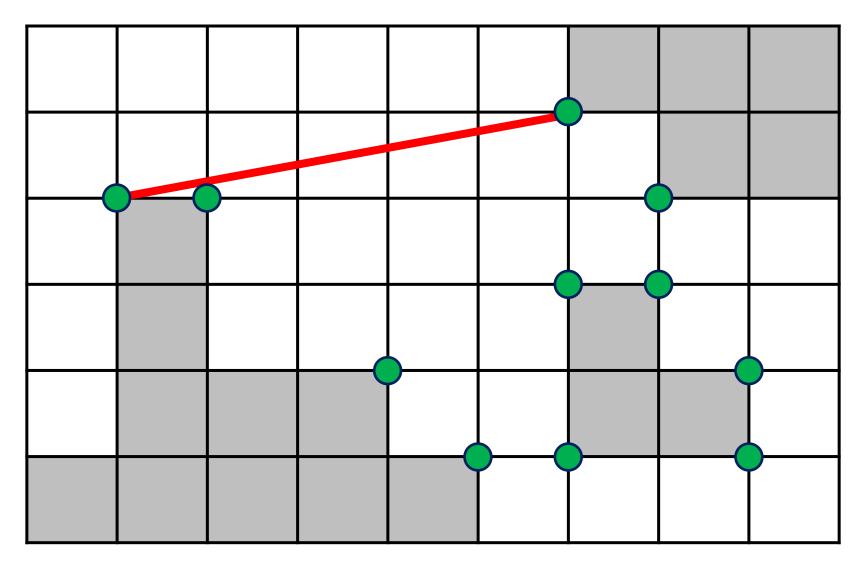


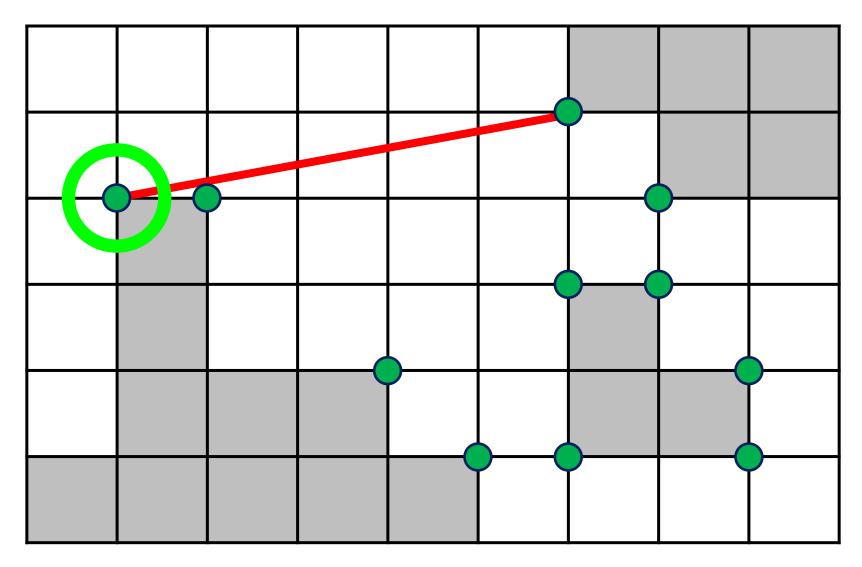


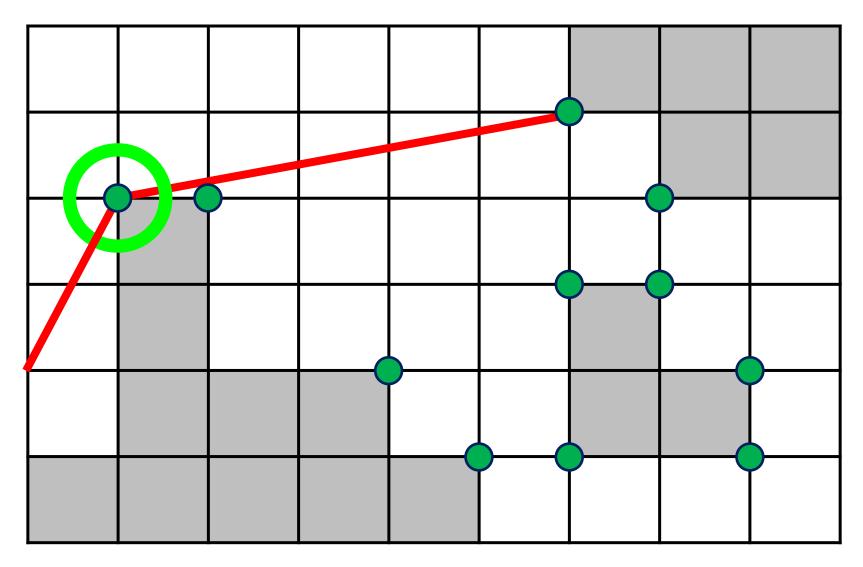


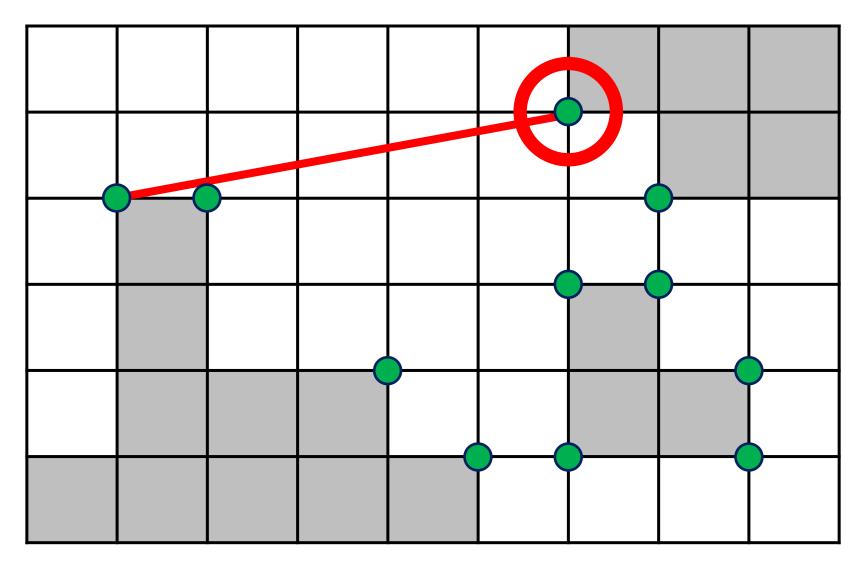


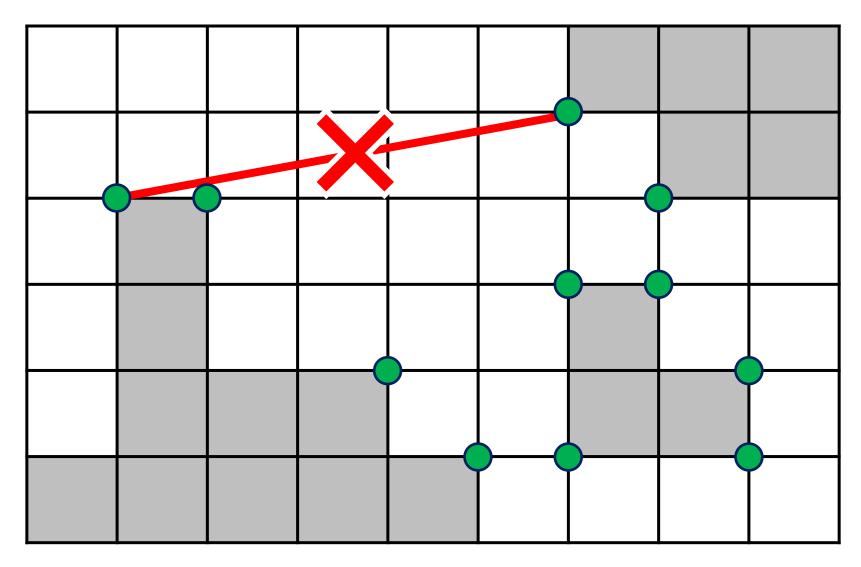


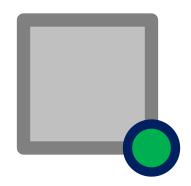


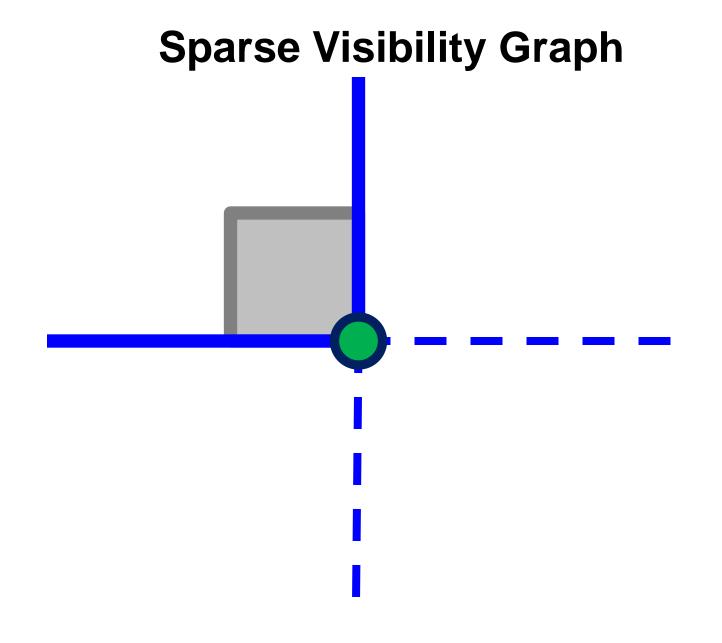


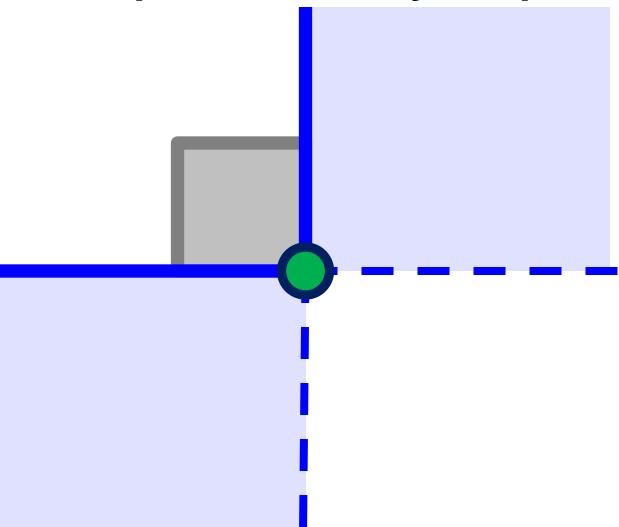


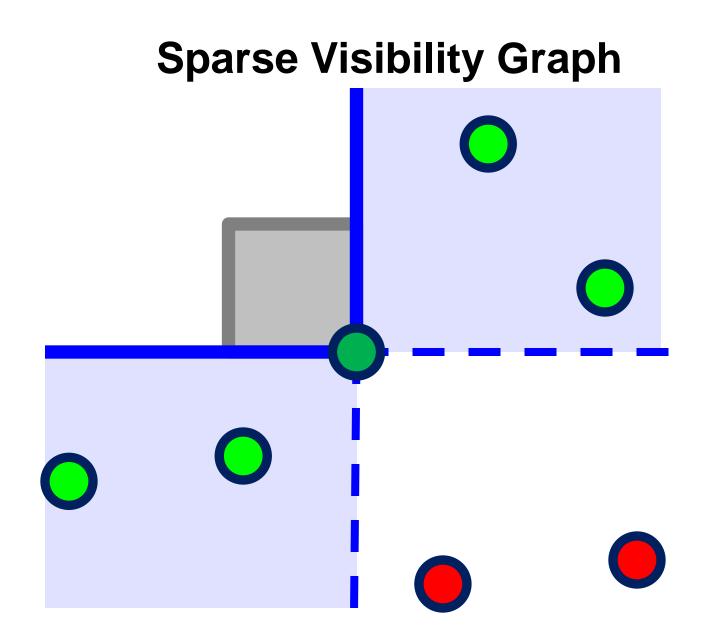


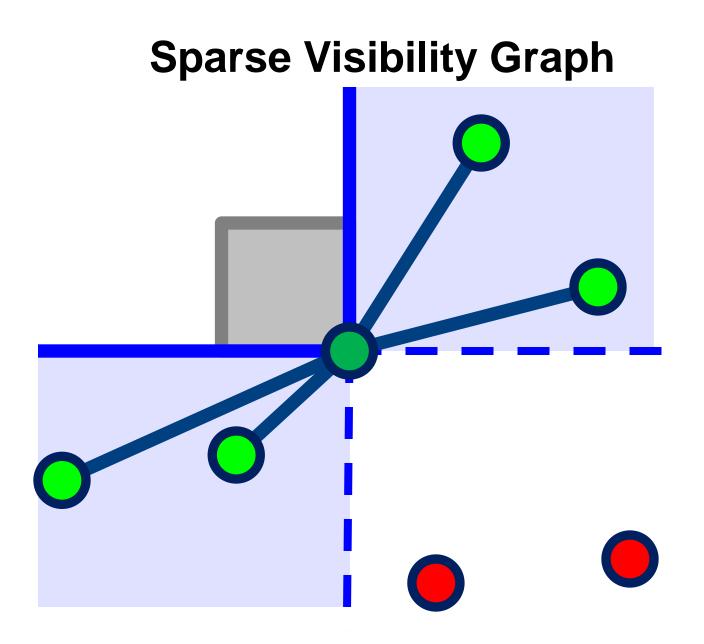


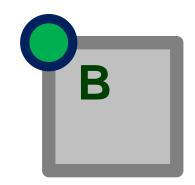


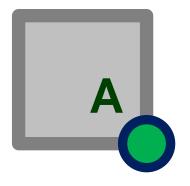


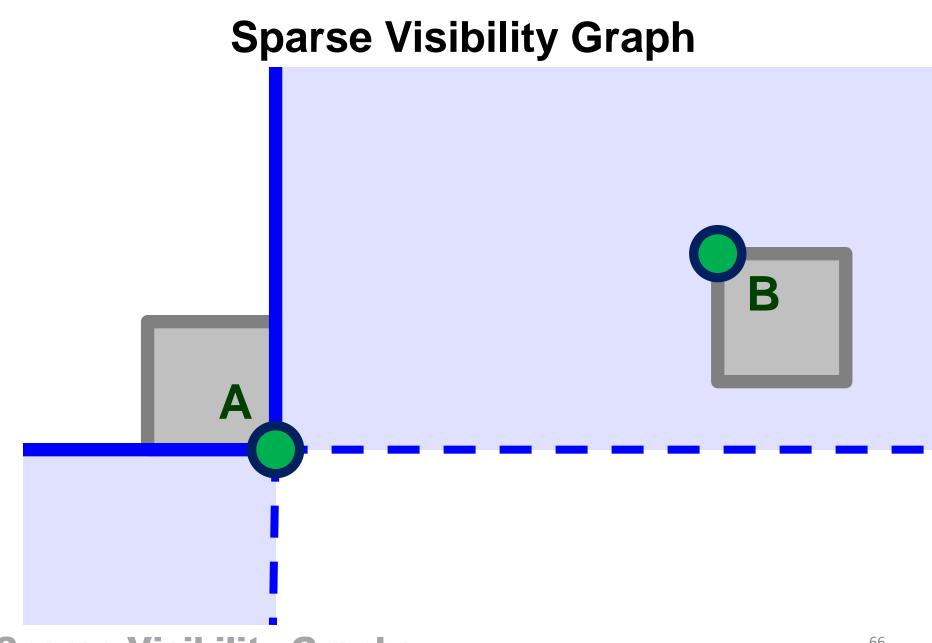


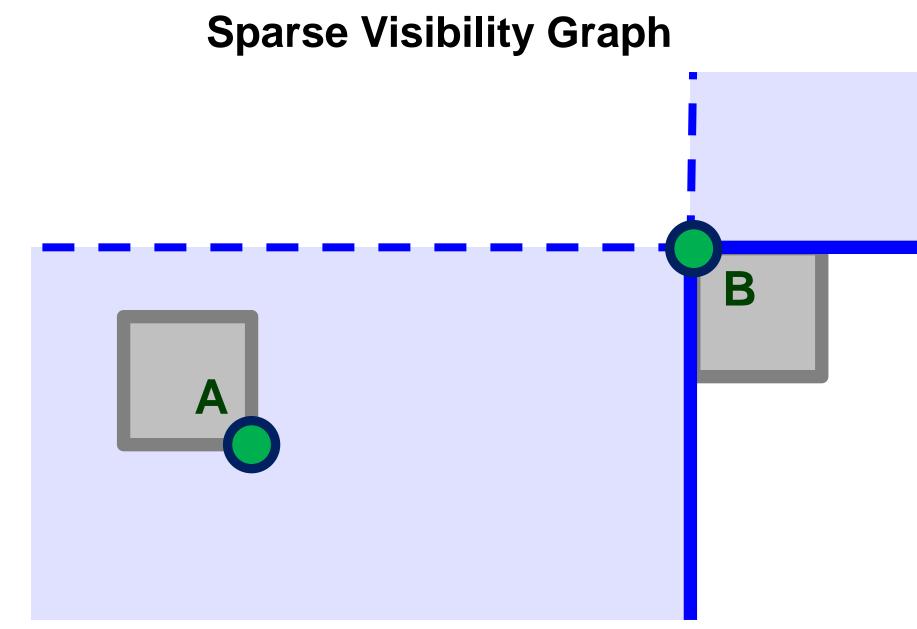


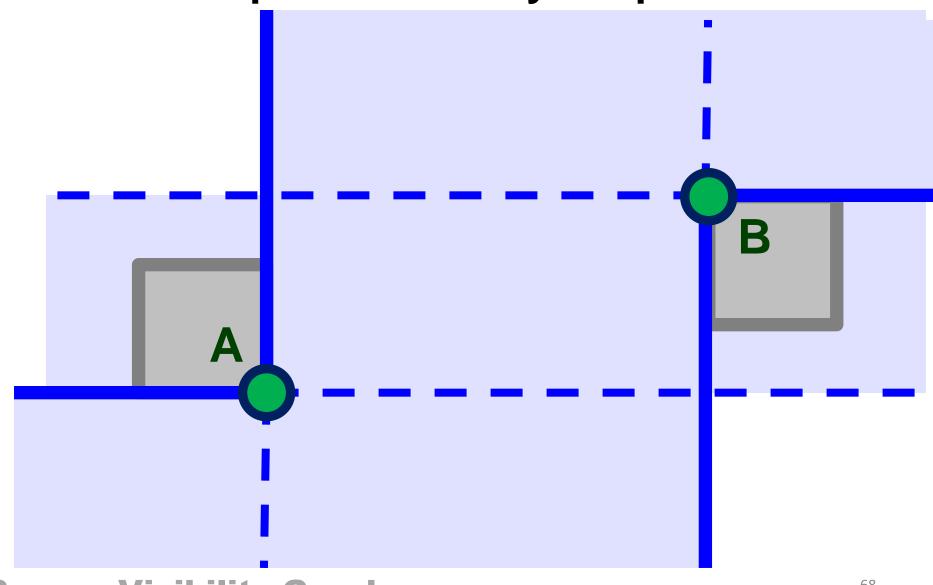


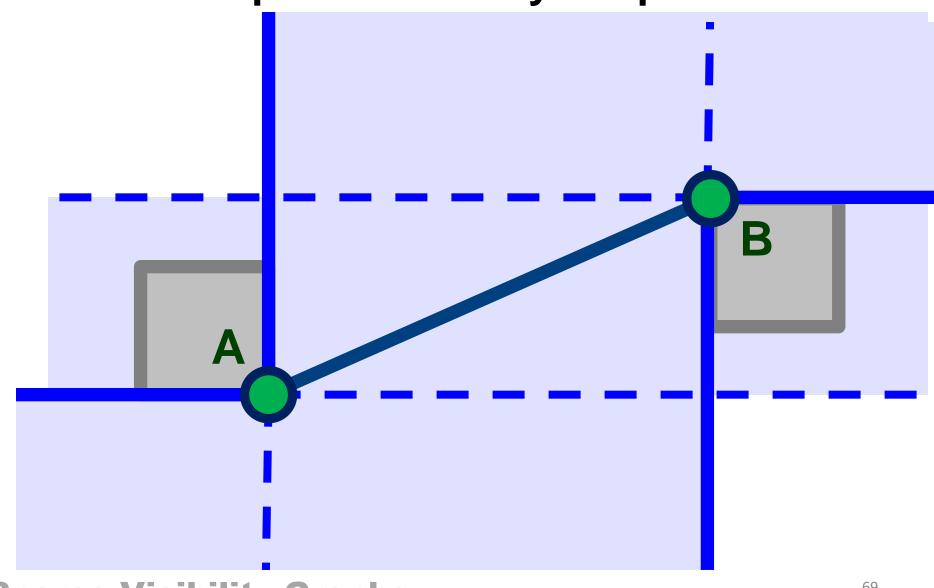




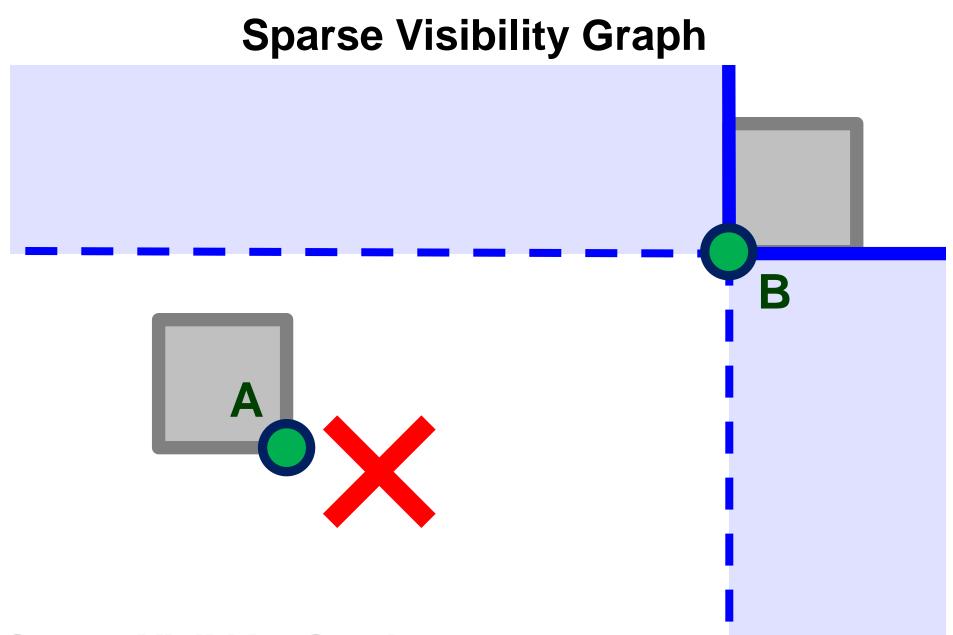


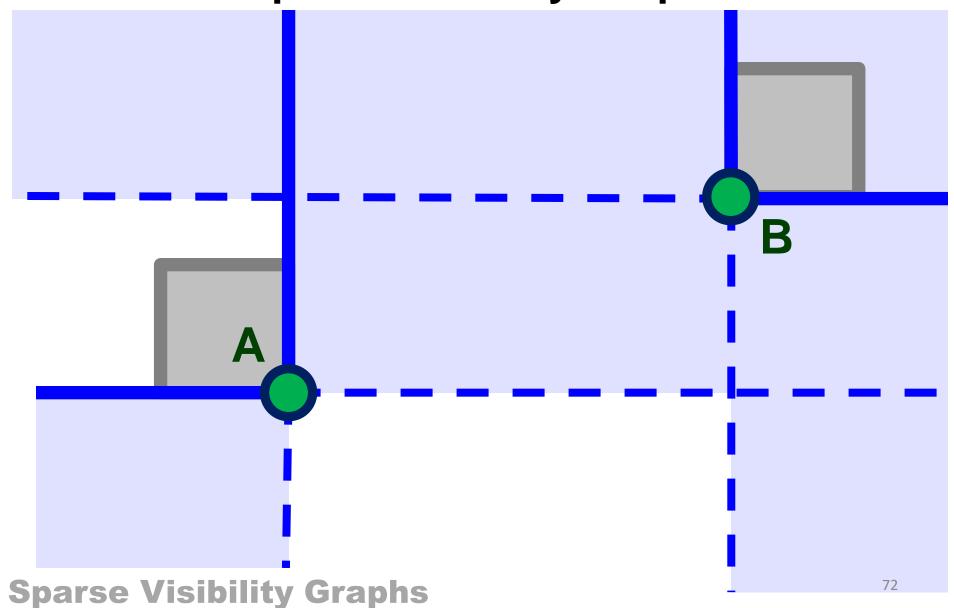


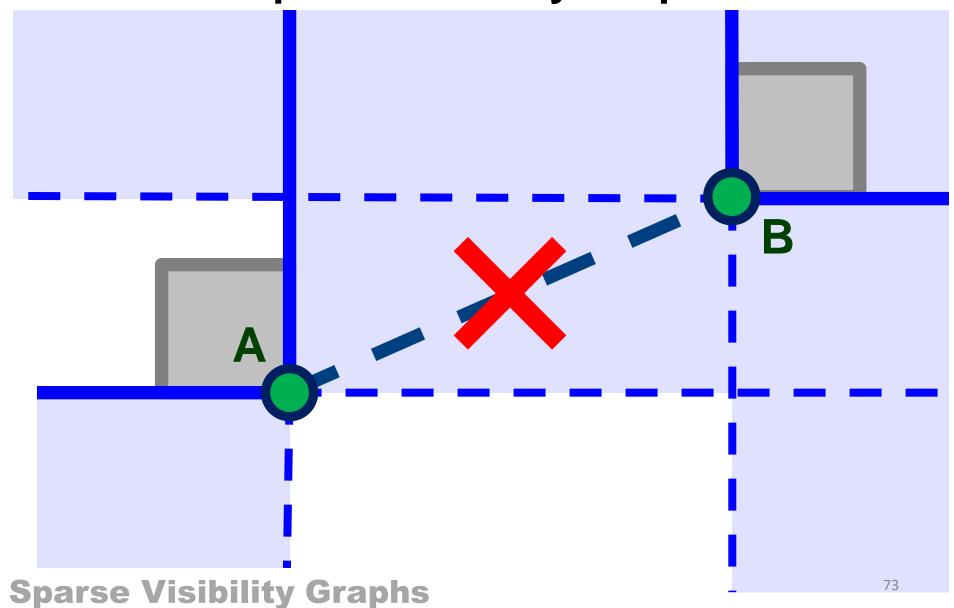




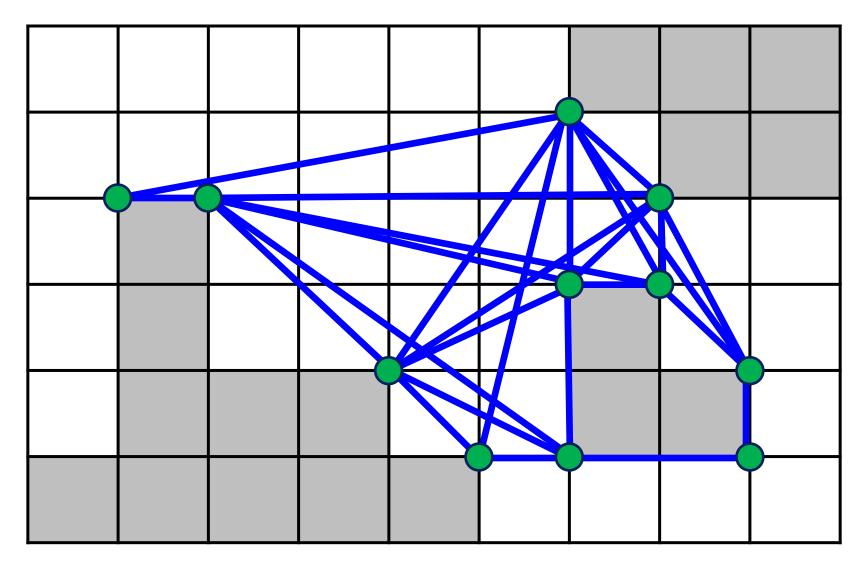
Sparse Visibility Graph R A **Sparse Visibility Graphs**

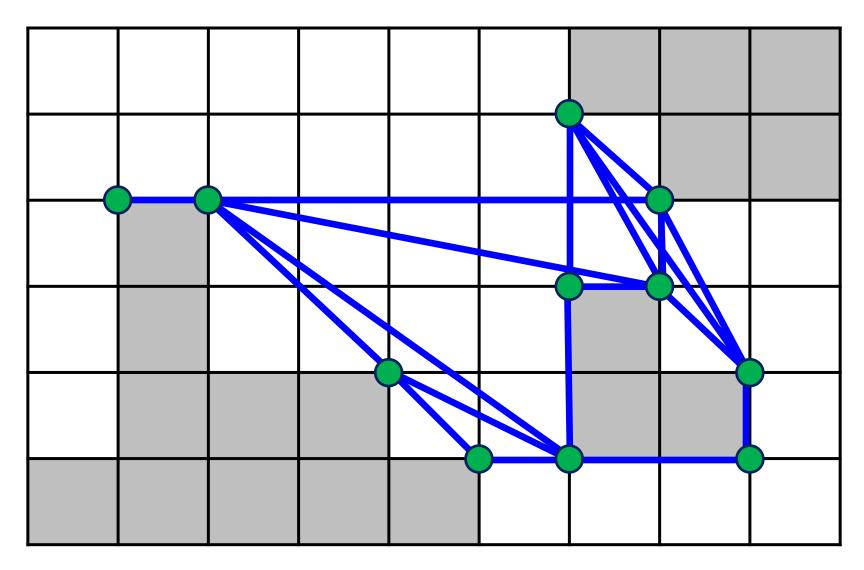


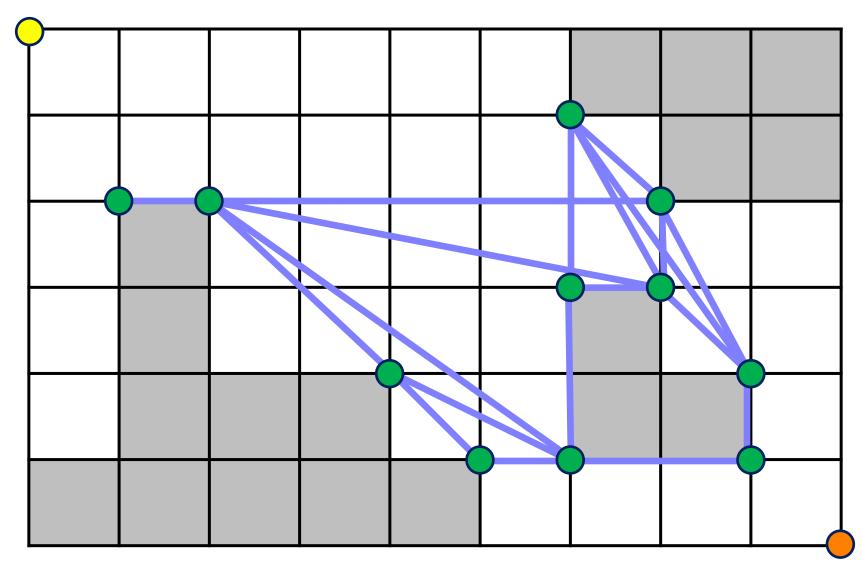


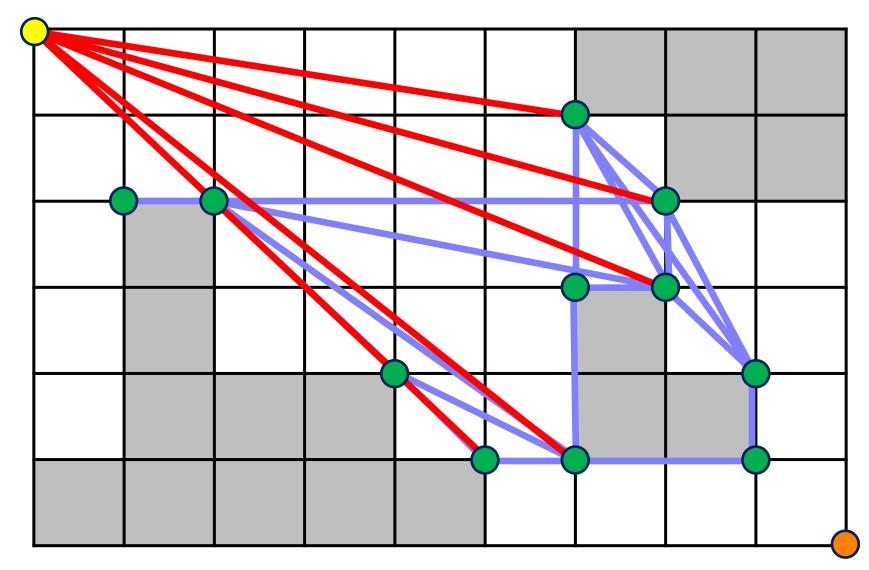


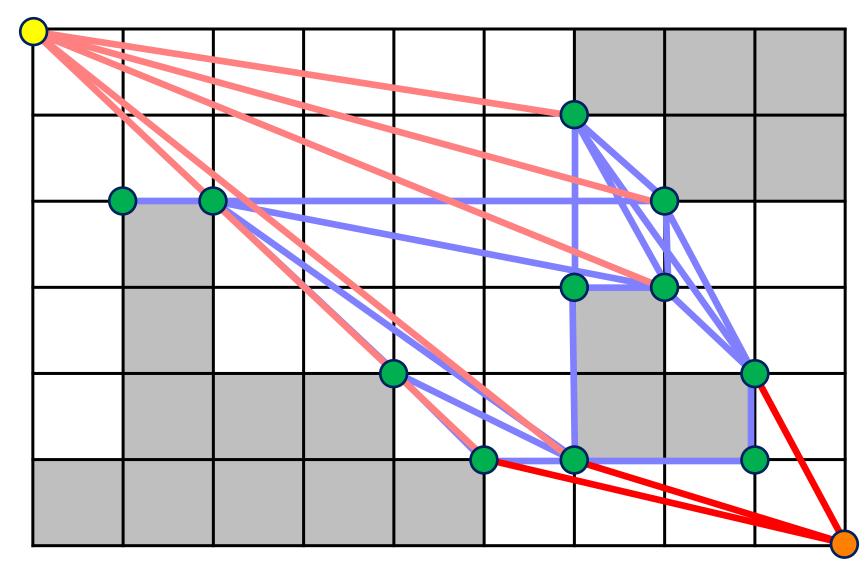
Visibility Graph

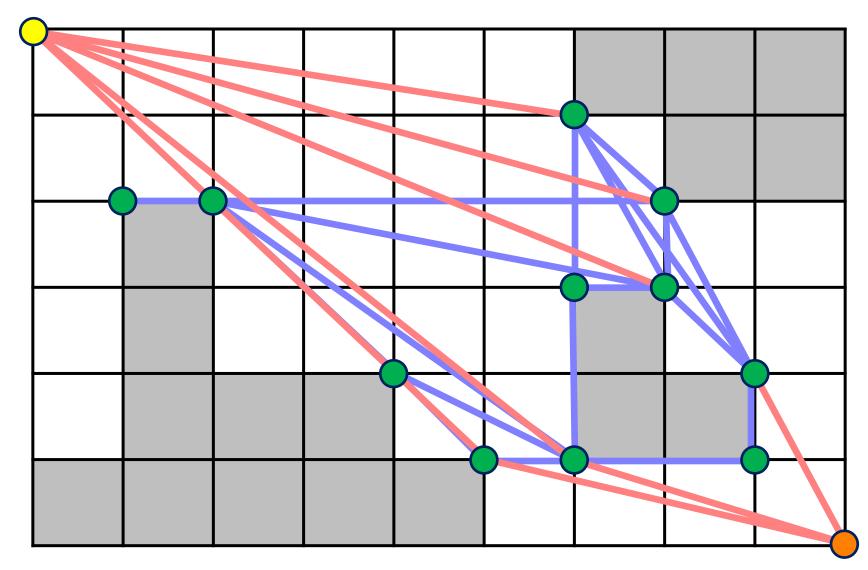


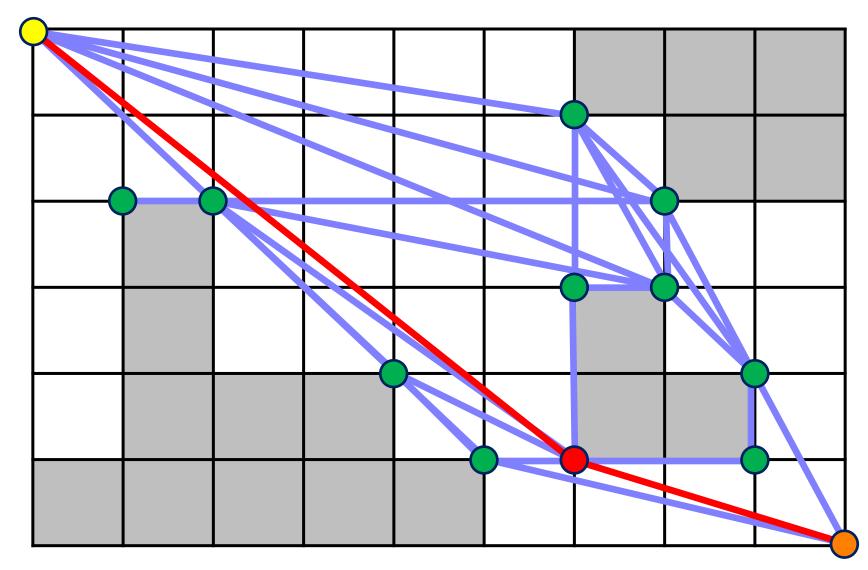


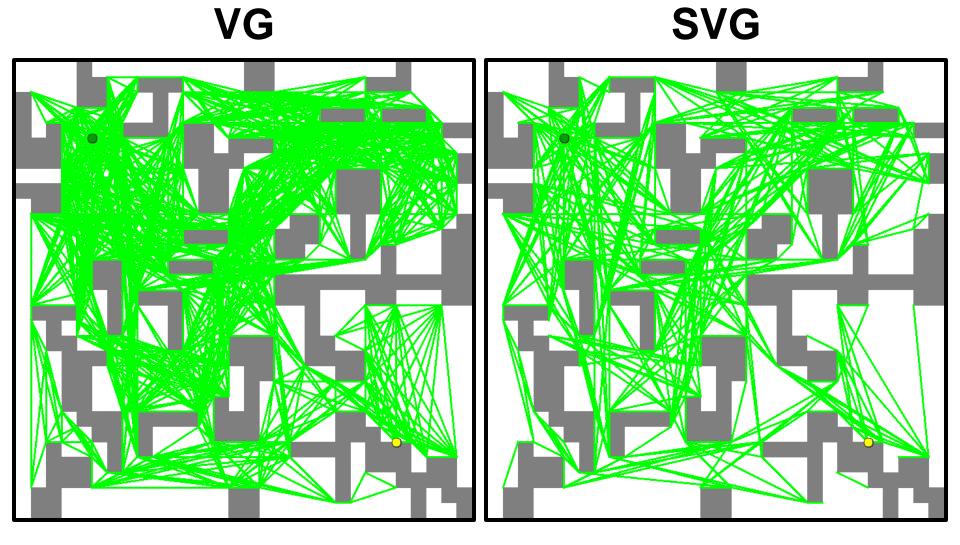






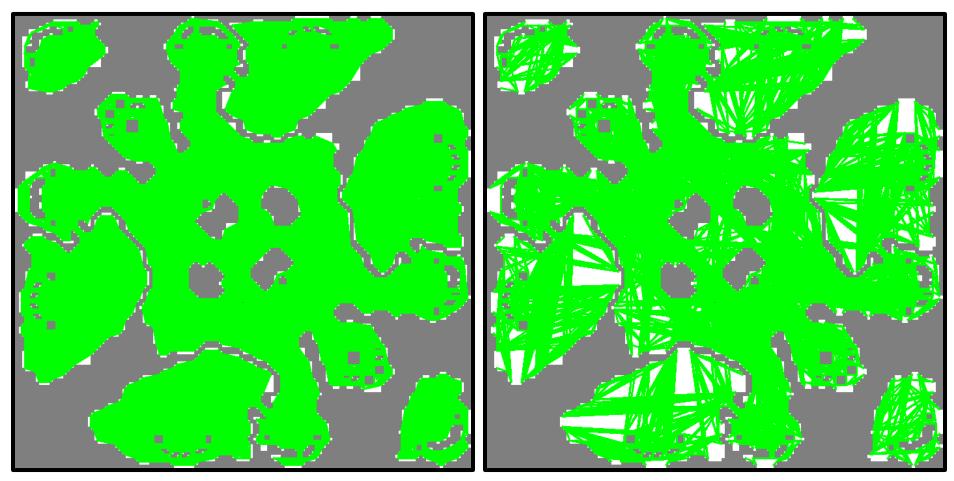






VG





Theorem 1

Every optimal path can be represented using only edges in the sparse visibility graph.

Theorem 2

Every edge in the sparse visibility graph is part of some optimal path between some pair of points.

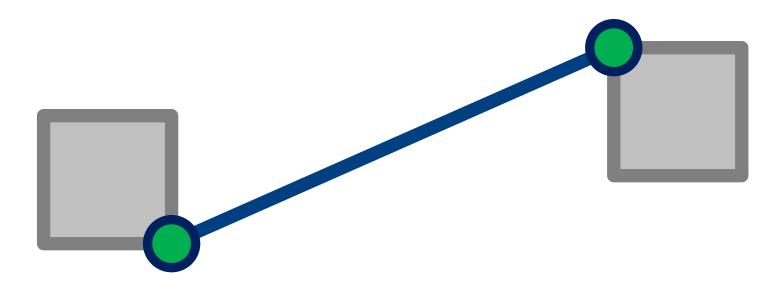
(where neither point is an endpoint of the edge)

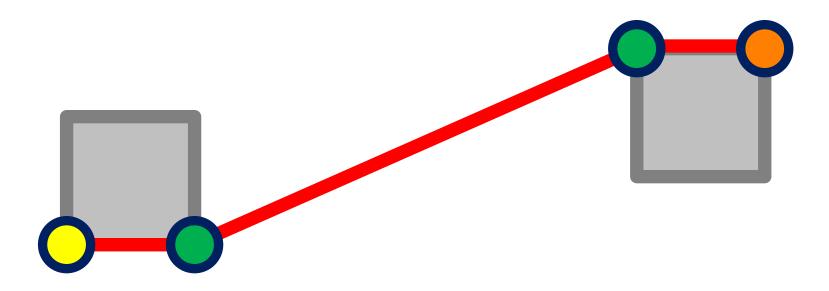
Theorem 2

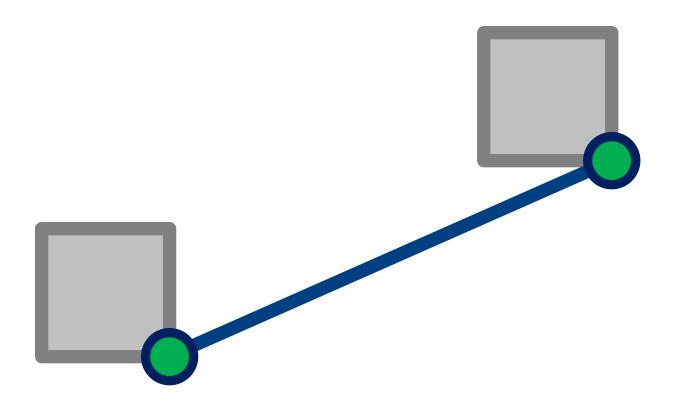
Every edge in the sparse visibility graph is part of some optimal path between some pair of points.

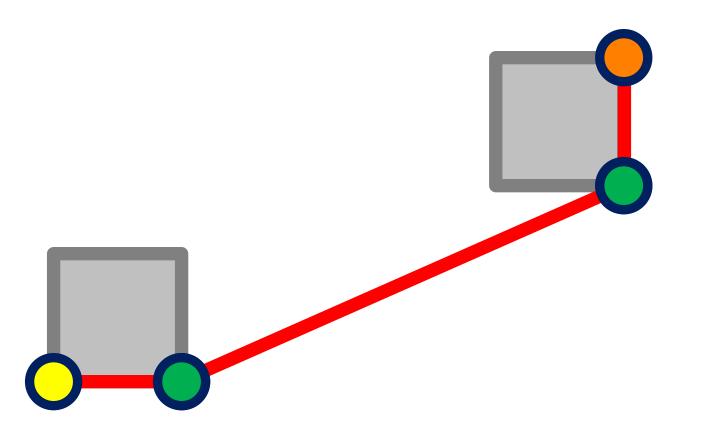
(where neither point is an endpoint of the edge)

i.e. every edge is "essential"

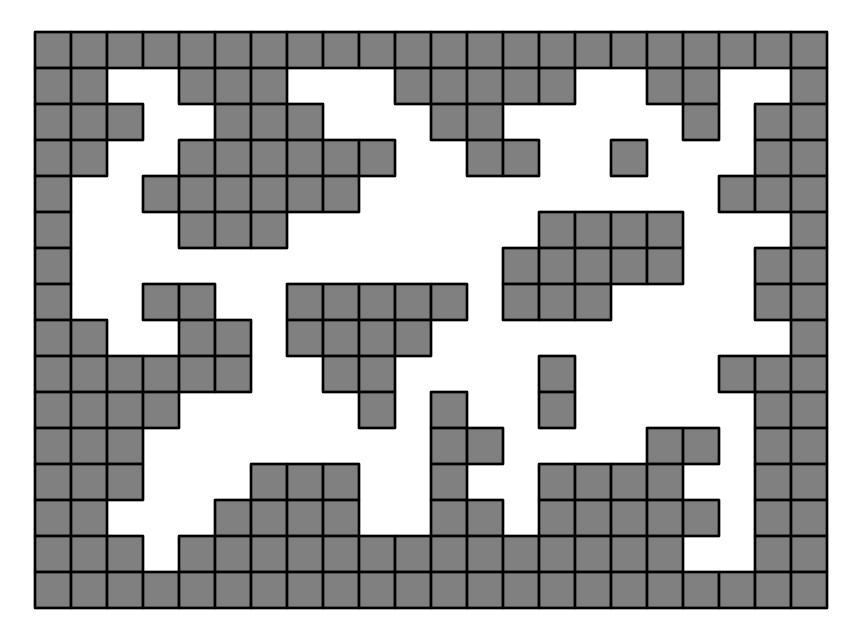


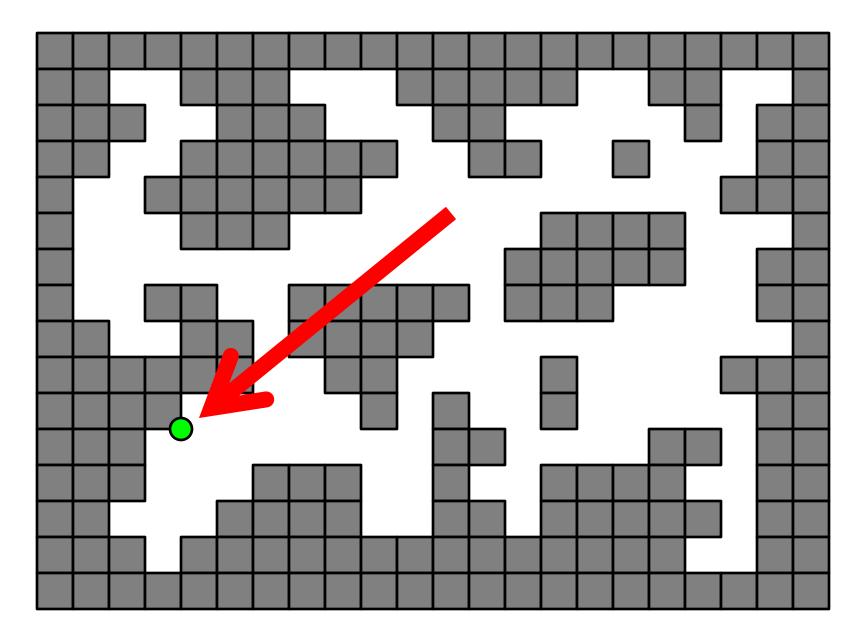


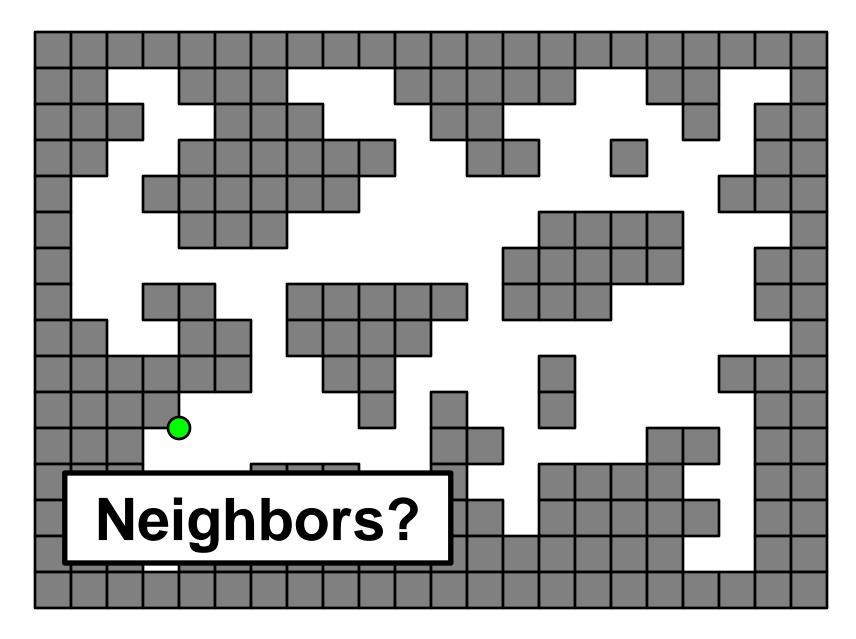


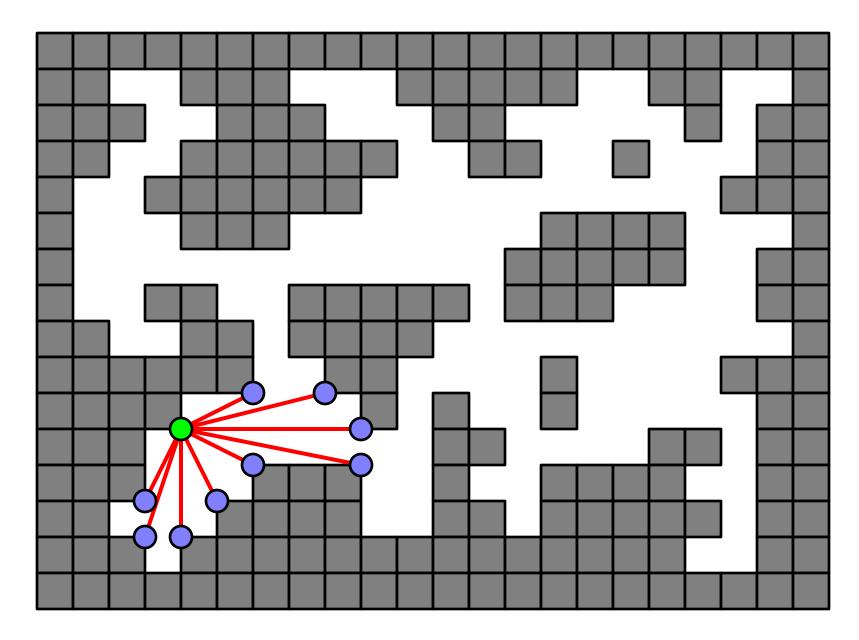


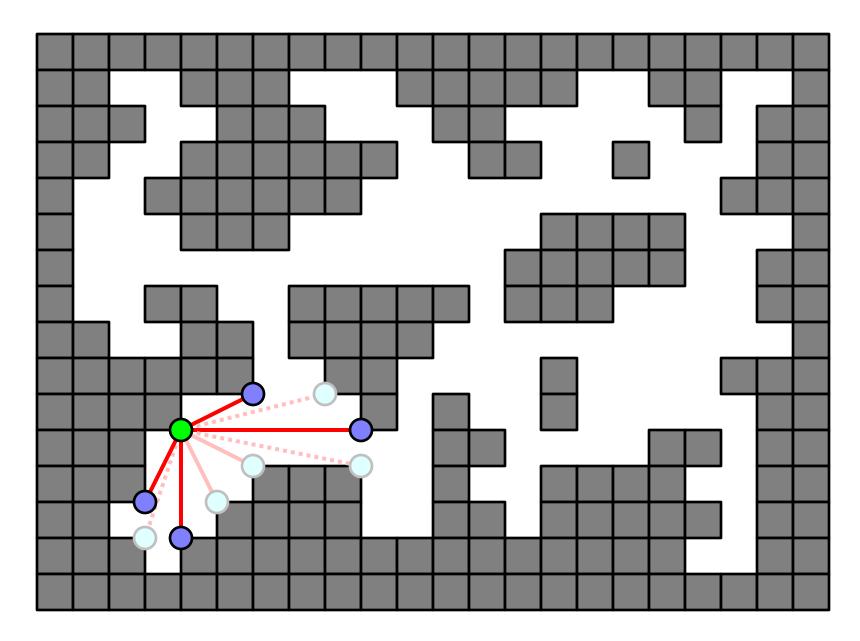
Finding Visible Neighbors with Line-of-Sight Scans

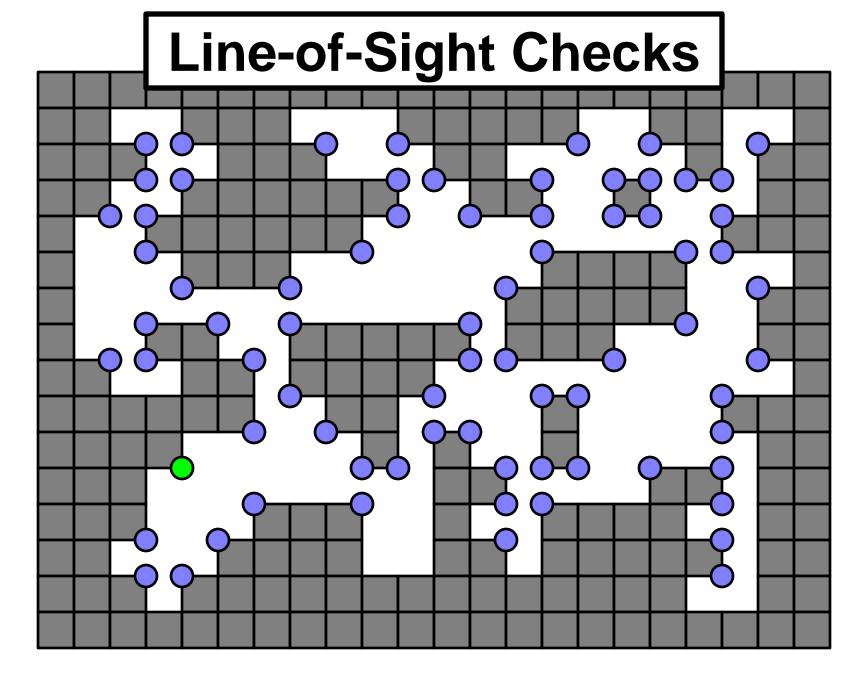


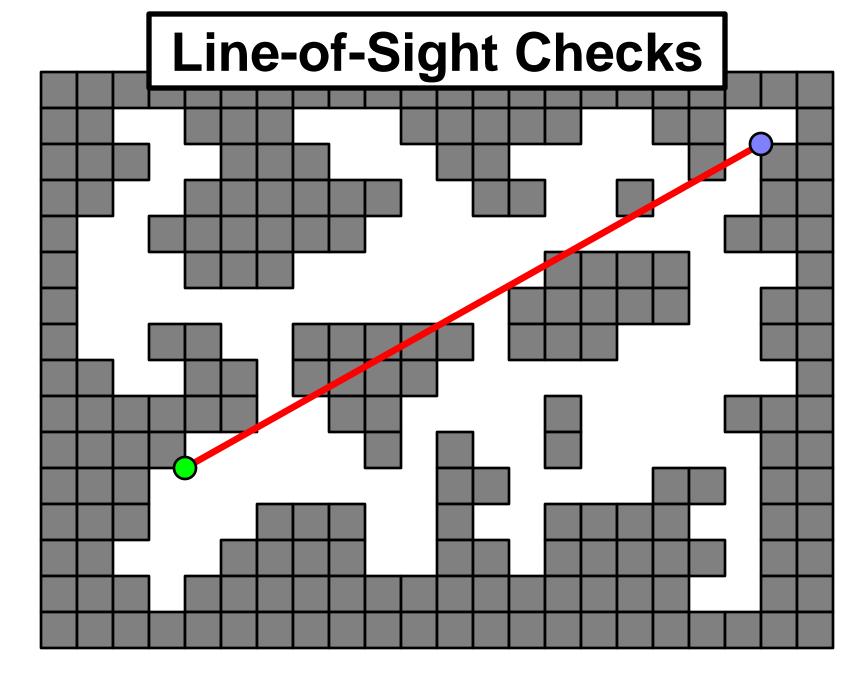




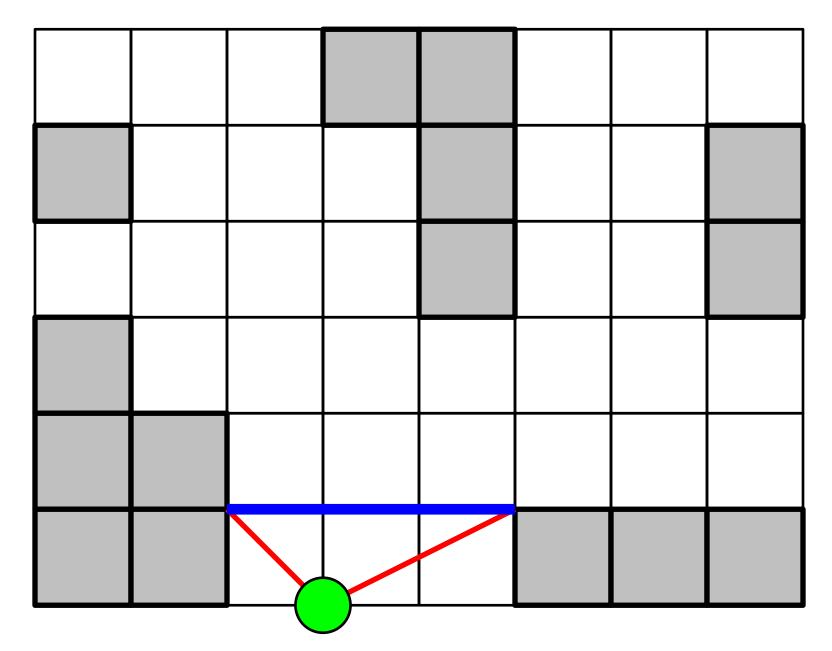


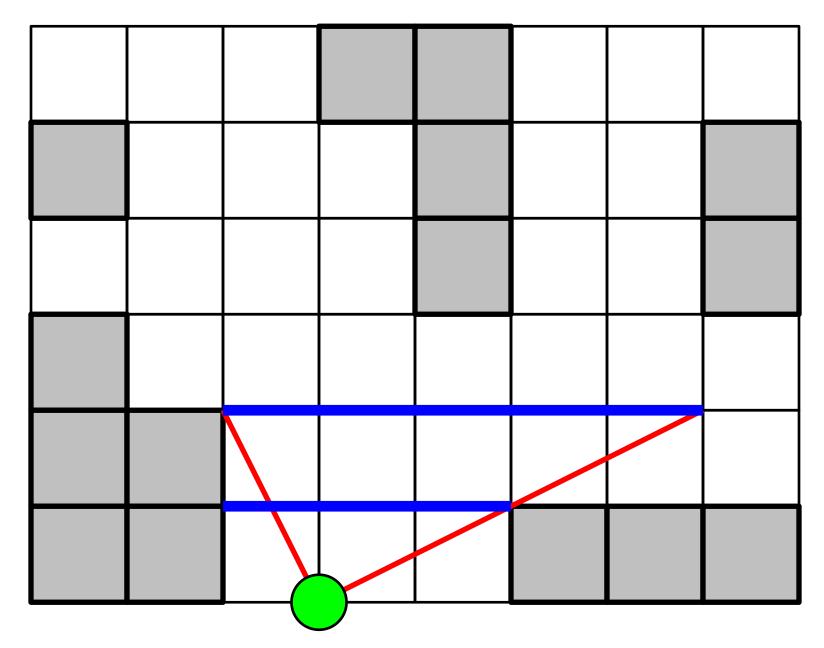


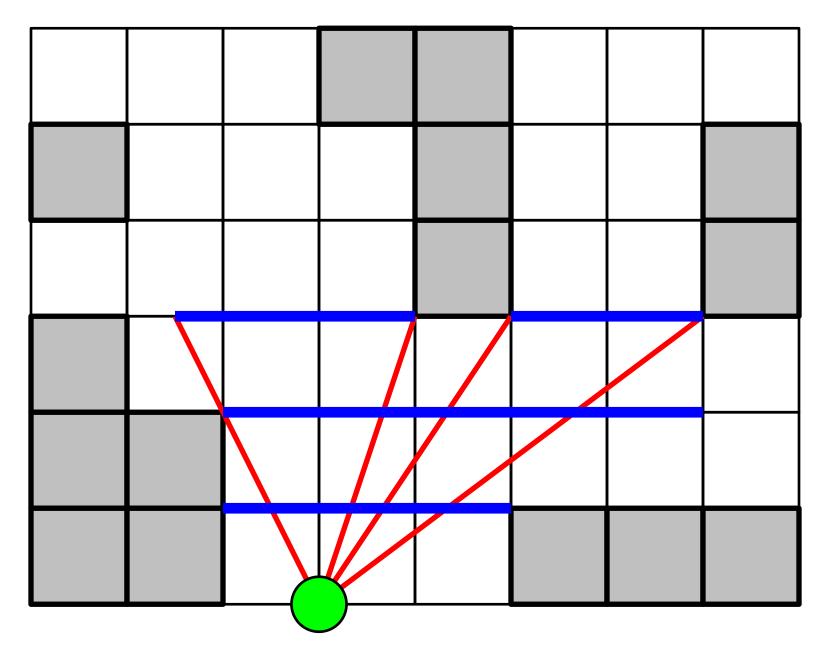


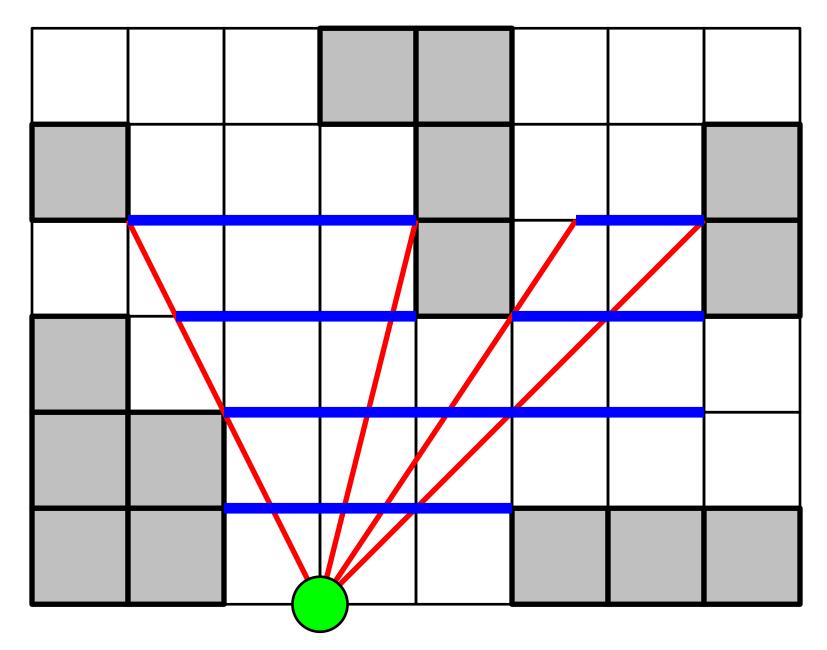


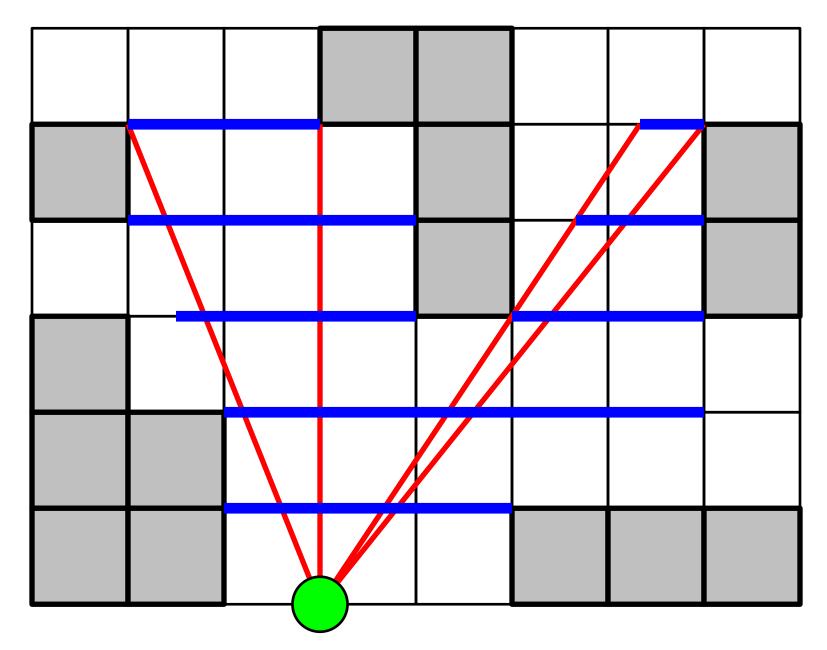
Line-of-Sight Scans Interval Search

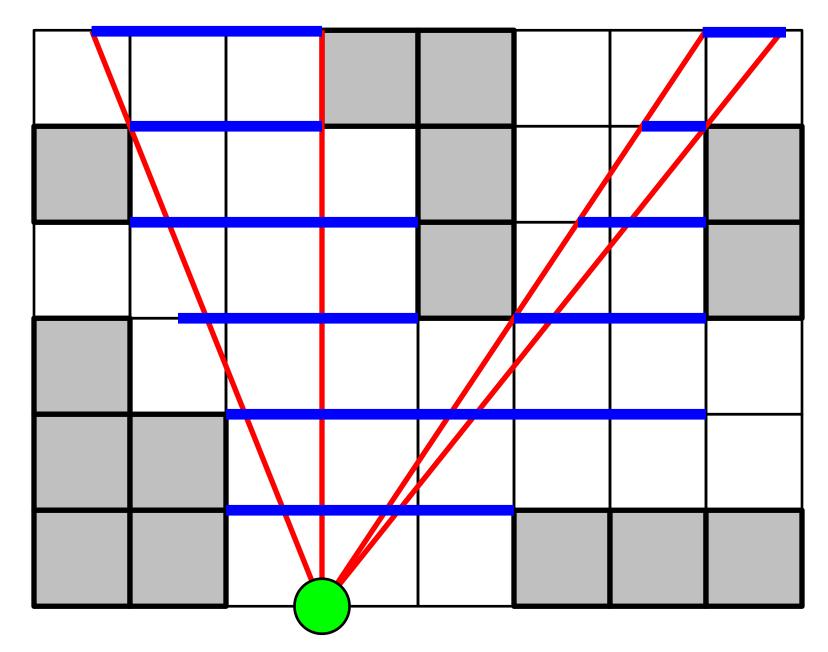


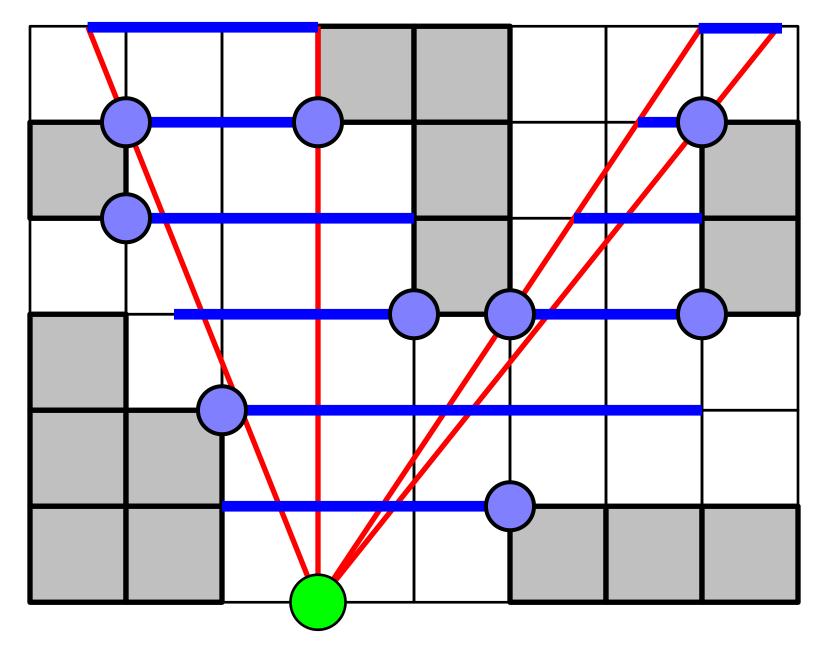


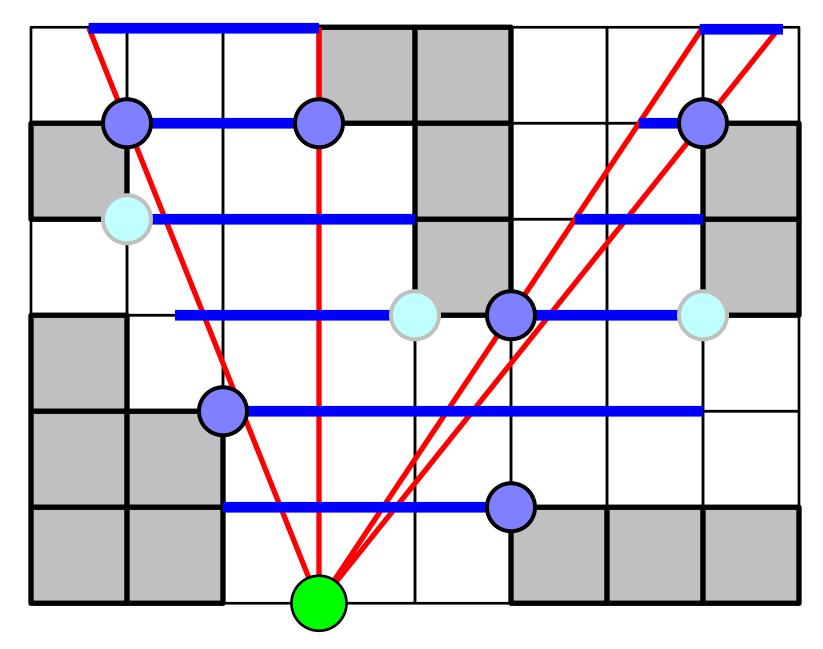










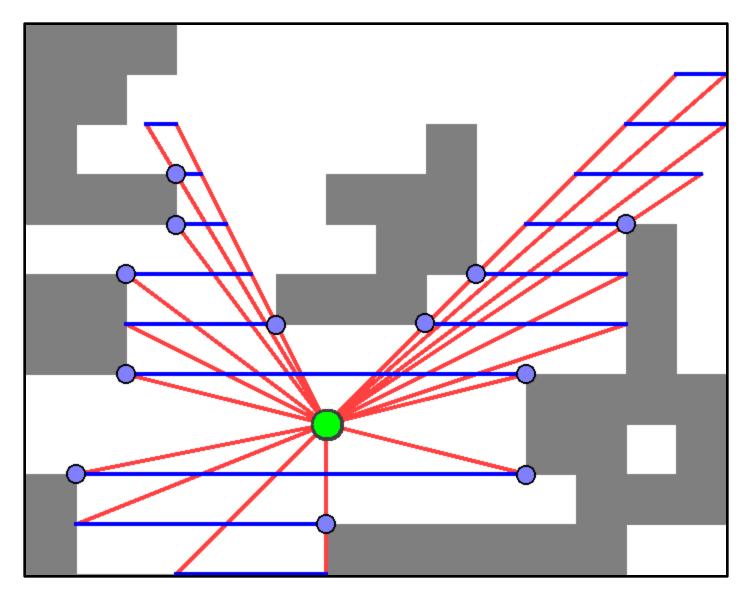


All-Direction Line-of-Sight Scans VS

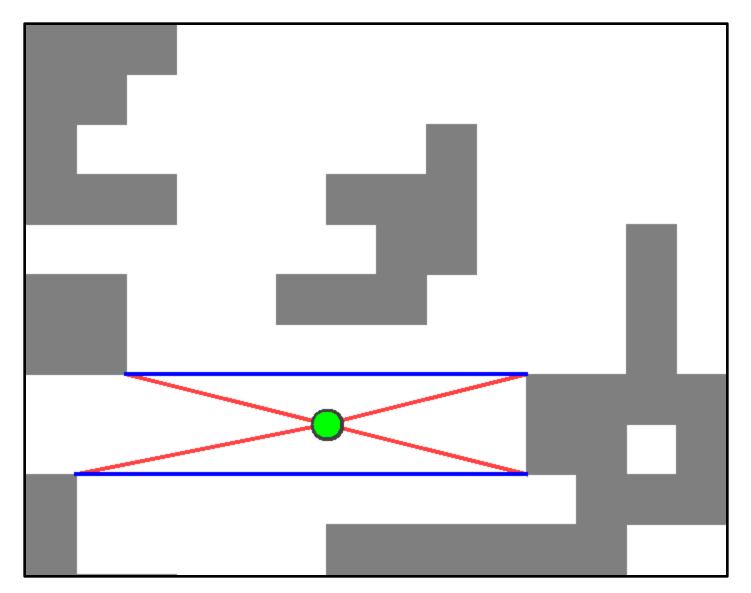
Taut-Direction Line-of-Sight Scans

All-Direction Line-of-Sight Scans

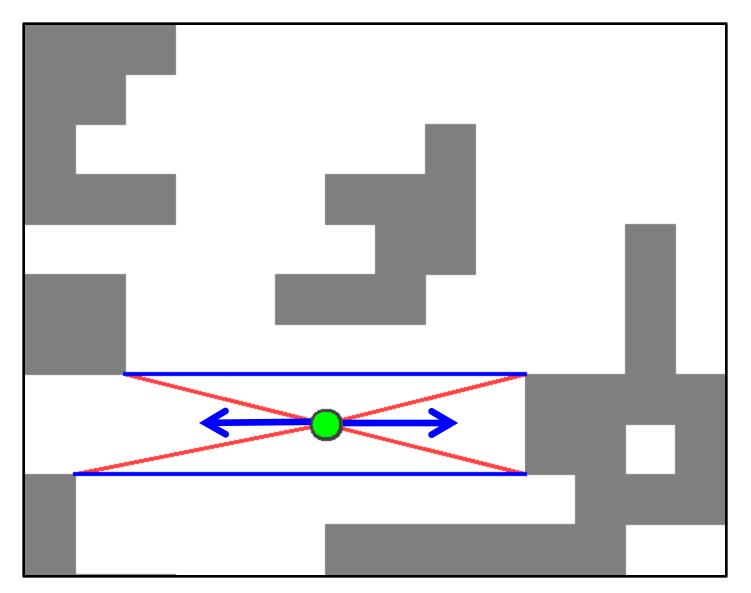
All-Direction Line-of-Sight Scans

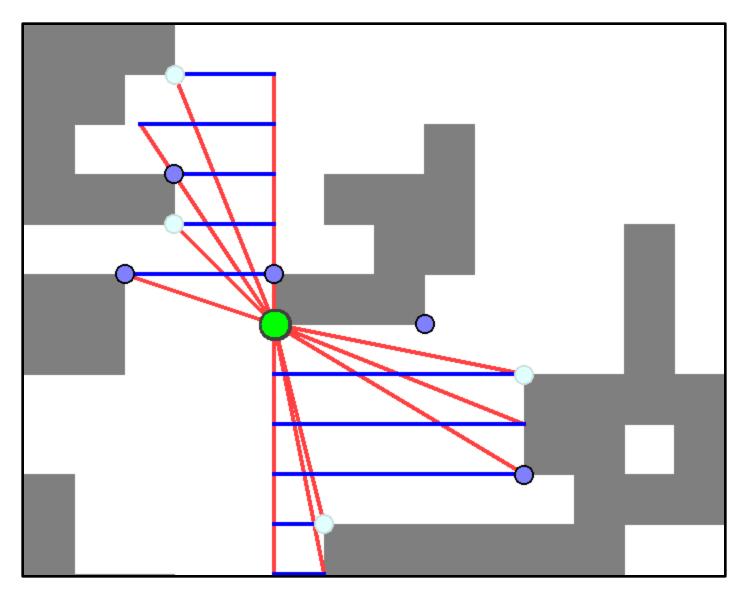


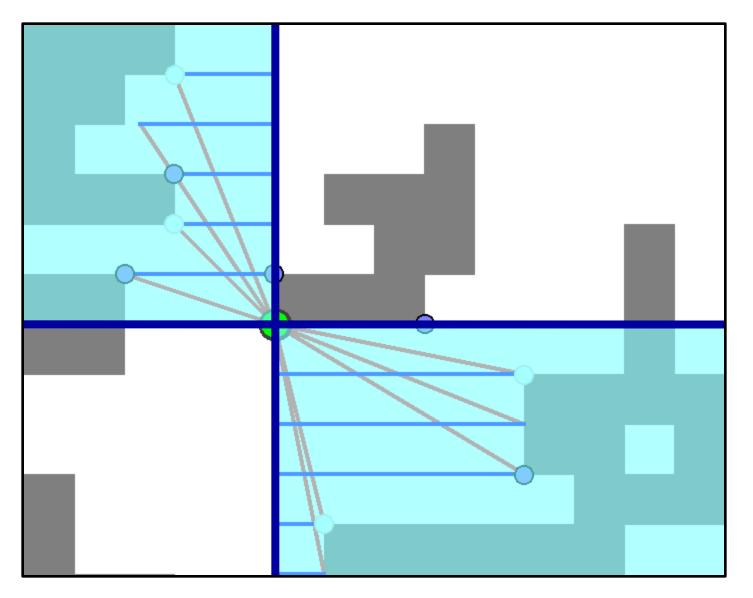
All-Direction Line-of-Sight Scans

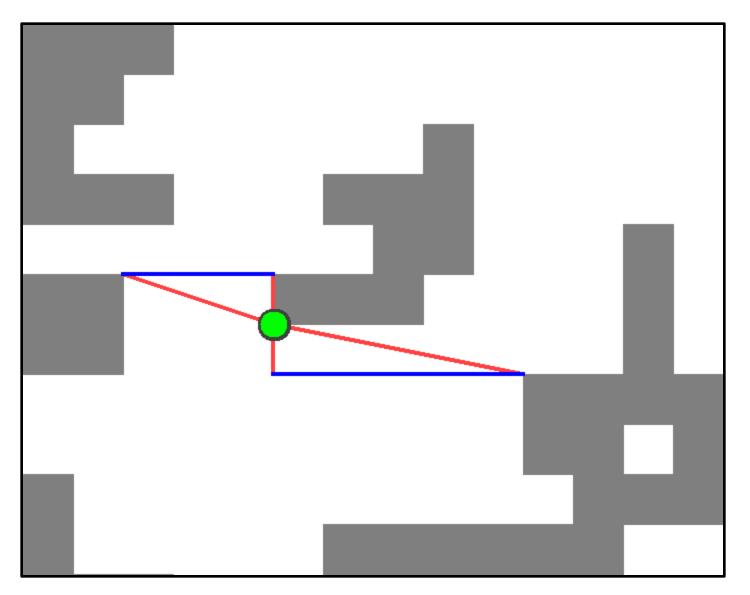


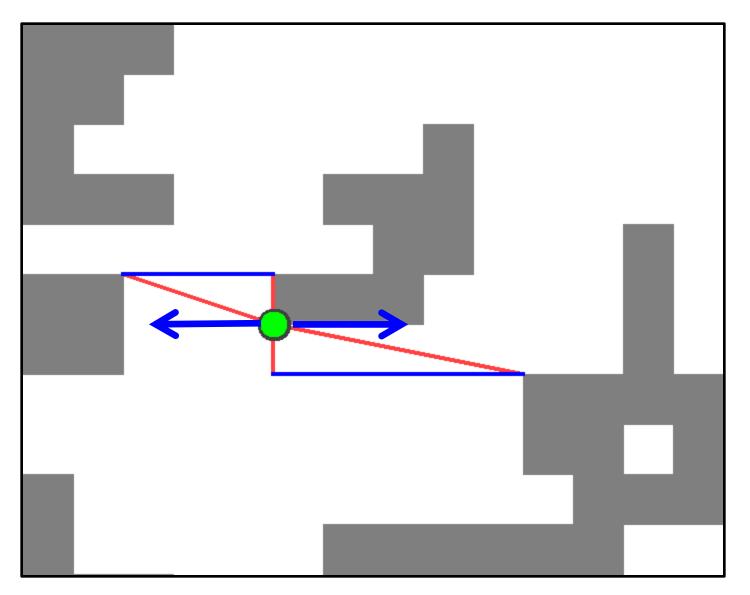
All-Direction Line-of-Sight Scans

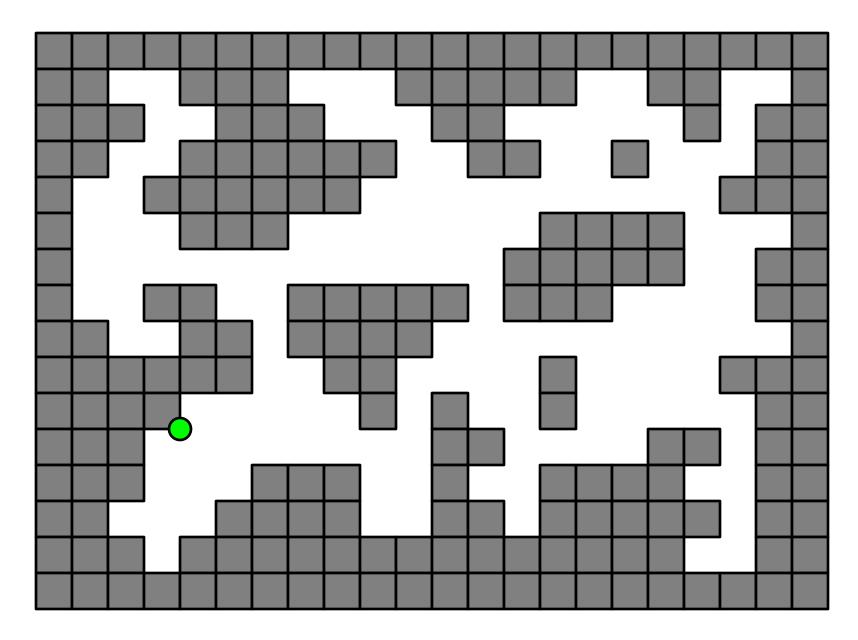


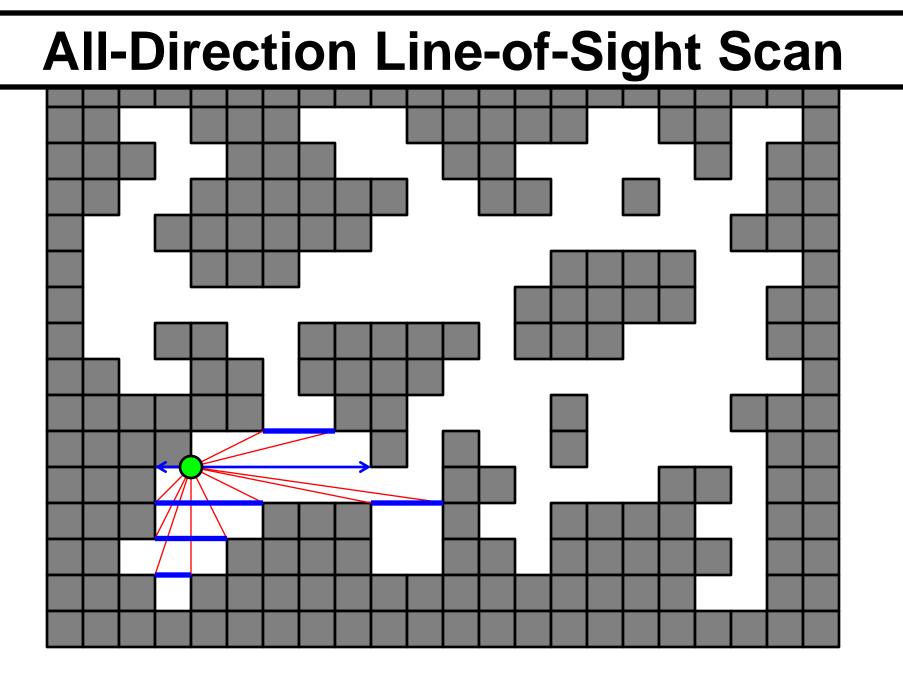


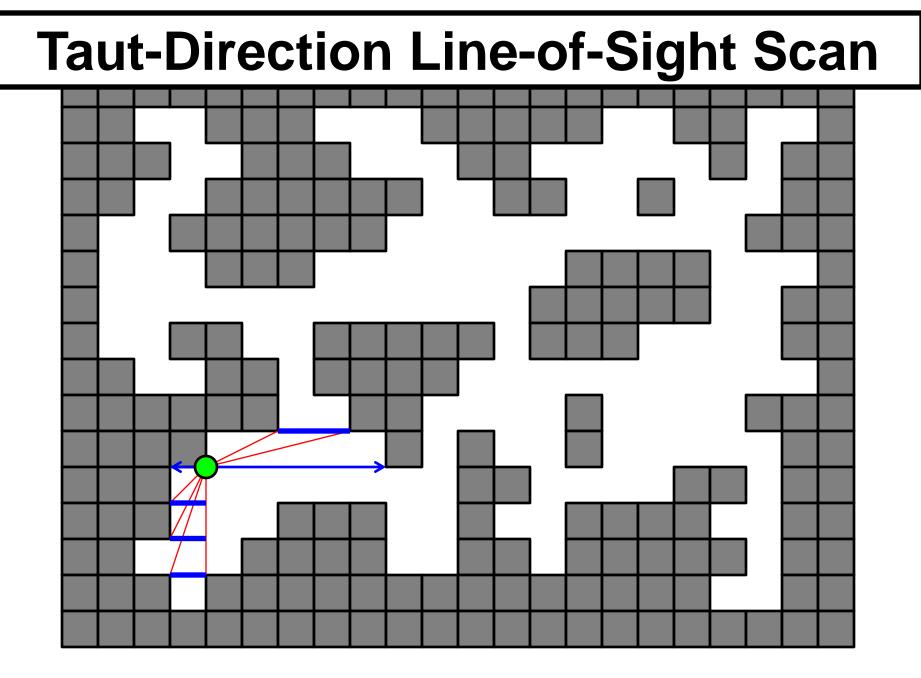


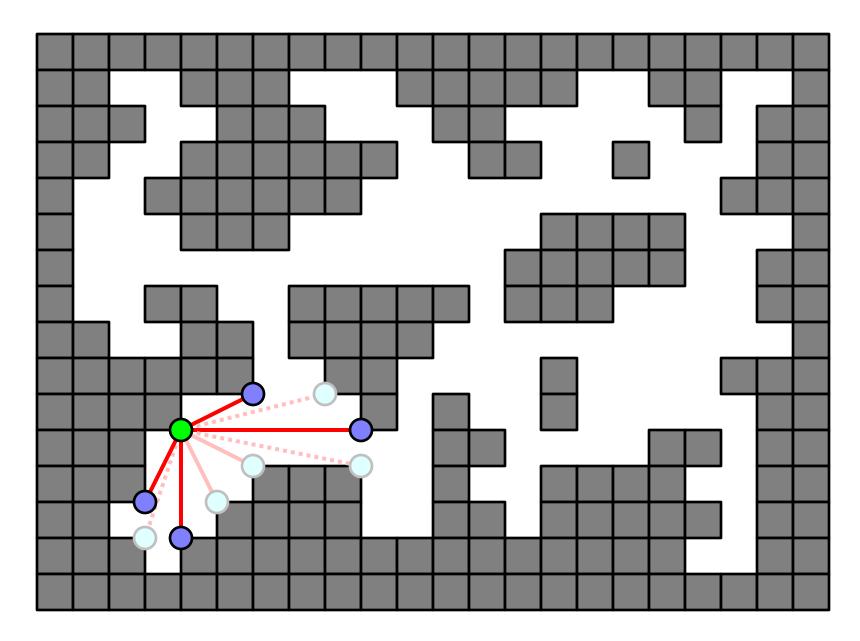




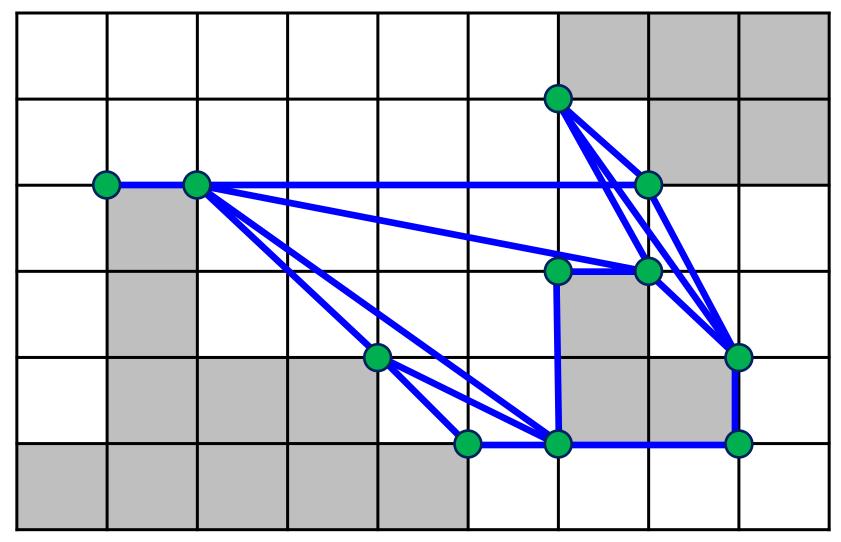




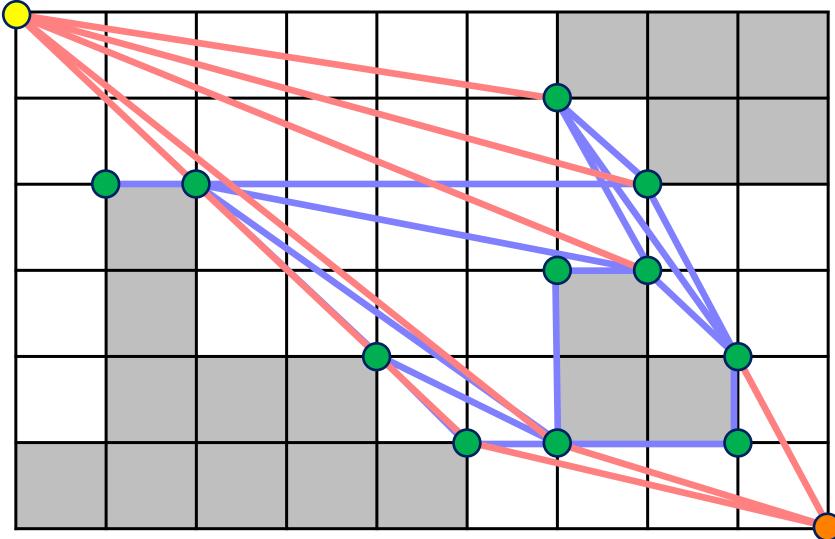




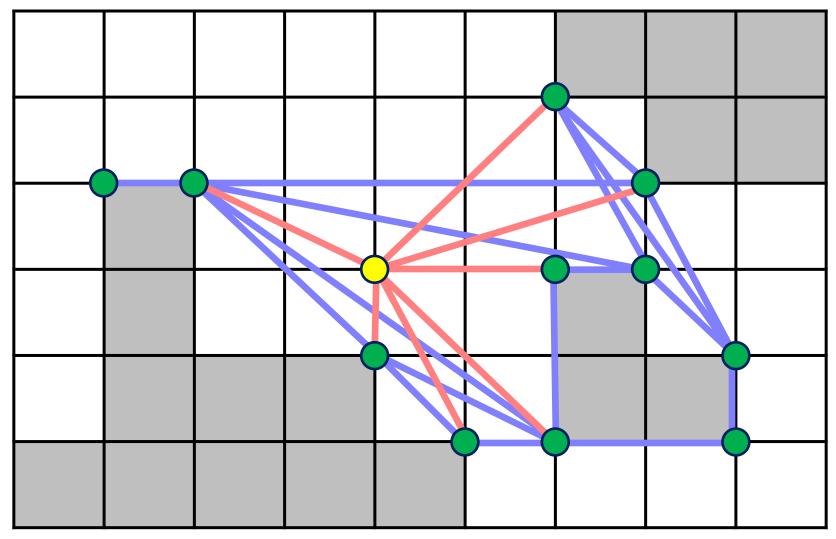
Sparse Visibility Graph Construction: Taut-Direction Line-of-Sight Scans



Inserting Start & Goal Points: All-Direction Line-of-Sight Scans



Inserting Start & Goal Points: All-Direction Line-of-Sight Scans



Comparison of Line-of-Sight Algorithms

Comparison of Line-of-Sight Algorithms (Using Construction Time)

Comparison of Line-of-Sight Algorithms

Visibility Graph

- VG_C Line-of-Sight Checks (All-Pairs)
- VG_RPS Rotational Plane Sweep Algorithm
- VG_S Line-of-Sight Scans

Comparison of Line-of-Sight Algorithms

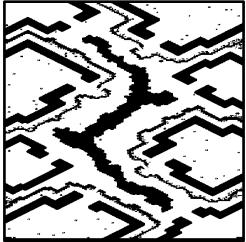
Visibility Graph

- **VG_C** Line-of-Sight Checks (All Pairs)
- VG_RPS Rotational Plane Sweep Algorithm
- VG_S Line-of-Sight Scans
- **Sparse Visibility Graph**
 - SVG Sparse Visibility Graph (Line-of-Sight Scans)

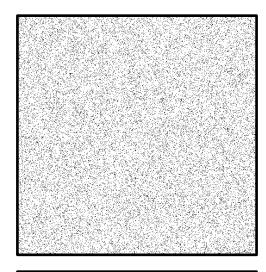
Benchmark Maps

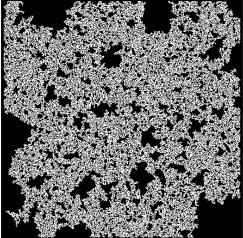
Game Maps





Random Maps

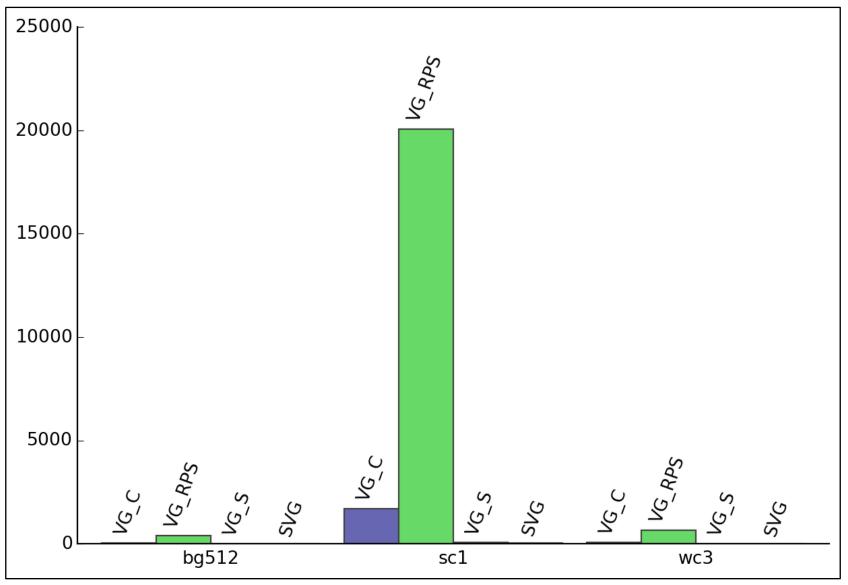




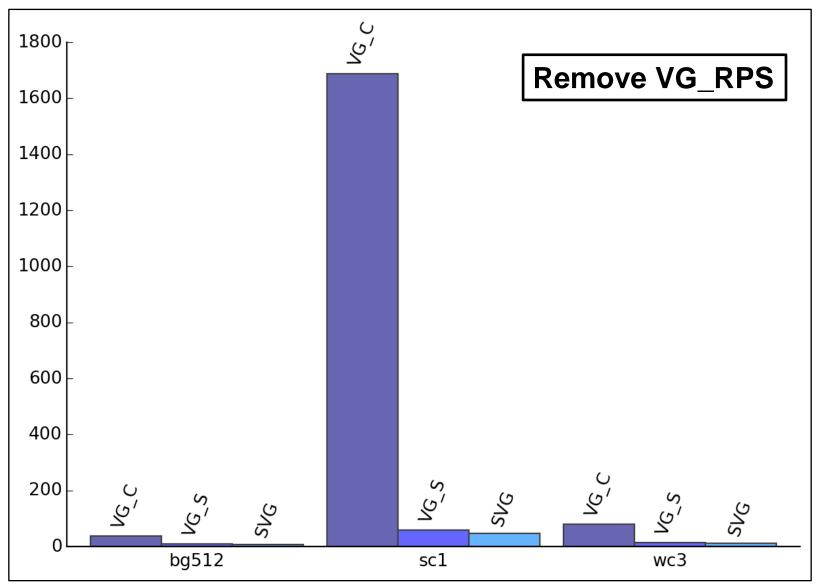
Construction Time

Benchmarks (Game) Construction Time

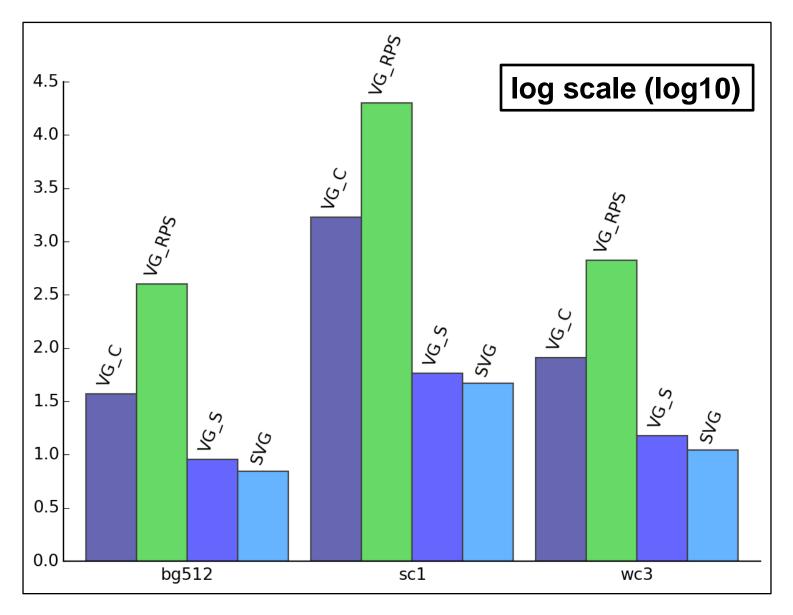
Construction Time (ms)



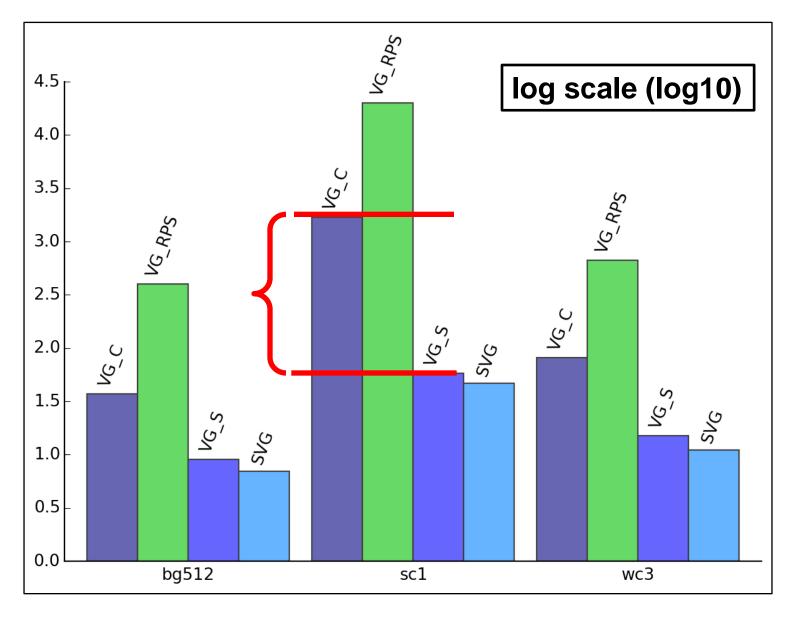
Construction Time (ms)



Construction Time (log10, ms)



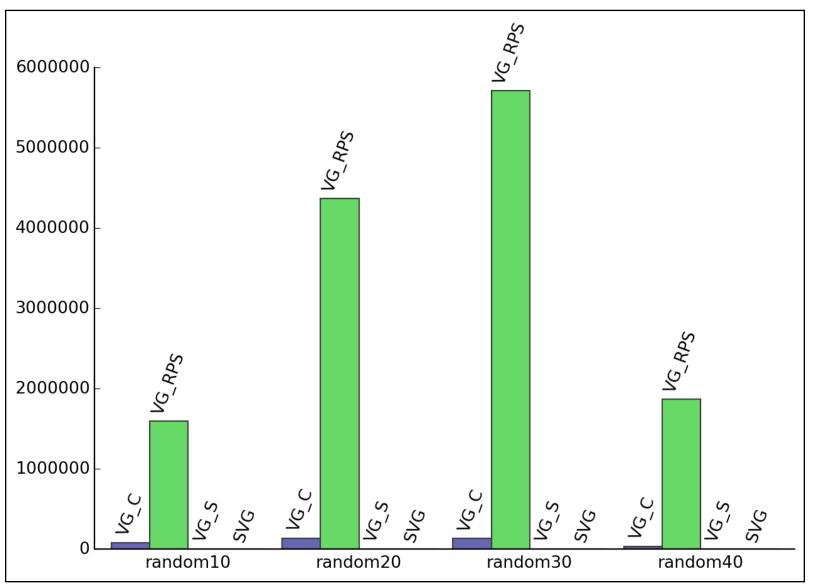
Construction Time (log10, ms)



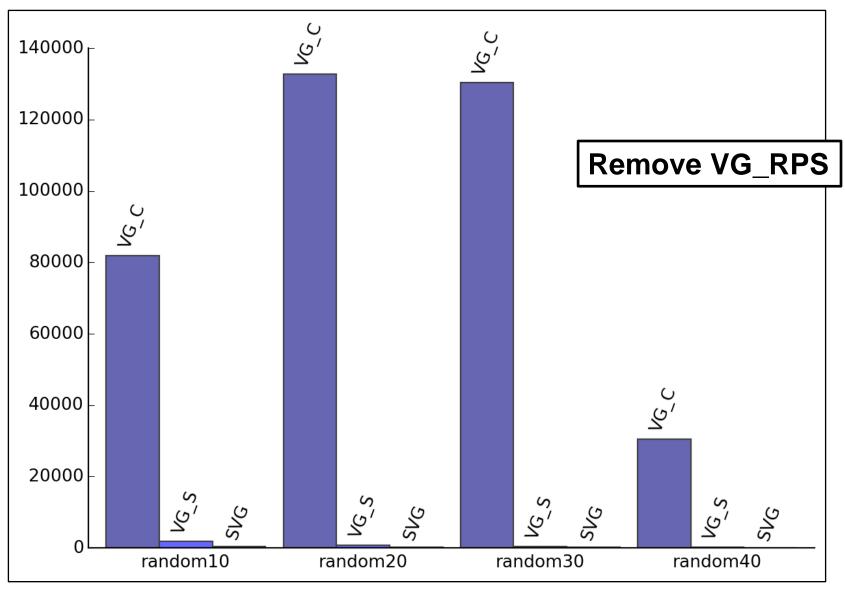
Construction Time

Benchmarks (Random) Construction Time

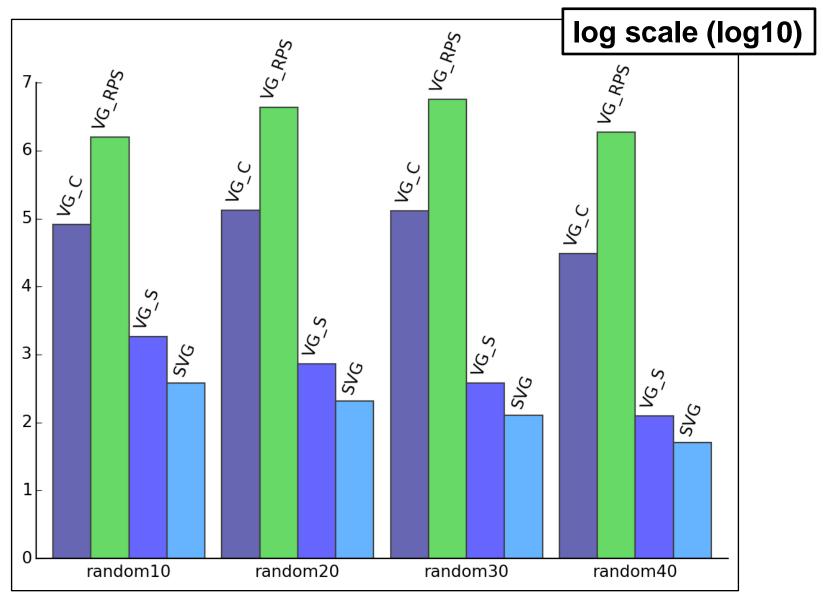
Construction Time (ms)



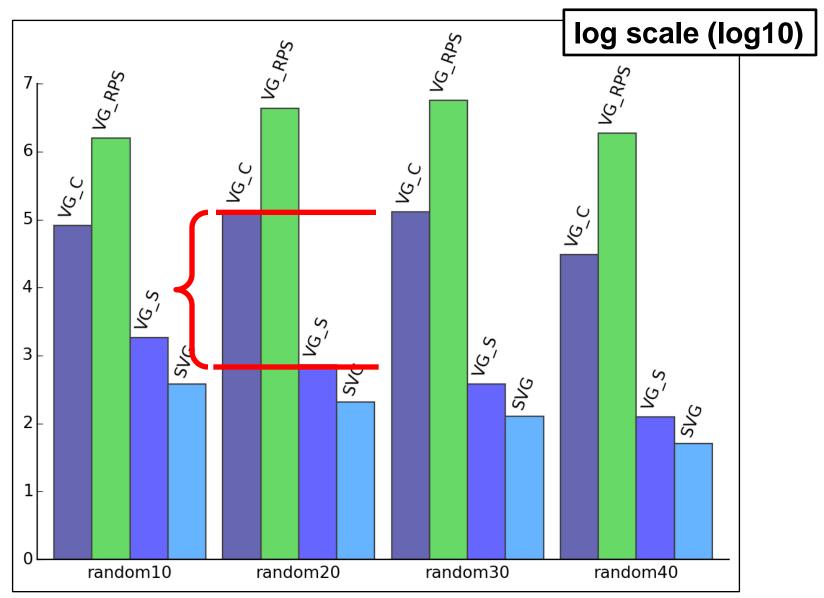
Construction Time (ms)



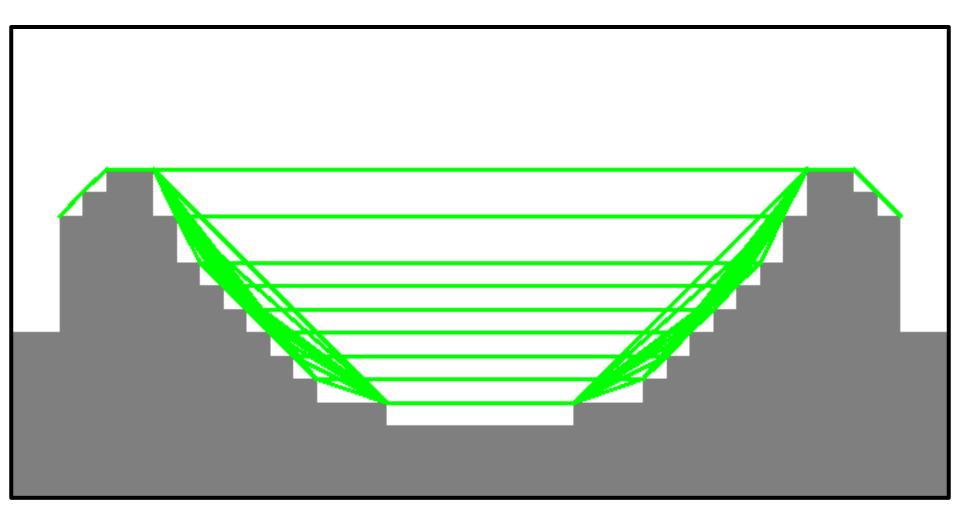
Construction Time (log10, ms)

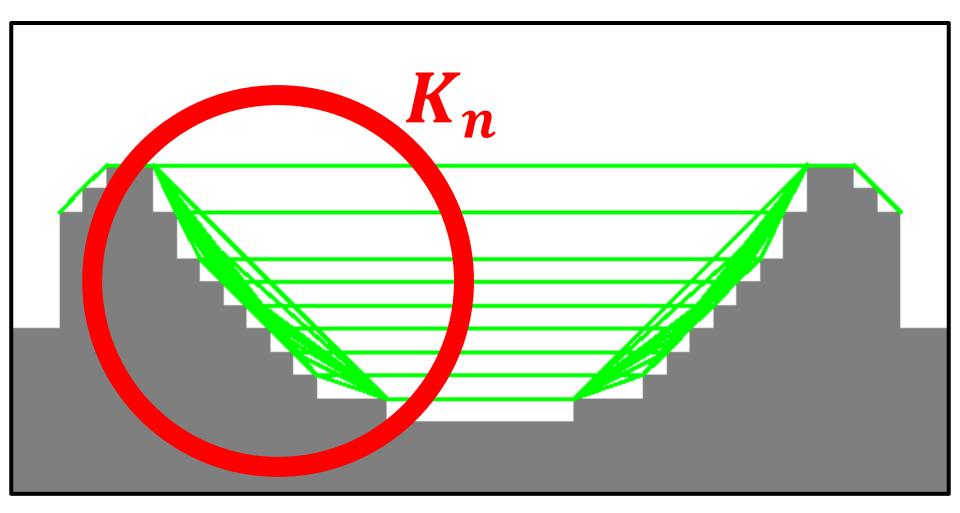


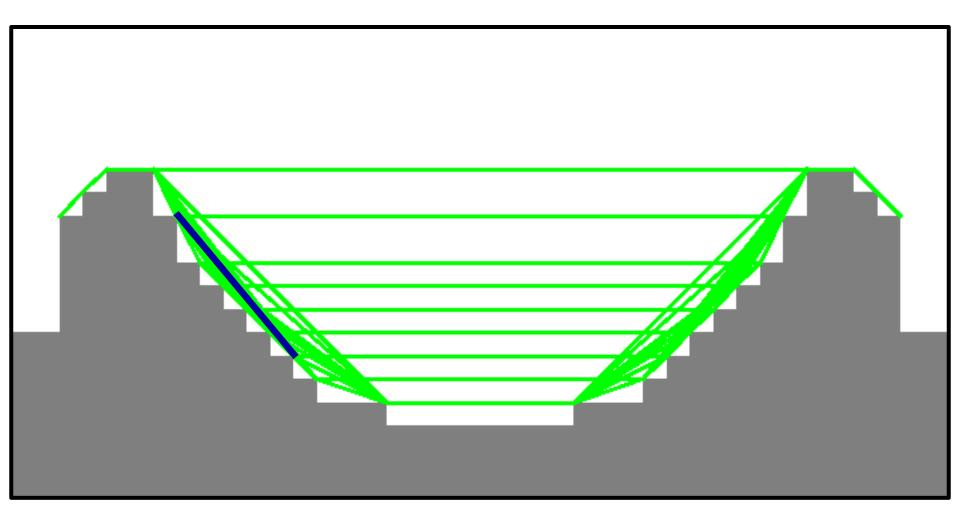
Construction Time (log10, ms)

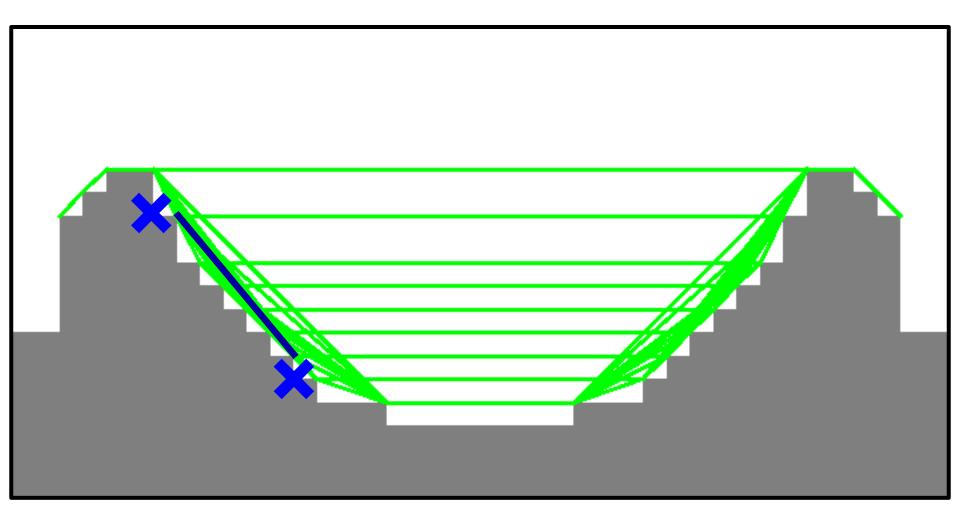


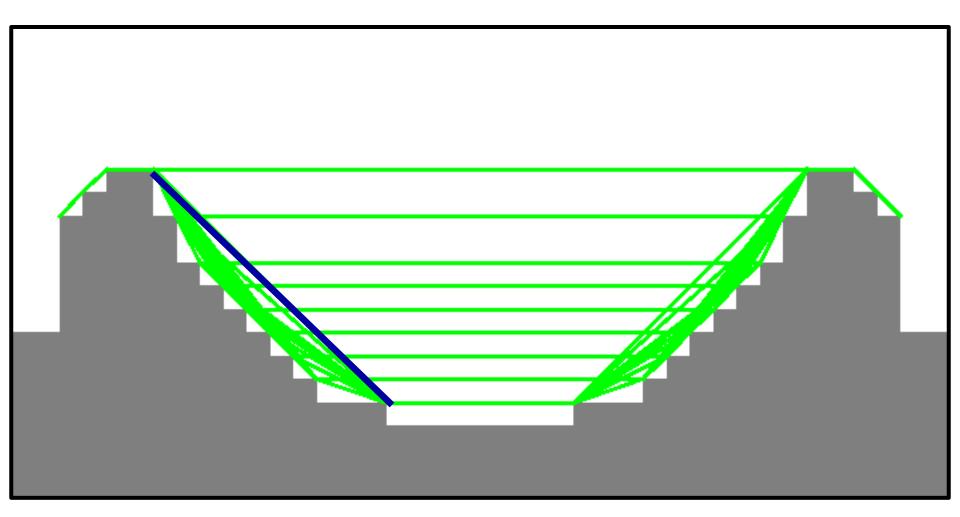
Edge N-Level Sparse Visibility Graphs



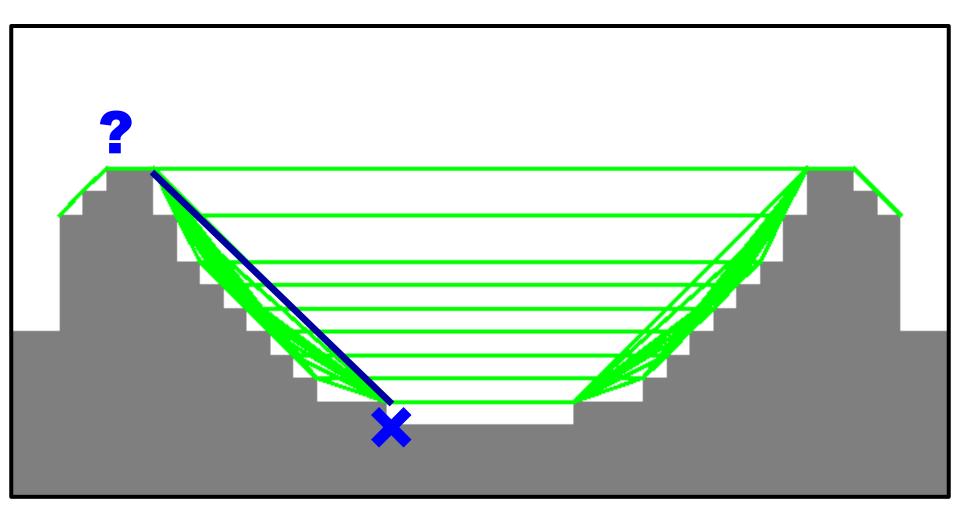








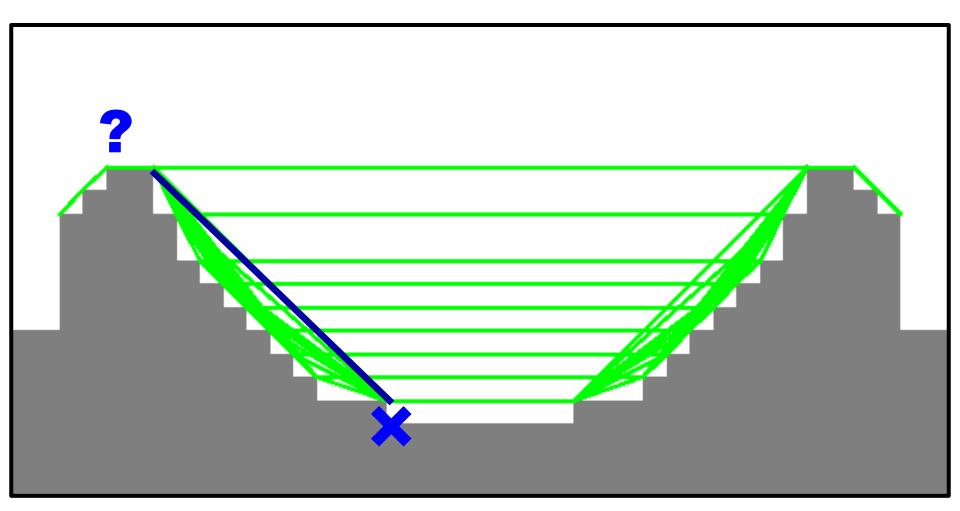
Sparse Visibility Graph



Idea:

Some edges are only used as a "Second hop from the start" or a "Second hop from the goal"

Sparse Visibility Graph



Thus:

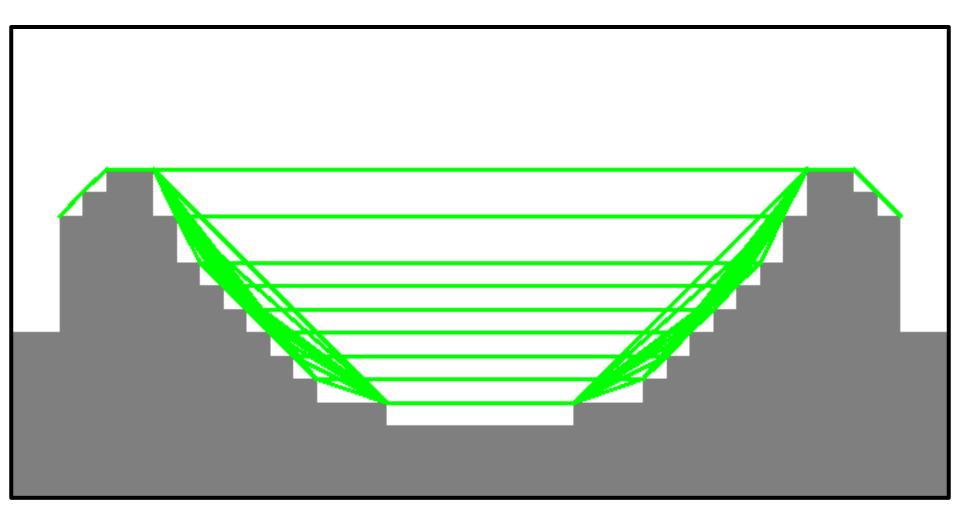
Prune away all edges that do not lead to another edge via a taut path

Thus:

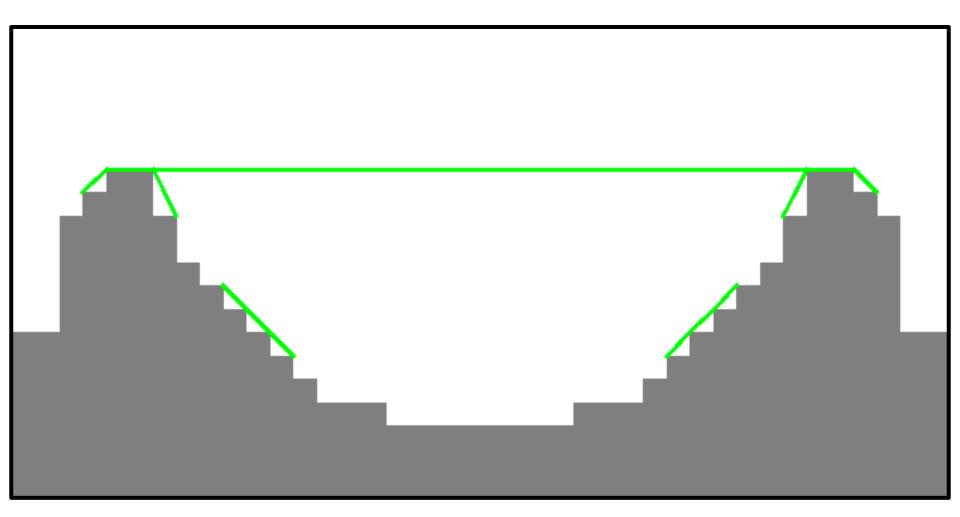
Prune away all edges that do not lead to another edge via a taut path

Mark them as "Level 1"

Sparse Visibility Graph



Sparse VG Algorithm



Then:

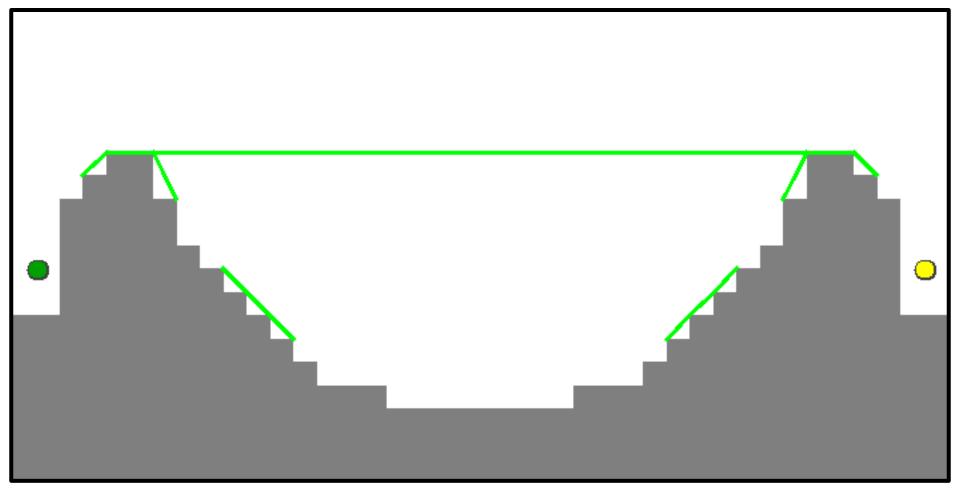
Do the same thing again. These edges are at most the third hop from the start or goal.

Then:

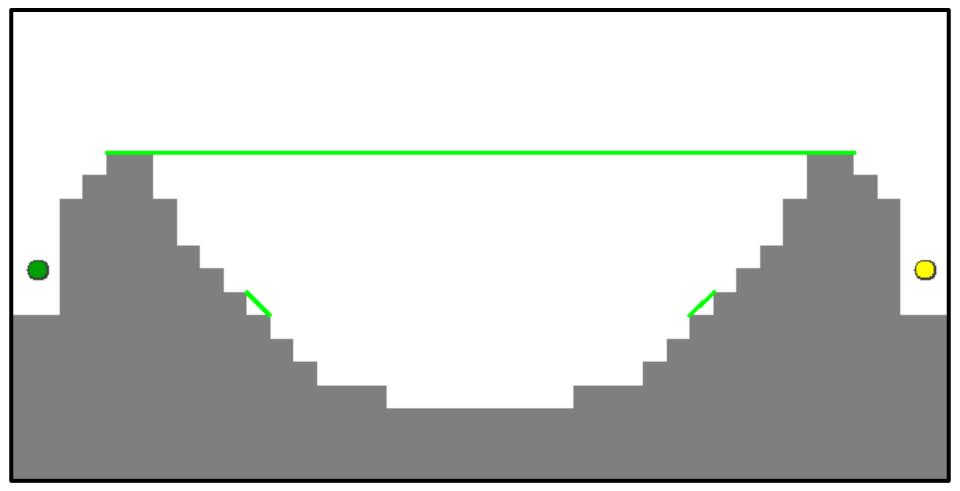
Do the same thing again. These edges are at most the third hop from the start or goal.

Mark them as "Level 2"

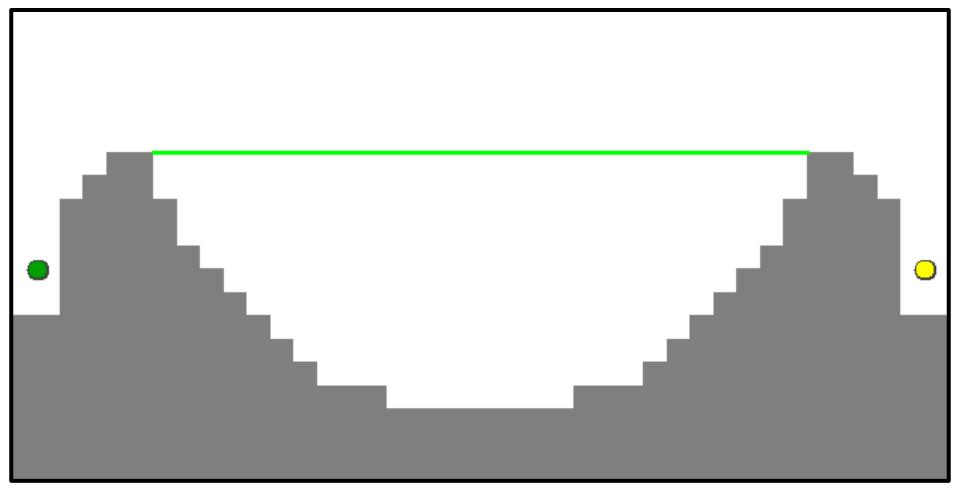
Level 1 edges removed



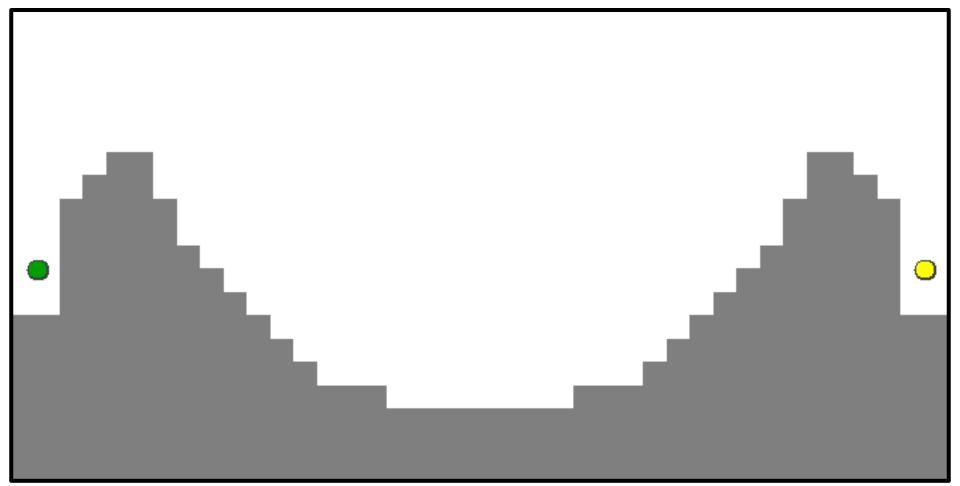
Level 2 edges removed



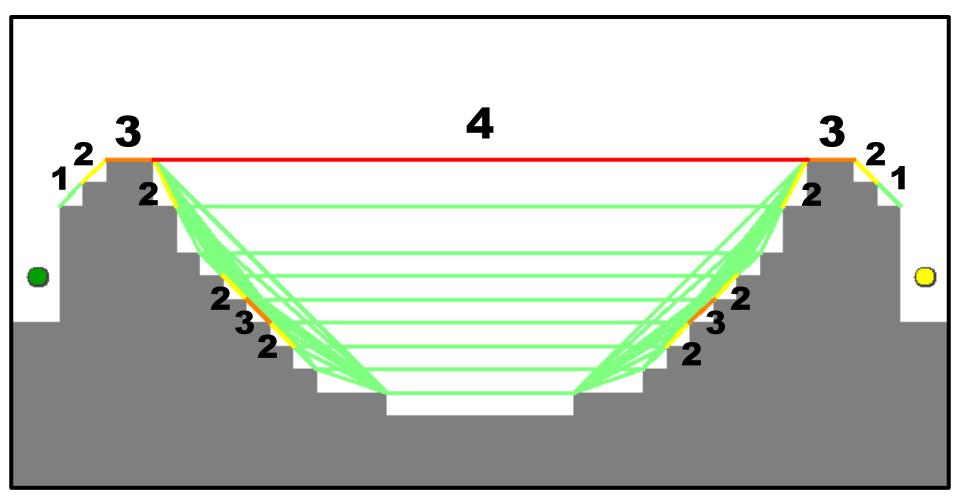
Level 3 edges removed



Level 4 edges removed



Edge N-Level SVG



Assuming all edges have finite level,

Assuming all edges have finite level,

Any taut path between the start and goal will be of the form:

Assuming all edges have finite level,

Any taut path between the start and goal will be of the form:

$$e_1e_2\cdots e_ke'_{k+1}\cdots e'_{n-1}e'_n$$

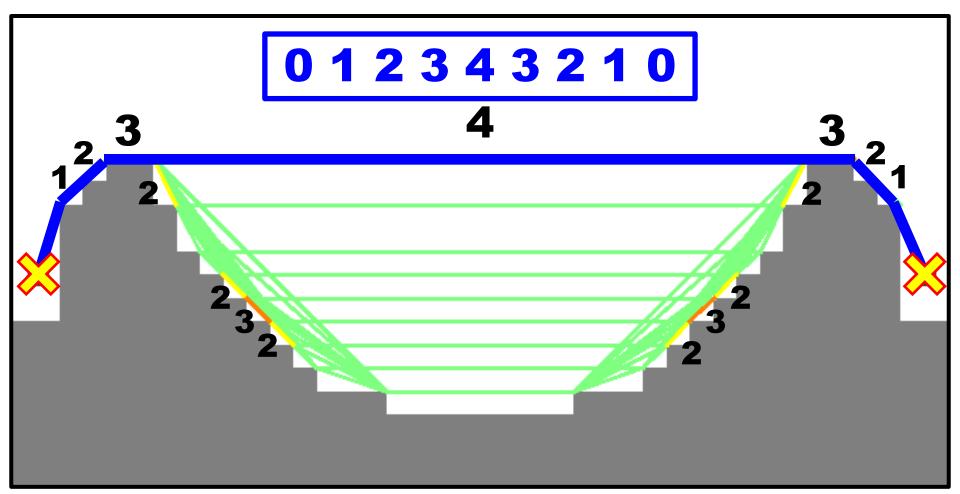
Assuming all edges have finite level,

Any taut path between the start and goal will be of the form:

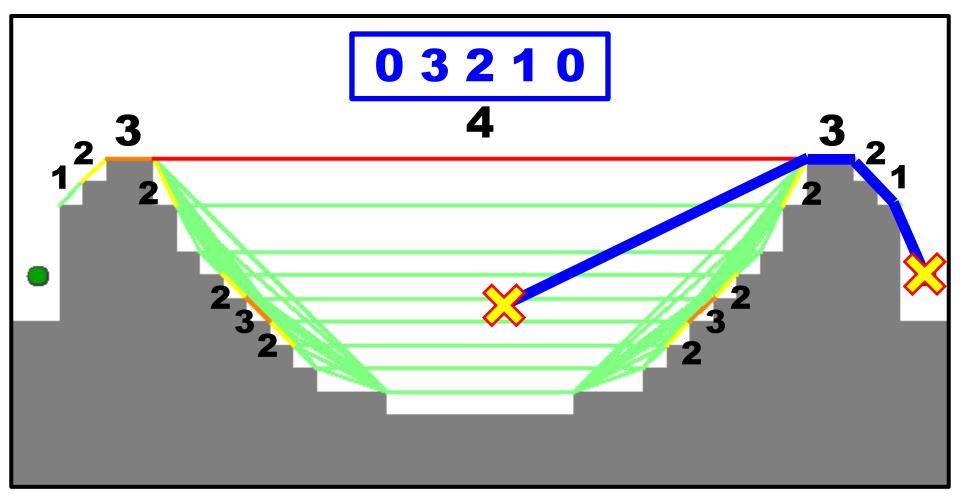
$$e_1e_2\cdots e_ke'_{k+1}\cdots e'_{n-1}e'_n$$

where $e_1e_2 \cdots e_k$ have strictly increasing levels, and $e'_{k+1} \cdots e'_n$ have strictly decreasing levels

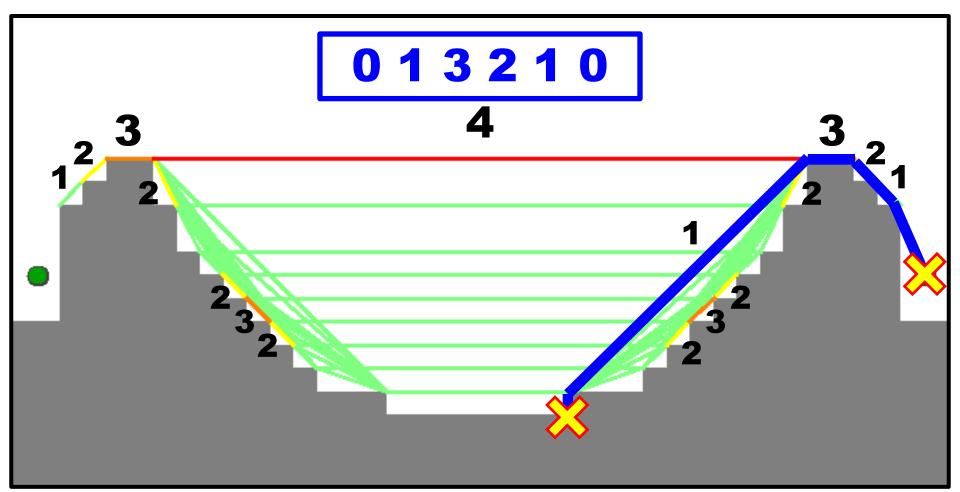
Edge N-Level Visibility Graph



Edge N-Level Visibility Graph



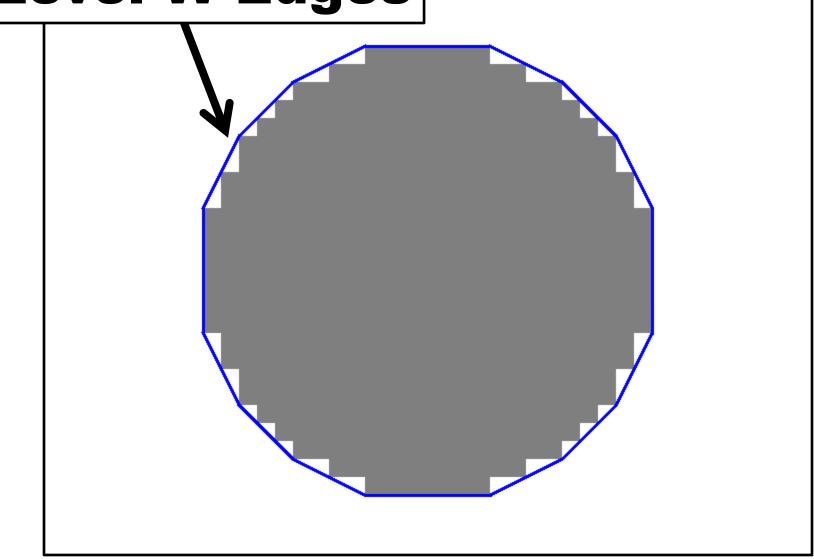
Edge N-Level Visibility Graph



Infinite-level edges?

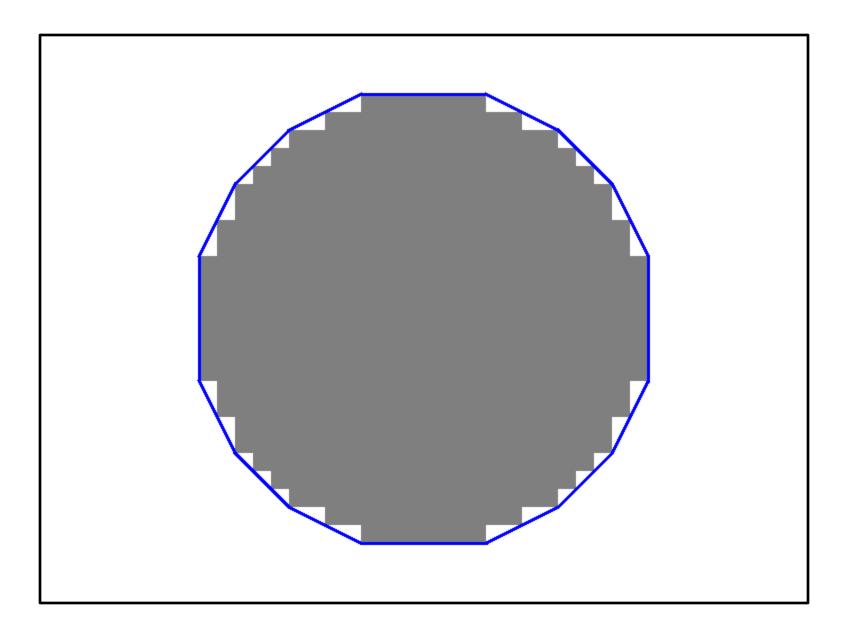
Not all edges can be pruned this way. The remaining edges are called Level-W edges

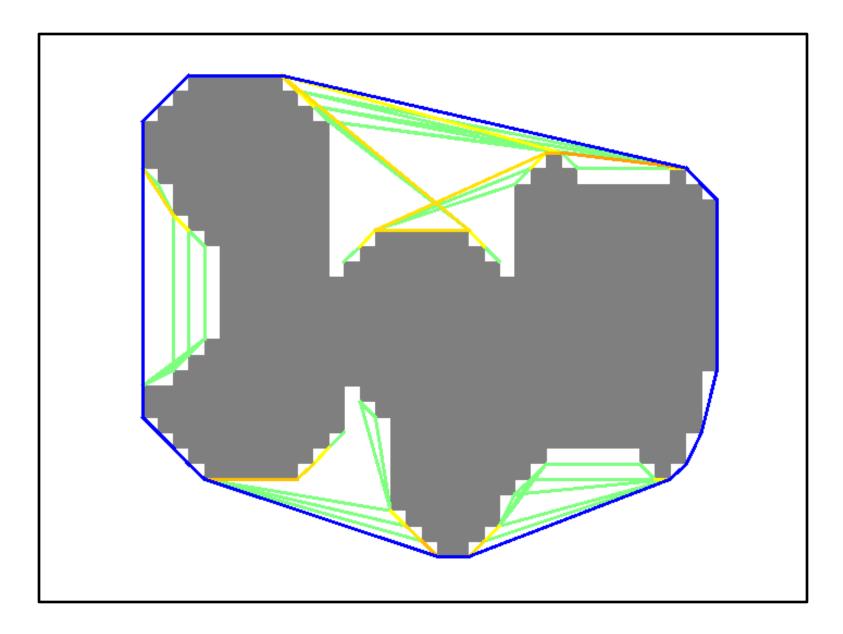
Level-W Edges

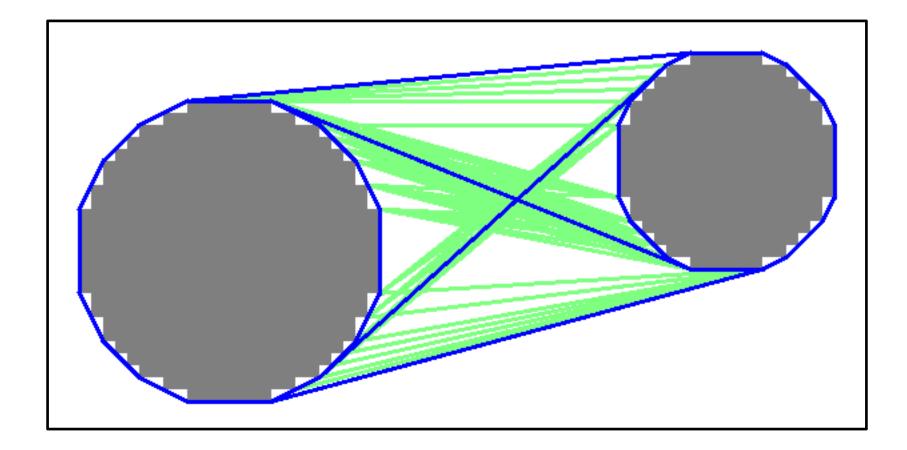


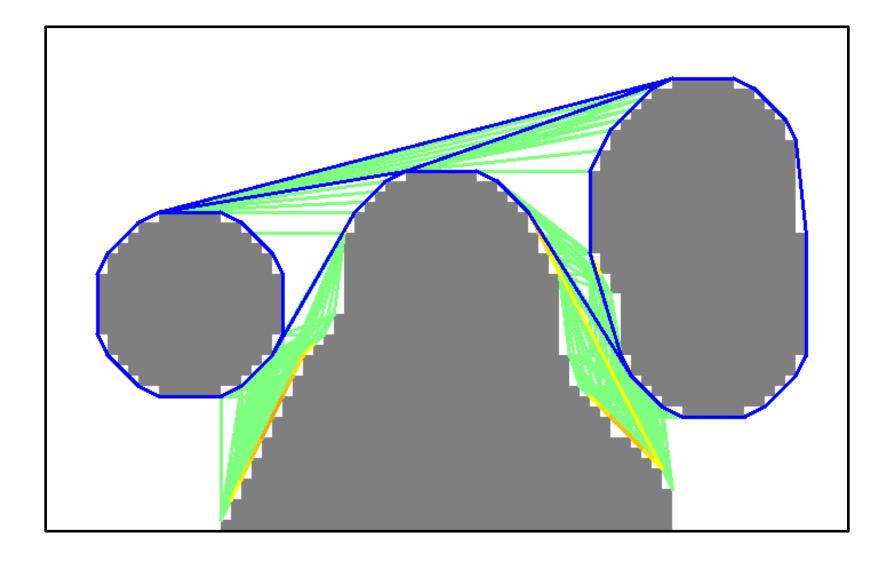
Note:

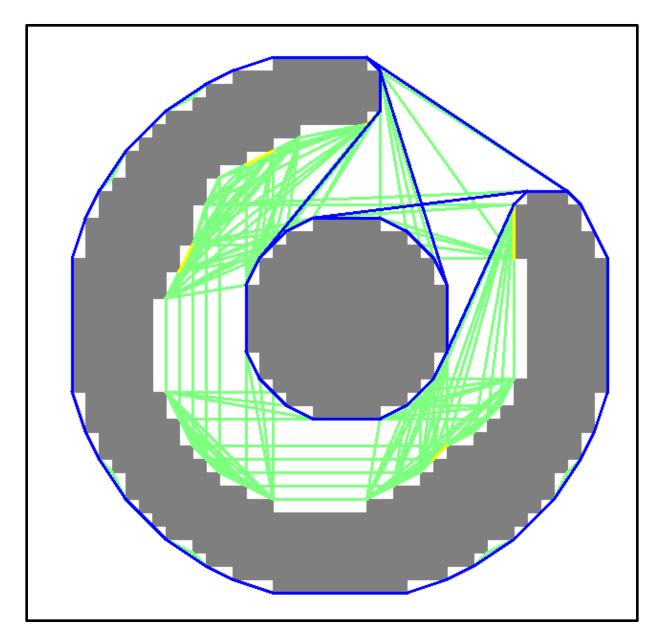
An edge is Level-W if and only if It is part of some taut cycle











Any taut path between the start and goal will be of the form:

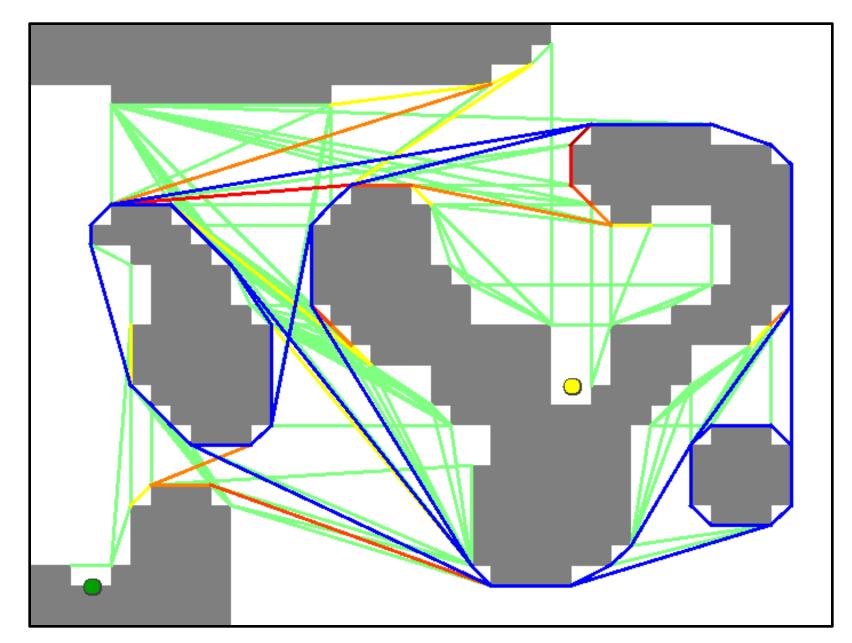
Any taut path between the start and goal will be of the form:

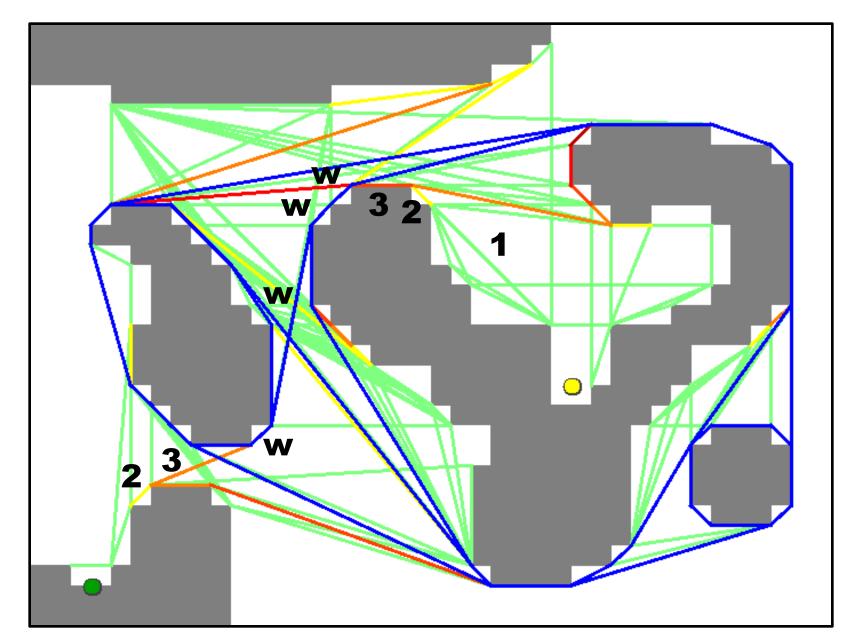
$$e_1 e_2 \cdots e_{k_1} w_{k_1+1} \cdots w_{k_2} e'_{k_2+1} \cdots e'_{n-1} e'_n$$

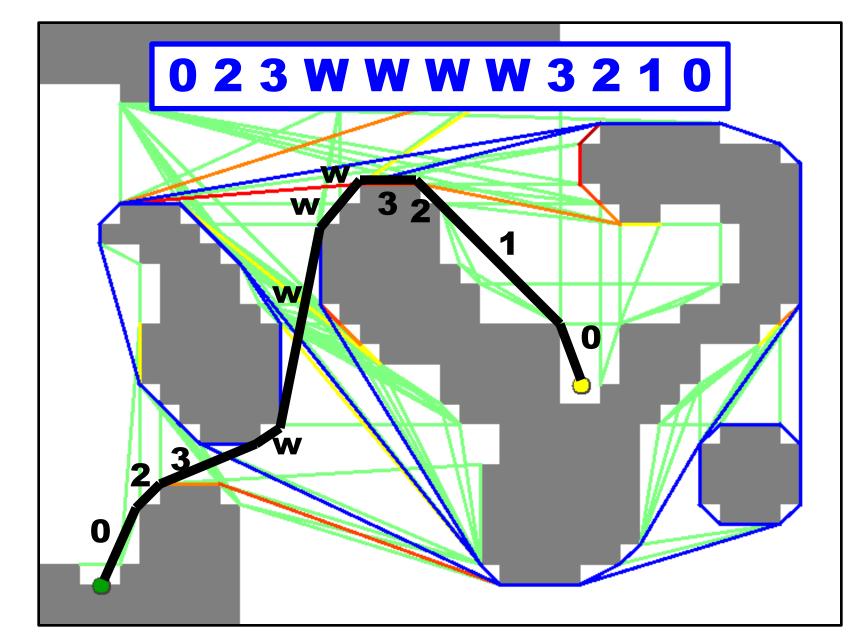
Any taut path between the start and goal will be of the form:

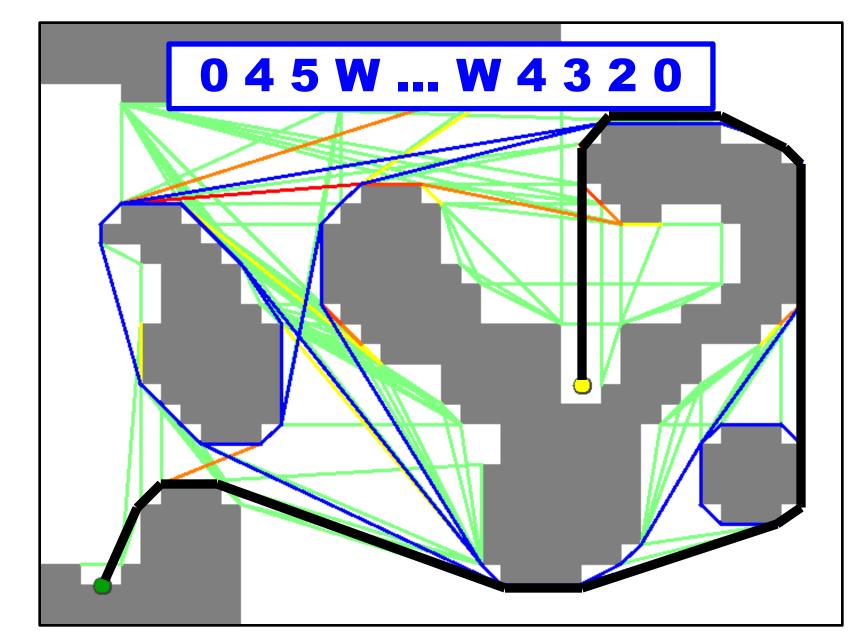
$$e_1 e_2 \cdots e_{k_1} w_{k_1+1} \cdots w_{k_2} e'_{k_2+1} \cdots e'_{n-1} e'_n$$

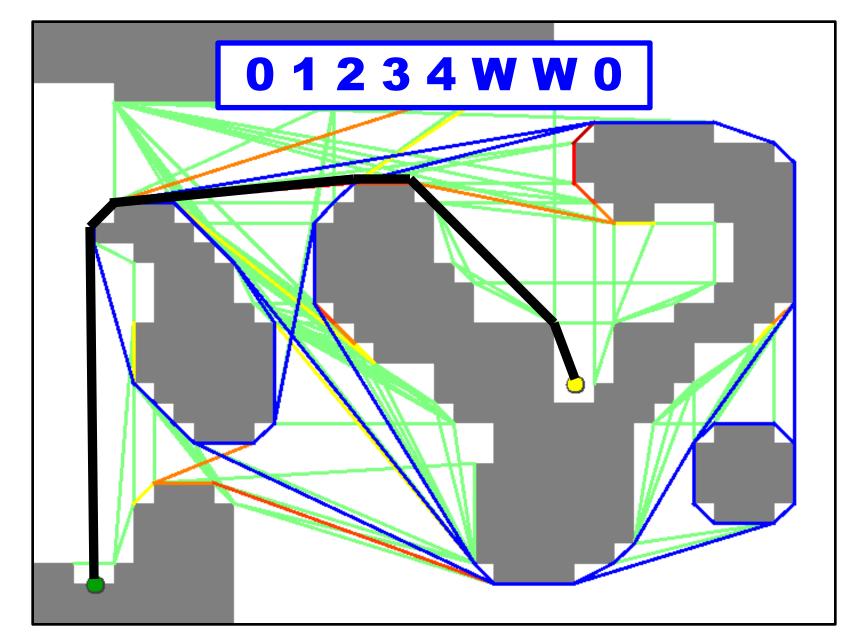
where $e_1e_2 \cdots e_k$ have strictly increasing levels, and $e'_{k_2+1} \cdots e'_n$ have strictly decreasing levels and $w_{k_1+1} \cdots w_{k_2}$ are all level-W



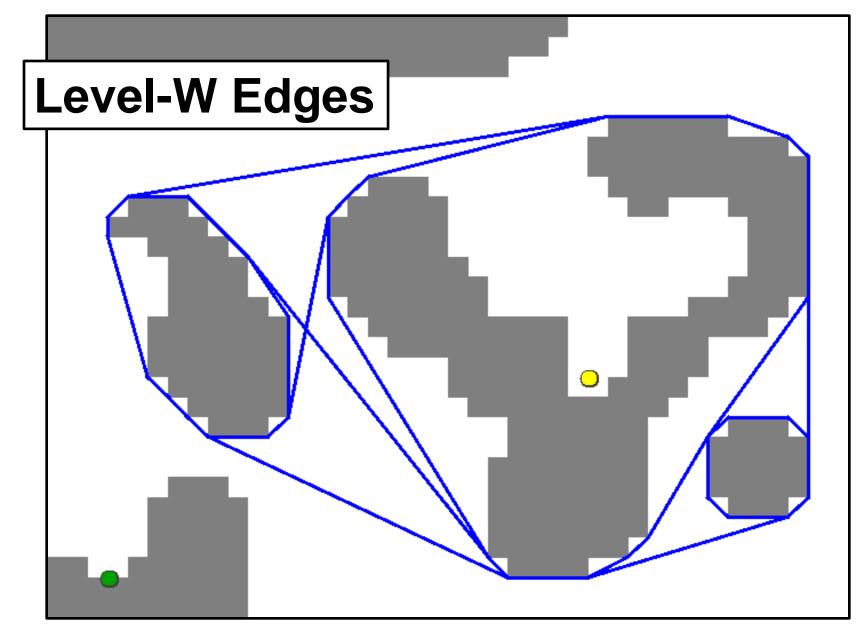


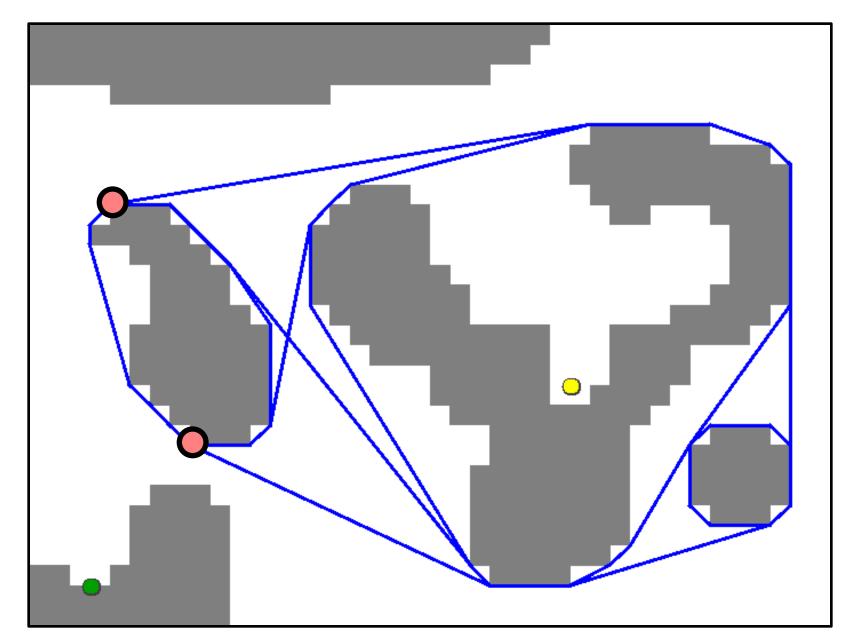


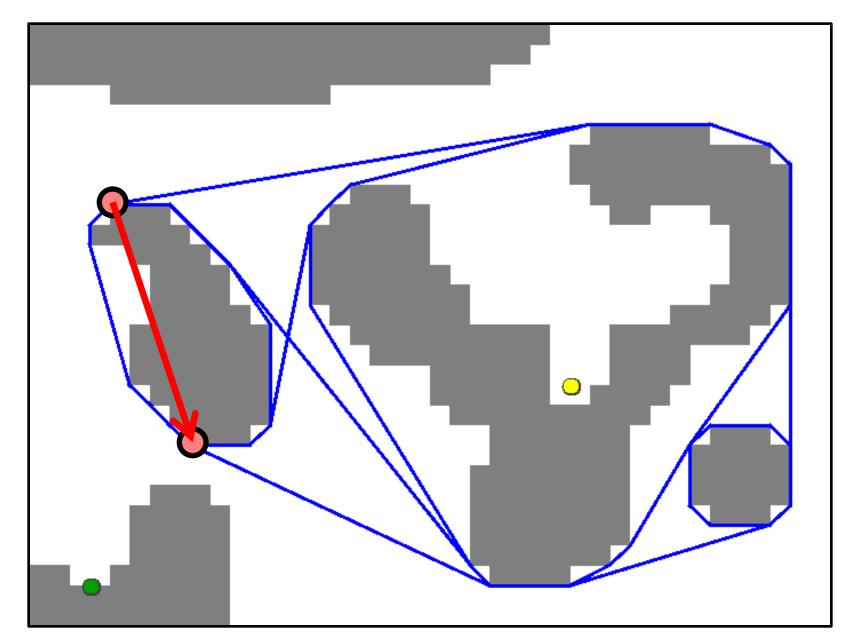


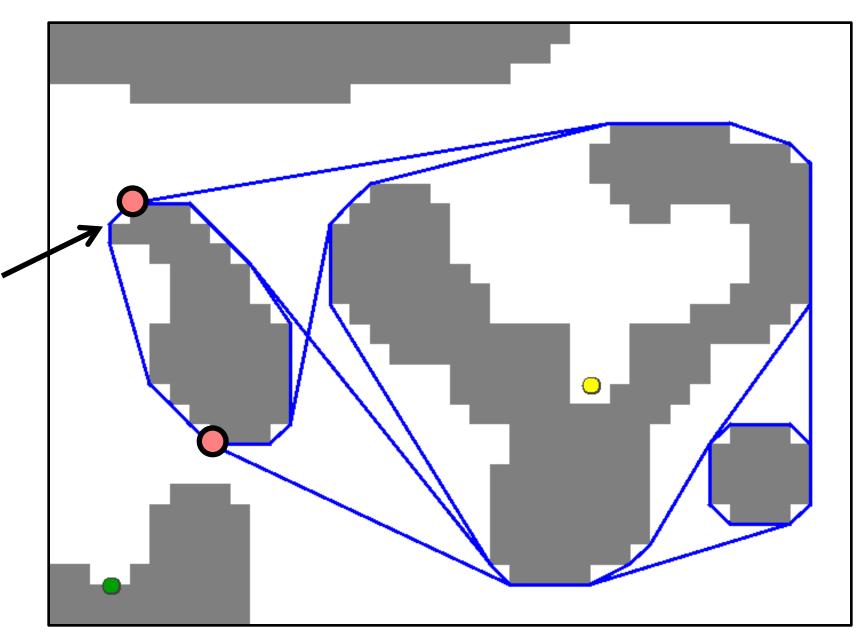


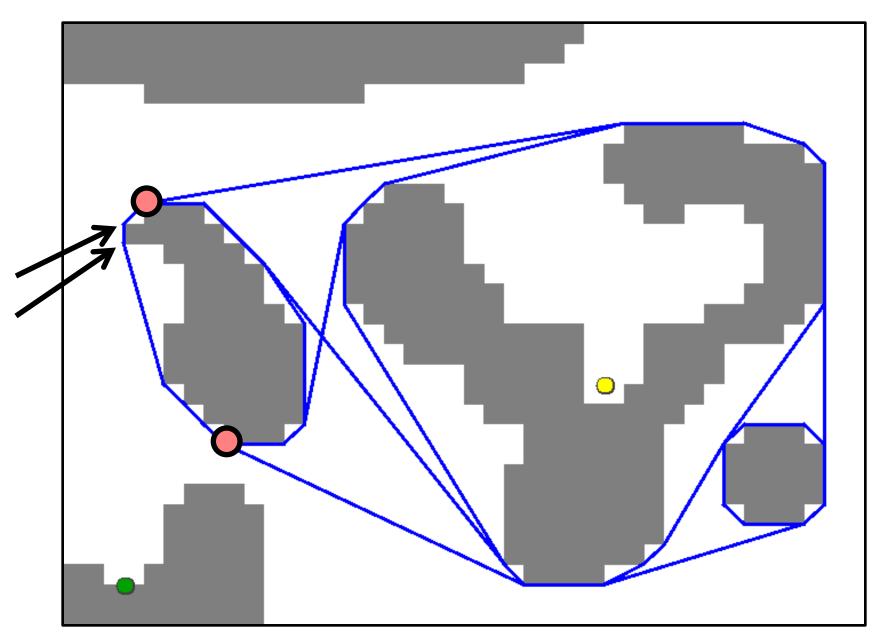
Skip Edges

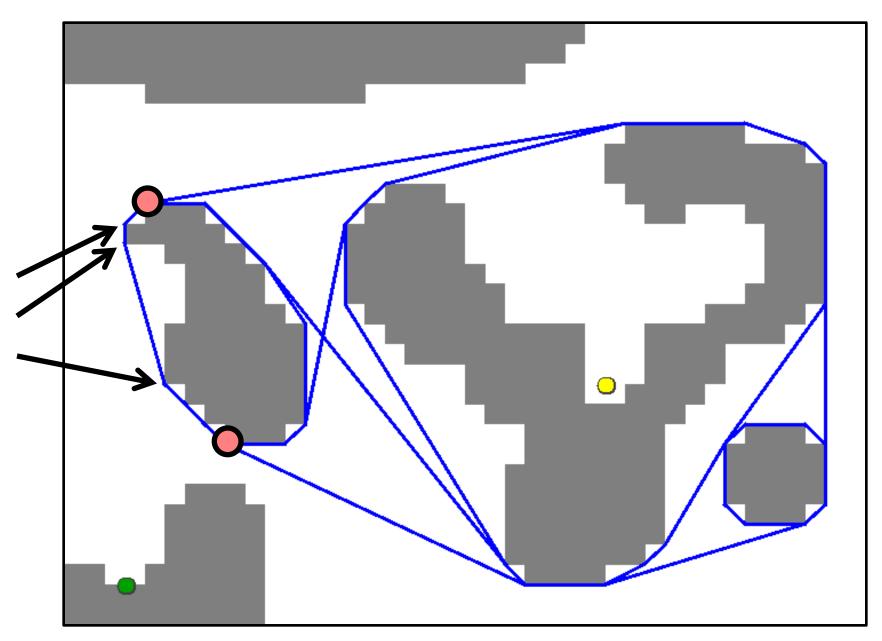


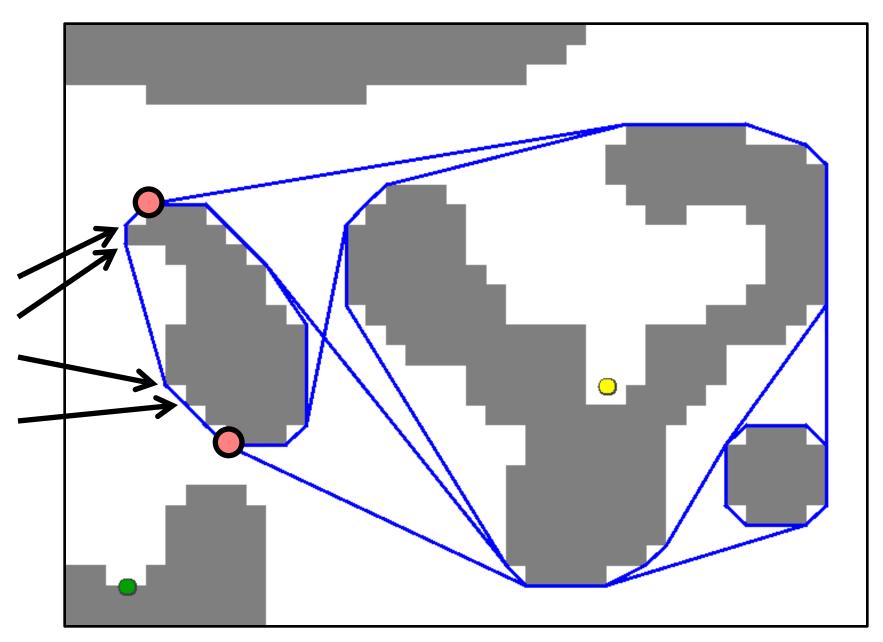


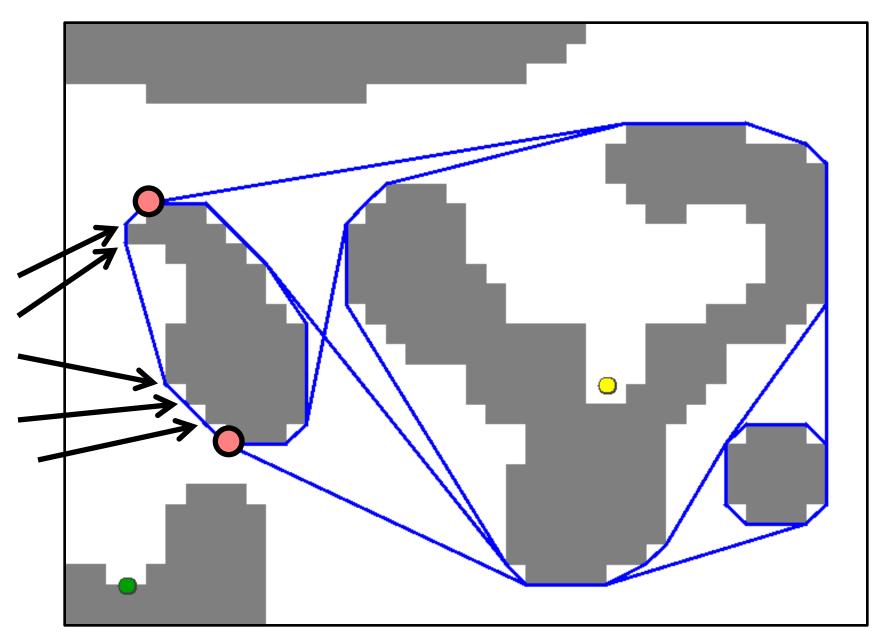


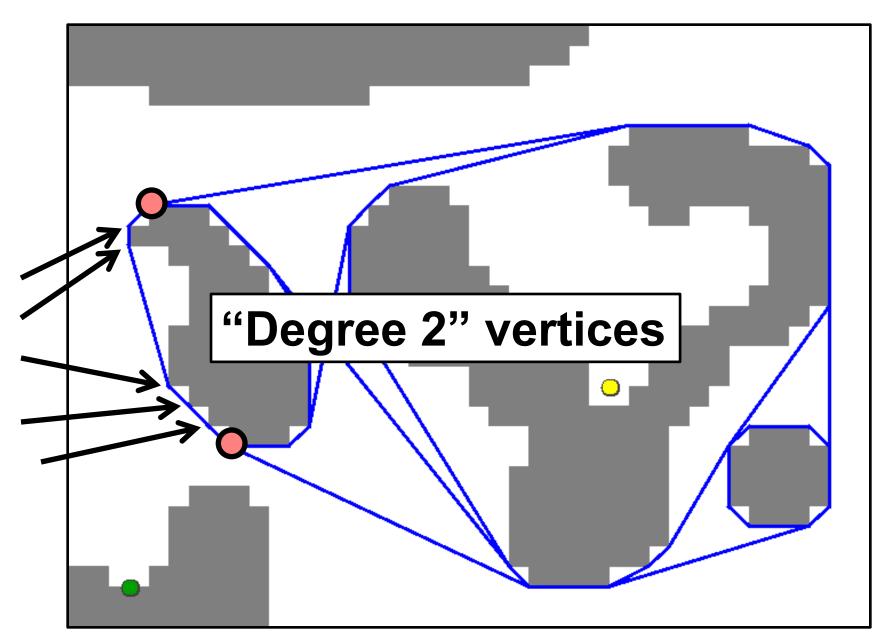


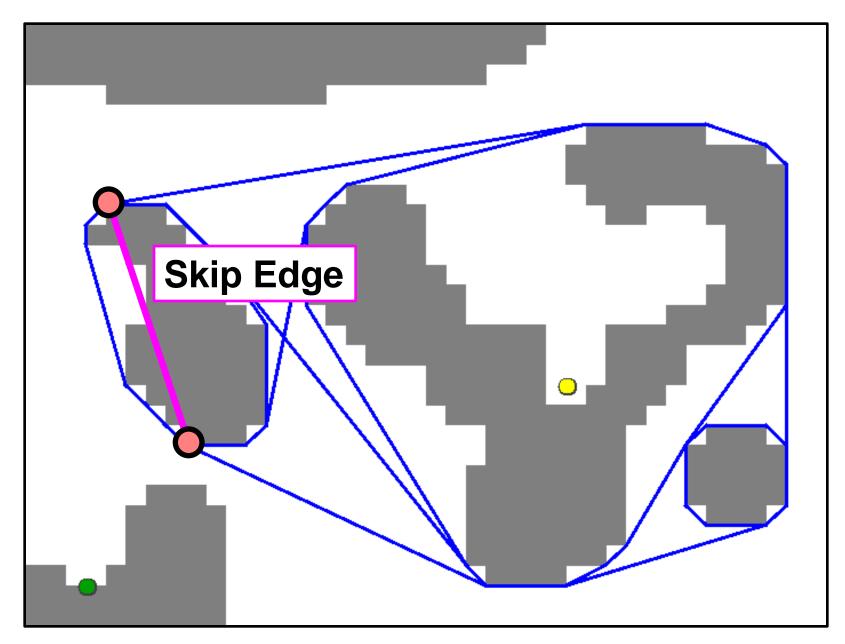


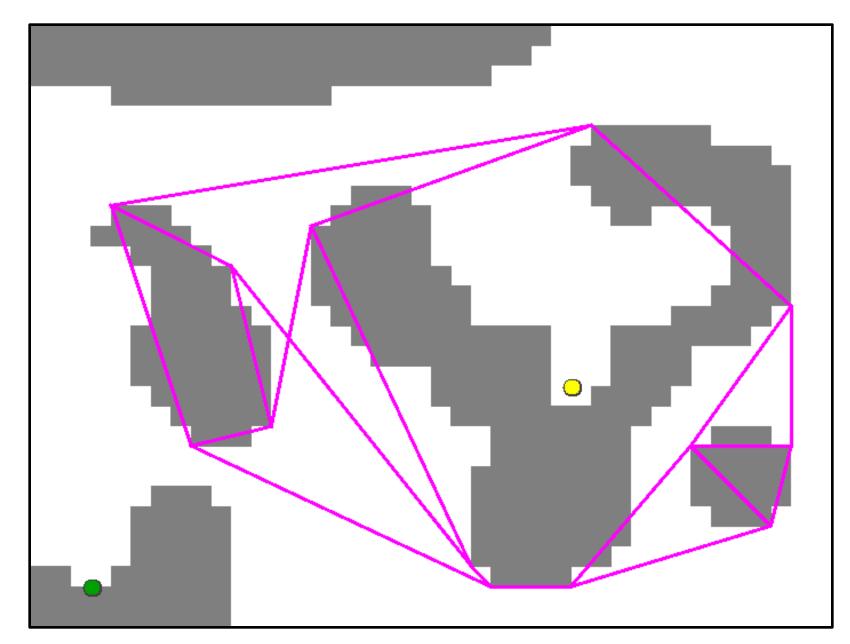


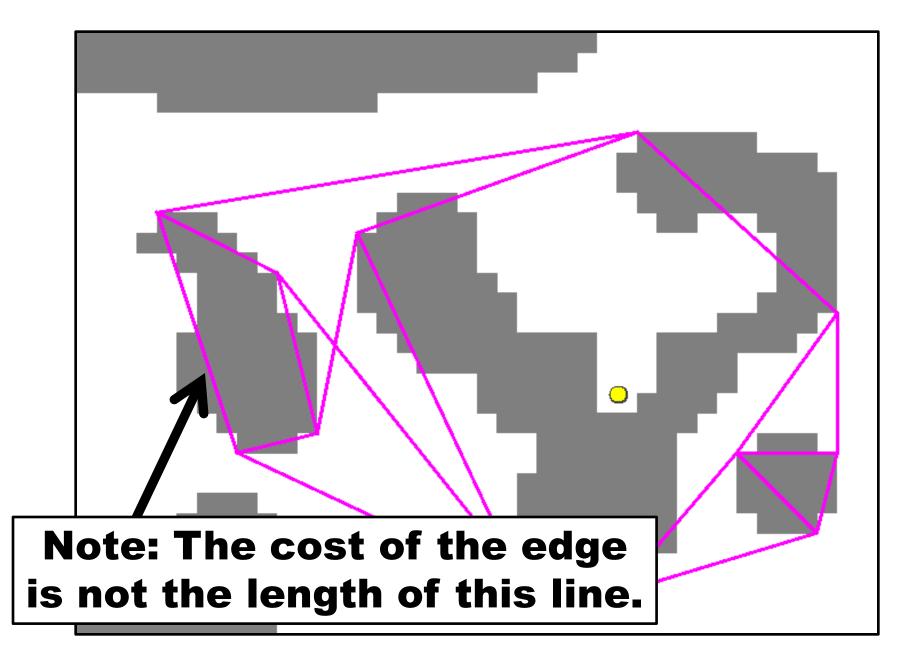


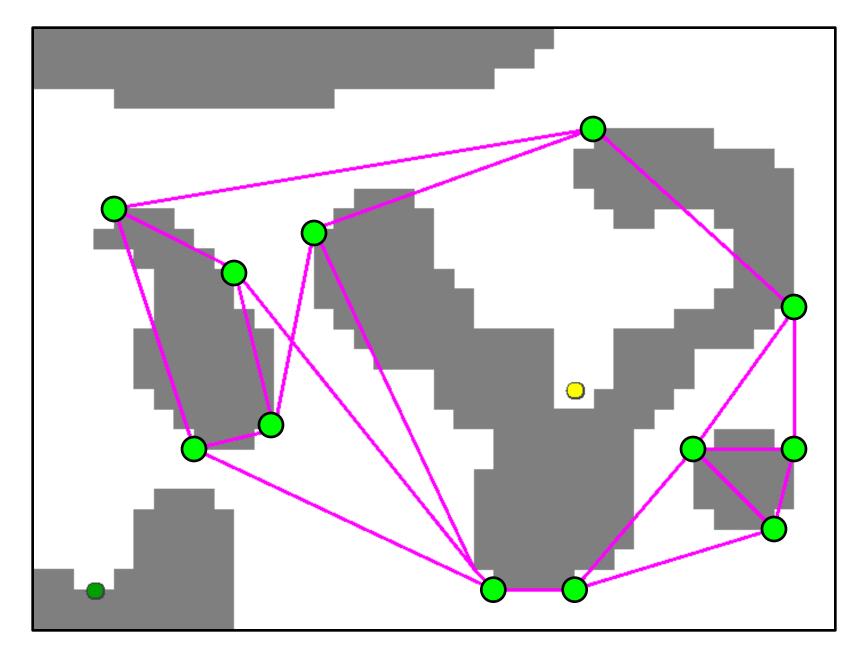


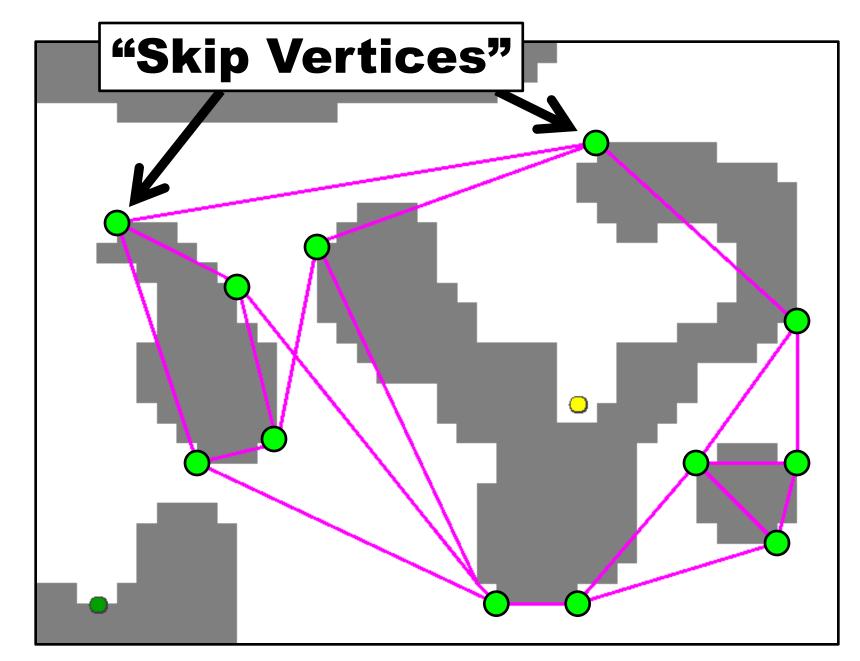


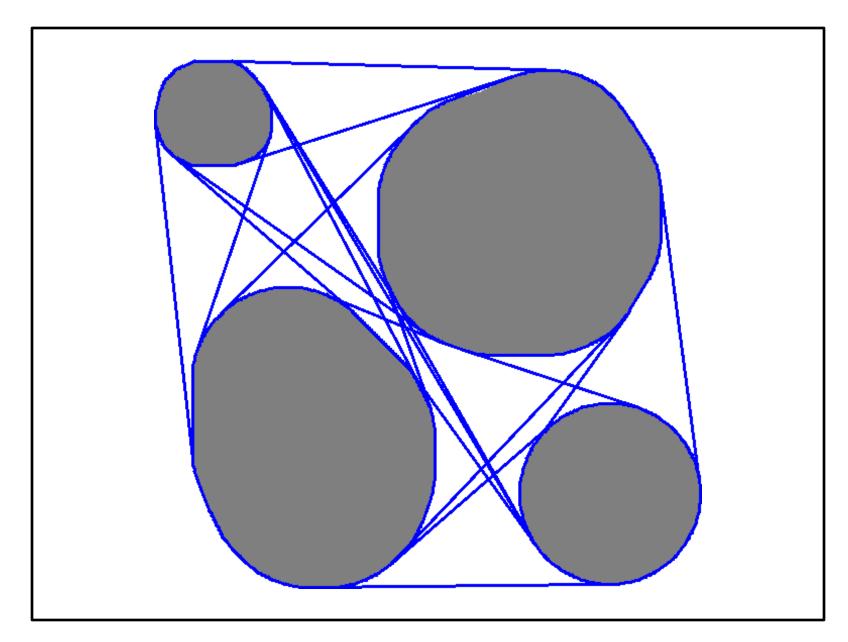


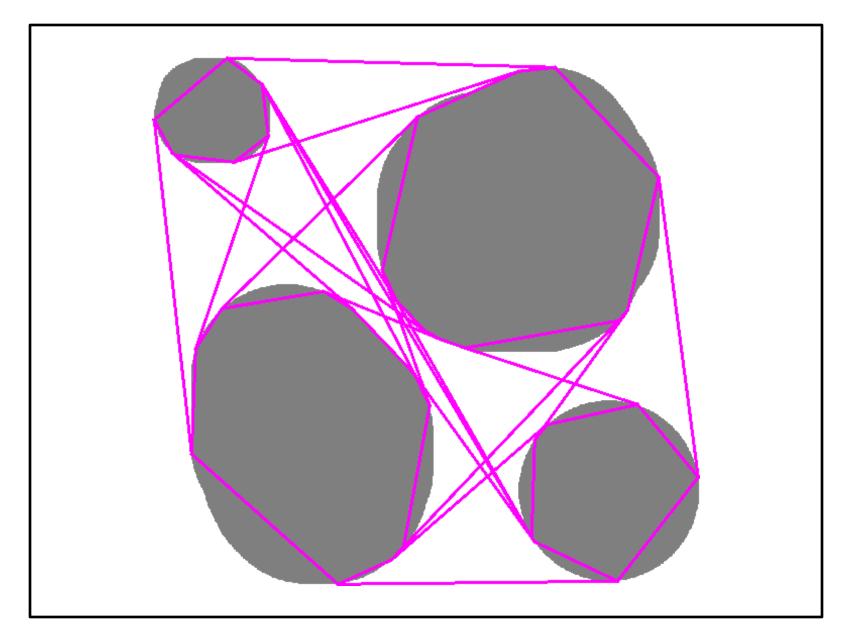












Algorithm

Three Phases

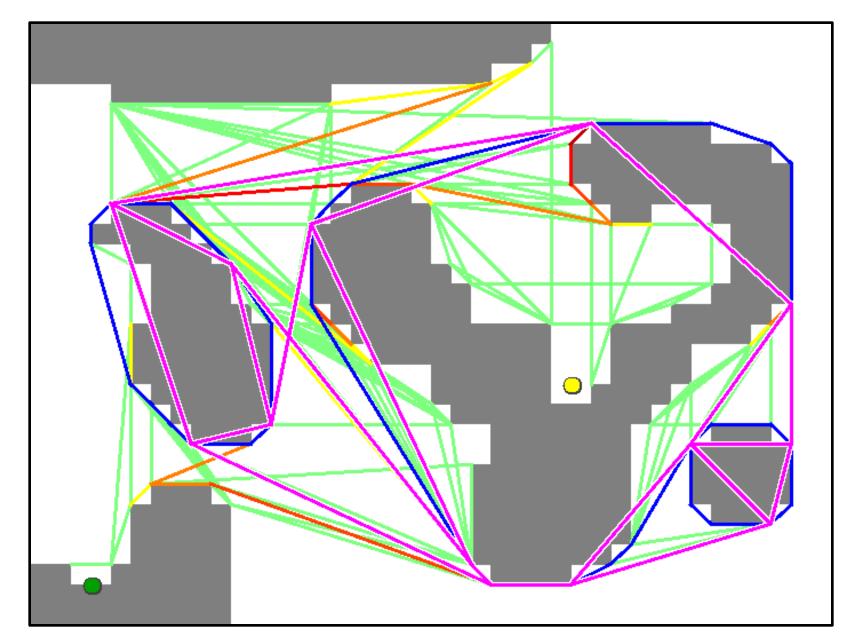
Step 1: Insertion

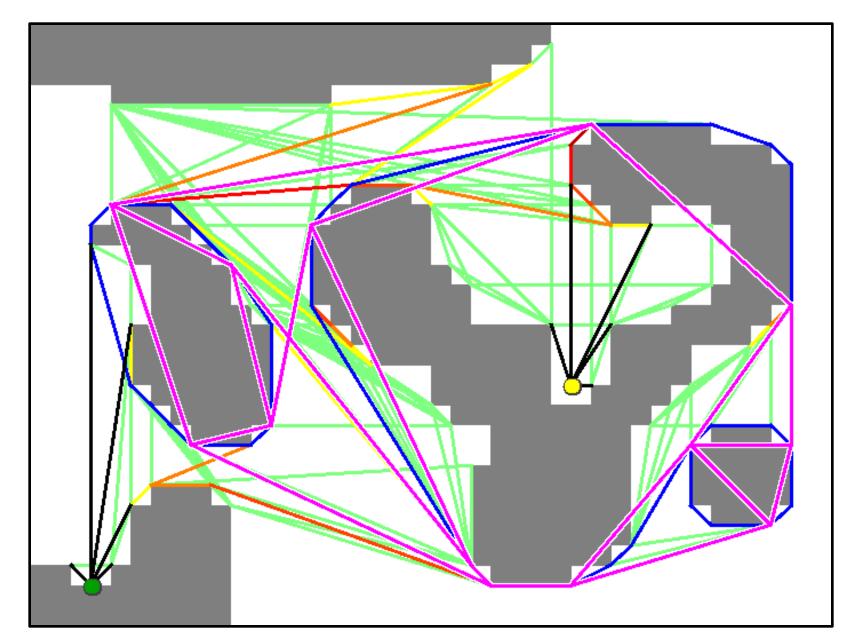
Step 2: Marking

Step 3: Search

Step 1: Insertion Connect Start and Goal

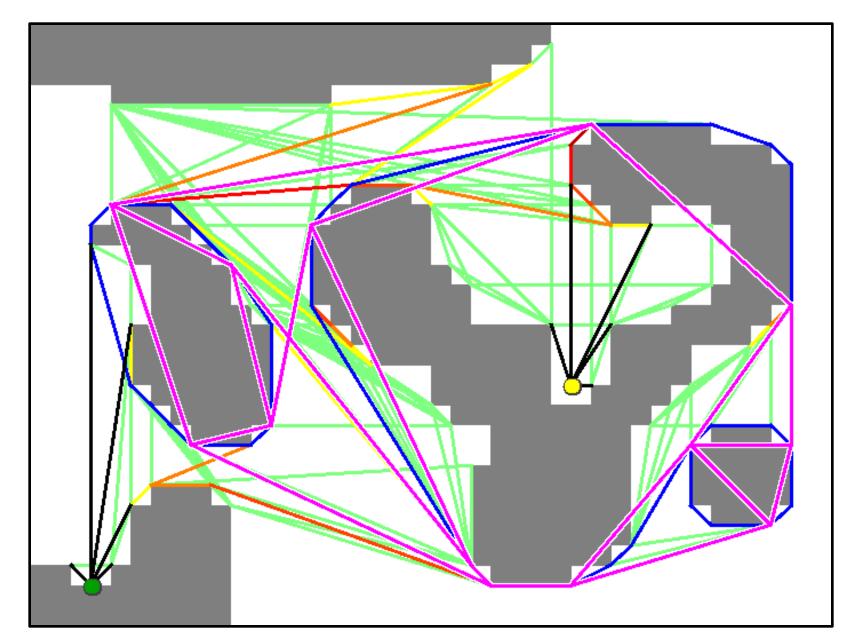
Step 1: Insertion Connect Start and Goal (using Line-of-Sight Scans)

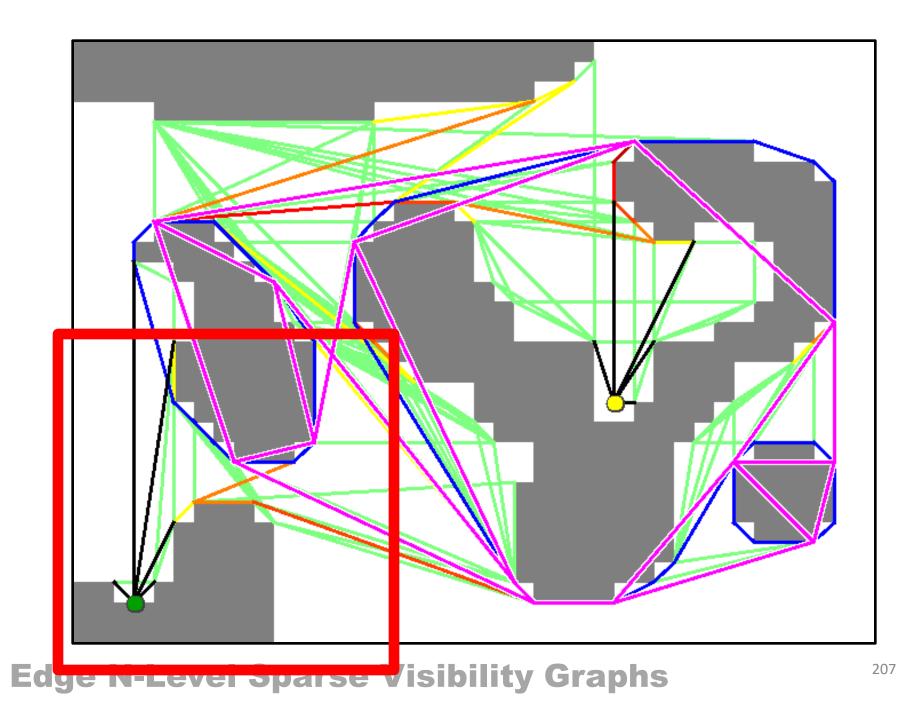


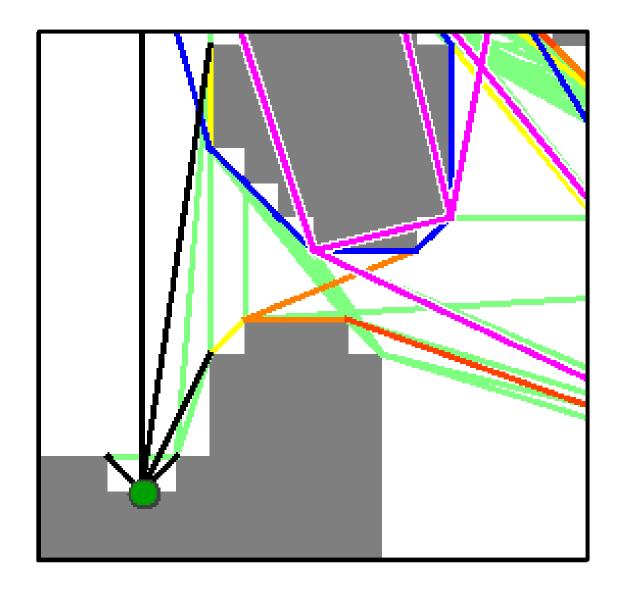


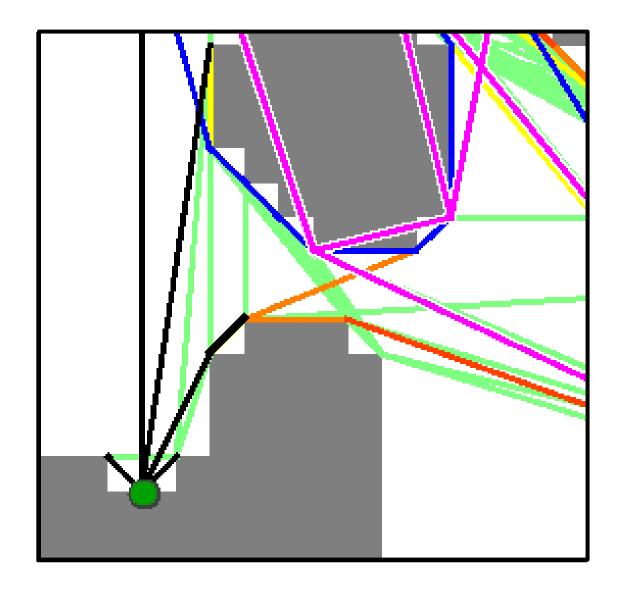
Step 2: Marking Mark all Edges Reachable by an Increasing Sequence or Level-W Edges.

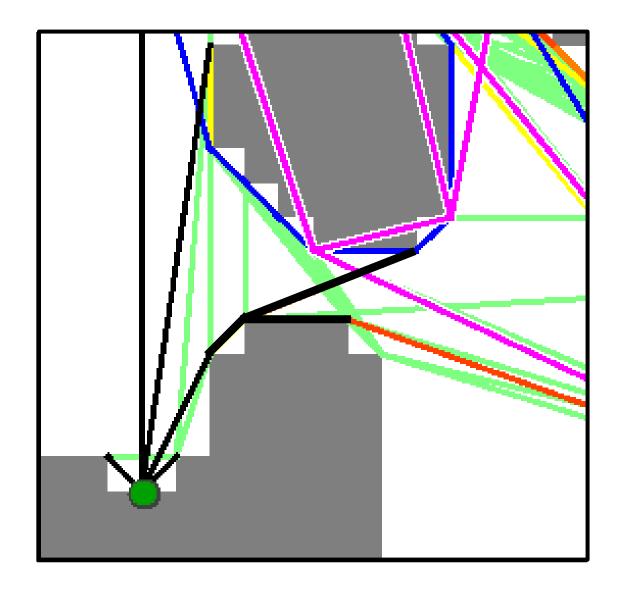
Stop when you reach any Skip Vertex.

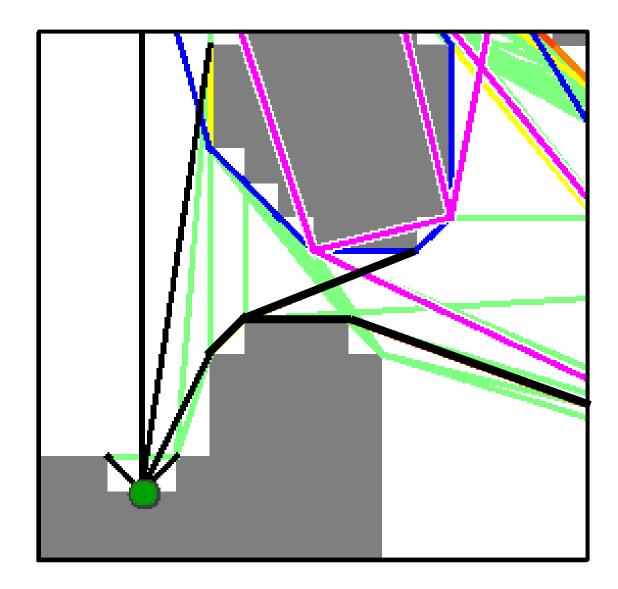


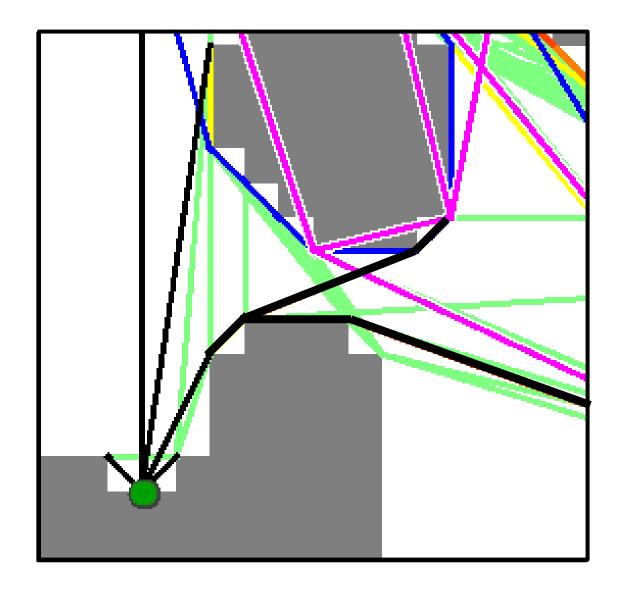


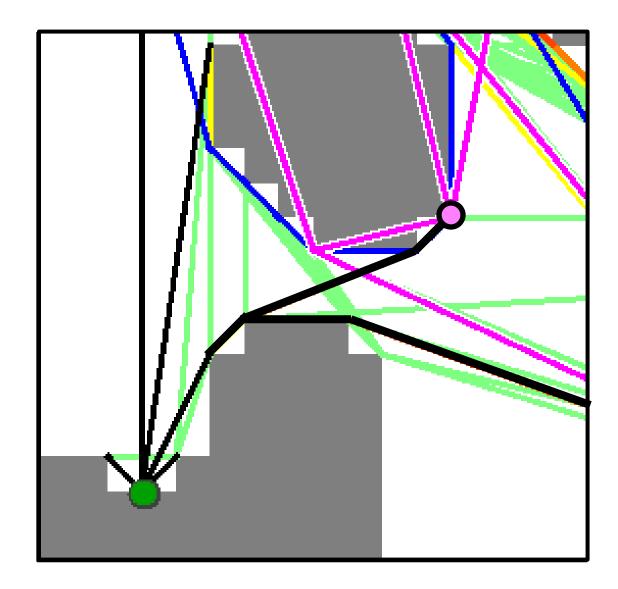


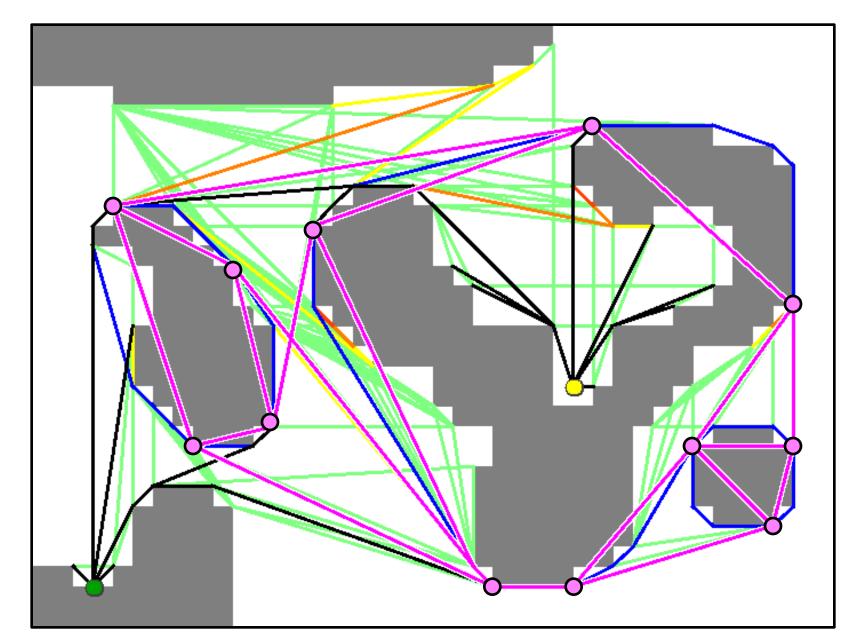


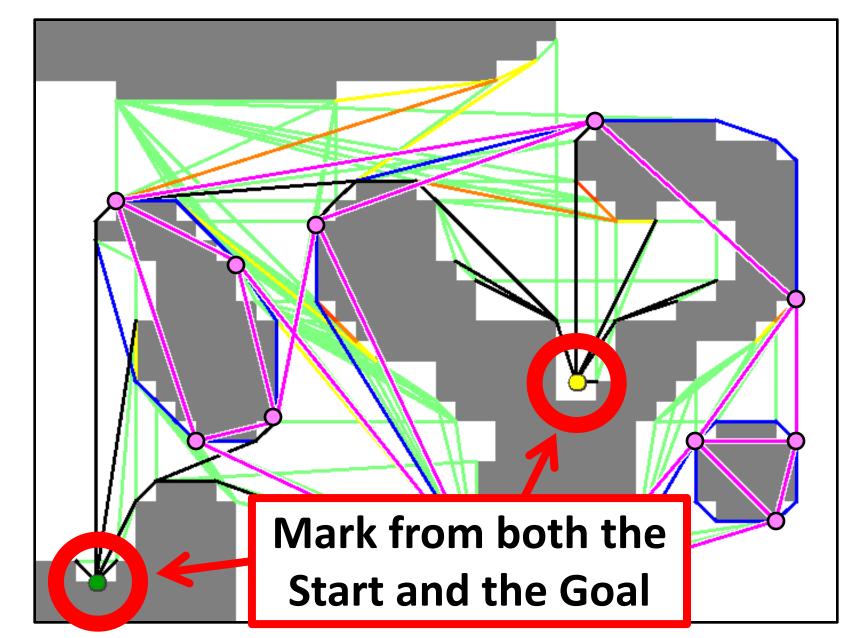






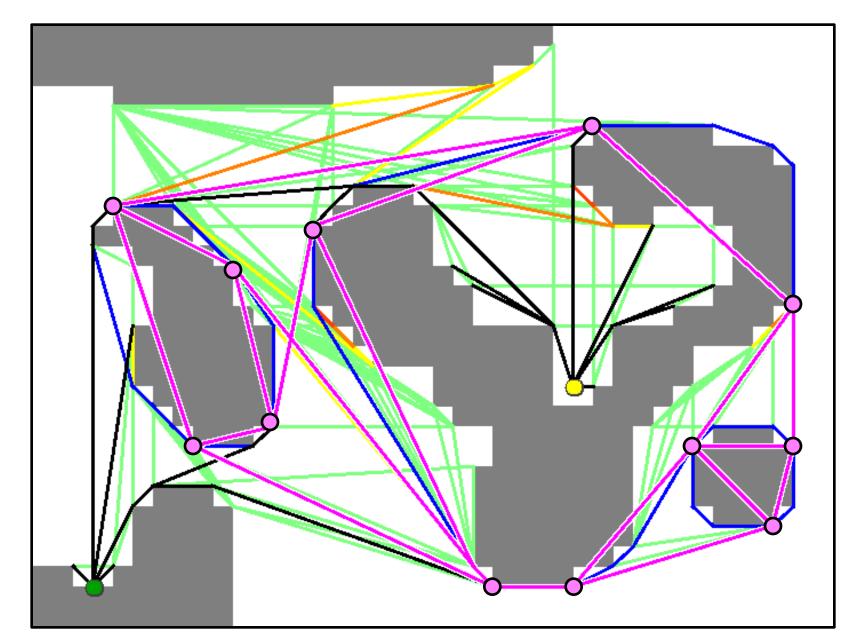


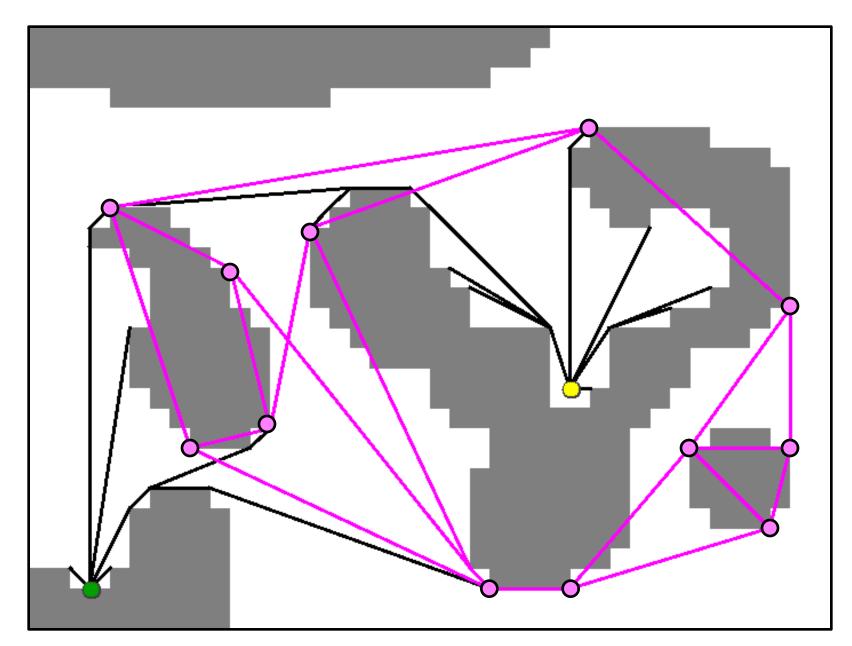


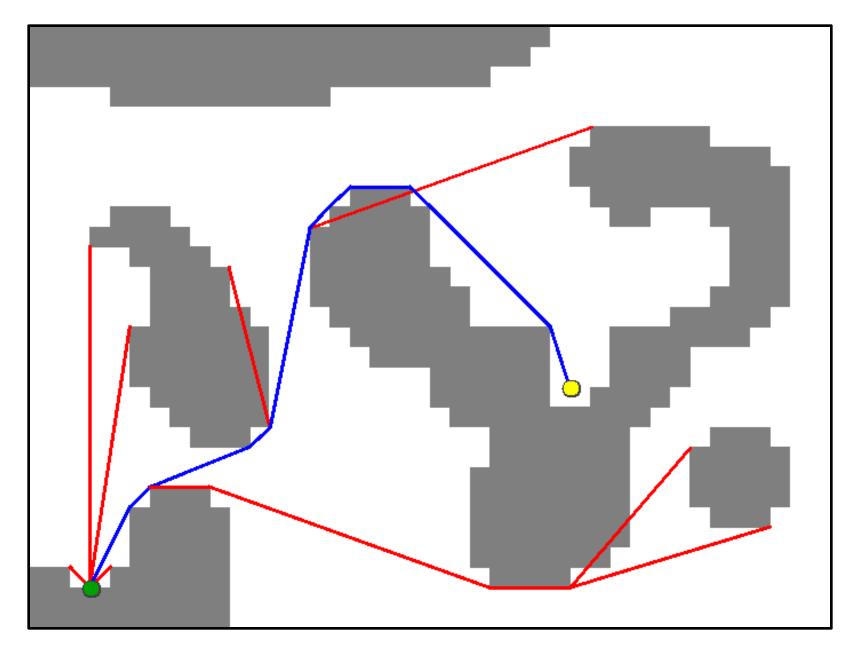


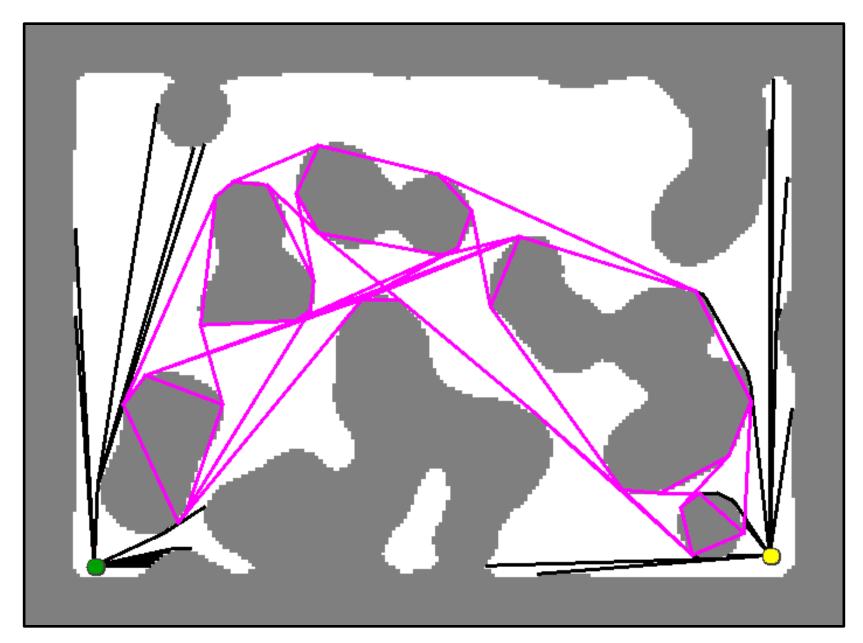
Step 3: Search Search Only Marked Edges & Skip Edges

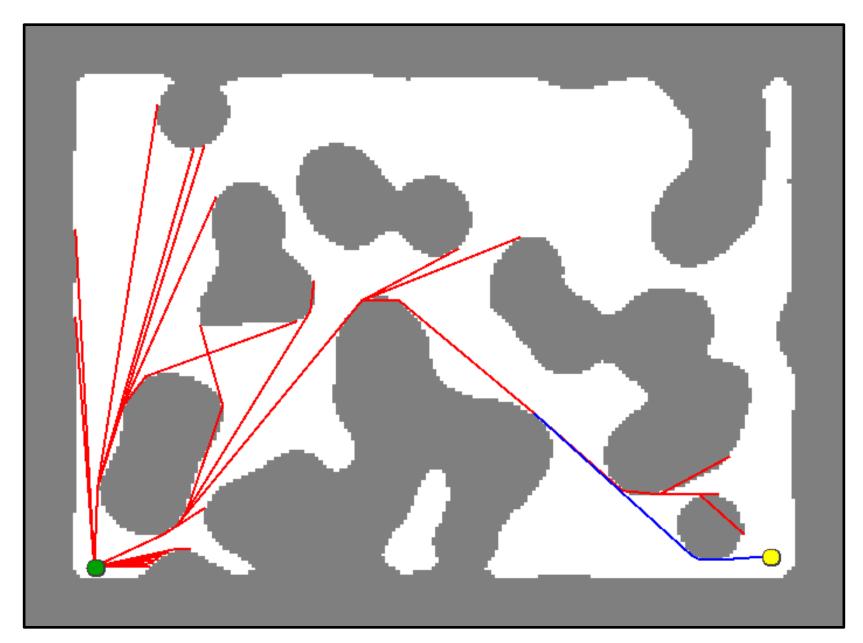
Note: Use Taut A* Search



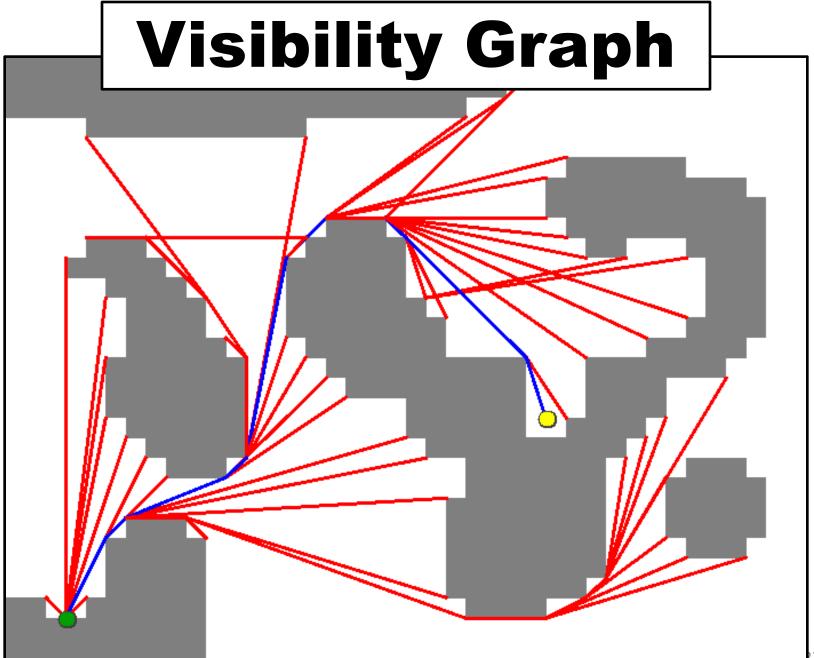


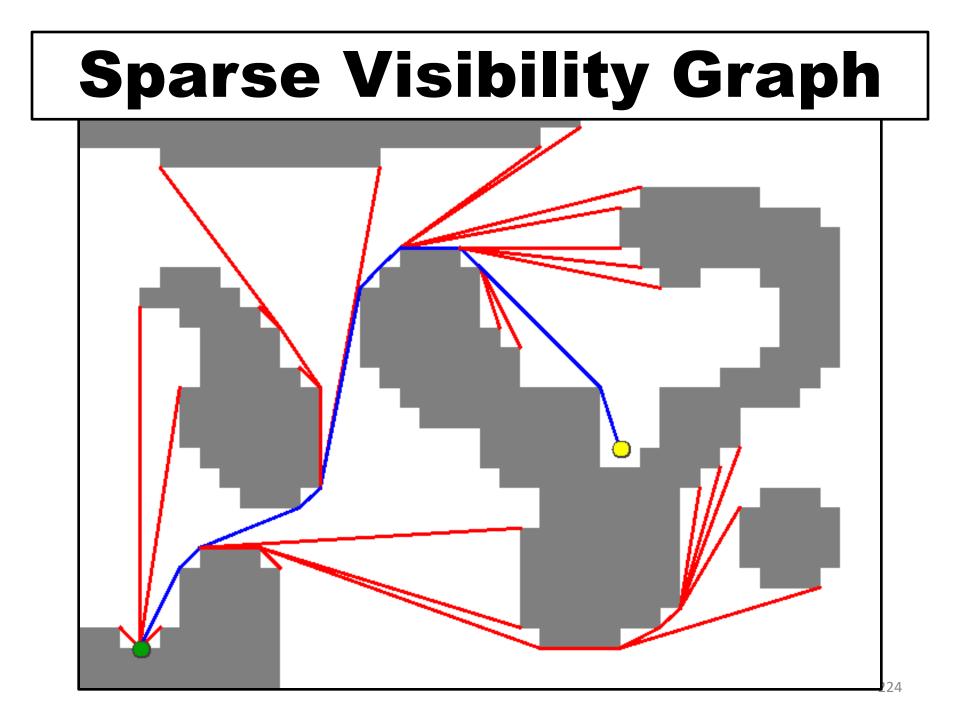


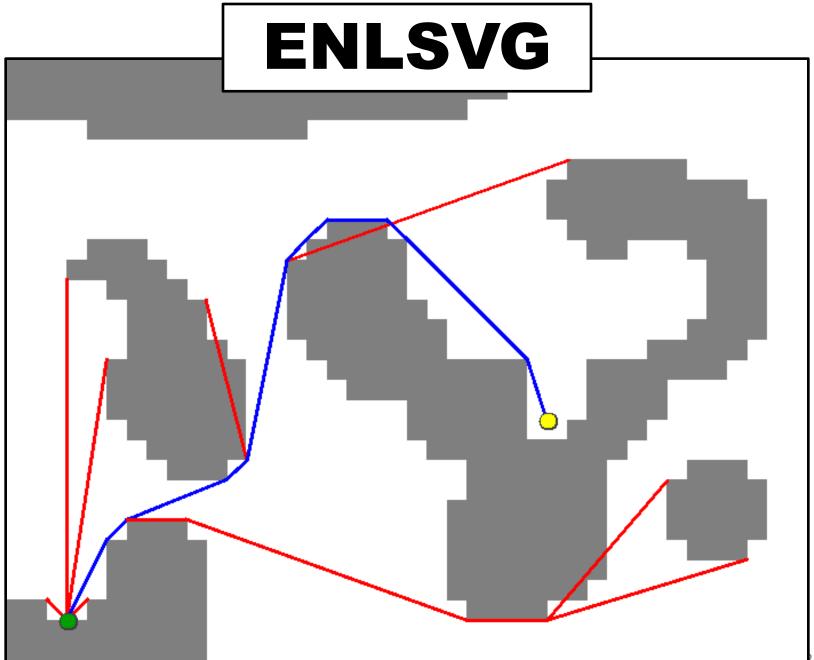


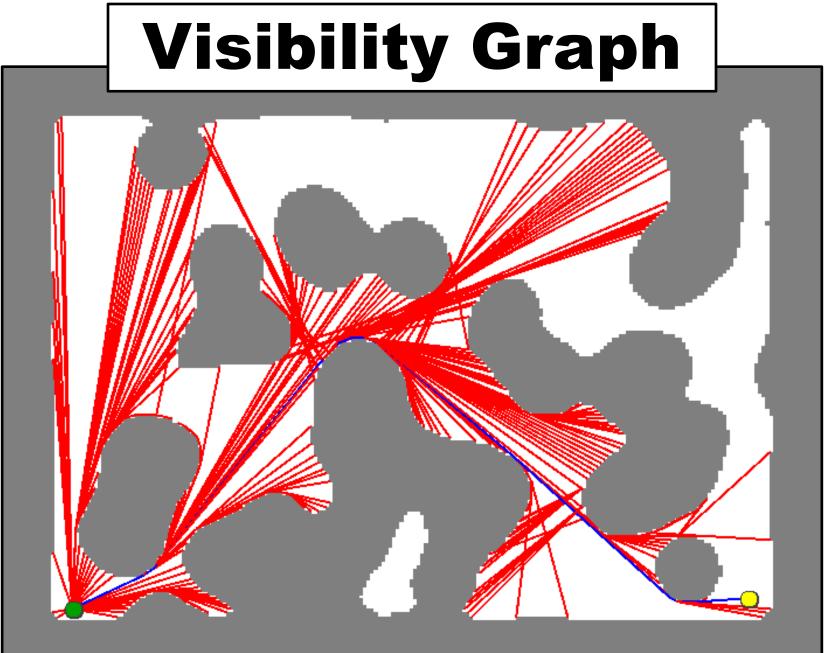


Search Tree Comparison

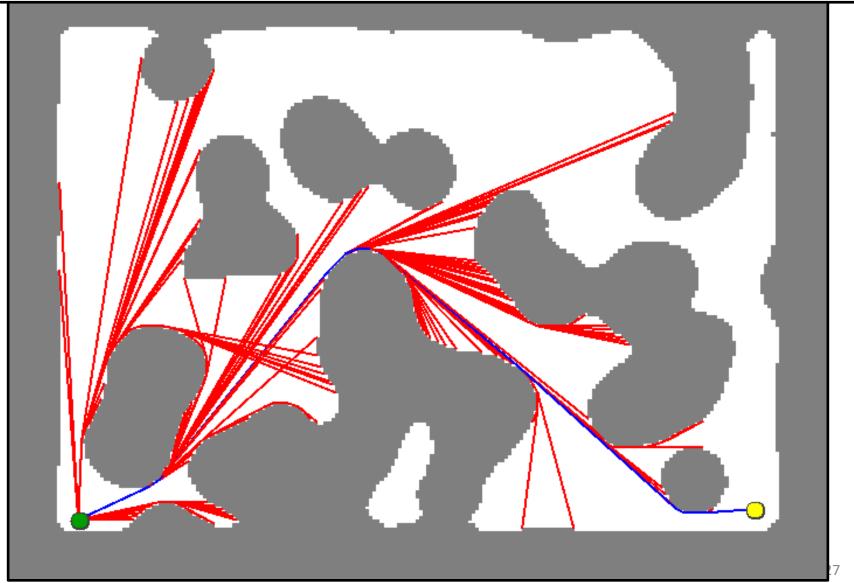


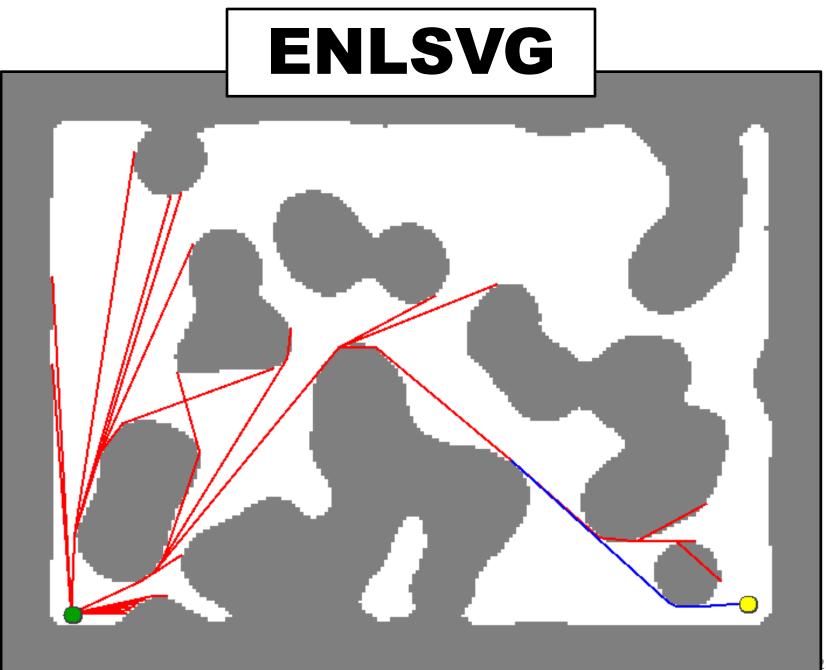




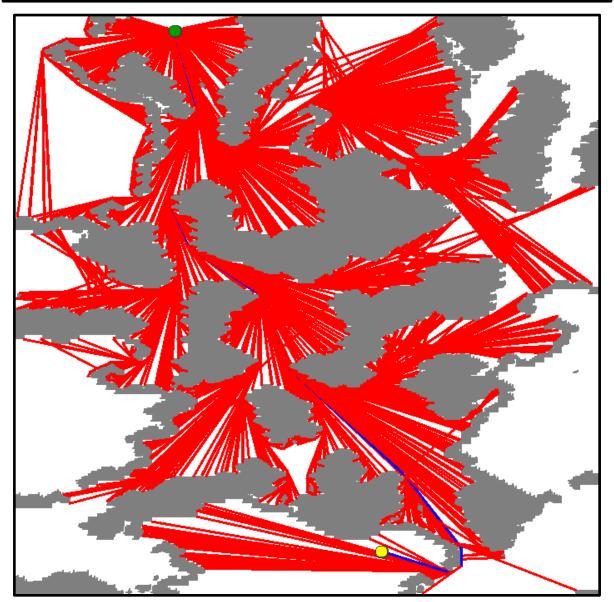


Sparse Visibility Graph

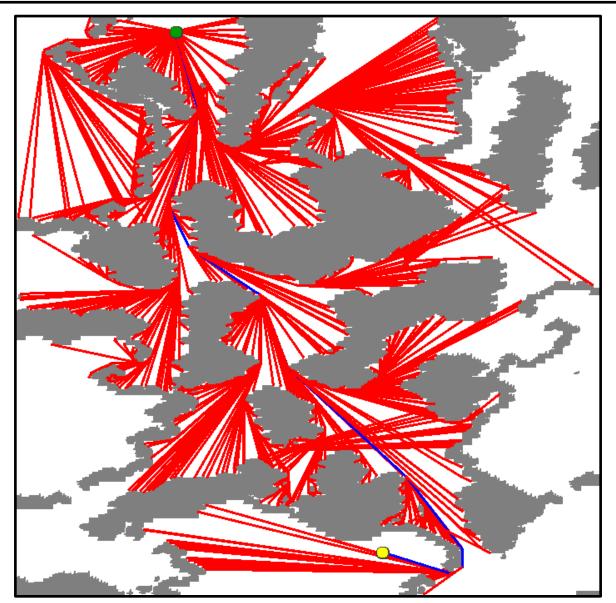




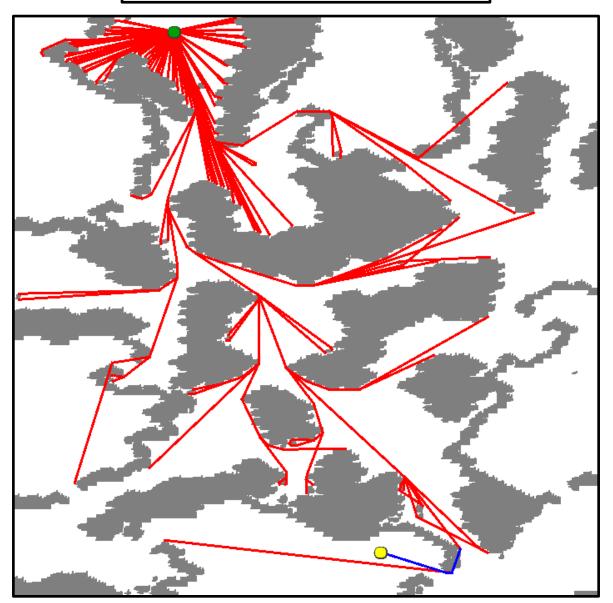
Visibility Graph



Sparse Visibility Graph





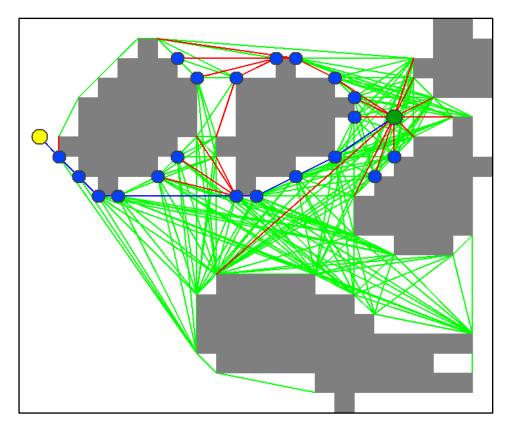


Experiments Running Time Comparison

VG C VG S SVG **ENLSVG** Theta* Anya16 **SG16**

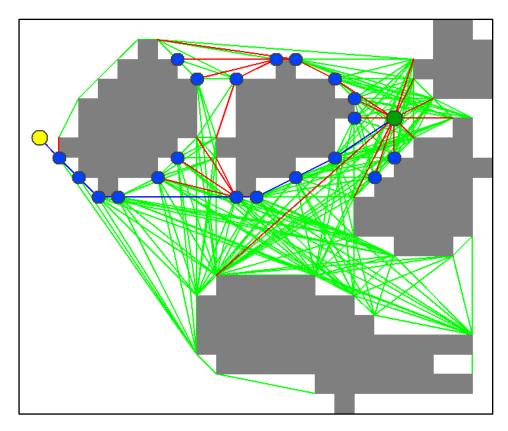
VG_C VG S SVG **ENLSVG** Theta* Anya16 **SG16**

Visibility Graphs Line-of-Sight Checks



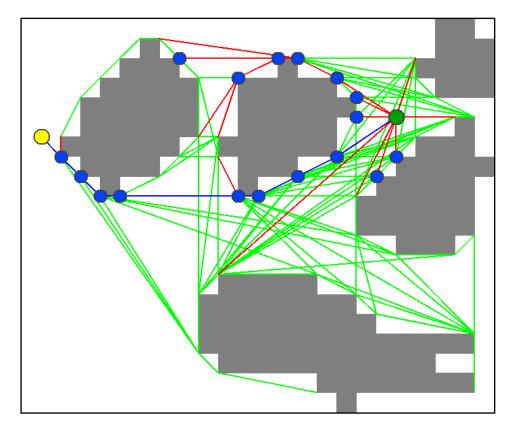
VG_C VG_S SVG **ENLSVG** Theta* Anya16 **SG16**

Visibility Graphs Line-of-Sight Scans



VG_C VG_S SVG **ENLSVG** Theta* Anya16 **SG16**

Sparse Visibility Graphs (Line-of-Sight Scans)

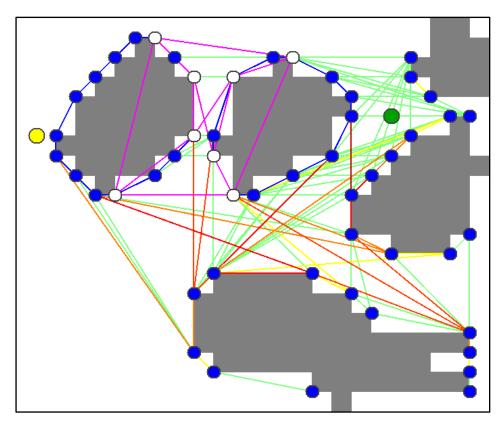


VG_C VG_S SVG

ENLSVG

Theta* Anya16 SG16

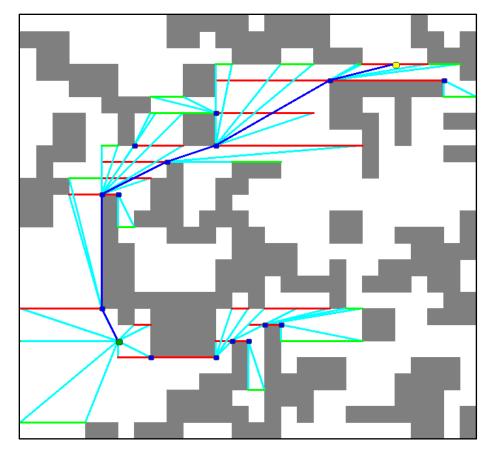
ENLSVGs (Line-of-Sight Scans)



Algorithms Compared Theta* Algorithm VG C Nash et al. 2007 VG S SVG **ENLSVG** Theta* Anya16 **SG16**

Anya (2016 Version)

Harabor et al. 2016

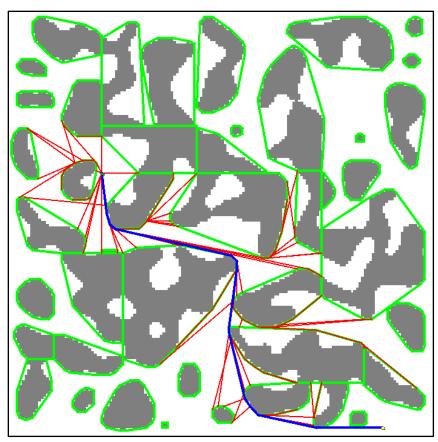


VG C VG S SVG **ENLSVG** Theta* Anya16 **SG16**

VG C VG S SVG **ENLSVG** Theta* Anya16 **SG16**

Speeding Up A* Search on Visibility Graphs Defined over Quadtrees

Shah & Gupta, 2016



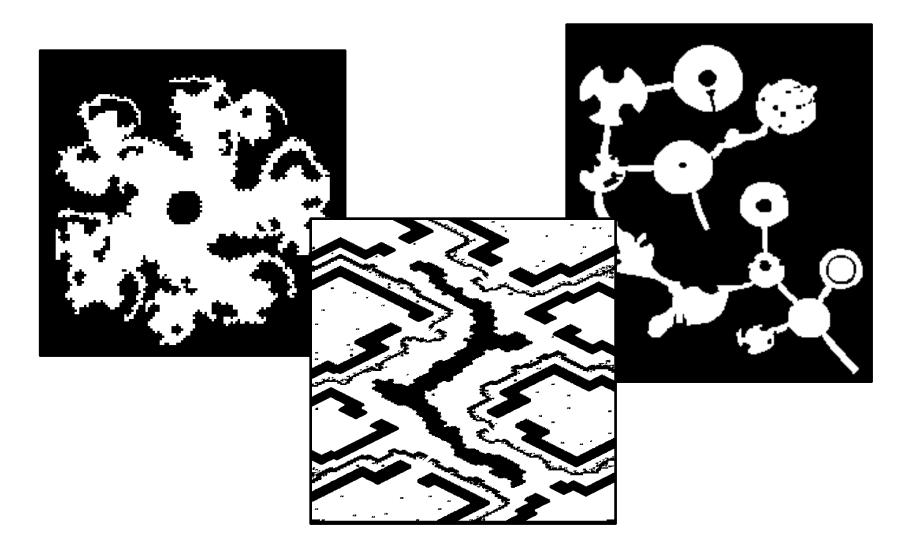
Running Time Comparison

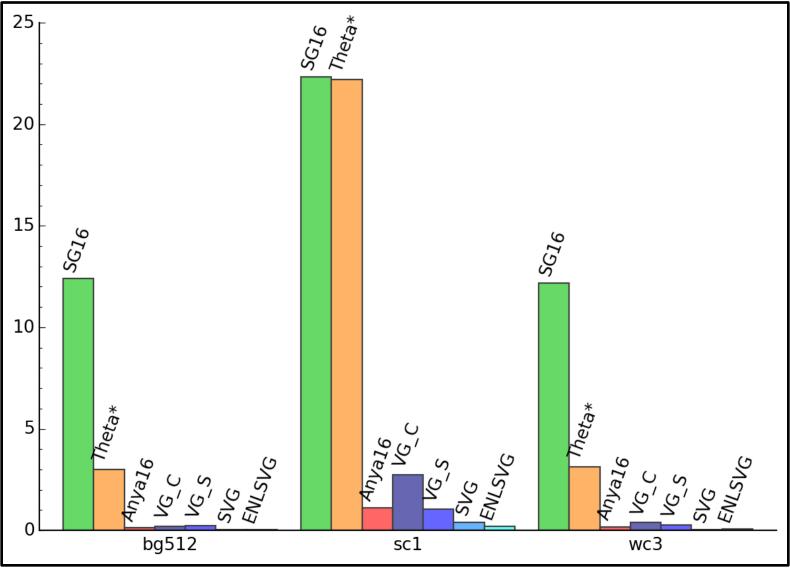
Running Time Comparison

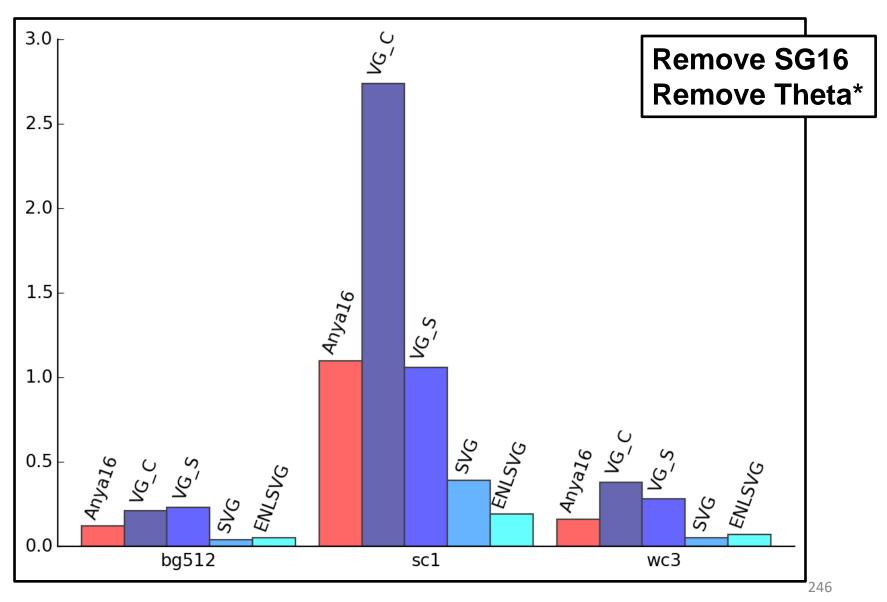
All running times are in milliseconds

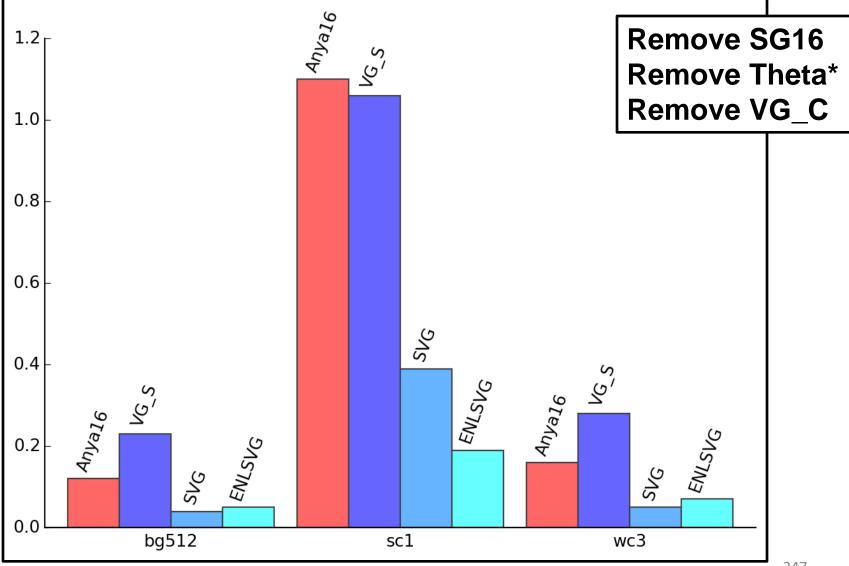
Running Time

Benchmarks (Game)









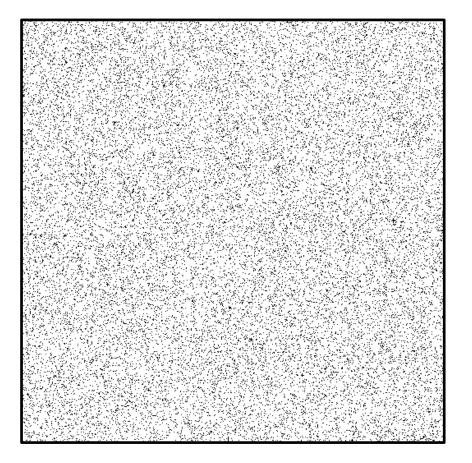
Running Time

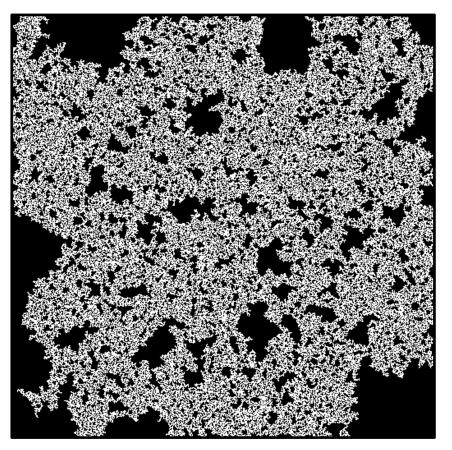
Benchmarks (Random)

Running Time Benchmarks (Random)

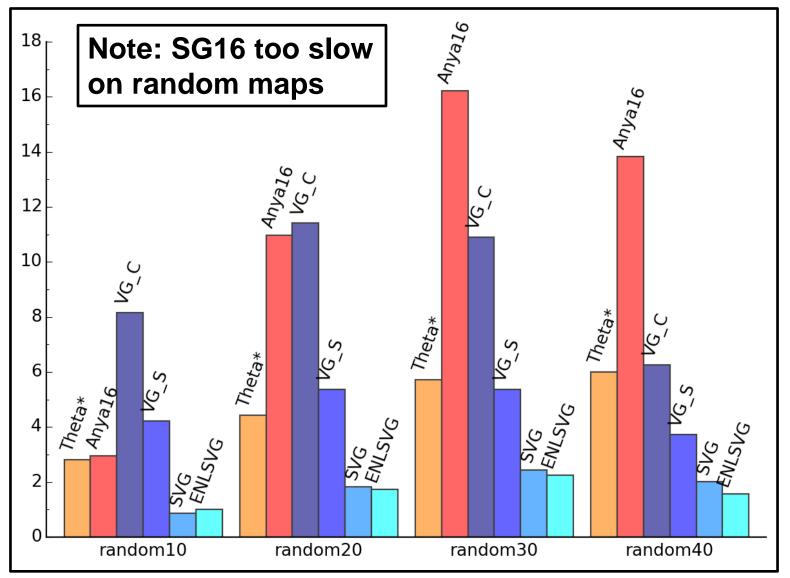
random10, 512x512

random40, 512x512





Running Time Benchmarks (Random)



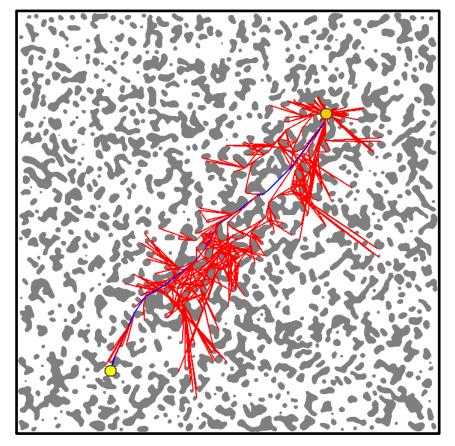
Running Time

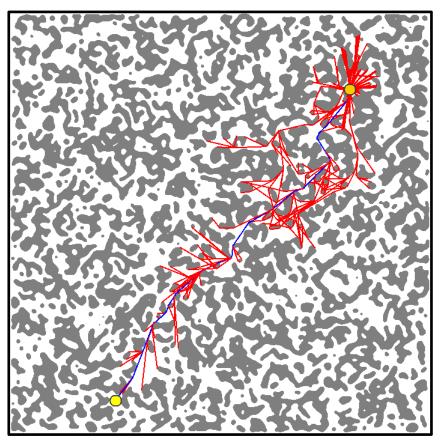
Cave Maps Generated using Cellular Automata (Johnson, Yannakakis, and Togelius 2010)

Running Time Cave Maps

30% Blocked 4000x4000

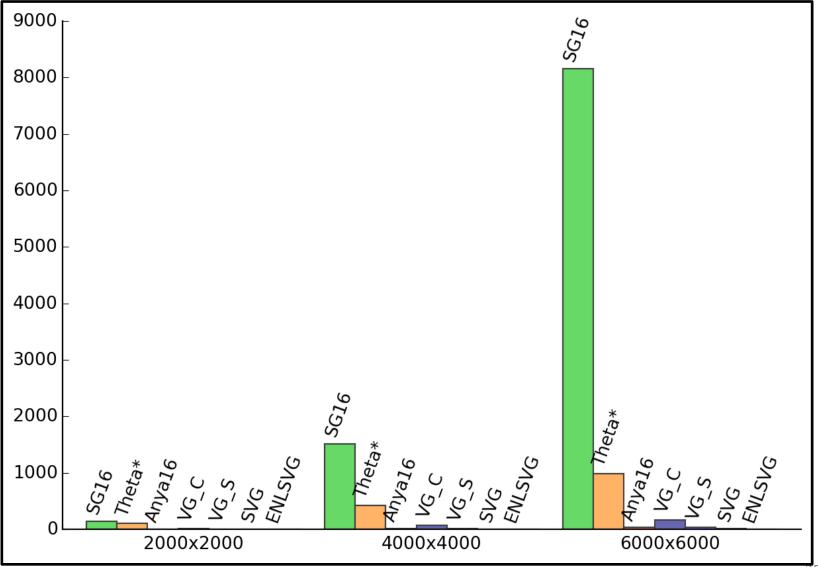
45% Blocked 4000x4000

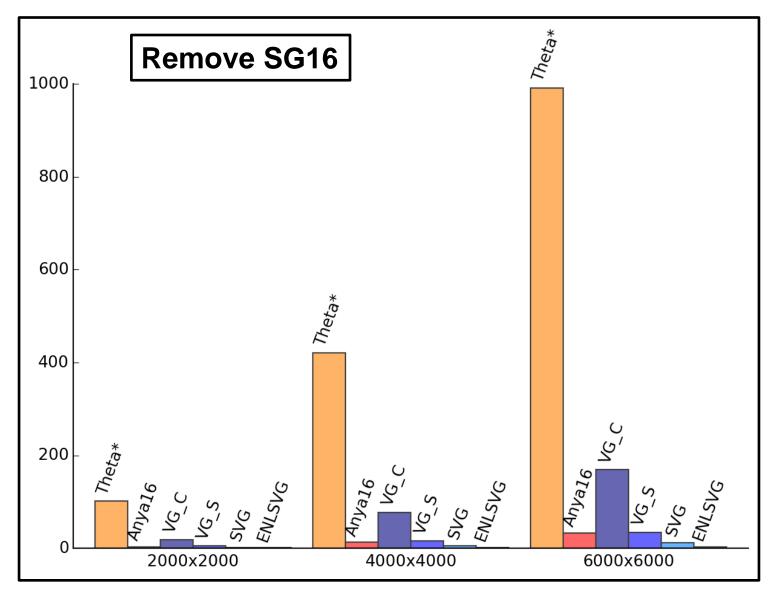


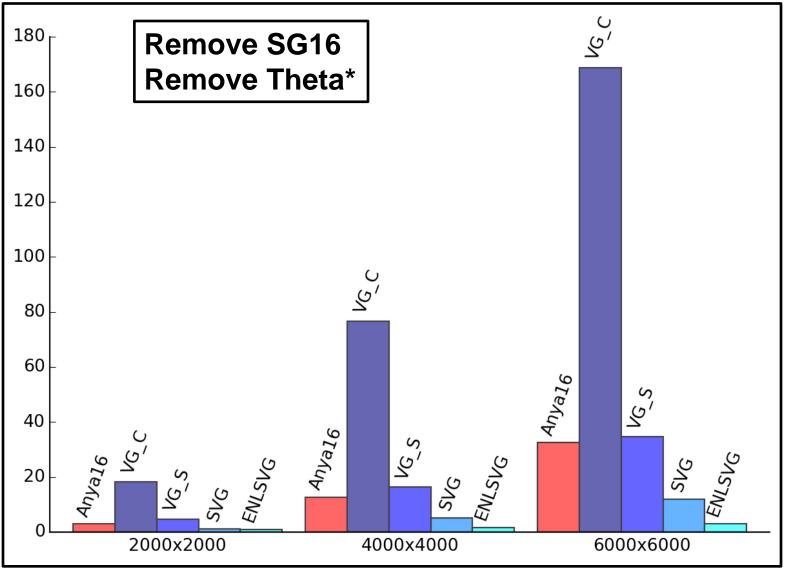


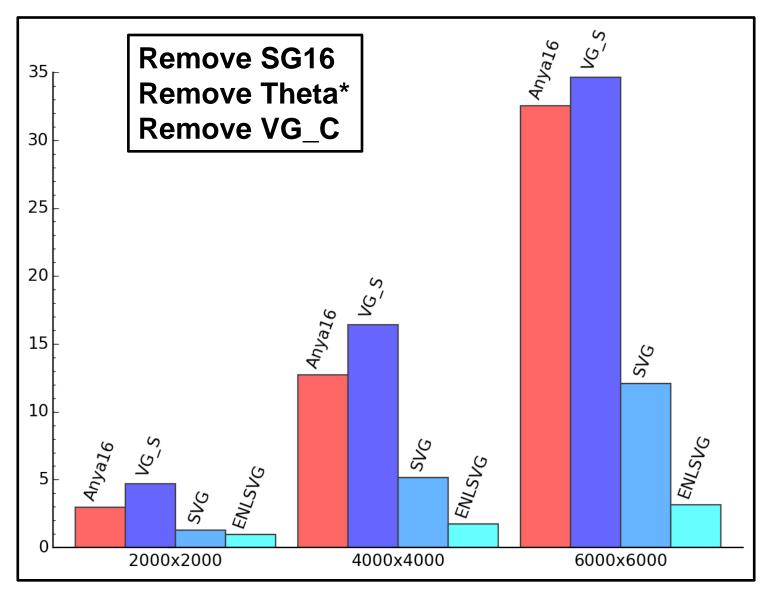
Running Time Cave Maps

30% Blocked



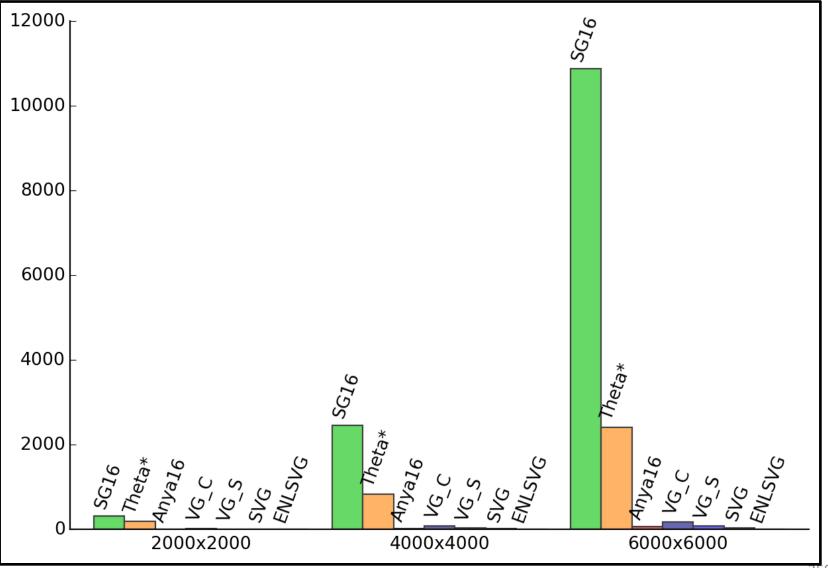


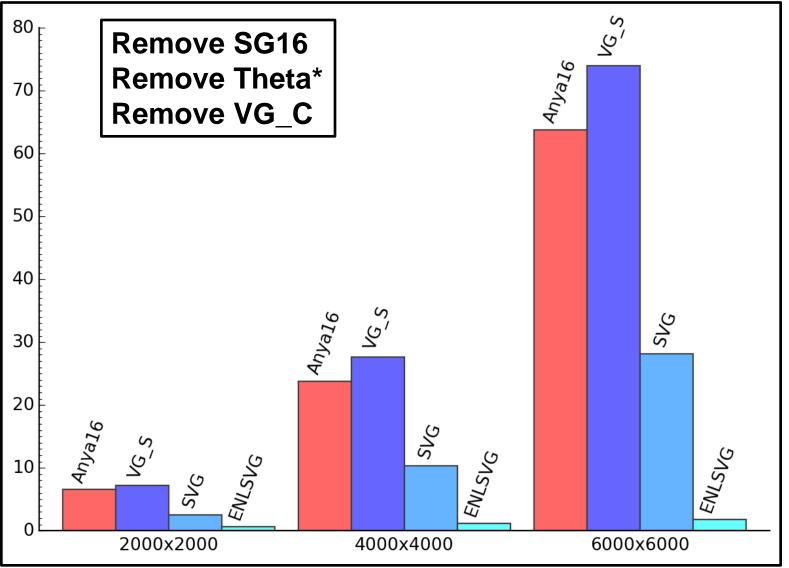




Running Time Cave Maps

45% Blocked





Running Time

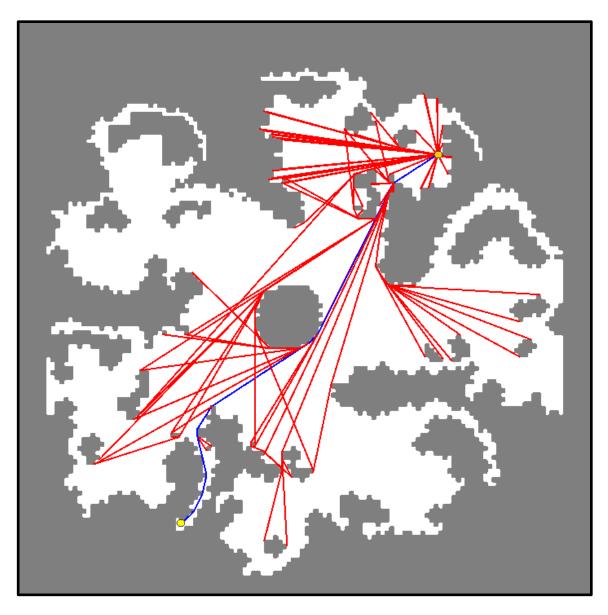
Upscaled Benchmarks

Running Time Upscaled Benchmarks

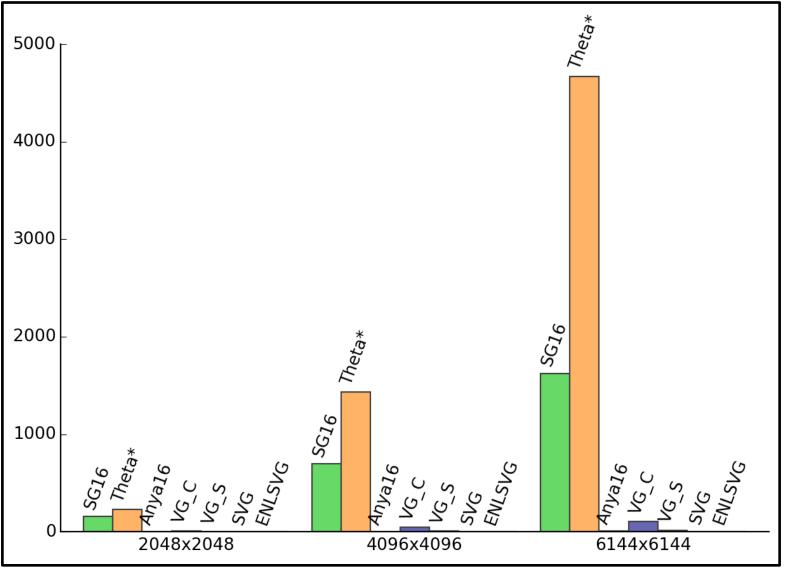
Upscaled Map (wc3, 512x512)

Map Size: 4096x4096

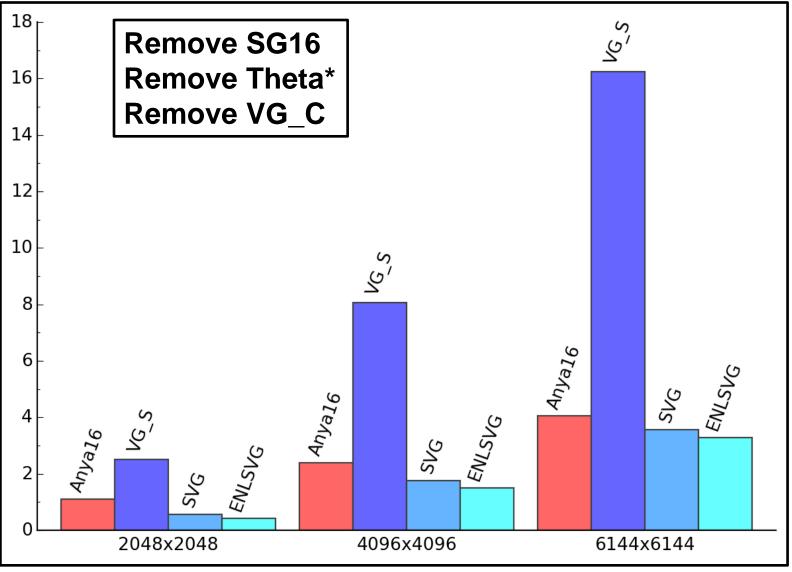
Cellular Automata used for smoothing



Running Time Upscaled Benchmarks



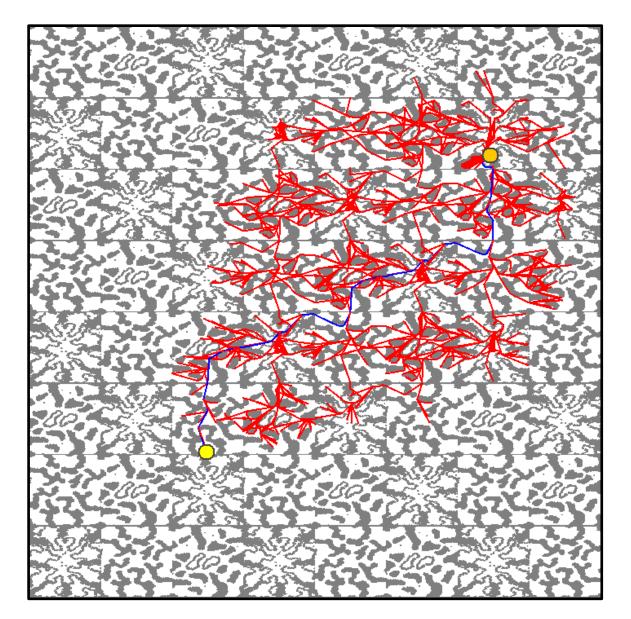
Running Time Upscaled Benchmarks



Running Time

Tiled Benchmarks

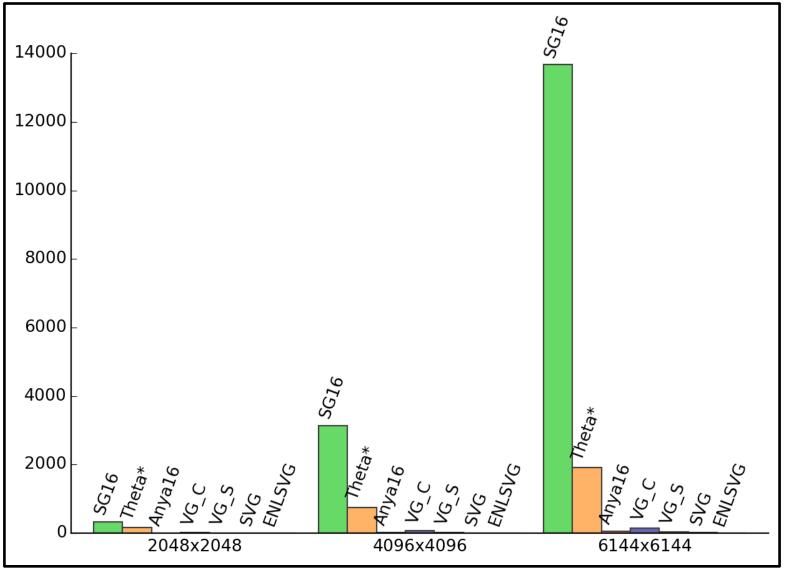
Running Time Tiled Benchmarks



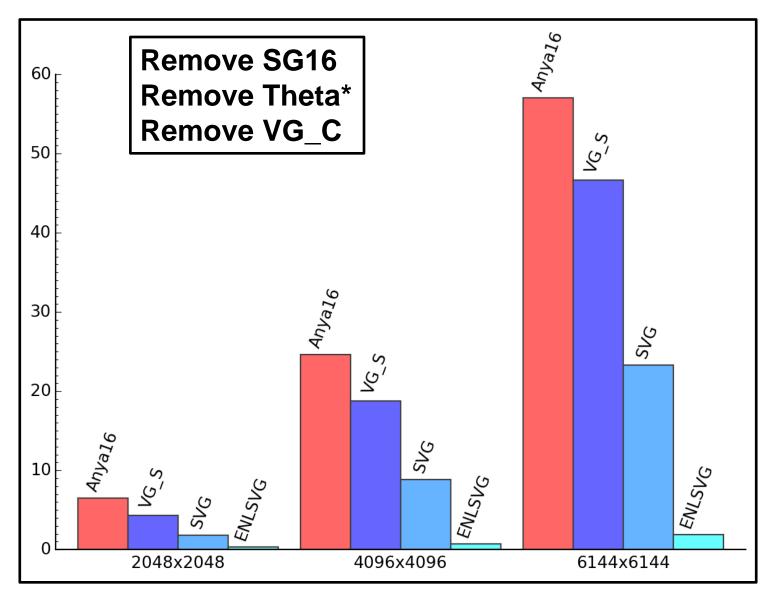
8x8 Tiled Map (wc3, 512x512)

Map Size: 4096x4096

Running Time Tiled Benchmarks



Running Time Tiled Benchmarks

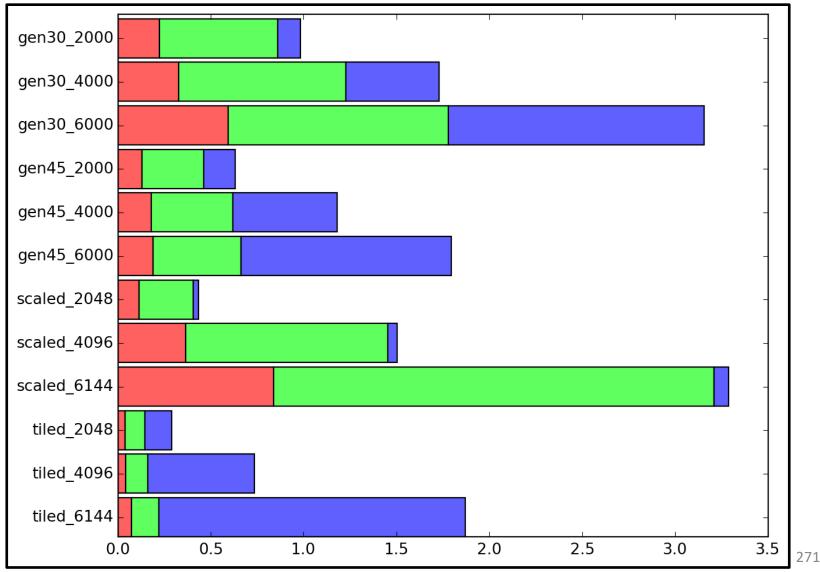


Three Main Components

Insertion

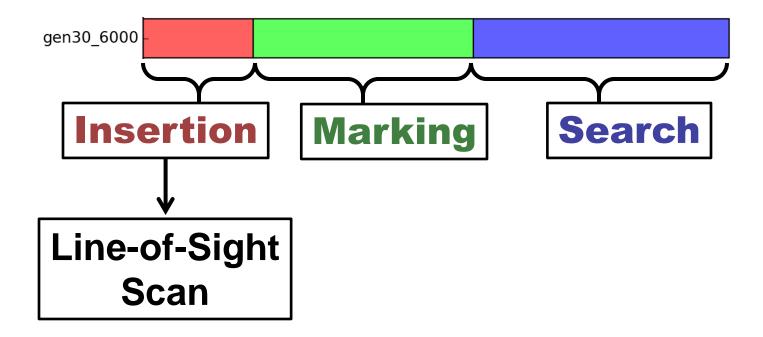
Marking

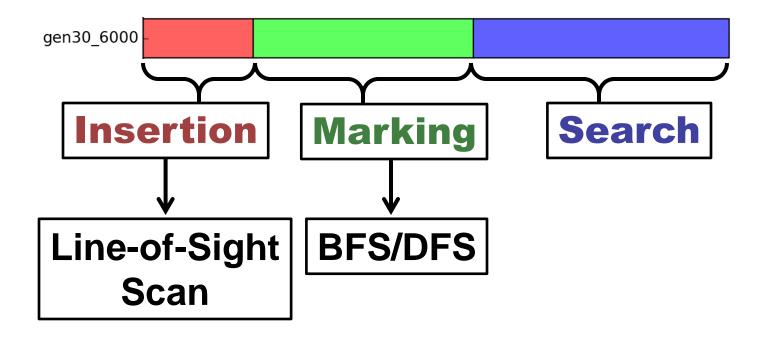
Search

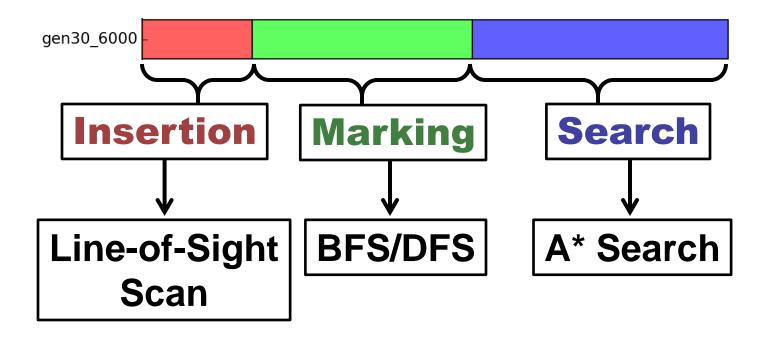


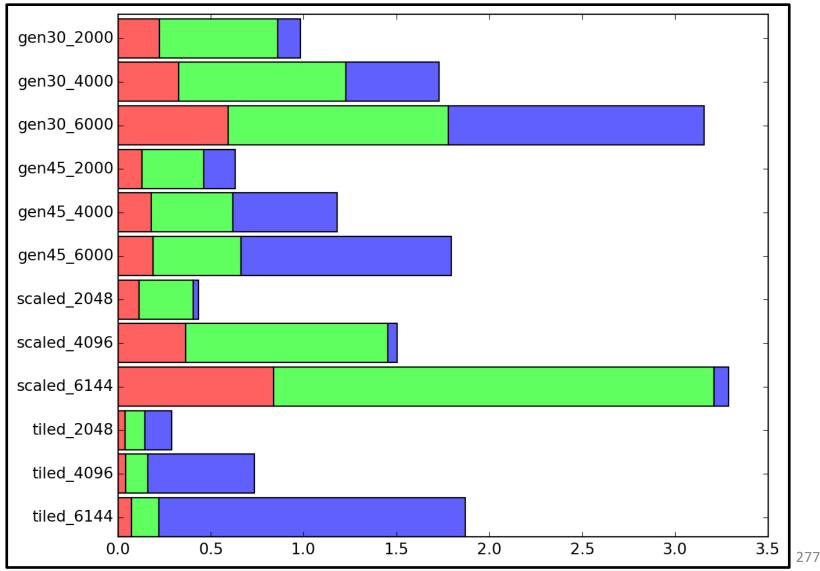


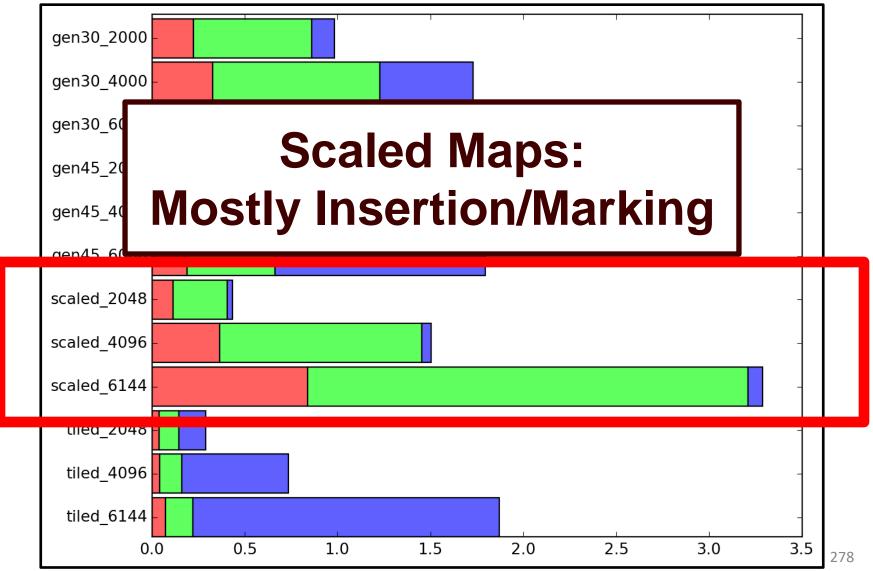


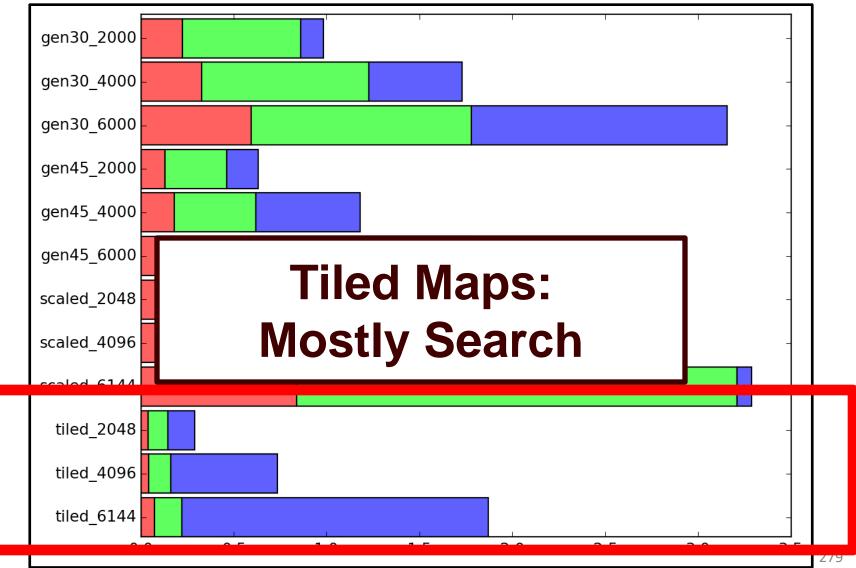












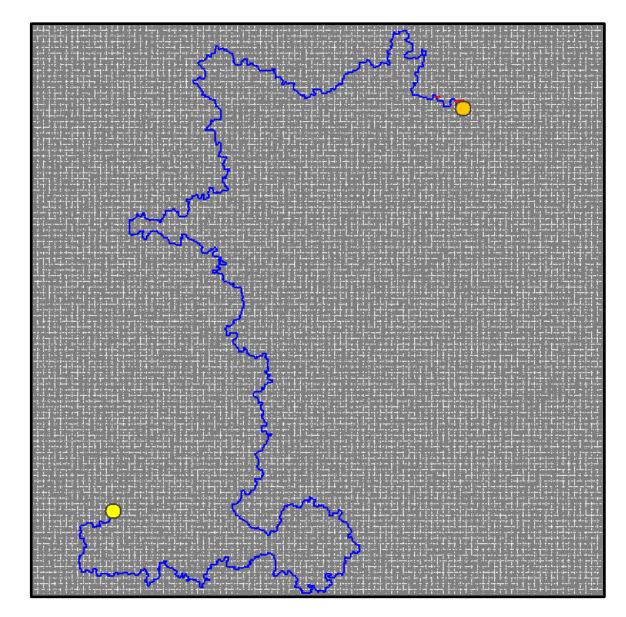
Running Time

Maze Maps

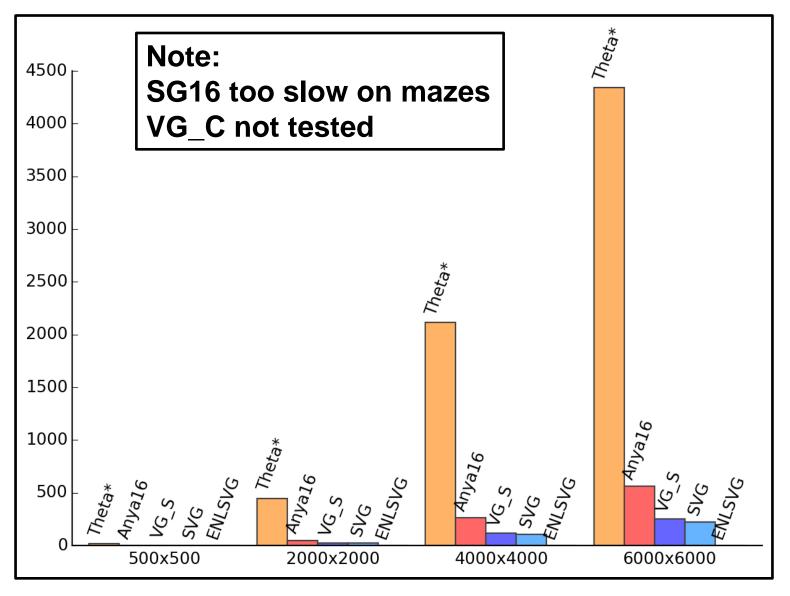
Running Time Maze Maps

2000x2000 Maze Corridor width 2

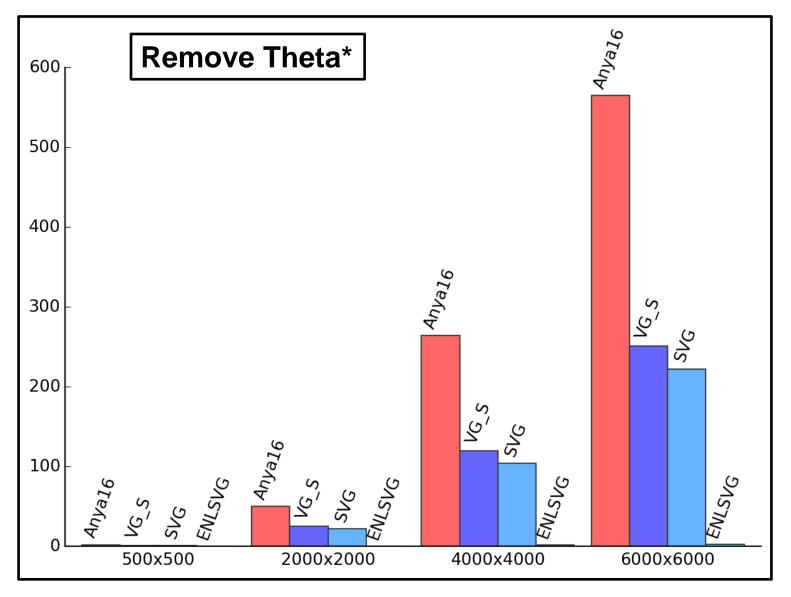
Spanning Tree n-1 edges



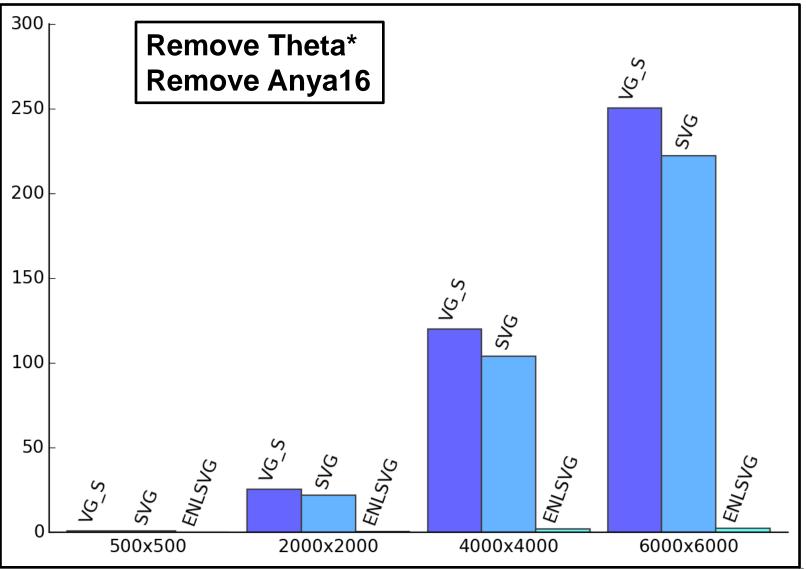
Running Time Maze Maps: Spanning Tree



Running Time Maze Maps: Spanning Tree



Running Time Maze Maps: Spanning Tree

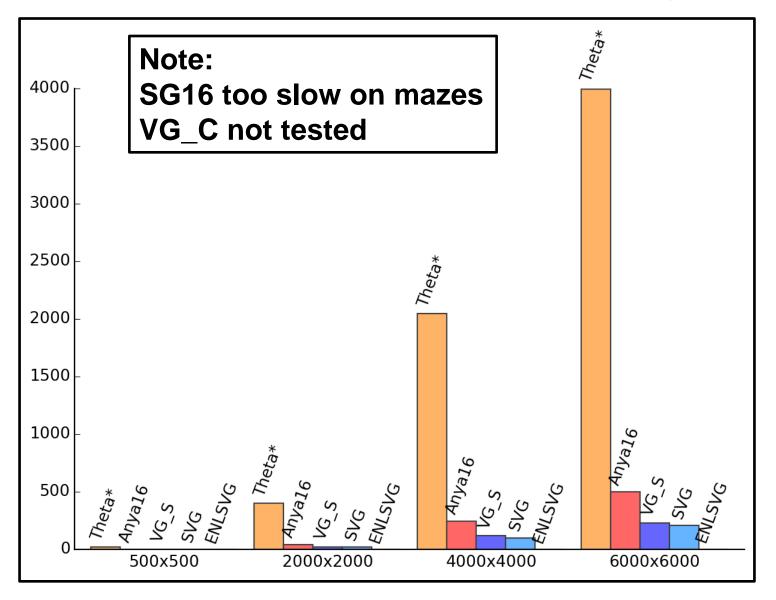


Running Time Maze Maps

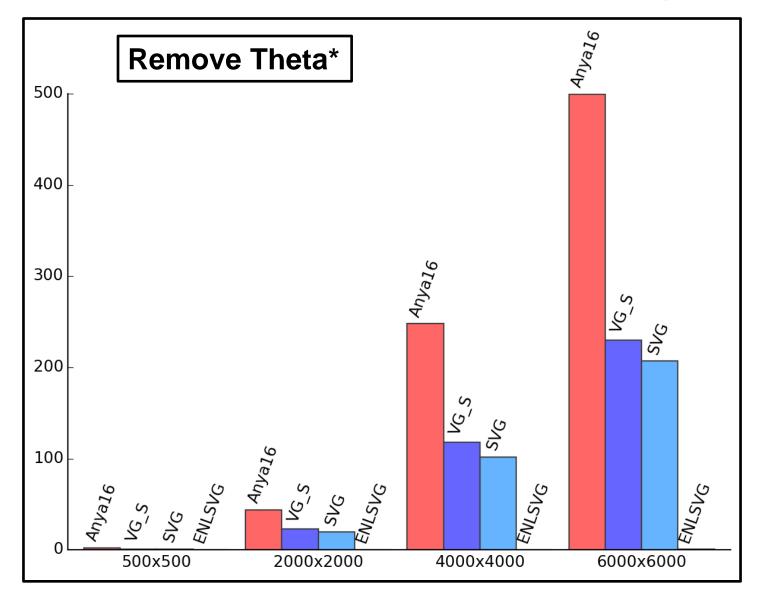
2000x2000 Maze Corridor width 2 Approximately 0.0001n + n - 1

edges

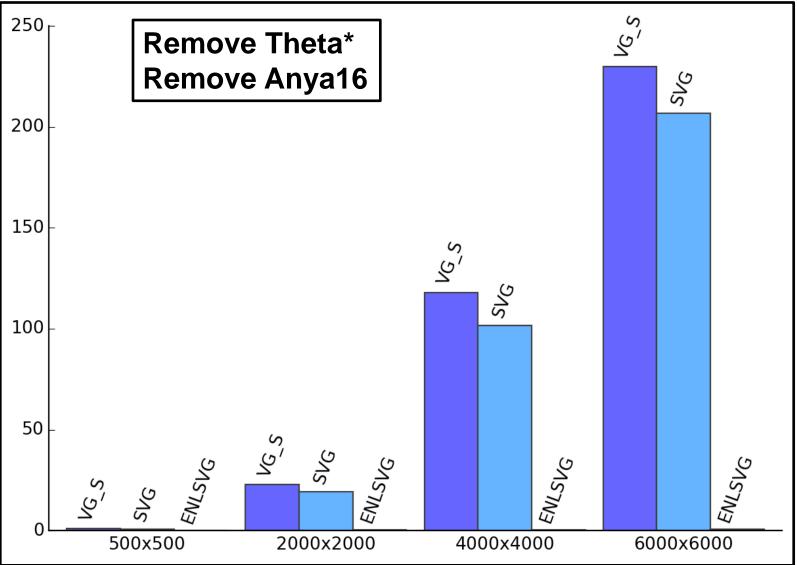
Running Time Maze Maps: +0.0001*n* edges



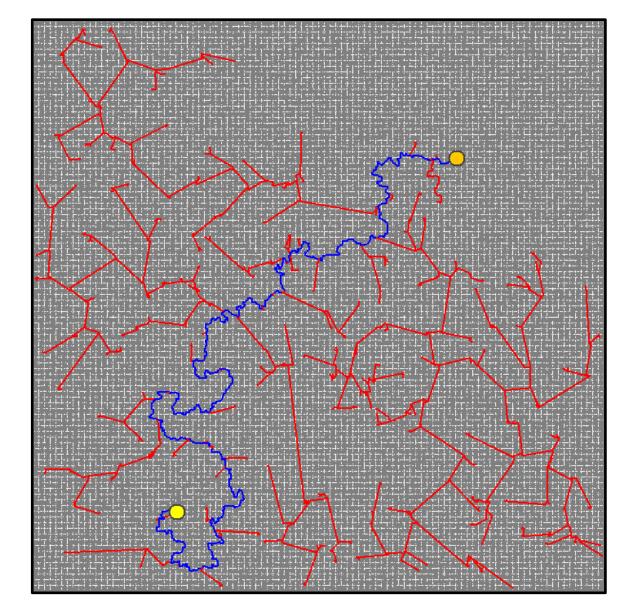
Running Time Maze Maps: +0.0001*n* edges



Running Time **Maze Maps:** +0.0001n edges



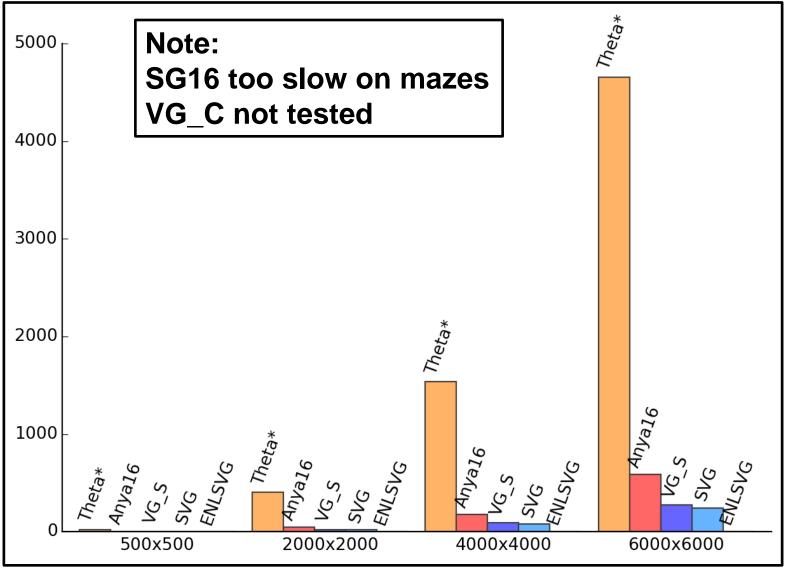
Running Time Maze Maps



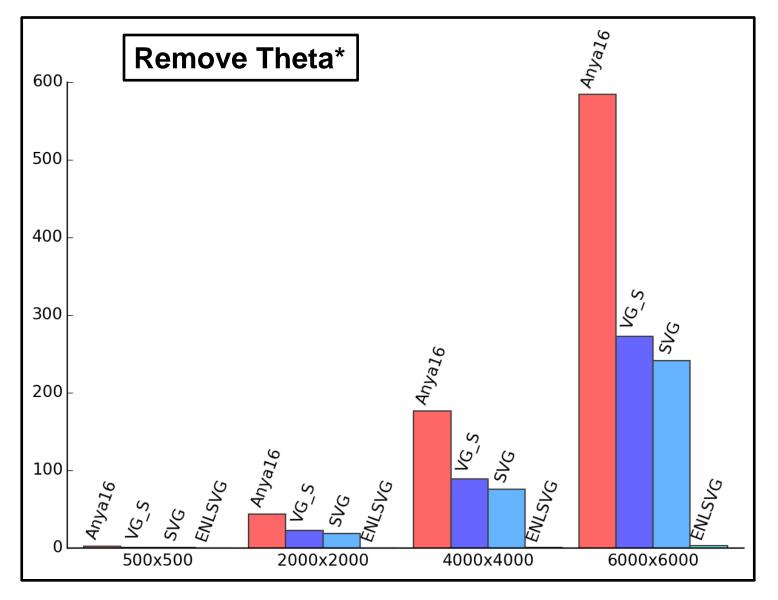
2000x2000 Maze Corridor width 2

Approximately 0.001n + n - 1 edges

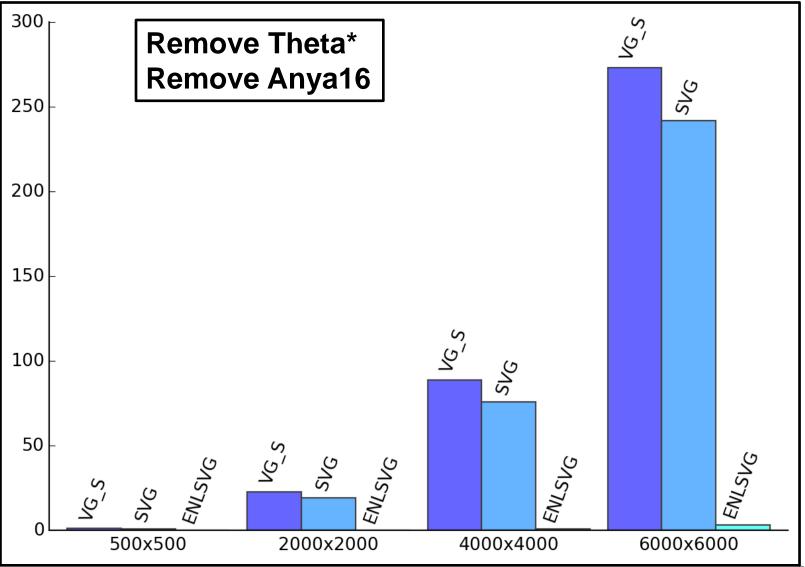
Running Time **Maze Maps:** +0.001n edges



Running Time **Maze Maps:** +0.001n edges



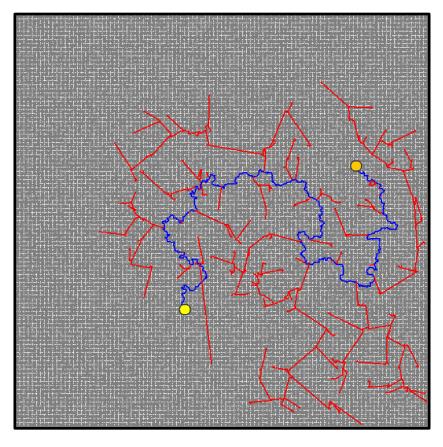
Running Time Maze Maps: +0.001*n* edges

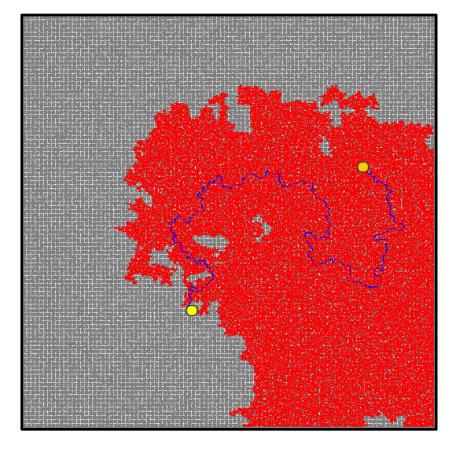


Running Time Maze Maps

ENLSVG Search Tree

SVG Search Tree







github.com/Ohohcakester/Any-Angle-Pathfinding

Implementation github.com/Ohohcakester/Any-Angle-Pathfinding **Google for "Any Angle Pathfinding"**



Edge N-Level Sparse Visibility Graphs: Fast Optimal Any-Angle Pathfinding Using Hierarchical Taut Paths

Shunhao Oh ohoh@u.nus.edu Hon Wai Leong leonghw@comp.nus.edu.sg

Department of Computer Science National University of Singapore

Implementation:

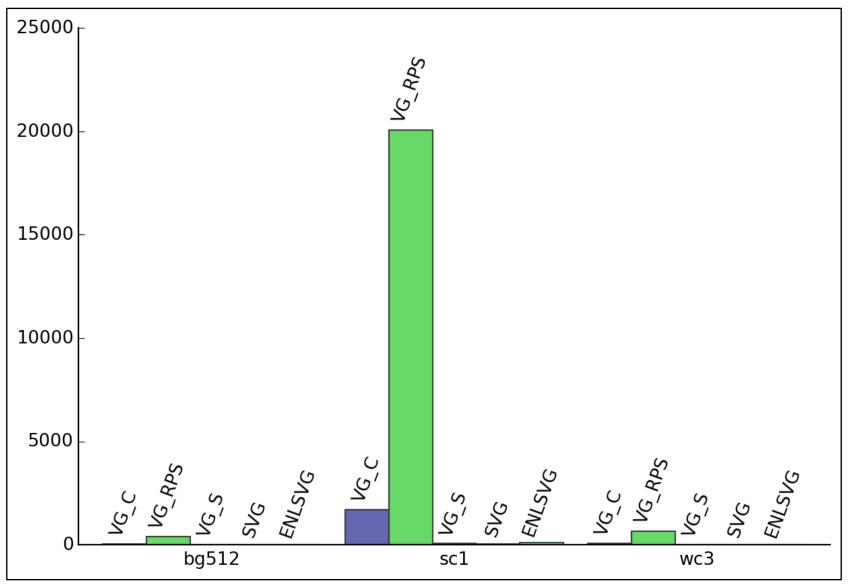
github.com/Ohohcakester/Any-Angle-Pathfinding

extra

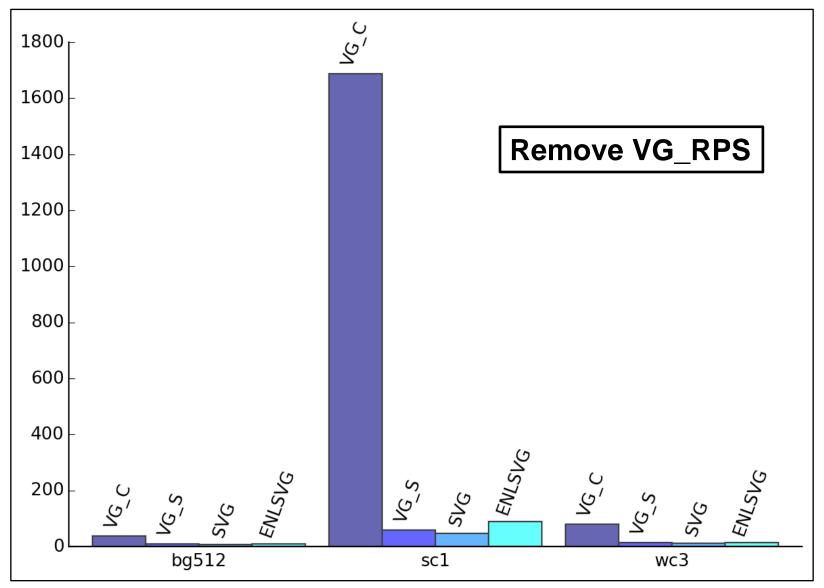
Construction Time

Benchmarks (Game) Construction Time

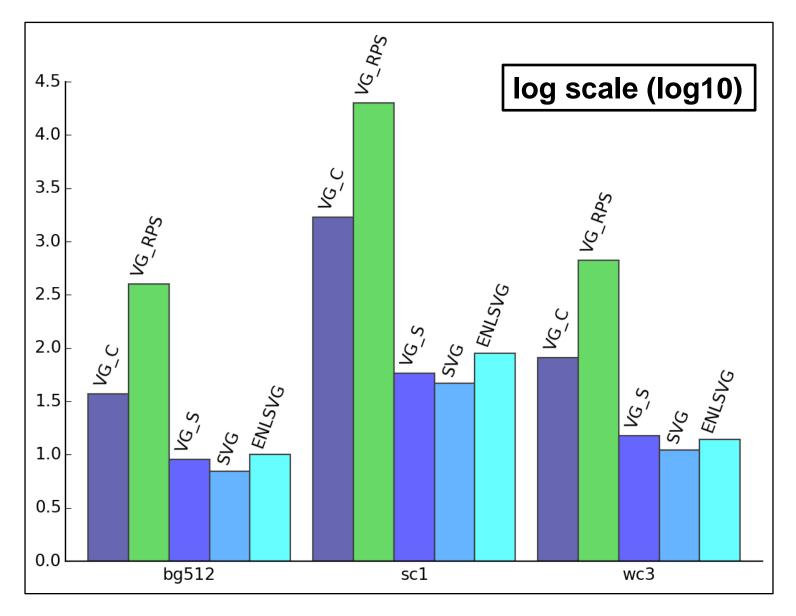
Construction Time (ms)



Construction Time (ms)



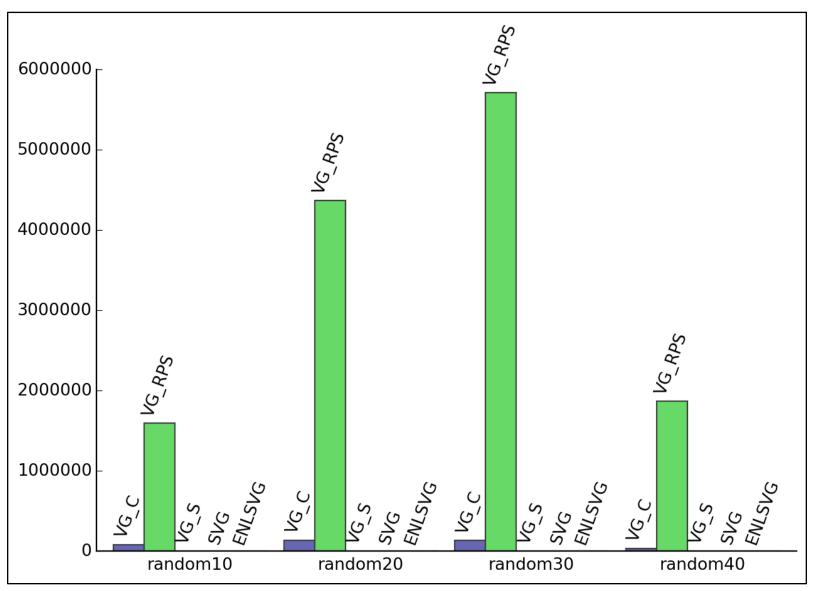
Construction Time (log10, ms)



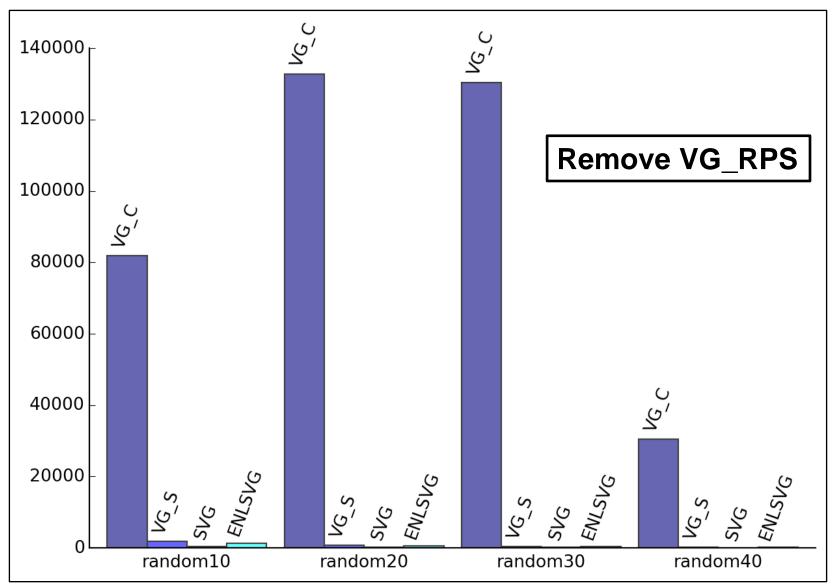
Construction Time

Benchmarks (Random) Construction Time

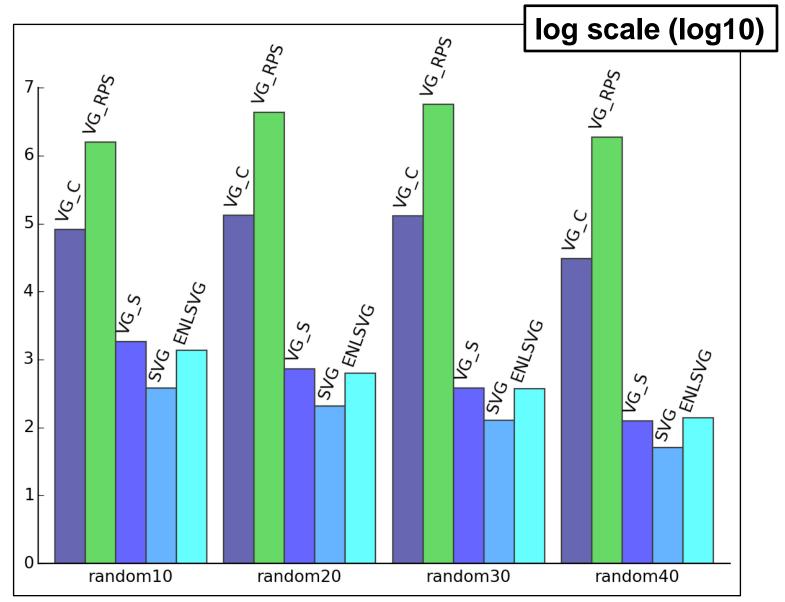
Construction Time (ms)



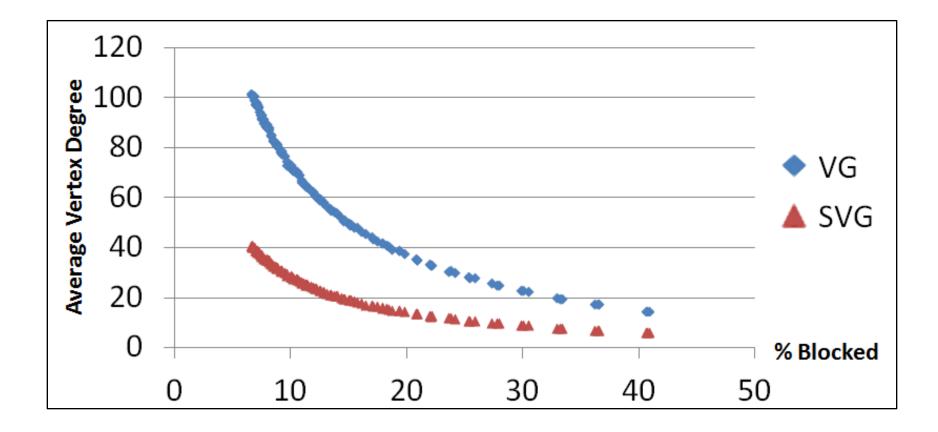
Construction Time (ms)



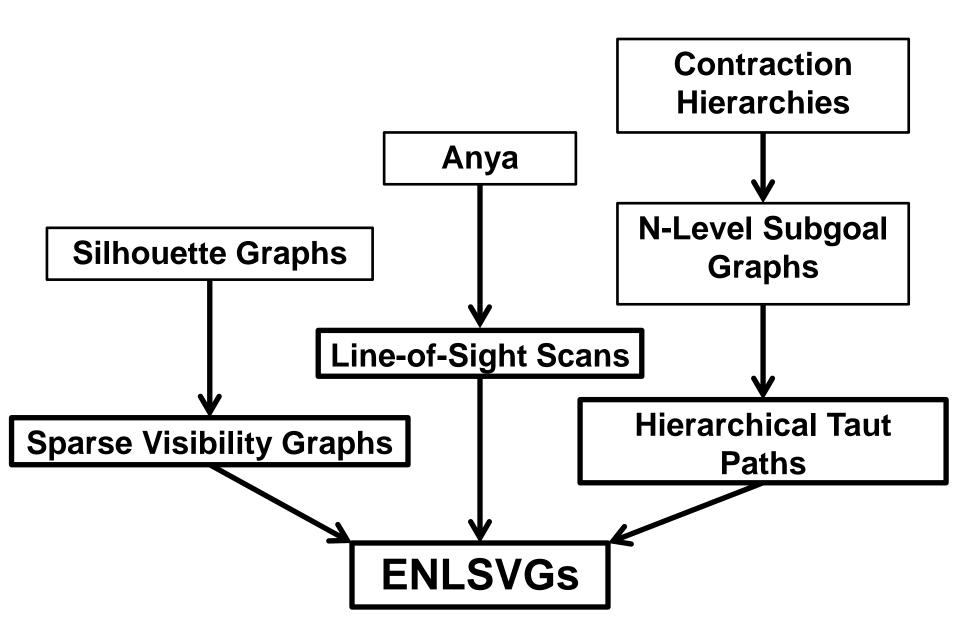
Construction Time (log10, ms)



Average Vertex Degree on Random Maps



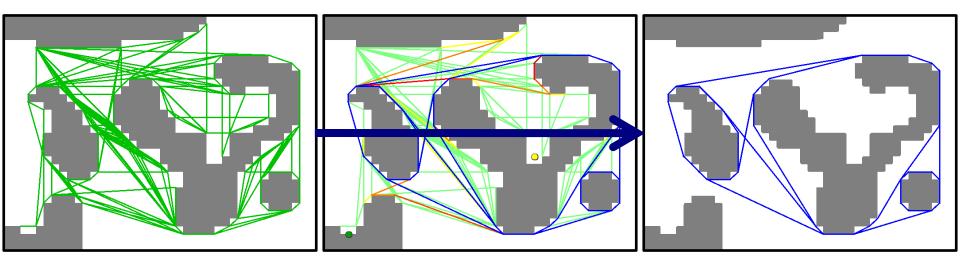
Roadmap



Roadmap

Silhouette Graphs

Tangent Graphs



Level ≥ 0

ENLSVG

Level W

ACKNOWLEDGEMENT

This presentation benefitted from **PowerPointLabs**

a PowerPoint plugin for creating better presentations with less effort.

PowerPointLabs is available for free at http://PowerPointLabs.info