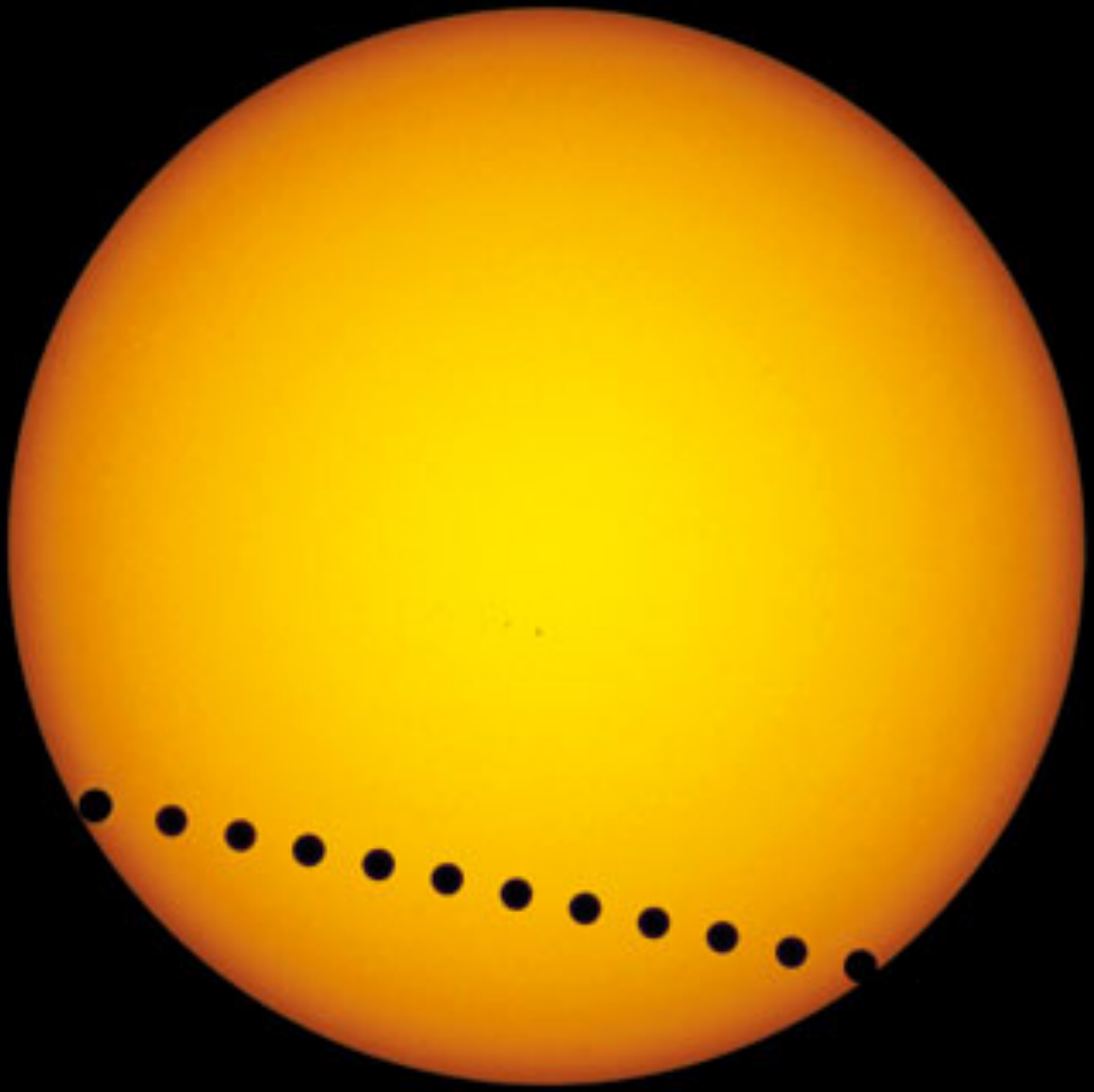
Figures for lecture notes

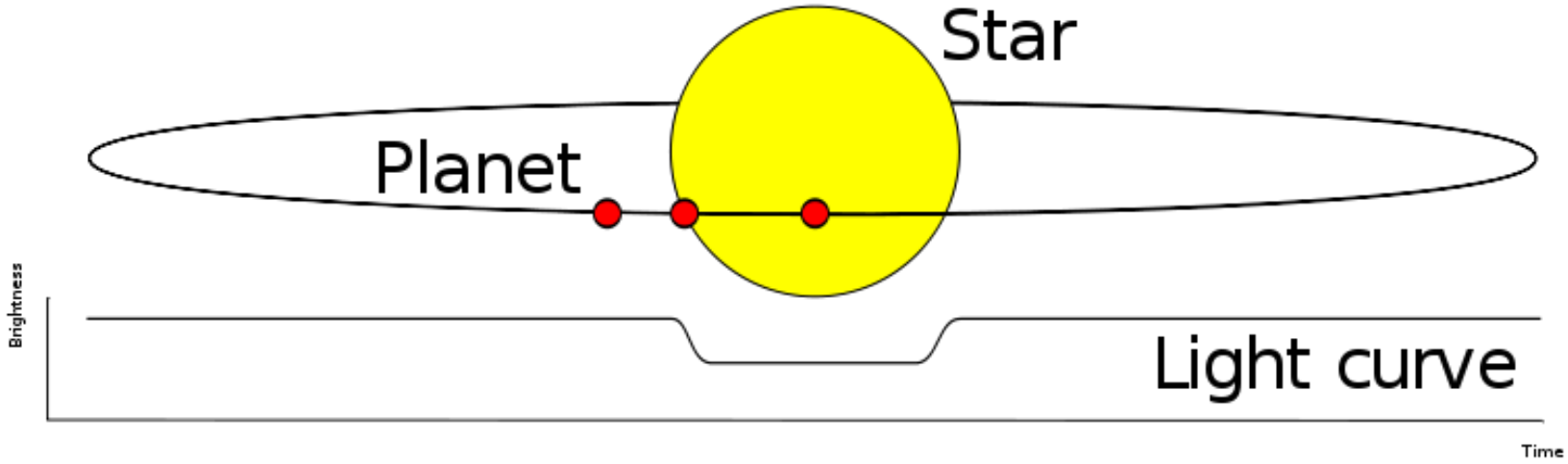
Daniel Alexander

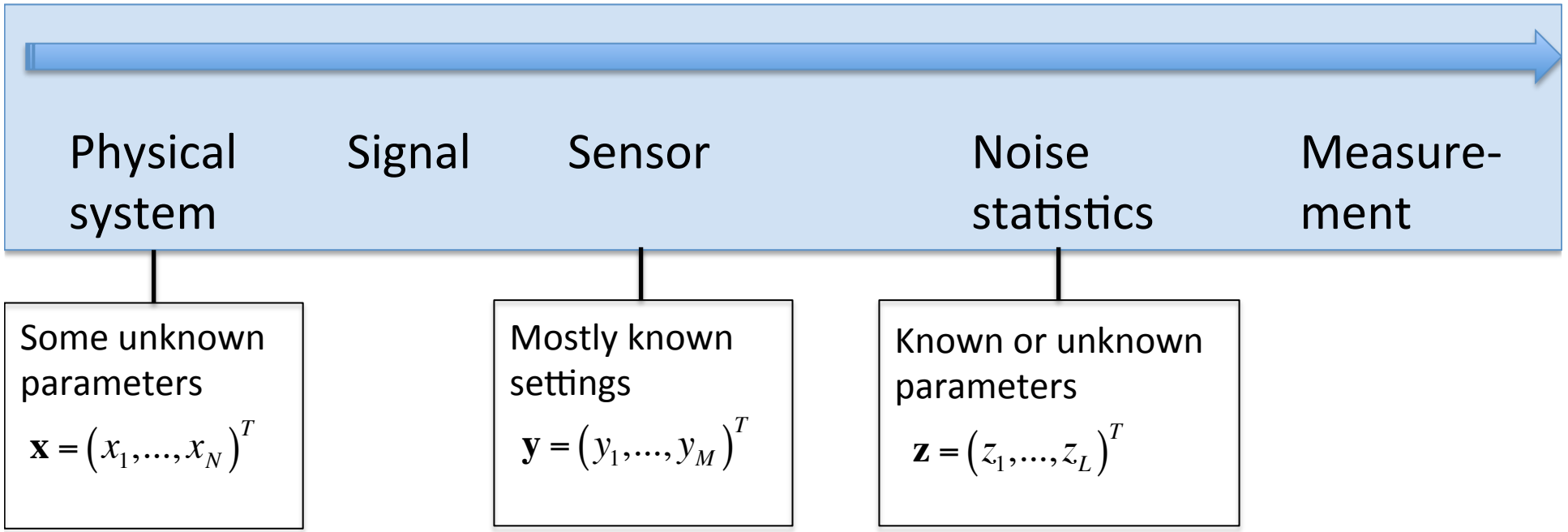
Venus transit of the sun.

Pasachoff
Nature **485**,
303–304, 2012.

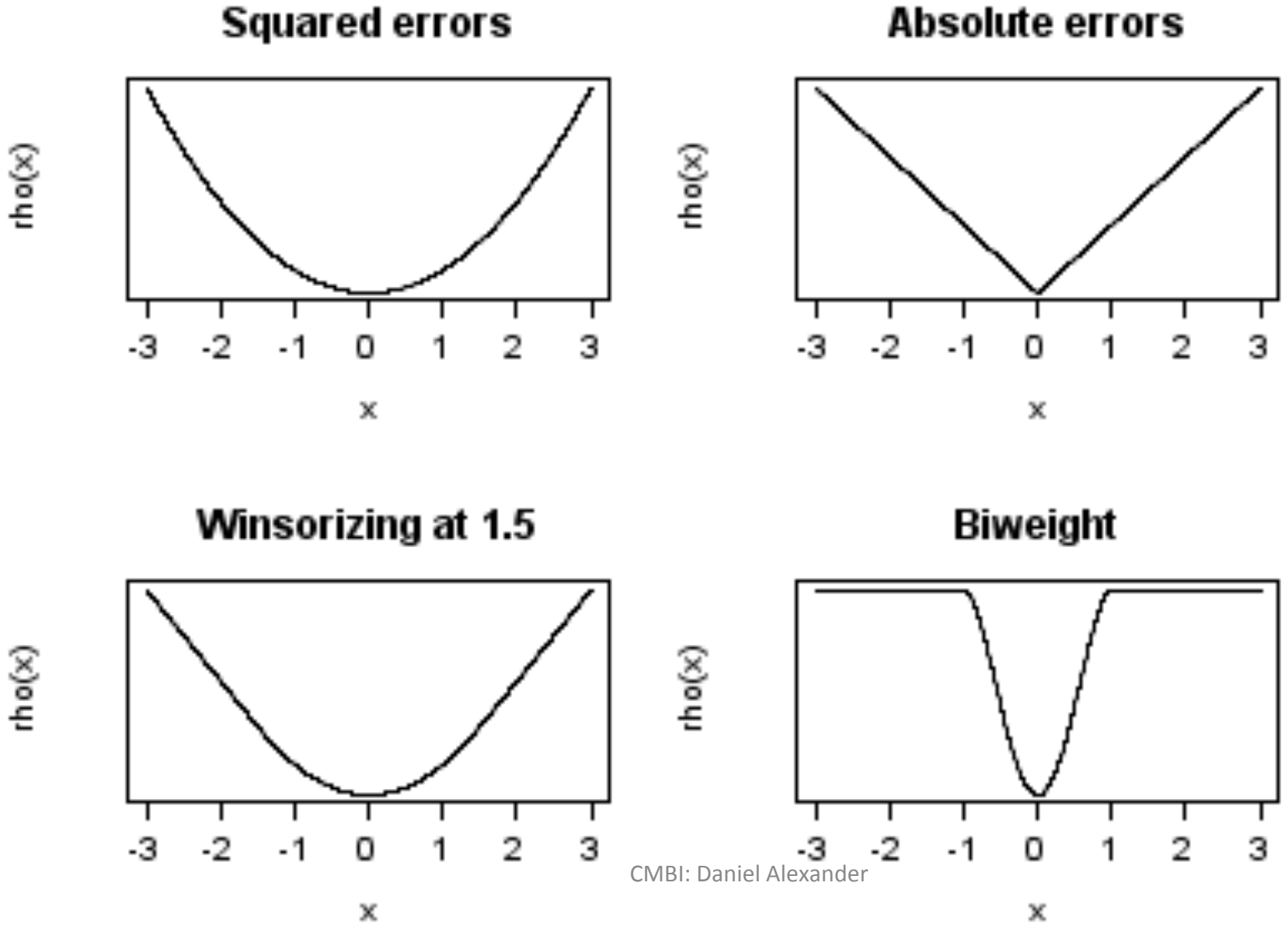


Example: exoplanets

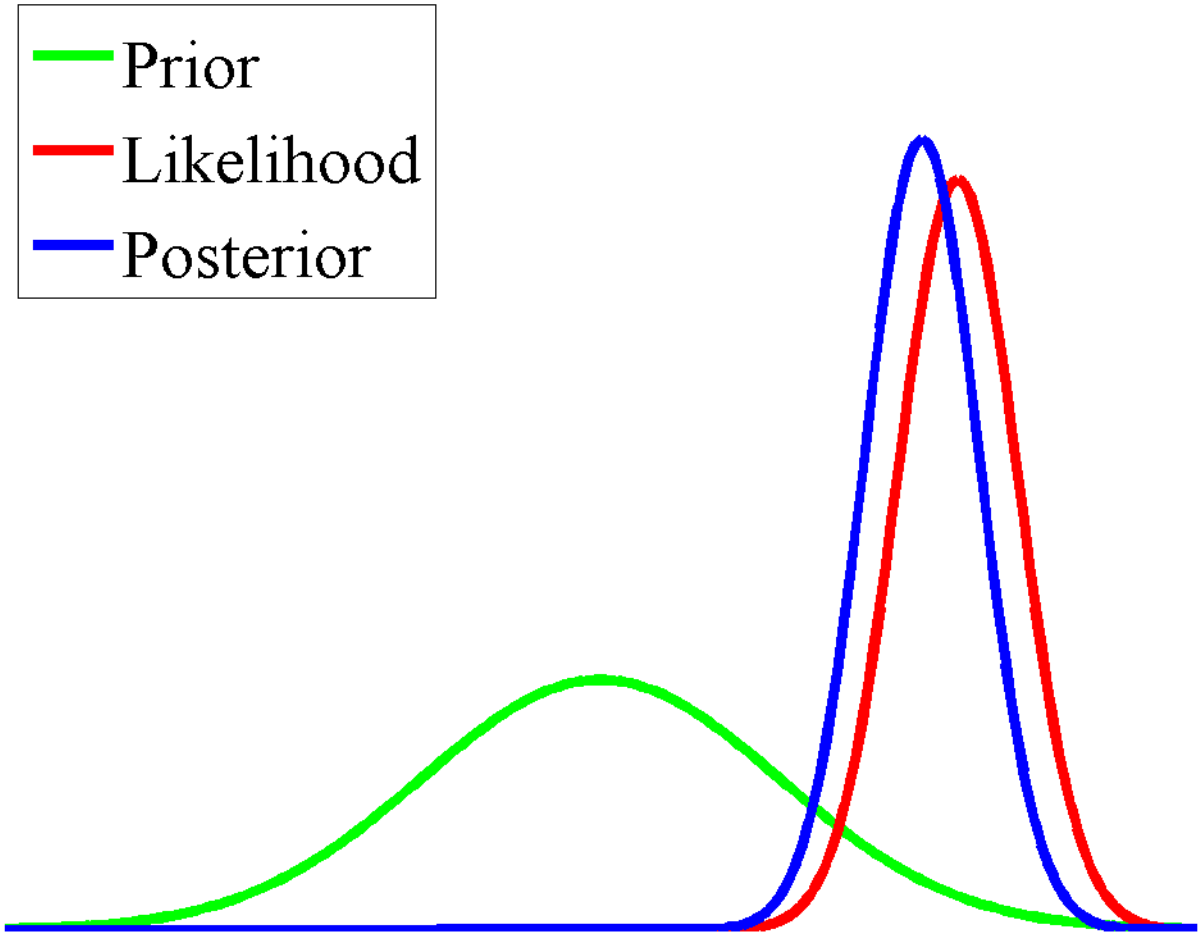



Physical system
Signal
Sensor
Noise statistics
Measure-ment
Some unknown parameters
$\mathbf{x} = (x_1, \ldots, x_N)^T$
Mostly known settings
$\mathbf{y} = (y_1, \ldots, y_M)^T$
Known or unknown parameters
$\mathbf{z} = (z_1, \ldots, z_L)^T$

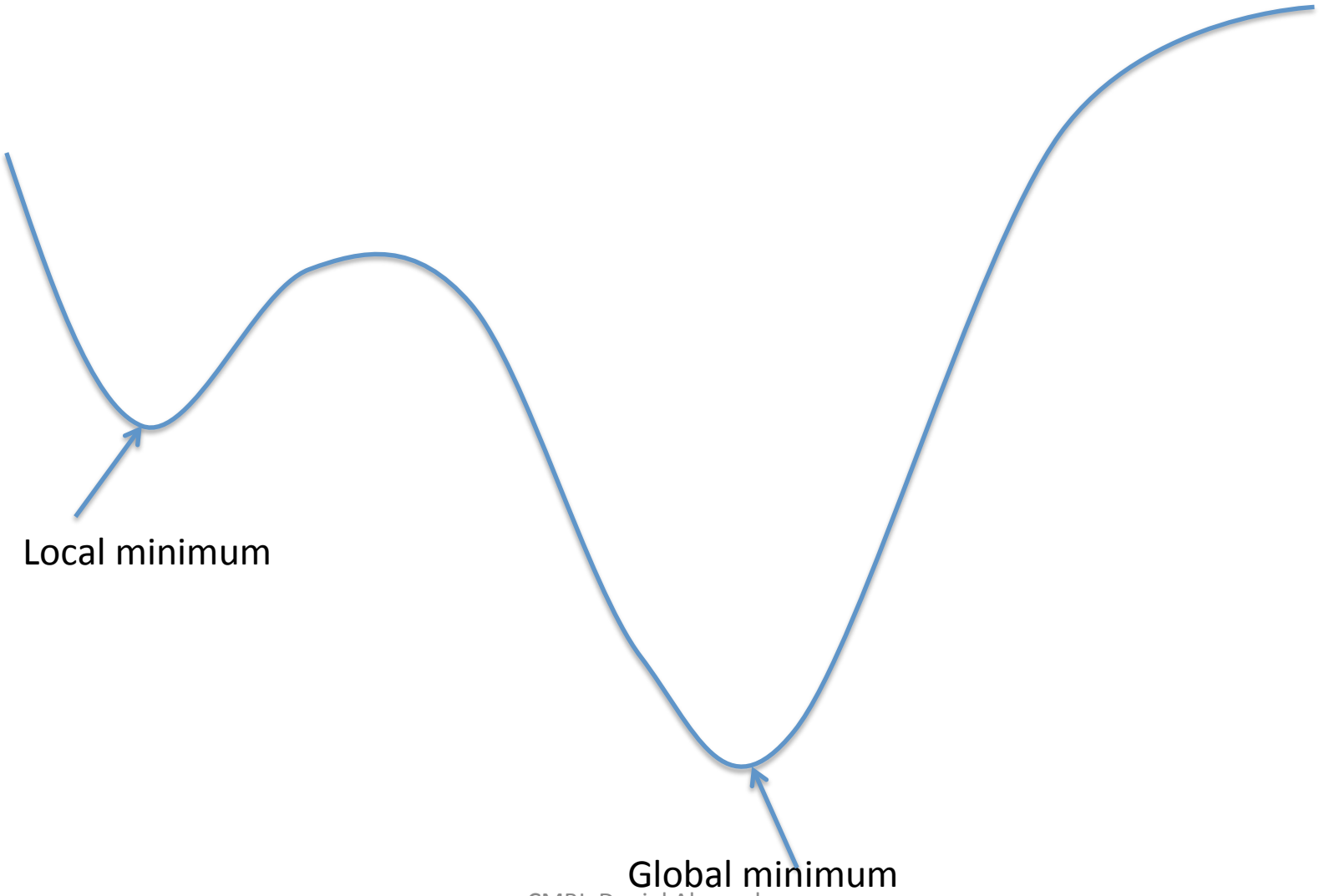
M-estimators



Uncertainty

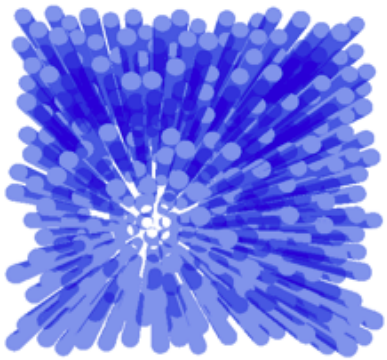


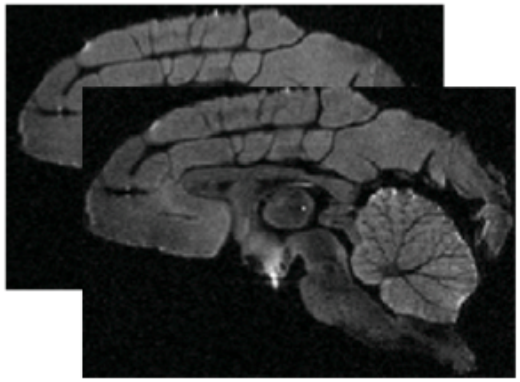
Global and local minima

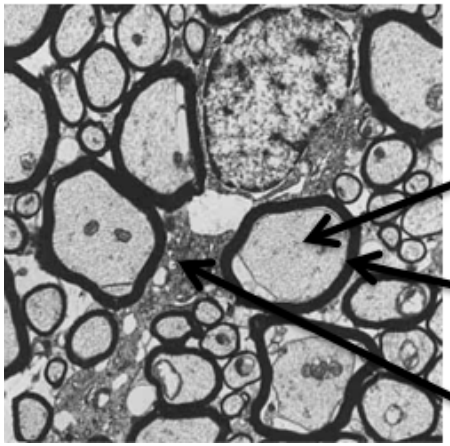


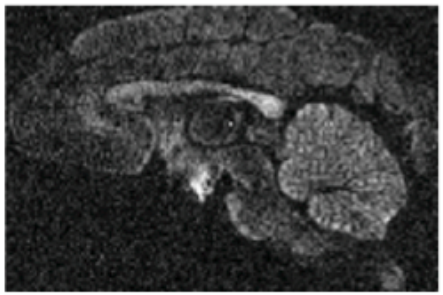
Global and local minima

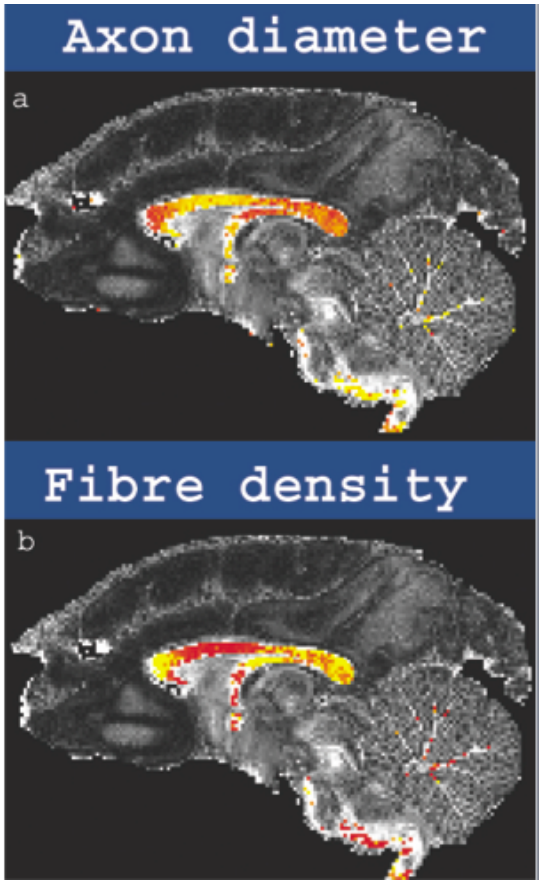
Global and local minima






Axon
Myelin sheath
Interstitium




Axon diameter
a
Fibre density
b

Water diffusion in Brain Tissue

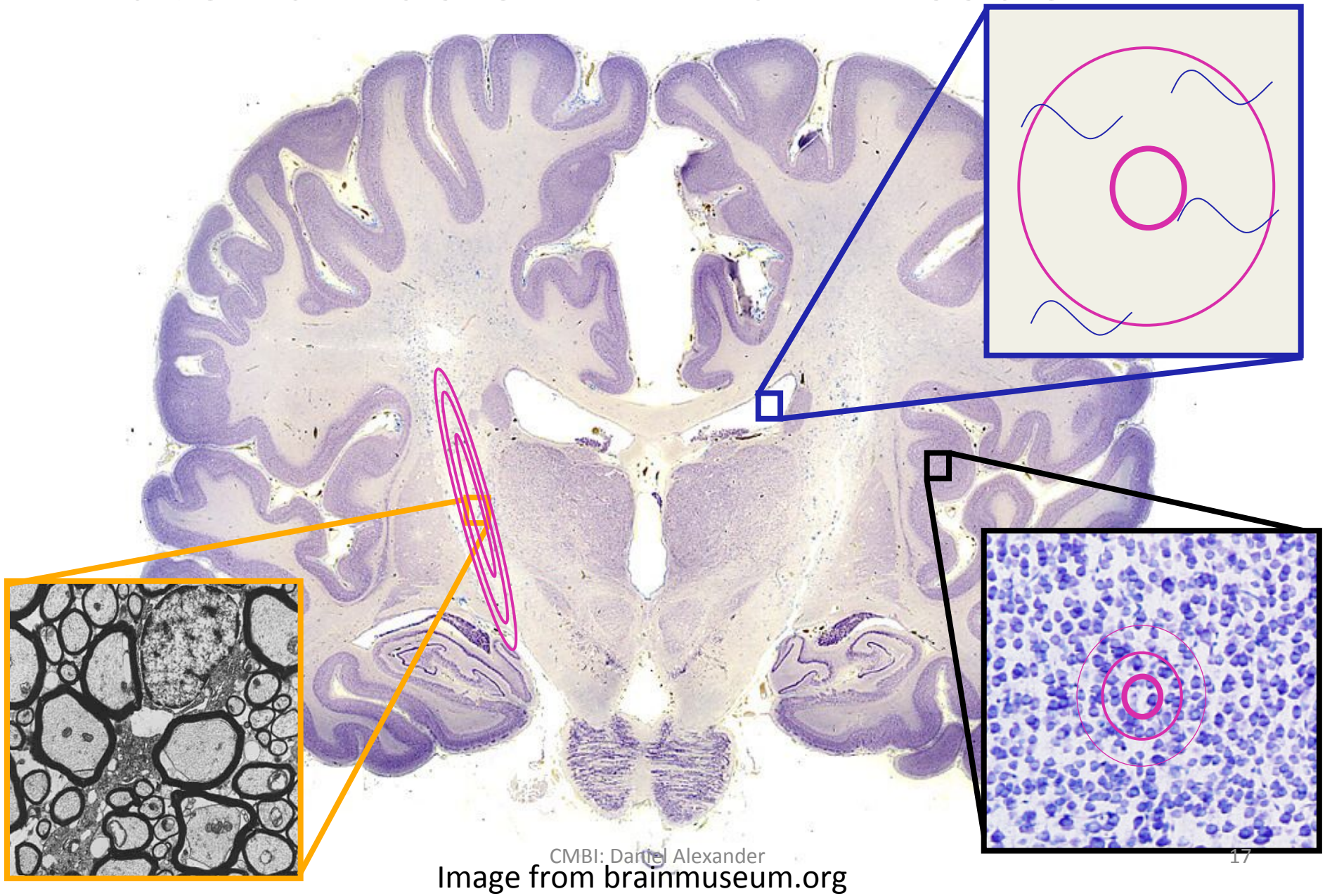
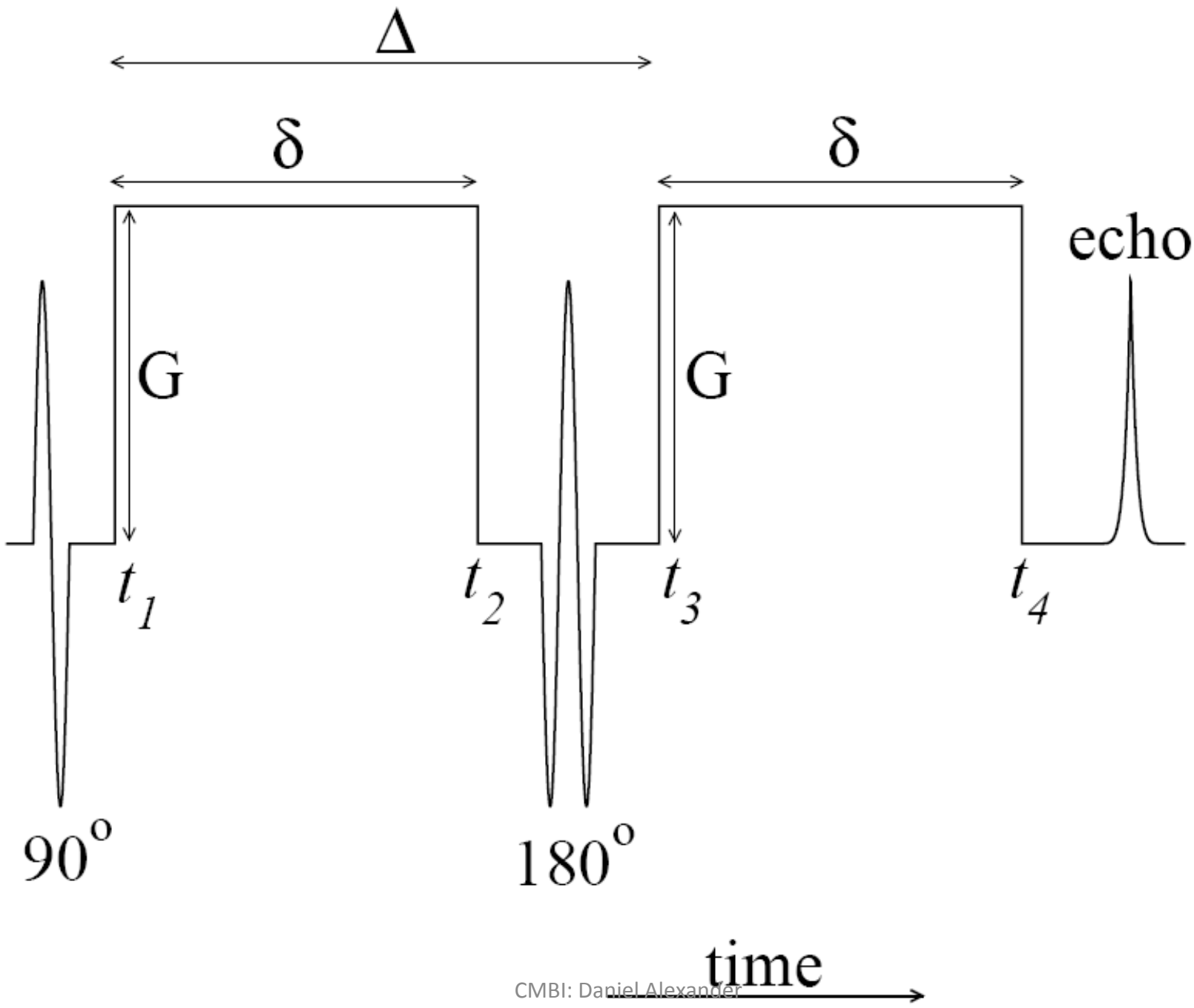
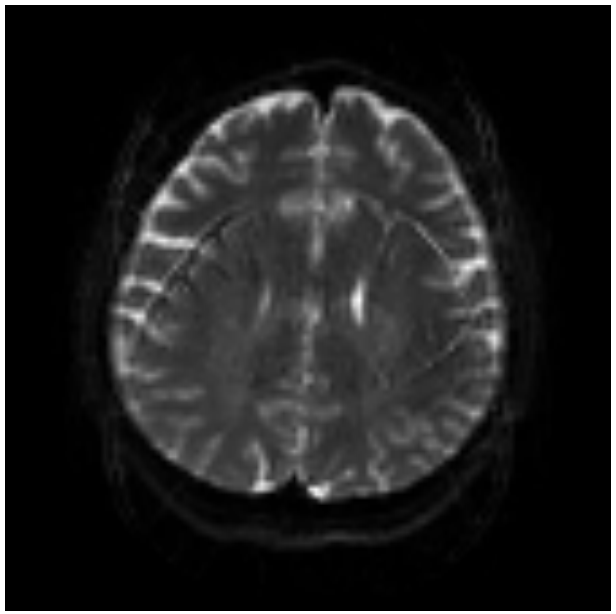


Image from brainmuseum.org

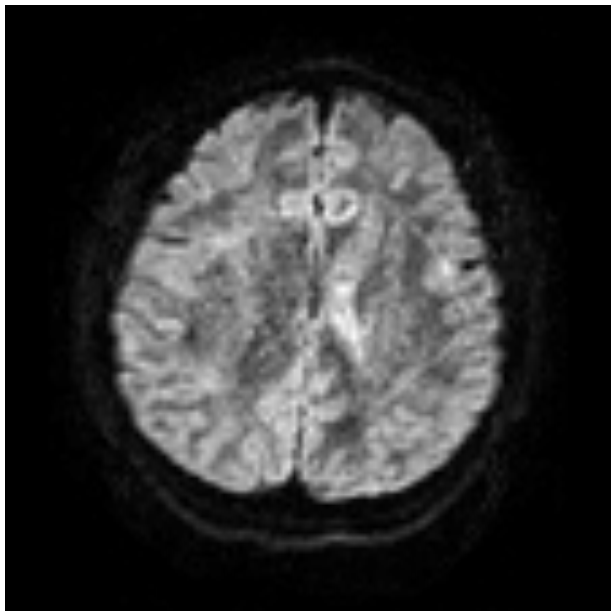

Δ
δ
δ
G
G
echo
t_1
t_2
t_3
t_4
90°
180°
time

Diffusivity estimate from two measurements

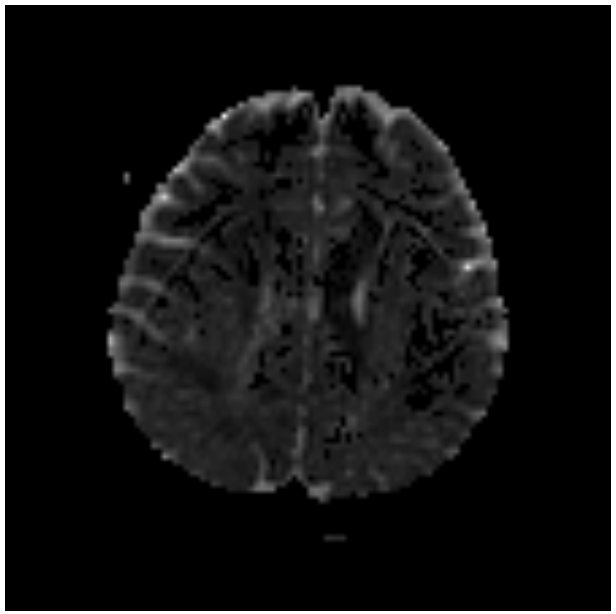
$b = 0$ image, i.e. $A(0)$



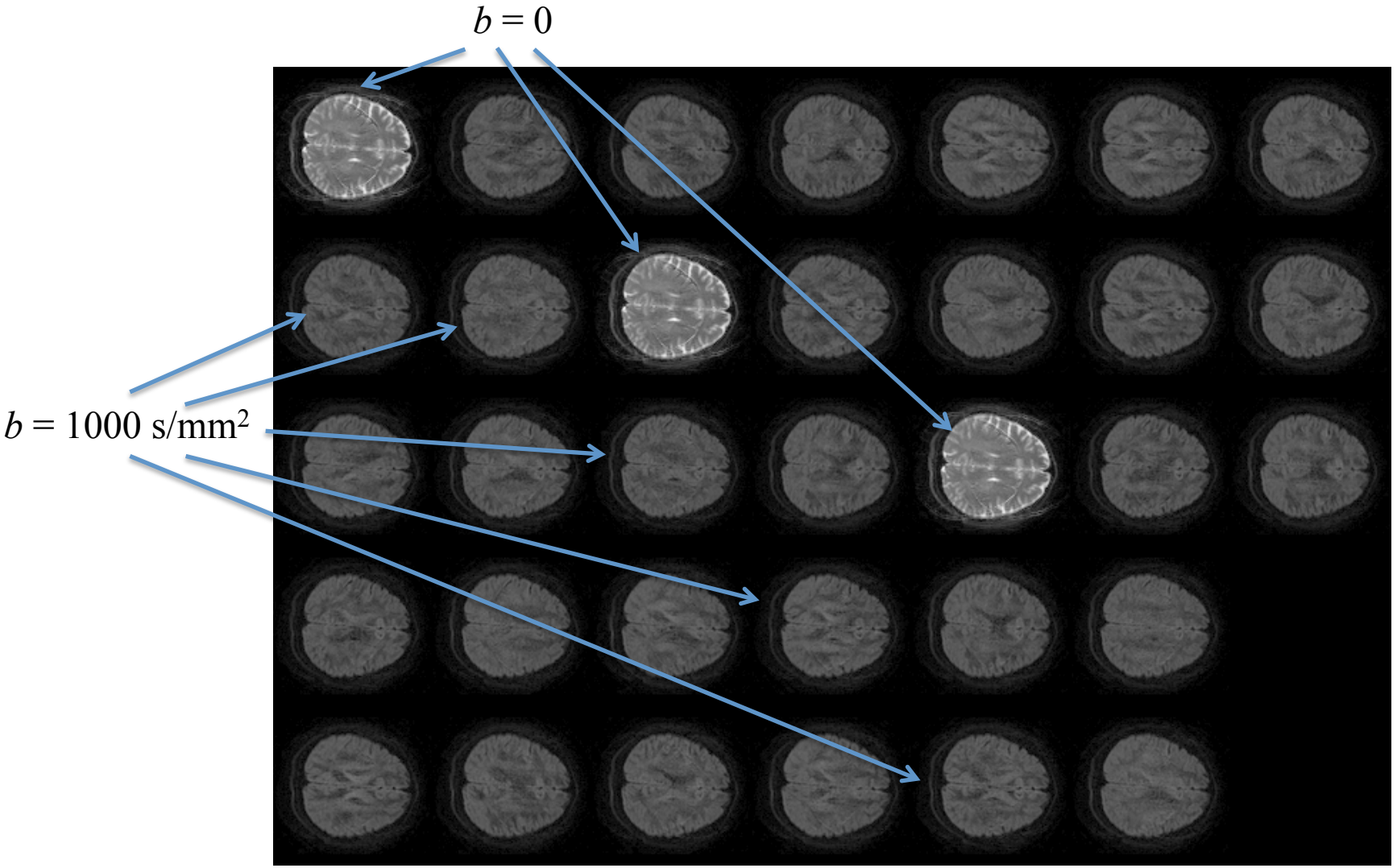
$b = 1000$ s/mm^2 image, i.e. $A(1000)$



Parametric map of d (mm^2/s)

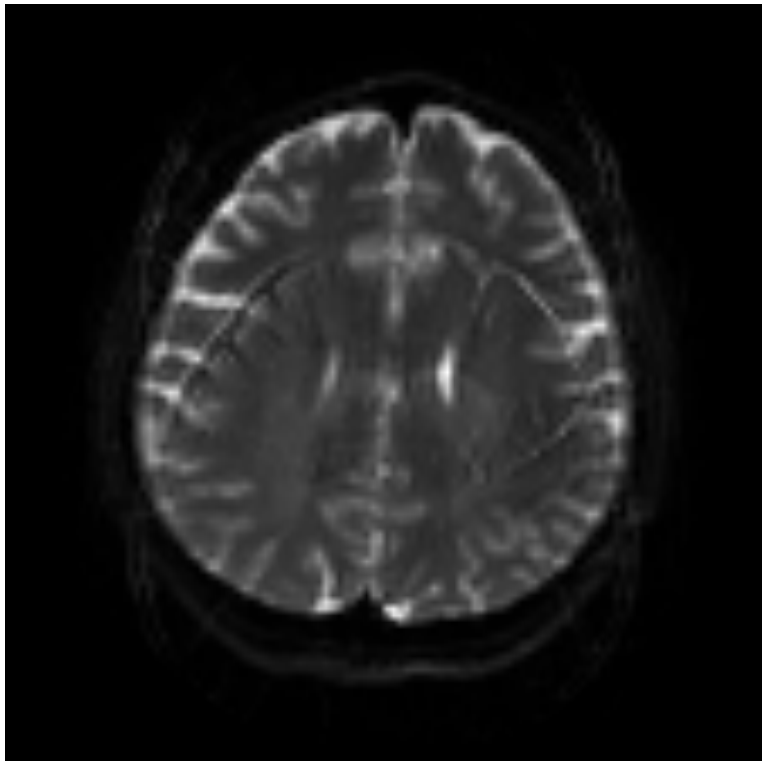


$$d = b^{-1}(\log A(0) - \log A(b))$$

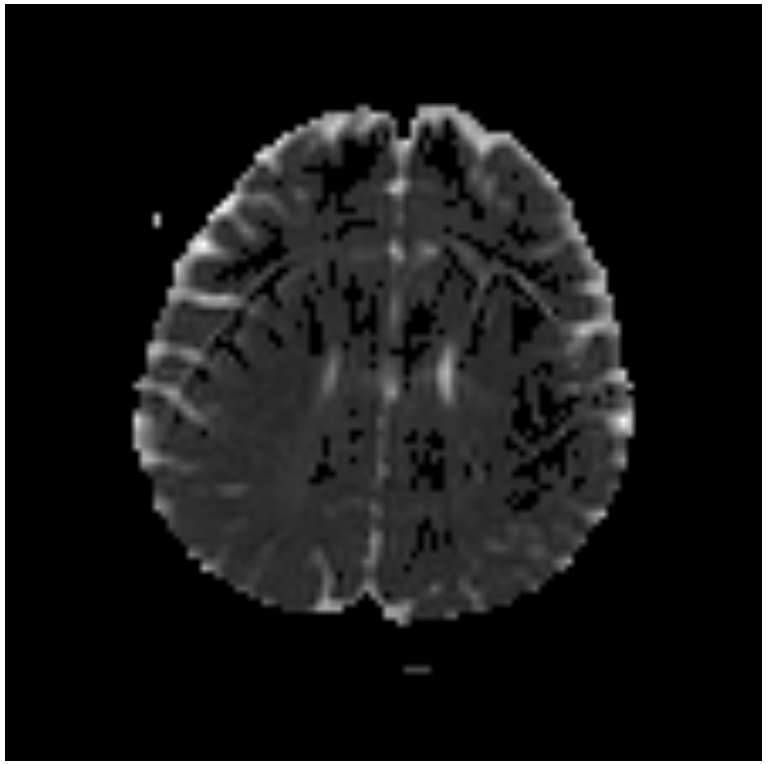

$b = 0$
$b = 1000$ s/mm^2

Parameter maps

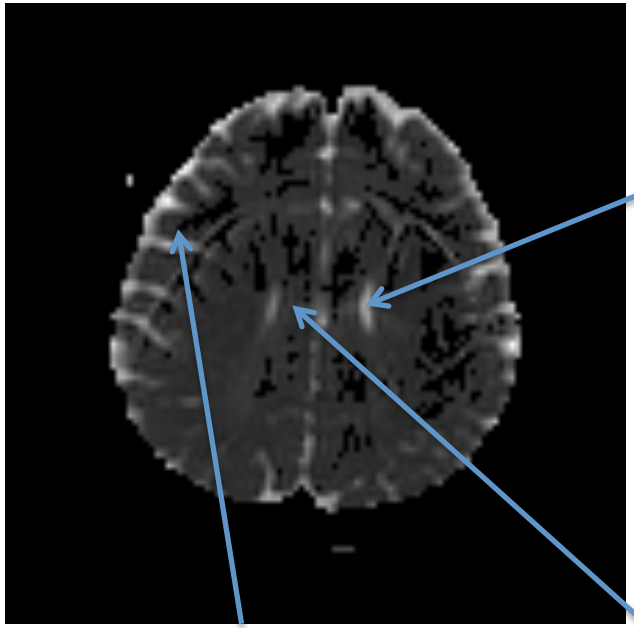
$S(0)$ a.u.

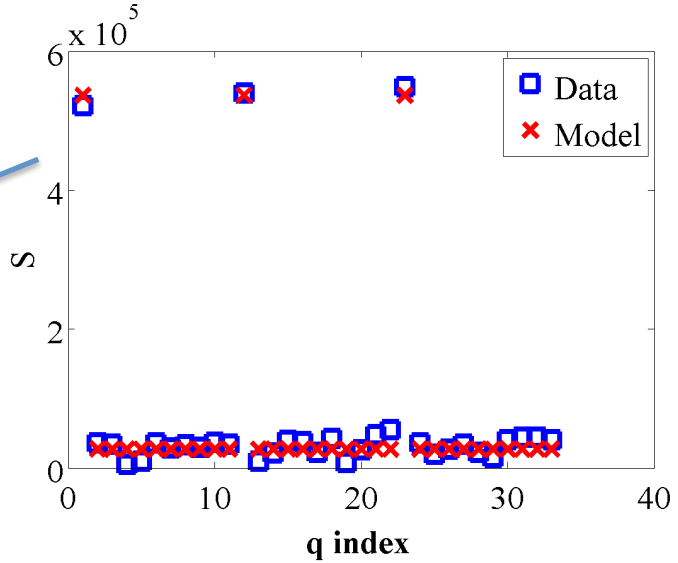


d (mm^2/s)

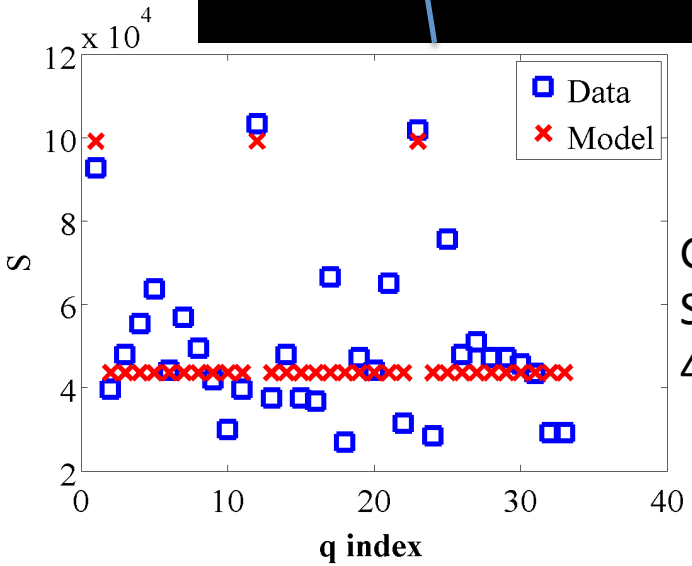


Eyeball the fit to the data

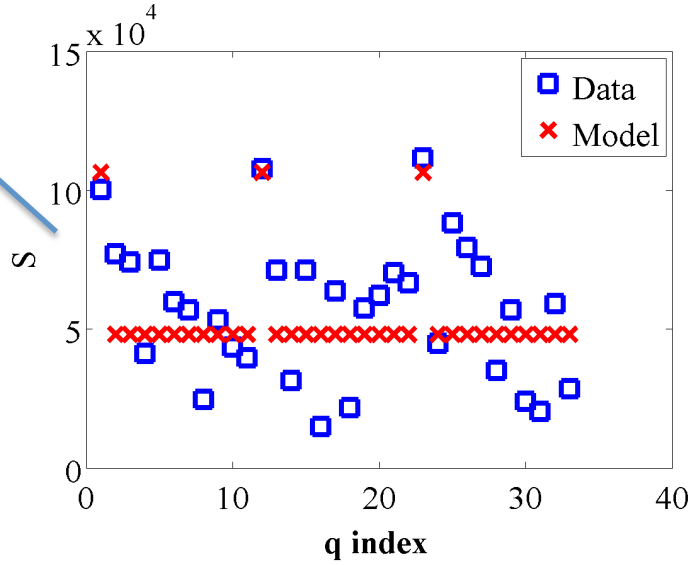




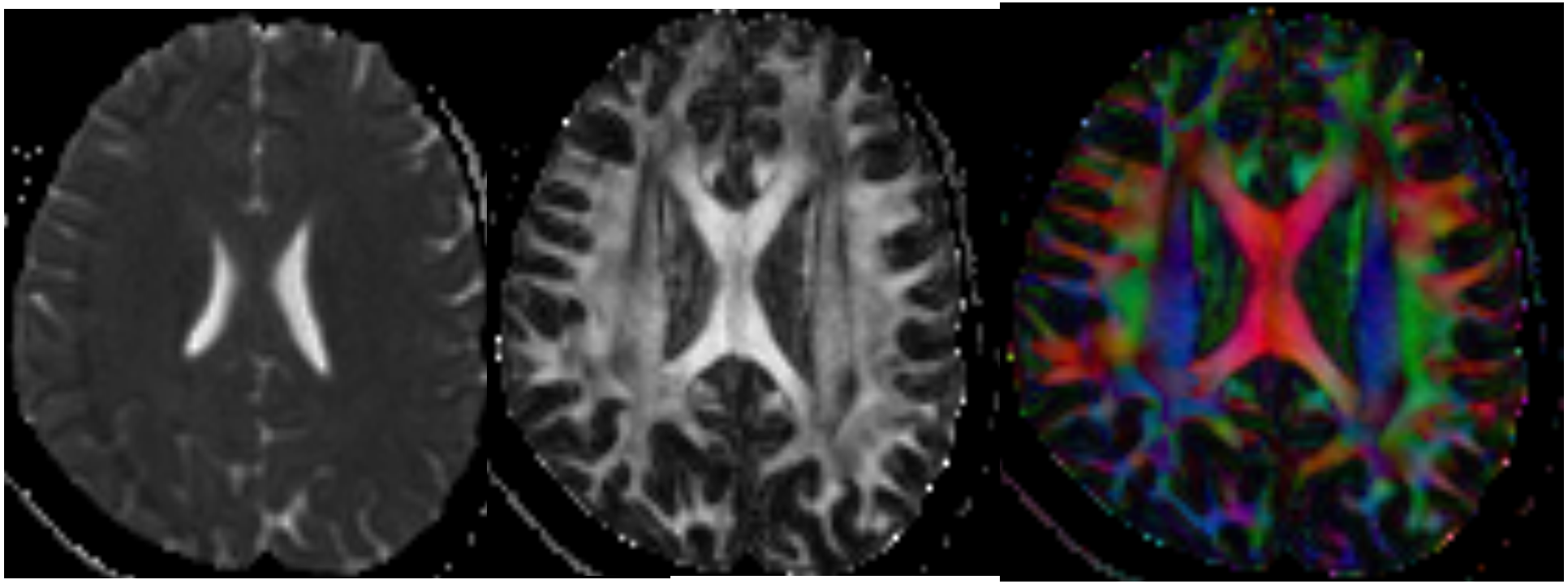
CSF Voxel
SSD =
5.4E9



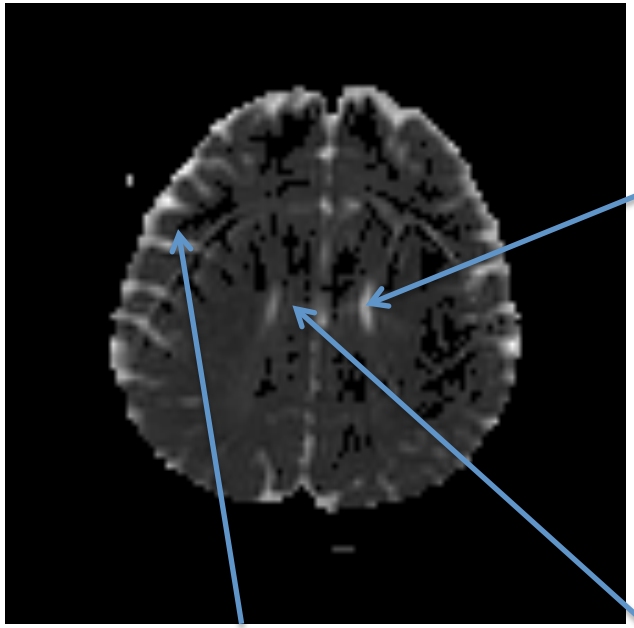
GM Voxel
SSD =
4.4E9

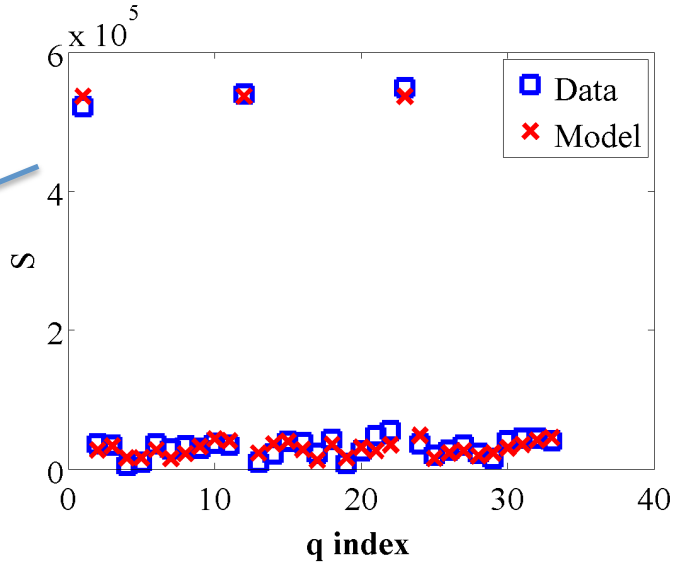


WM Voxel
SSD =
13.0E9

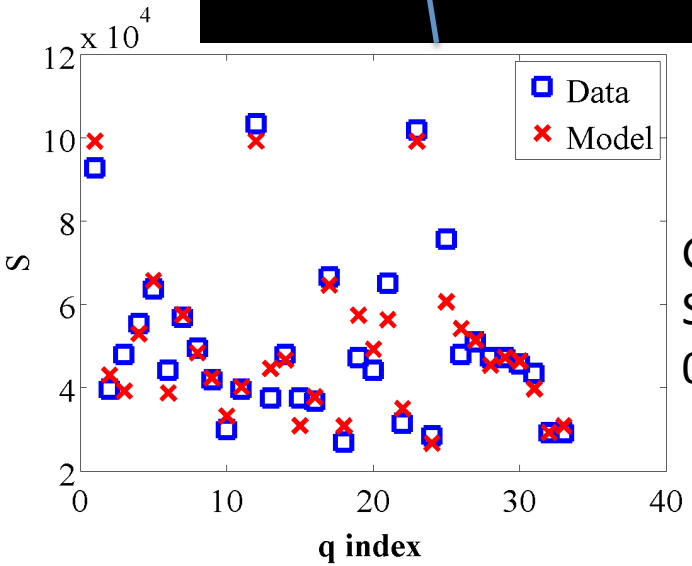


Non-linear DT fit

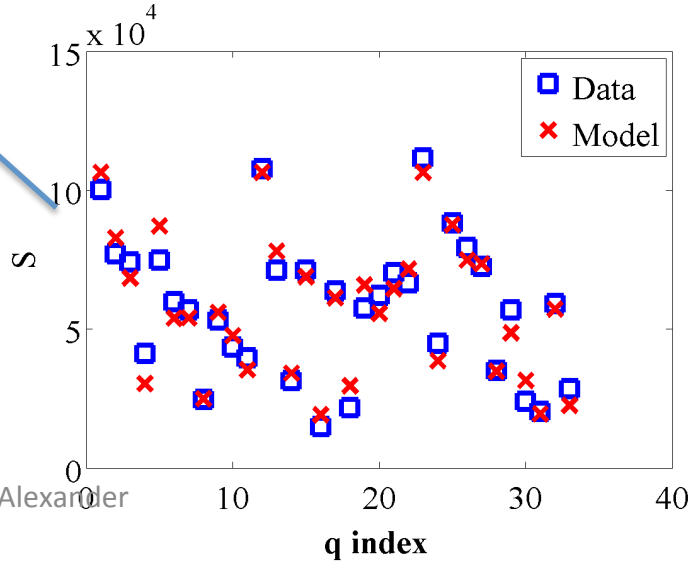




CSF Voxel
SSD =
3.0E9



GM Voxel
SSD =
0.8E9



WM Voxel
SSD =
1.0E9

Eg: quantitative magnetization transfer study in the normal human brain.

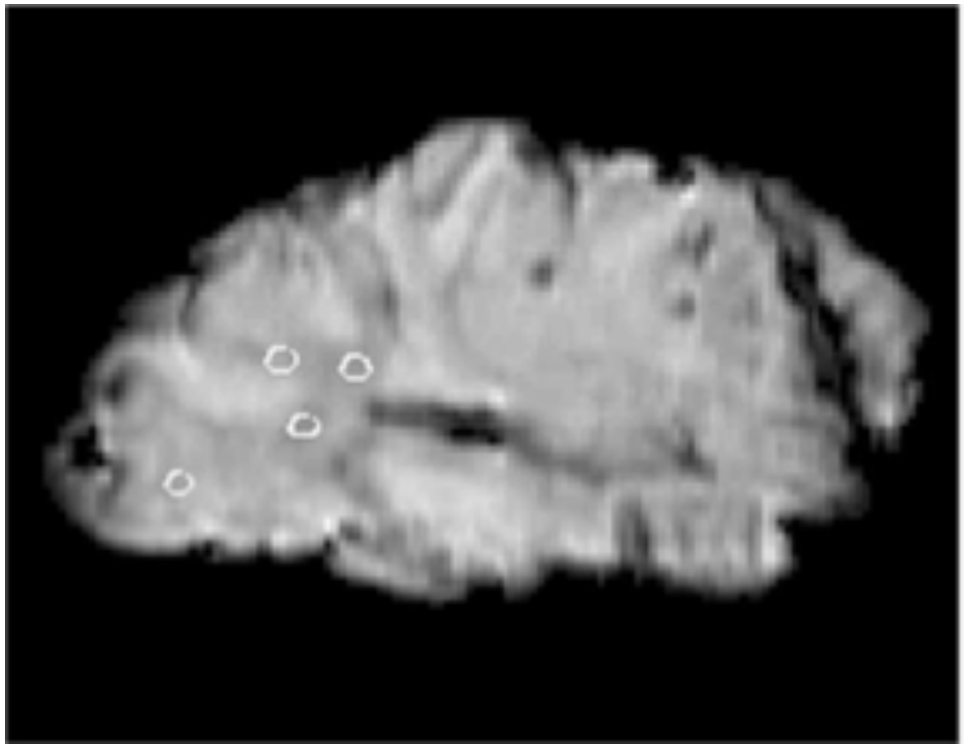
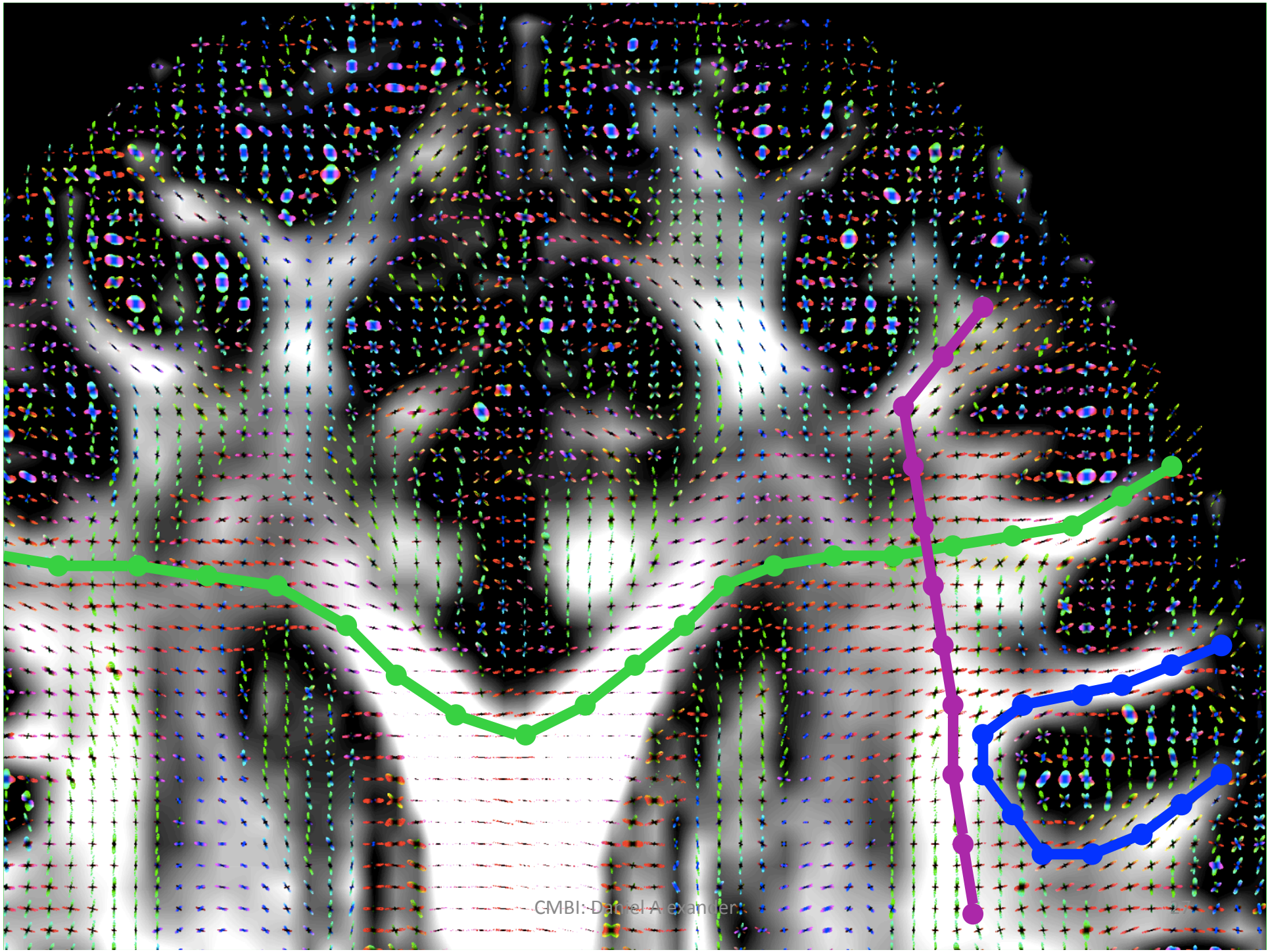


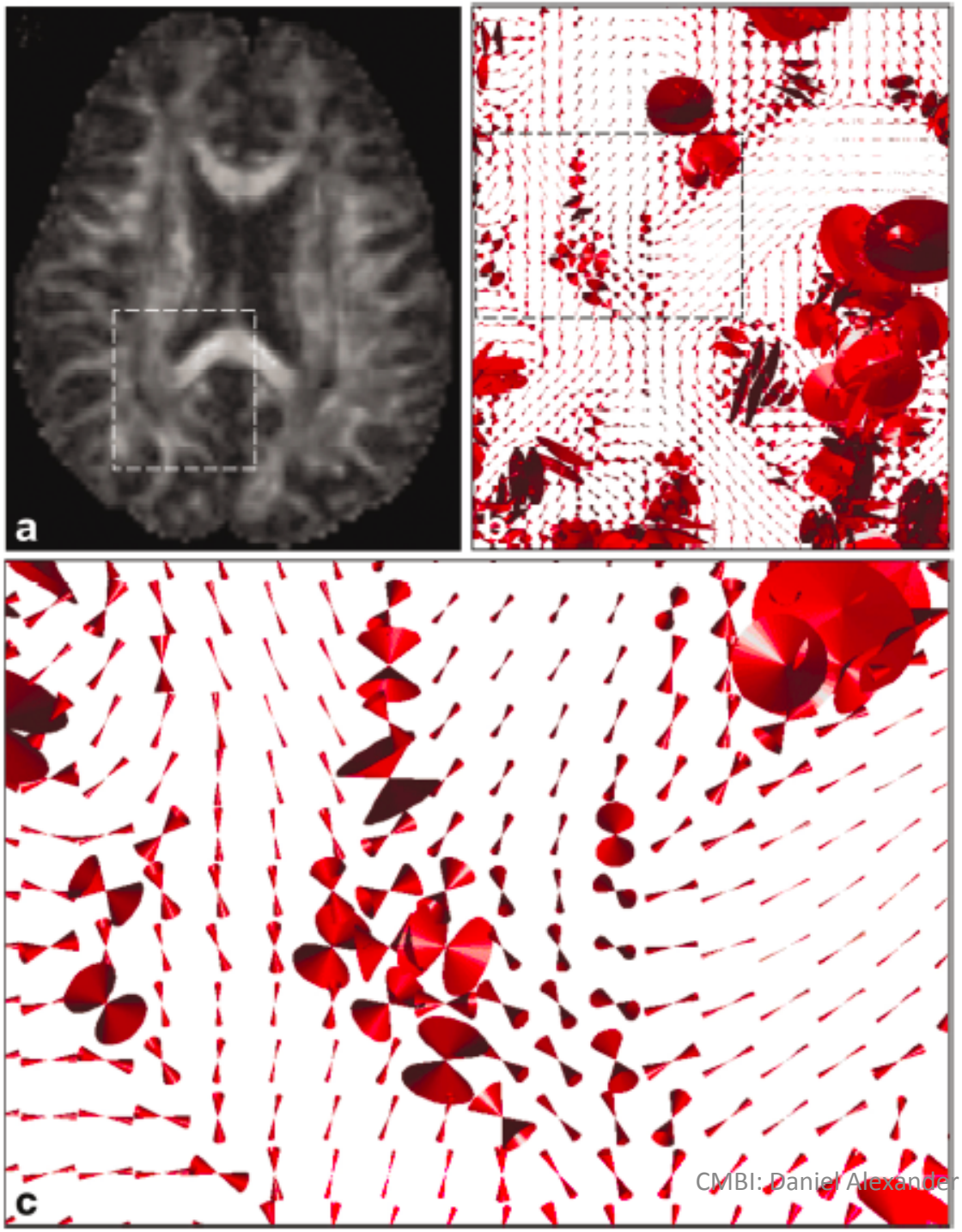
Table 1
Average quantitative MT parameters from the 3D-MTSPGR sequence, measured in the brain of six healthy volunteers

Area			f [%]	T_2^B [μs]	T_1^A [ms]	T_{1obs} [ms]	T_2^A [ms]
Corpus callosum	Genu	Mean	10.1	11.4	726	728	56.7
		SD	0.8	0.8	68	68	10.2
	Splenium	Mean	9.7	11.9	711	713	73.0
		SD	1.2	0.4	83	83	43.4
	Body	Mean	8.0	11.4	853	854	58.3
		SD	0.8	1.0	74	74	18.5
Internal capsule	Genu	Mean	9.0	12.7	749	751	54.8
		SD	1.2	1.2	67	67	12.7
	Posterior limb	Mean	9.3	13.8	734	736	56.7
		SD	1.1	8.9	47	47	4.9
Corona radiata		Mean	8.8	13.7	730	732	66.2
		SD	0.7	7.9	46	46	7.8
Optic radiation		Mean	9.4	12.3	661	663	61.1
		SD	0.9	8.4	44	44	7.7
Pedunculus cerebri		Mean	9.2	13.4	745	747	72.5
		SD	1.3	0.9	61	60	29.9
Cerebellar WM		Mean	9.4	11.8	718	719	59.0
		SD	0.8	0.5	51	51	3.8
Pons		Mean	8.1	11.7	825	827	77.3
		SD	0.9	1.1	70	70	17.3
Frontal WM		Mean	9.1	13.4	719	720	53.5
		SD	0.6	0.8	41	40	5.0
Thalamus		Mean	6.6	11.8	975	976	69.8
		SD	0.7	0.8	61	61	8.5
Caudate		Mean	5.0	11.4	1164	1164	82.8
		SD	0.6	0.8	117	117	19.0
Putamen		Mean	5.2	10.9	1092	1092	78.6
		SD	0.7	0.6	85	85	9.9
Frontal GM		Mean	4.7	11.8	1279	1279	107.6
		SD	0.6	1.6	128	128	26.2

f = macromolecular proton fraction; A = free pool; B = restricted pool; SD = standard deviation; GM = grey matter; WM = white matter. See text for further details.
With the exception of the corpus callosum and the pons, the values were measured bilaterally.

Tracto-
graphy


CMBI: Daniel Alexander

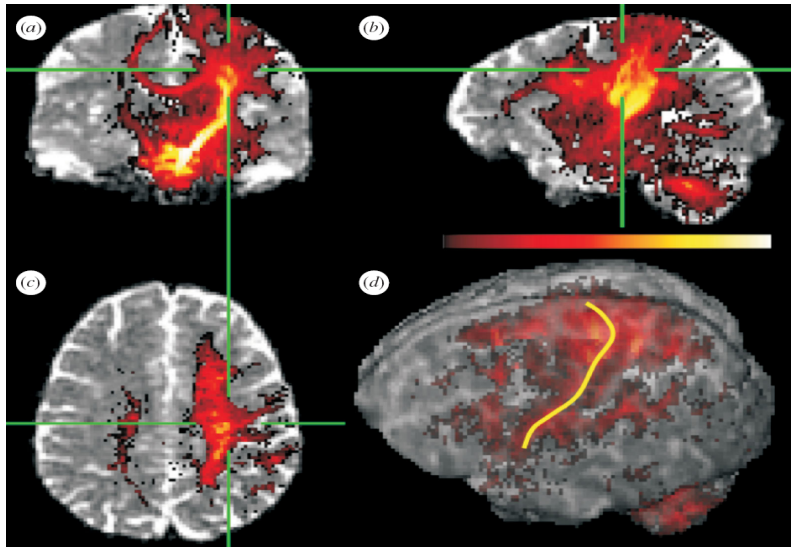


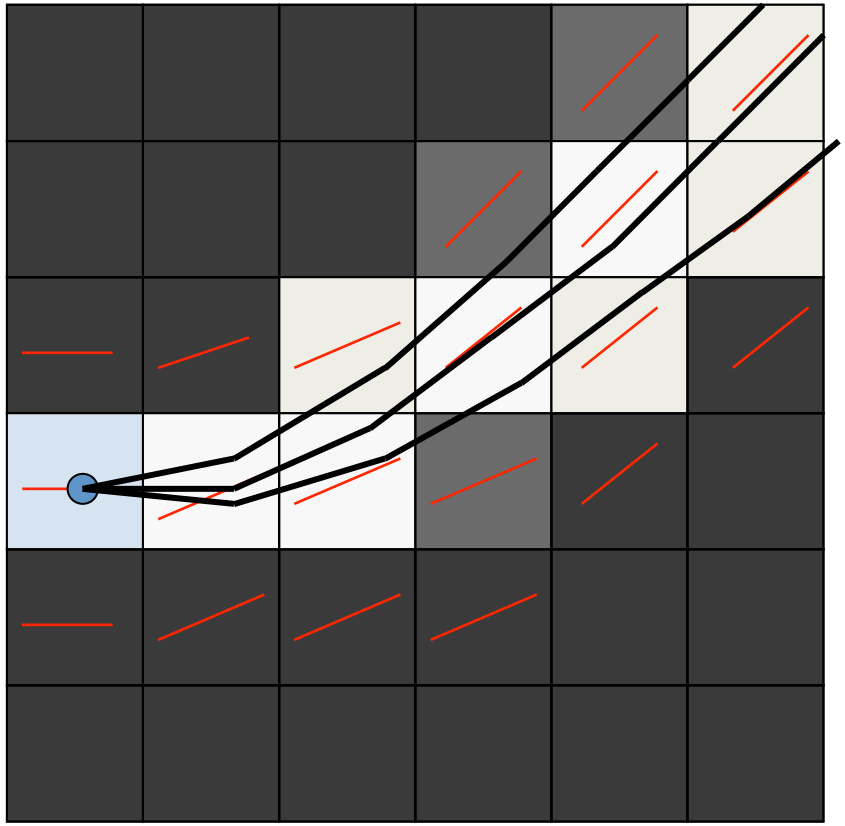
D.K. Jones
Determining and Visualizing Uncertainty in Estimates of Fiber Orientation From Diffusion Tensor MRI
Magnetic Resonance in Medicine 2004.

FIG. 2. Cones of uncertainty (showing the 95% confidence angle) at the level of the splenium of the corpus callosum. **a:** Fractional anisotropy. **b:** Cones of uncertainty in the region indicated by the dashed lines in **a**. This region is further magnified in **c**. The zoomed area highlights a region where fibers cross and the uncertainty in ε_1 is large.

Probabilistic Tractography

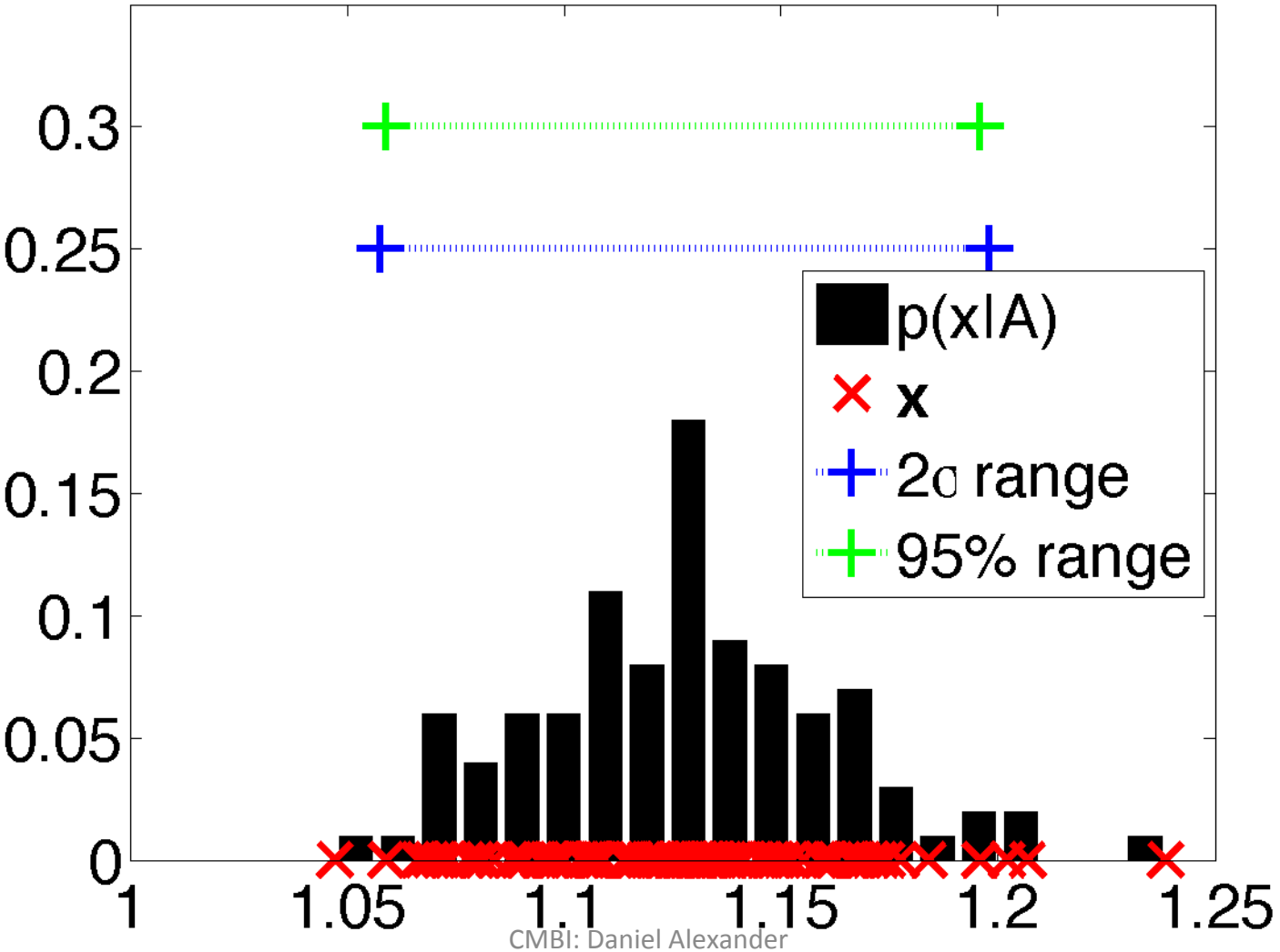
- Model uncertainty of fiber orientations
- Repeat:
  - Sample orientations
  - Track a streamline
- Count streamlines in each voxel



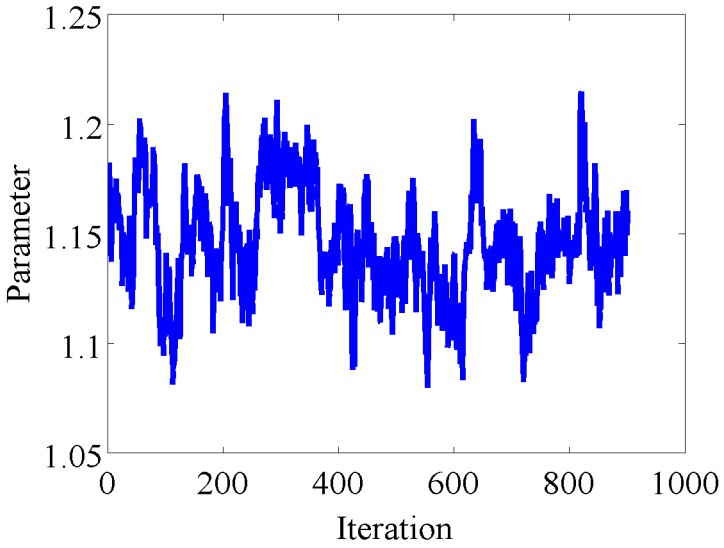


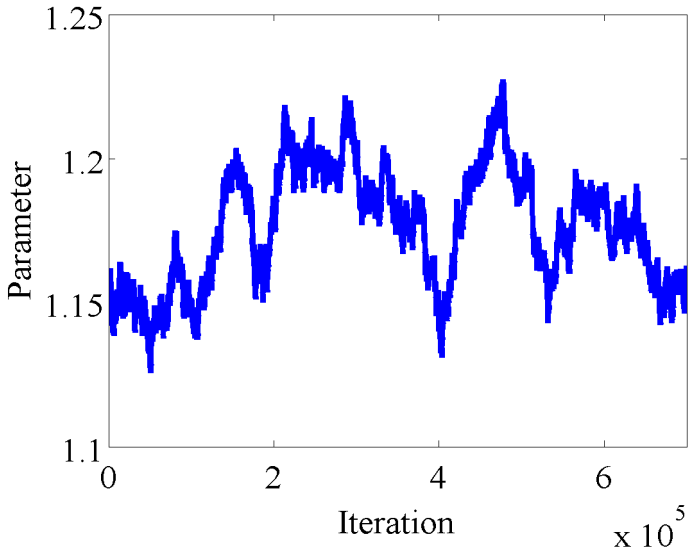
This is the basic PICo algorithm by Parker et al, JMRI 2003.

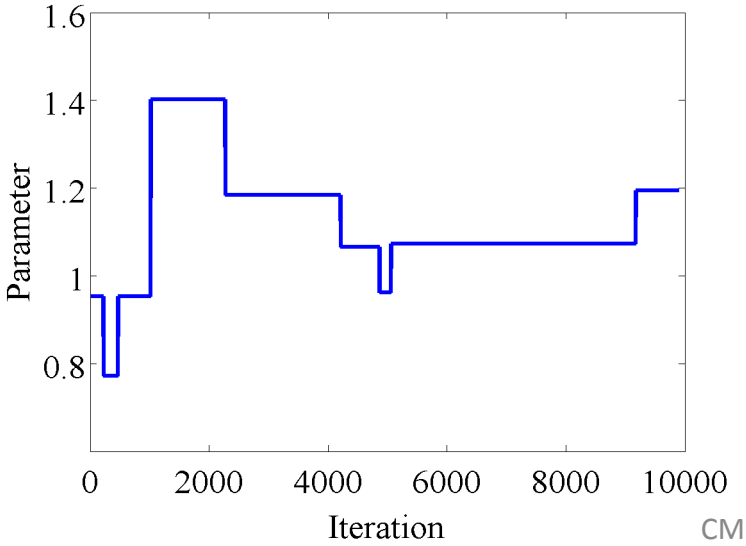
Example output

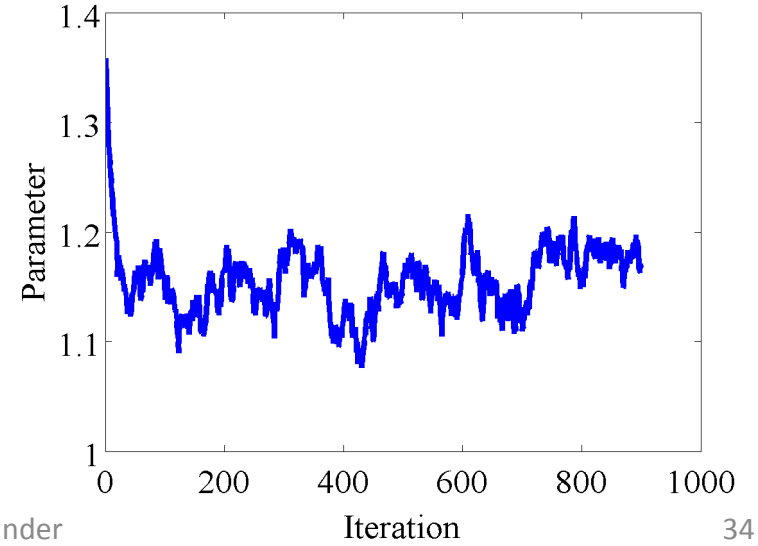


Common problems



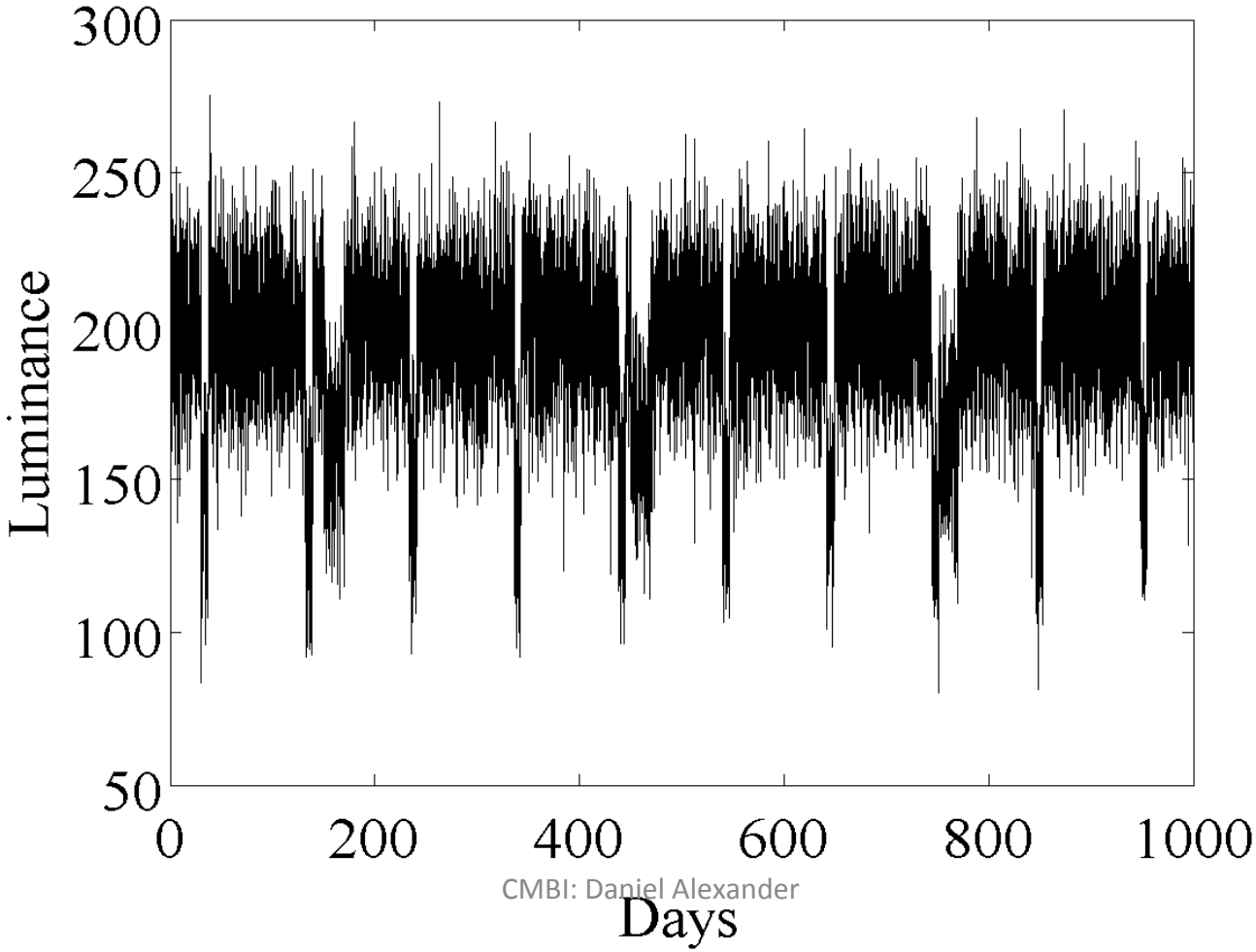






What is the problem?

- How many planets around this star?



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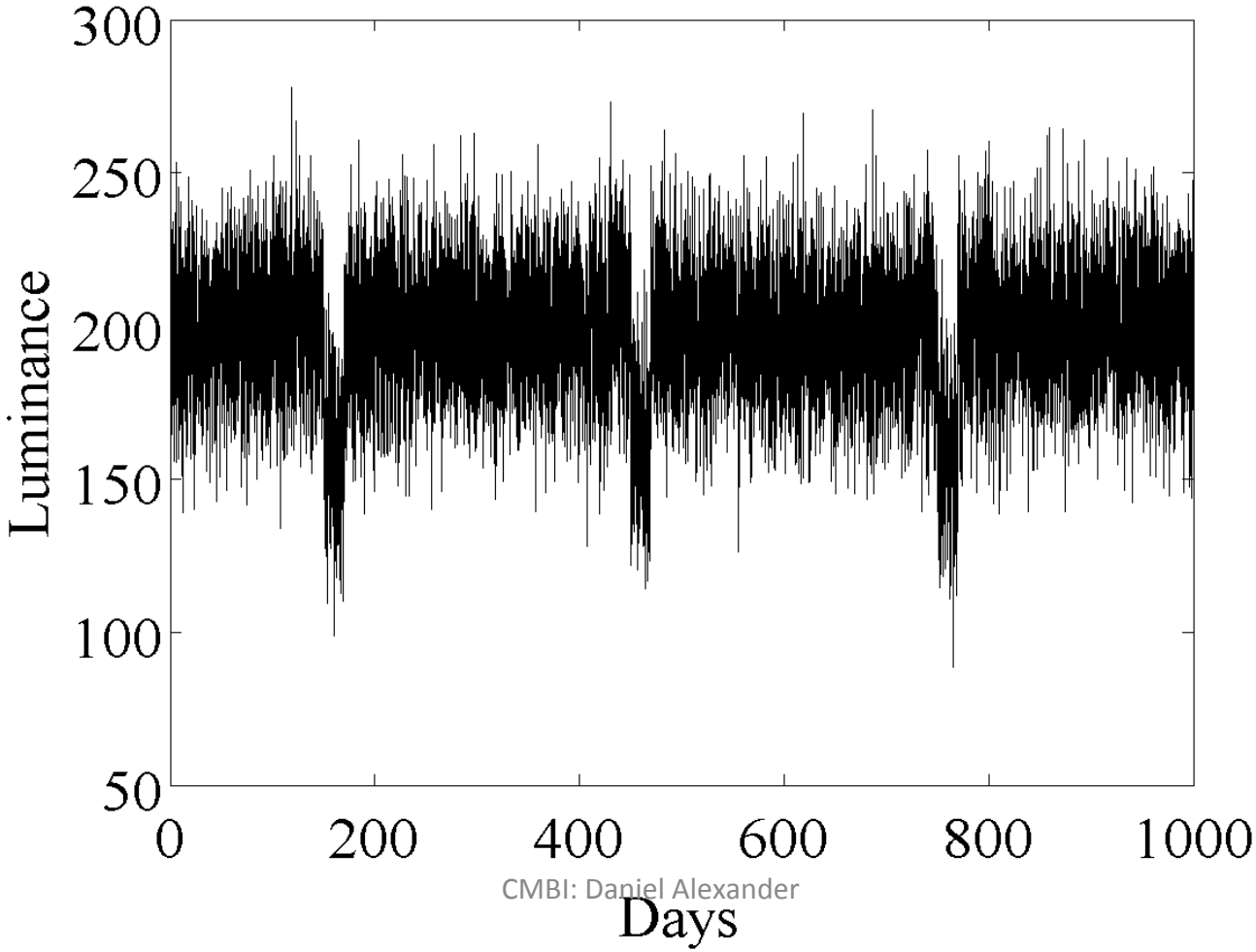
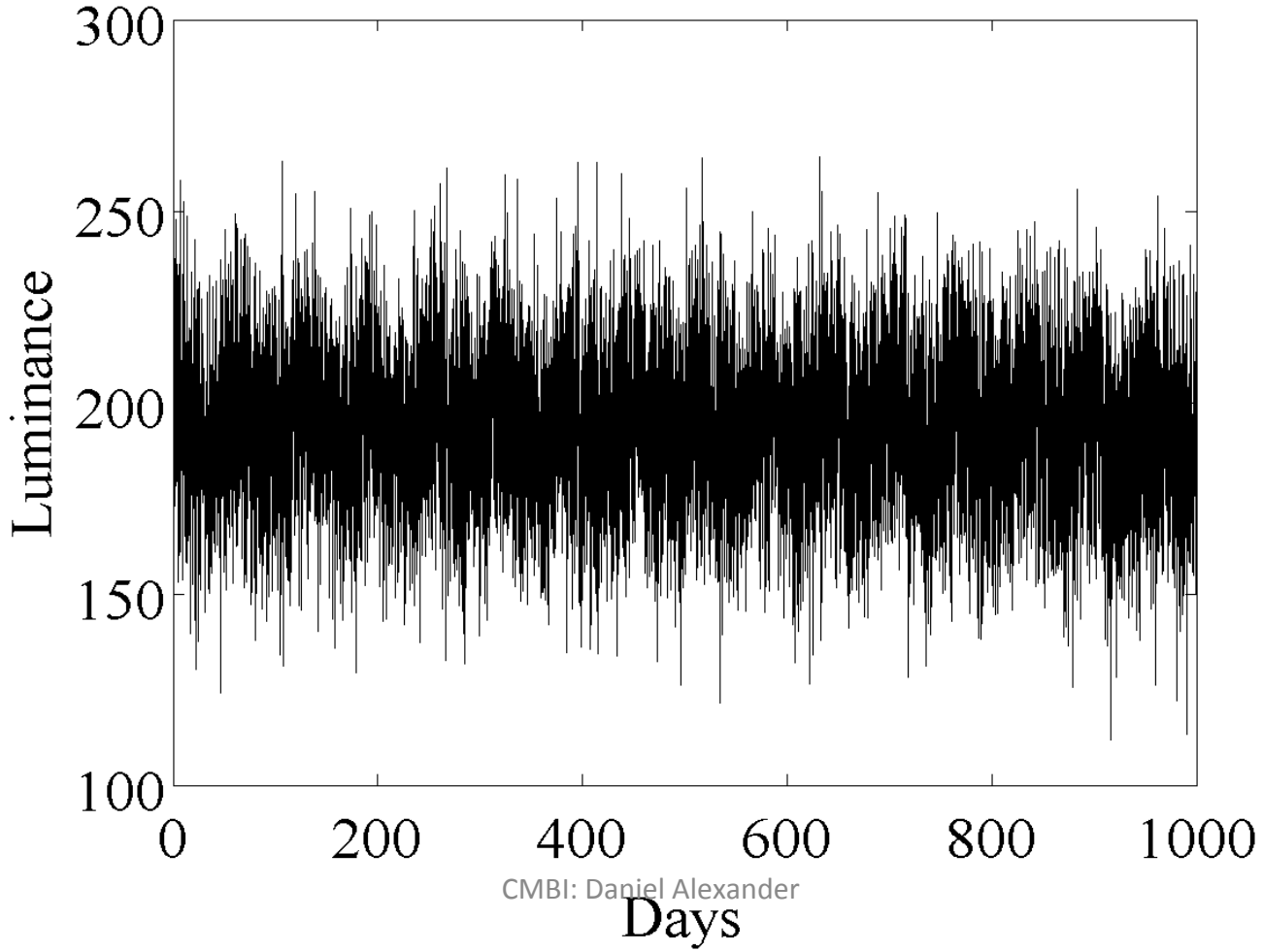
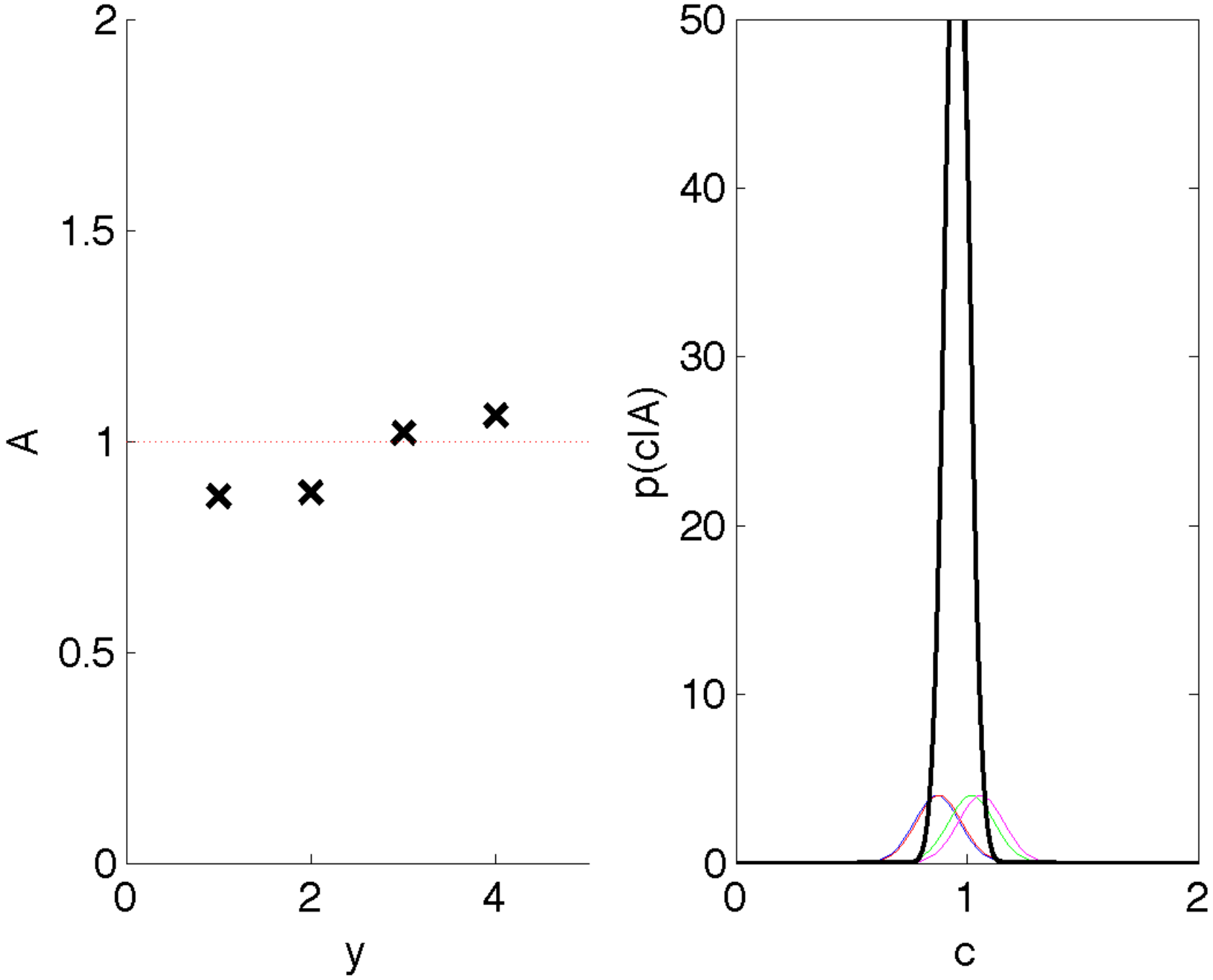
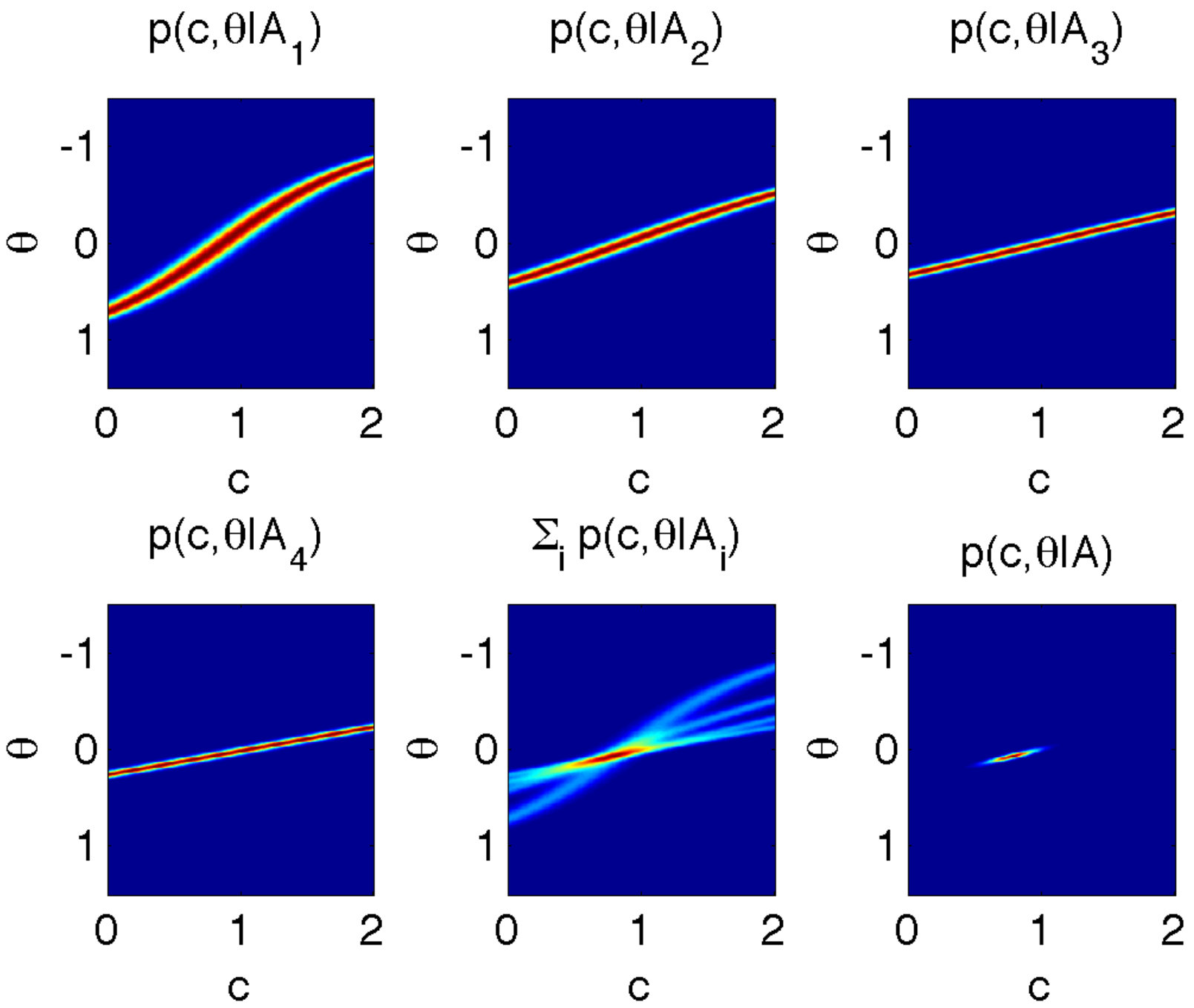


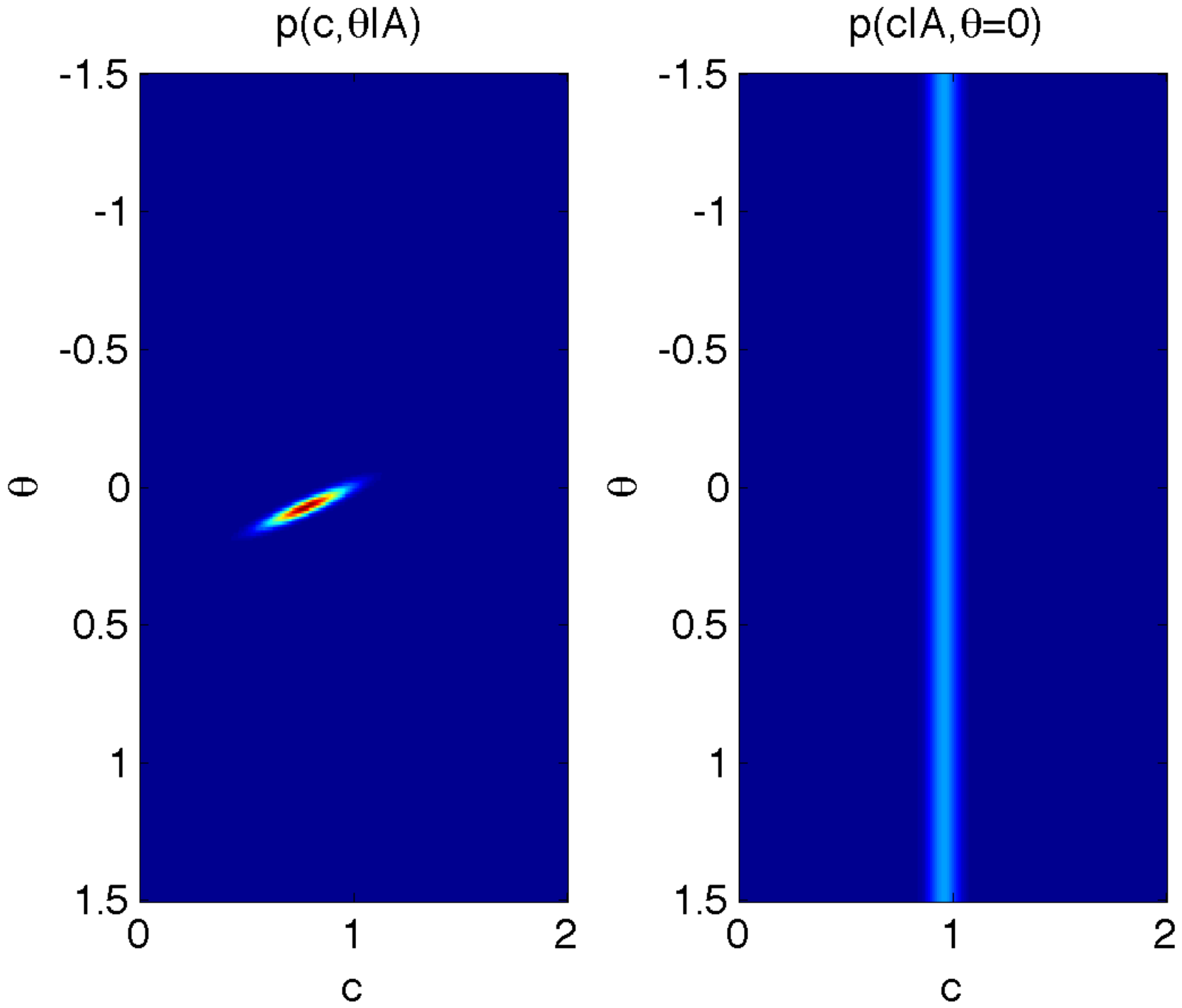
What is the problem?

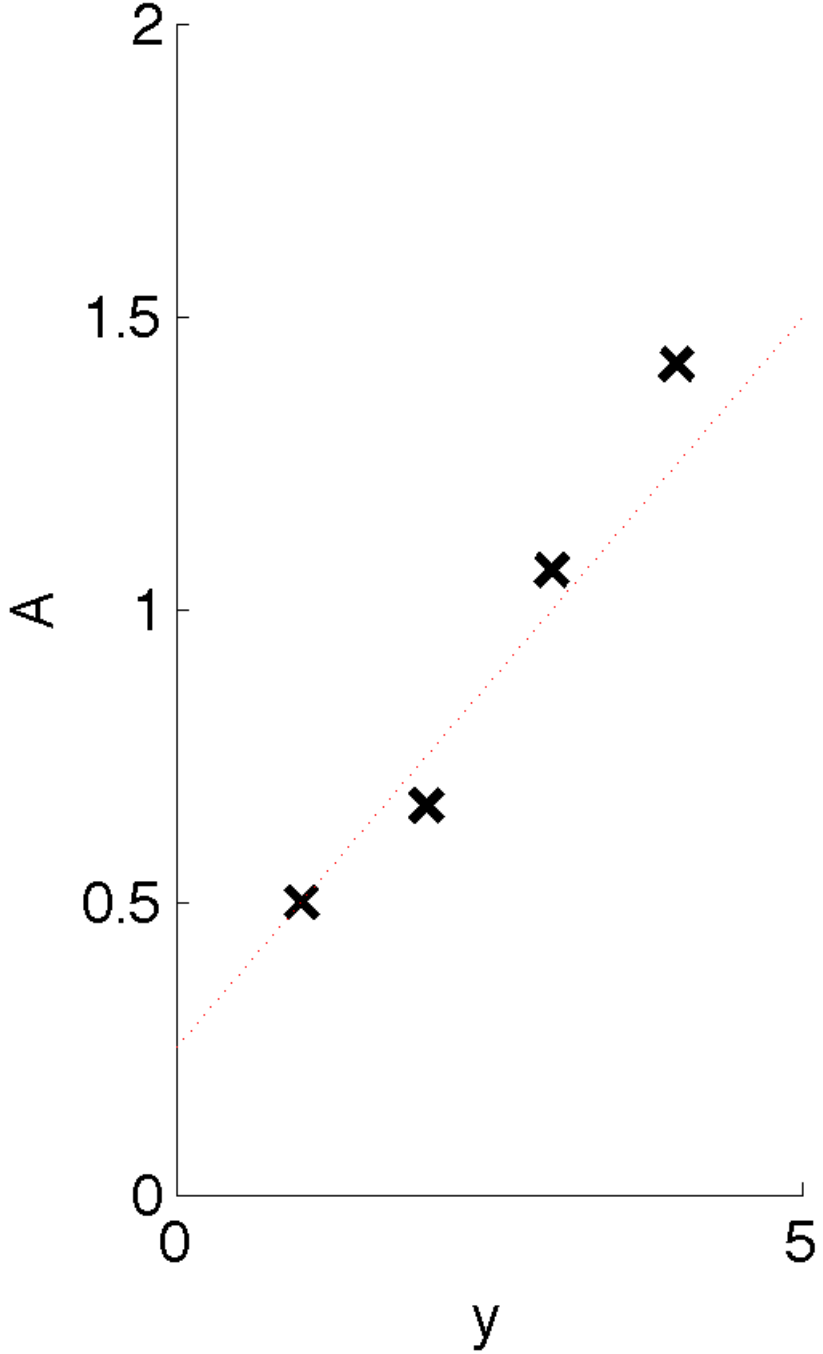
- How many planets around this star?

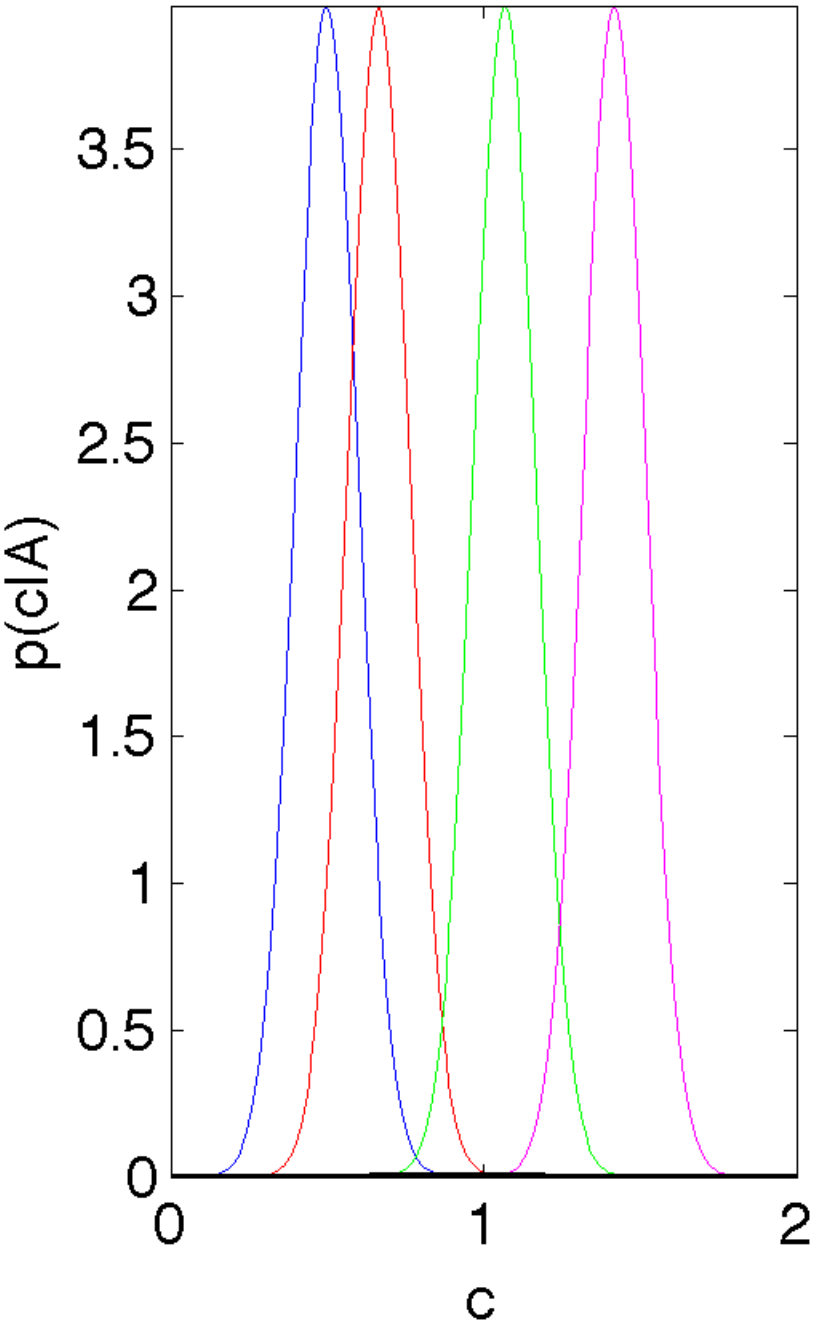


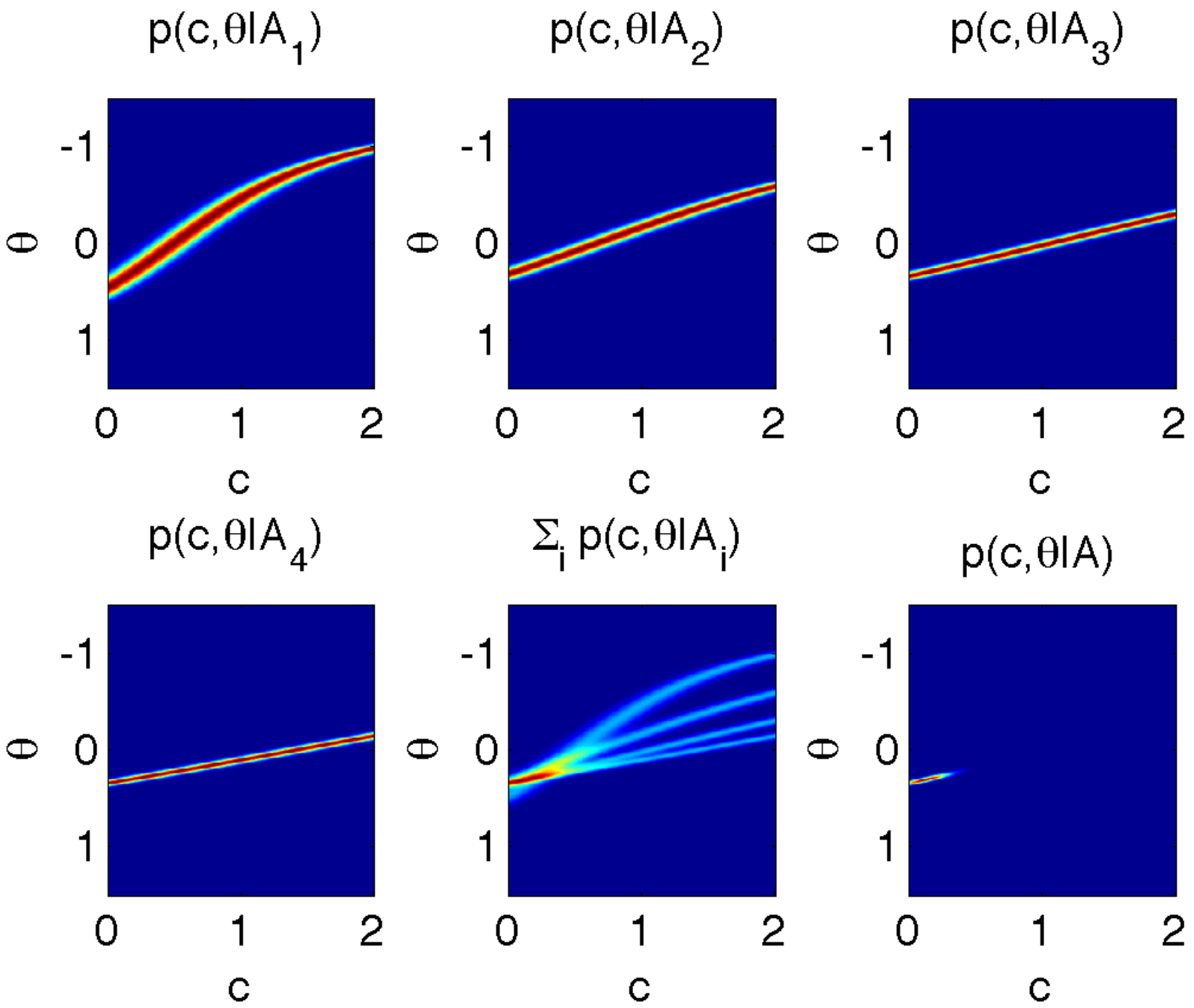

2
1.5
1
0.5
0
A
0
2
4
y
50
40
30
20
10
0
p(c|A)
0
1
2
c

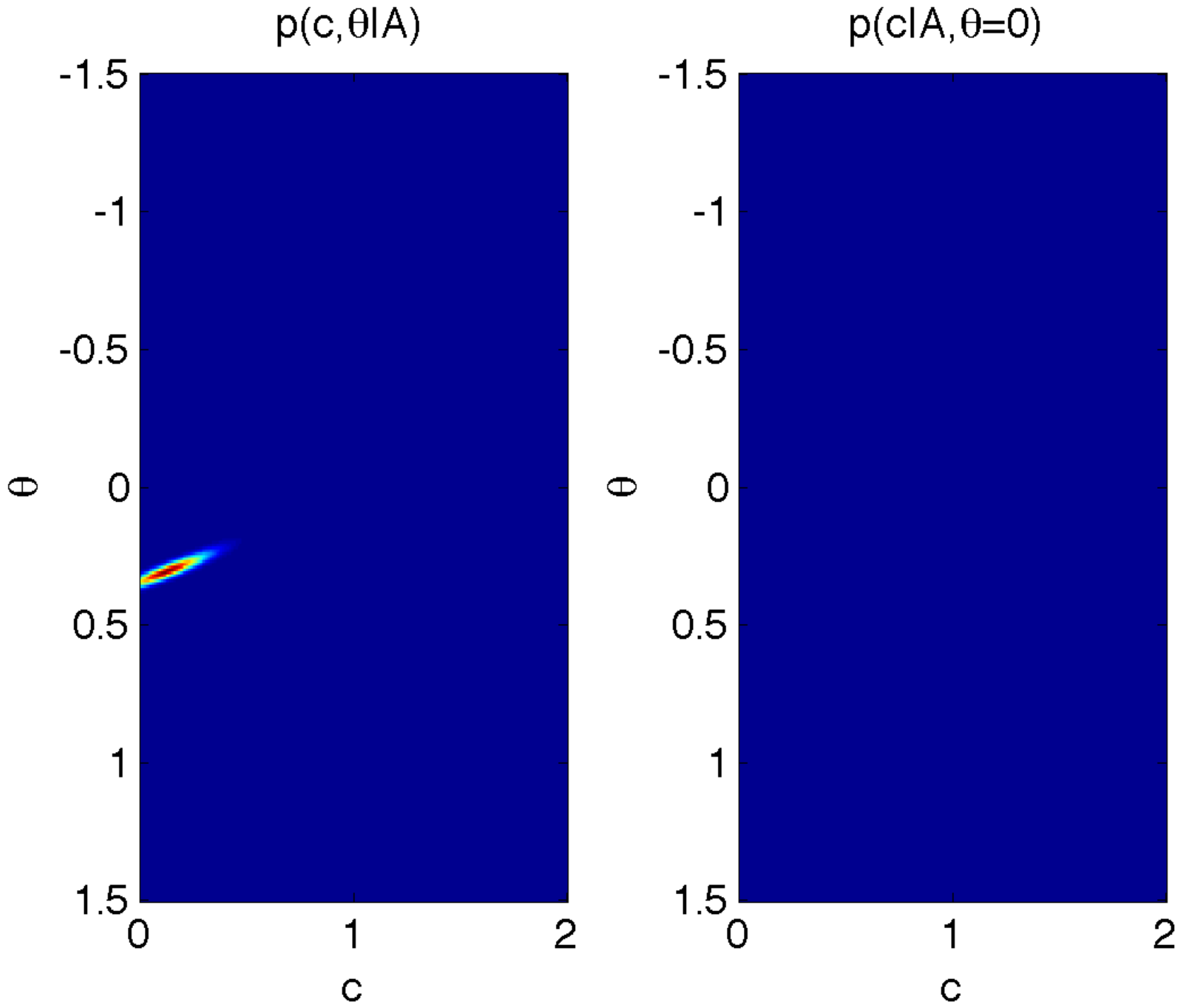

p(c,θ|A1)
p(c,θ|A2)
p(c,θ|A3)
p(c,θ|A4)
Σi p(c,θ|Ai)
p(c,θ|A)
θ
c
-1
0
1
0
1
2


p(c,θ|A)
p(c|A,θ=0)
-1.5
-1
-0.5
0
0.5
1
1.5
θ
0
1
2
c


2
1.5
1
0.5
0
A
0
5
y


3.5
3
2.5
2
1.5
1
0.5
0
p(c|A)
0
1
2
c

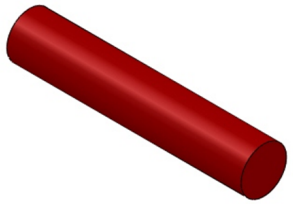

p(c,θ|A1)
p(c,θ|A2)
p(c,θ|A3)
p(c,θ|A4)
Σi p(c,θ|Ai)
p(c,θ|A)
θ
c
-1
0
1
0
1
2


p(c,θ|A)
p(c|A,θ=0)
-1.5
-1
-0.5
0
0.5
1
1.5
θ
0
1
2
c

Panagiotaki Models

Panagiotaki et al Neuroimage 2012.

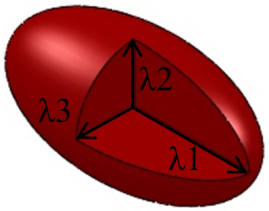
[1: Anisotropic Restricted]



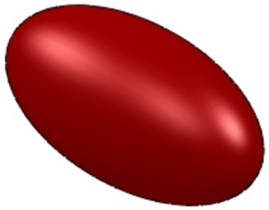
Cylinder (C)

Stick (S)

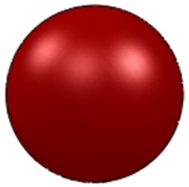
[2: Anisotropic Hindered]



Tensor (T)



Zeppelin (Z)

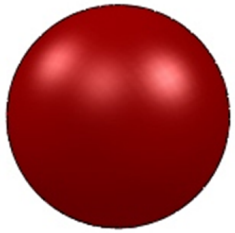


Ball (B)

[3: Isotropic restricted]

Astrocylinders (AC)

Astrosticks (AS)



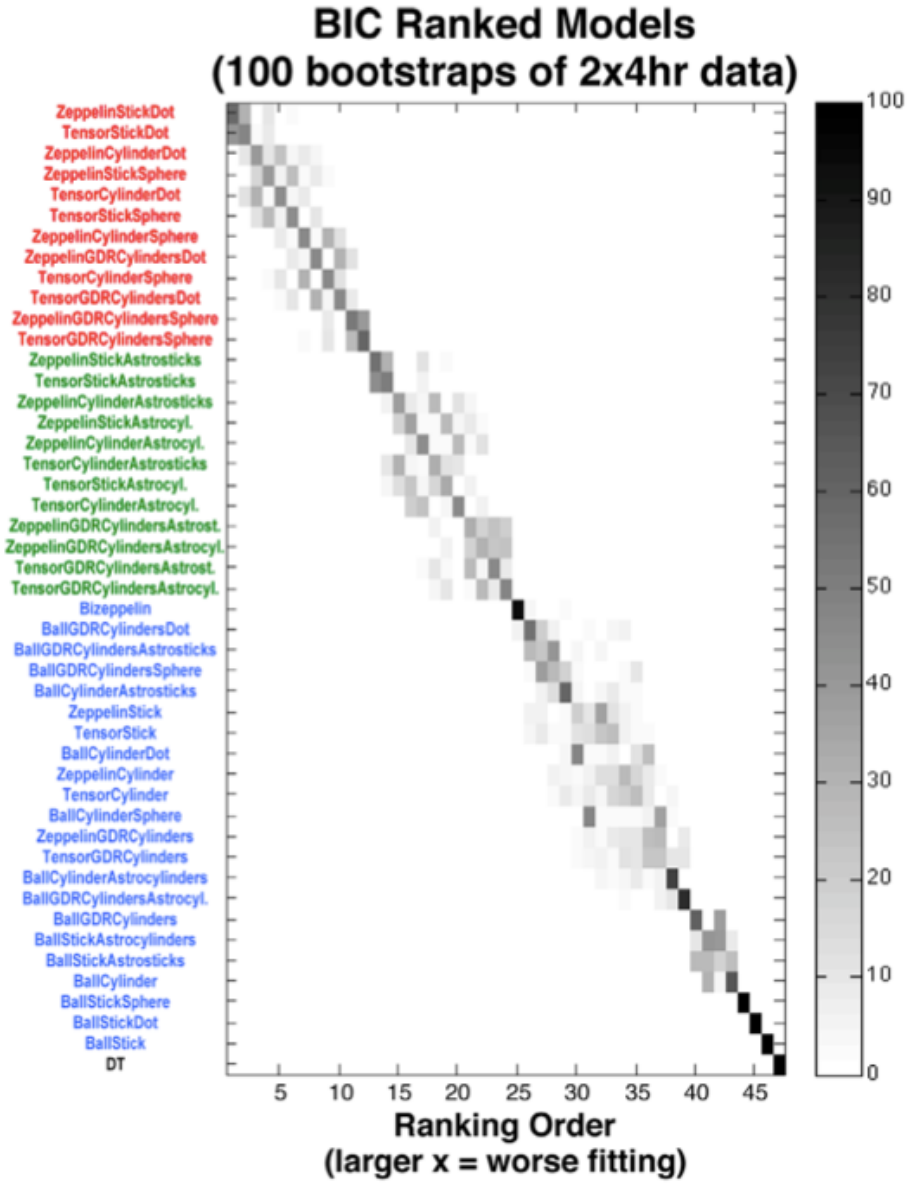
Sphere (SP)

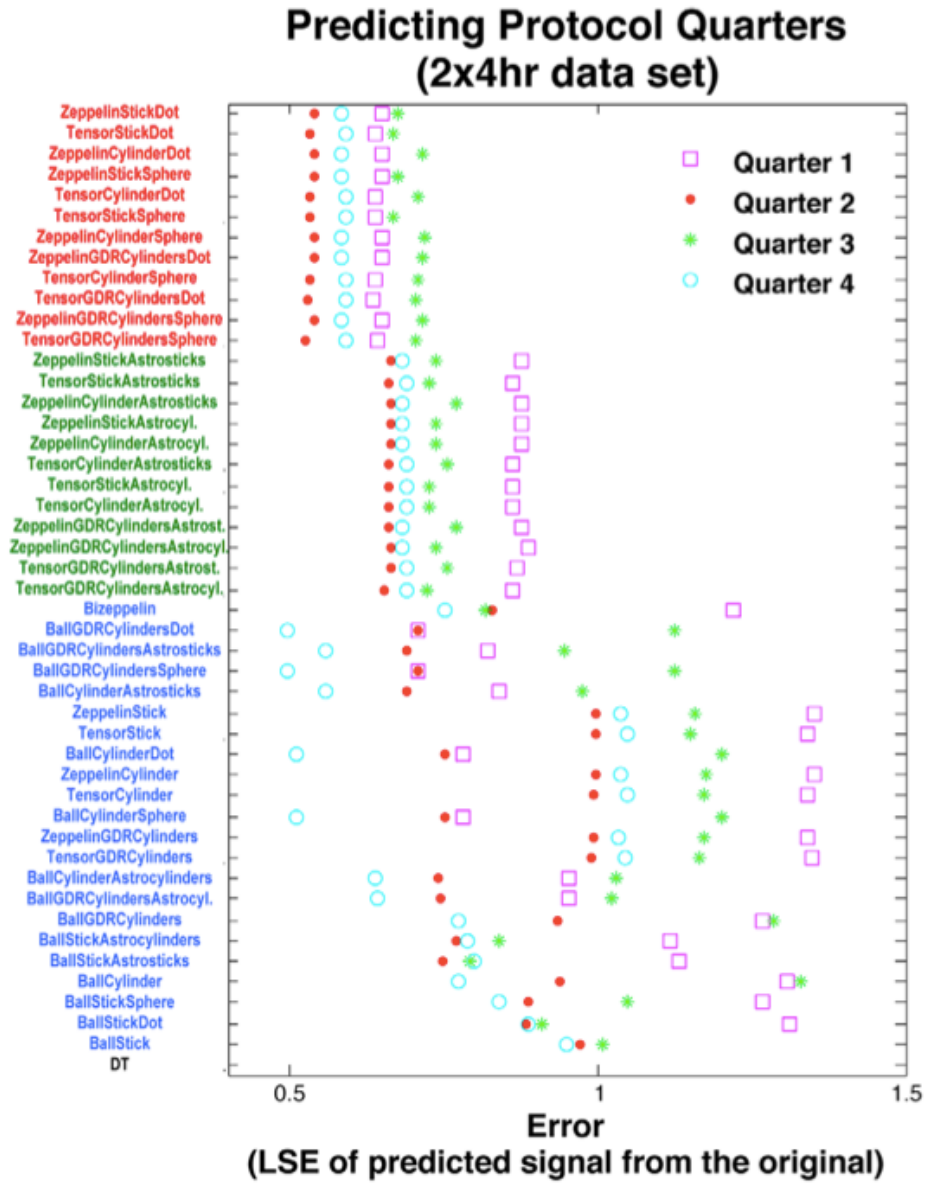
Dot (D)

Picture from Richardson MRM 2013

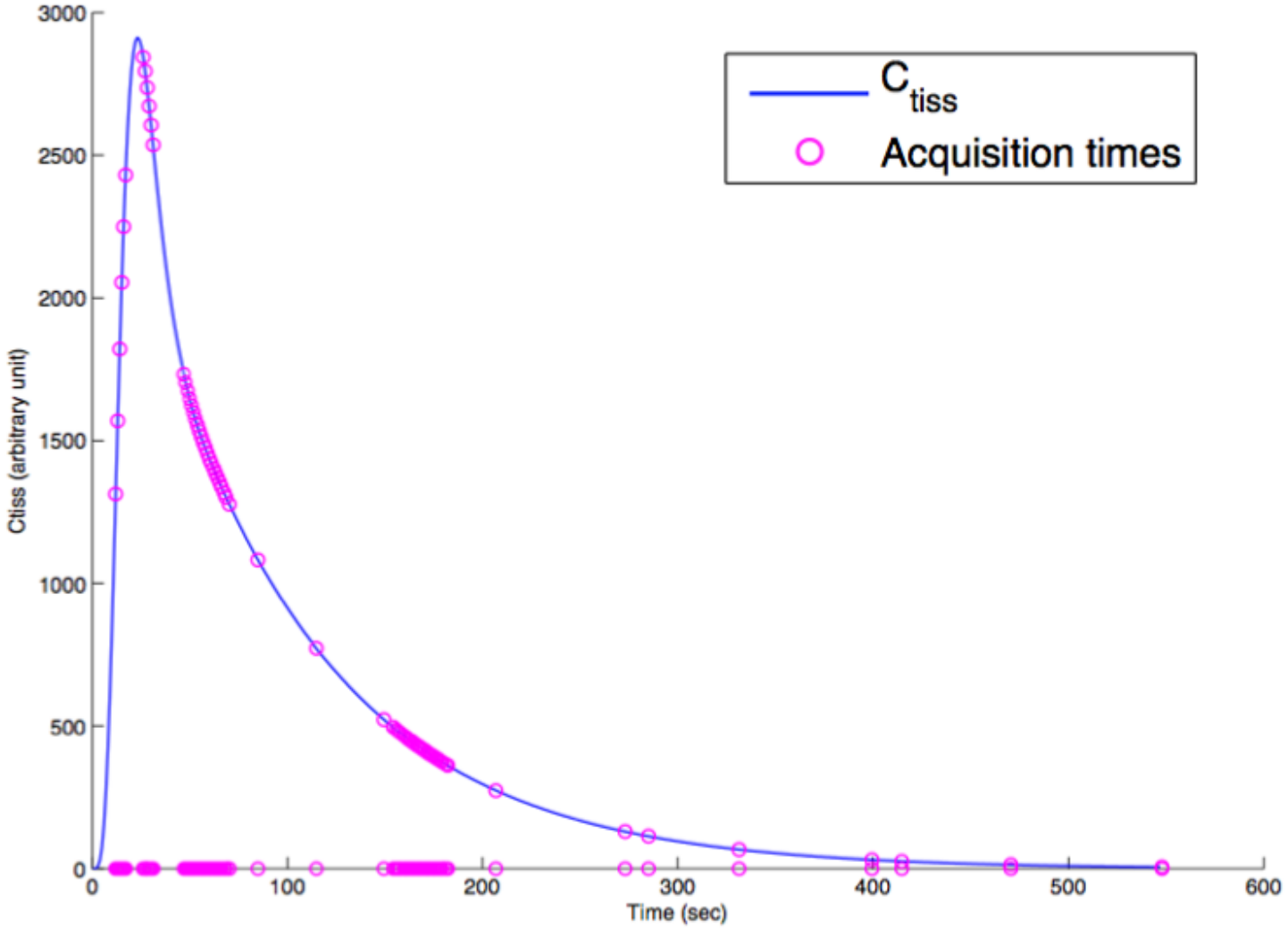
Ferizi et al
MRM 2014.

Ferizi Ranking



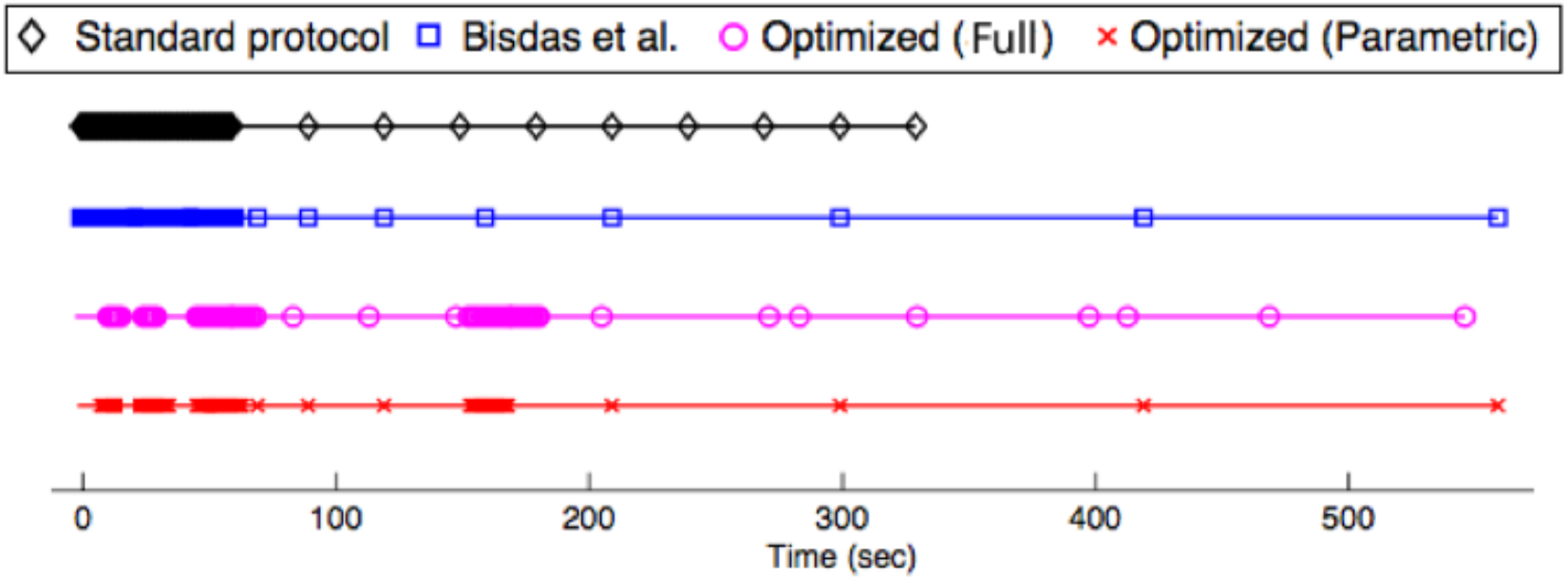


General optimized experiment design



Parametrized optimization

- Four groups of consecutive measurements
- One group of increasingly spaces measurements.



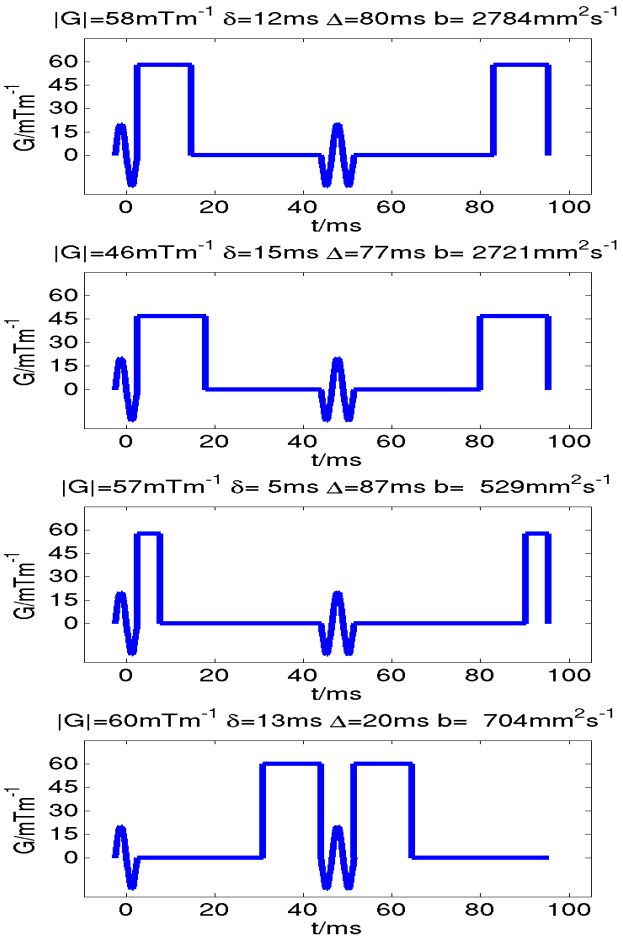
Standard protocol	Bisdas et al. protocol	Full optimized	Parametric optimized
9.44	10.9	9.03	6.81

Evaluation

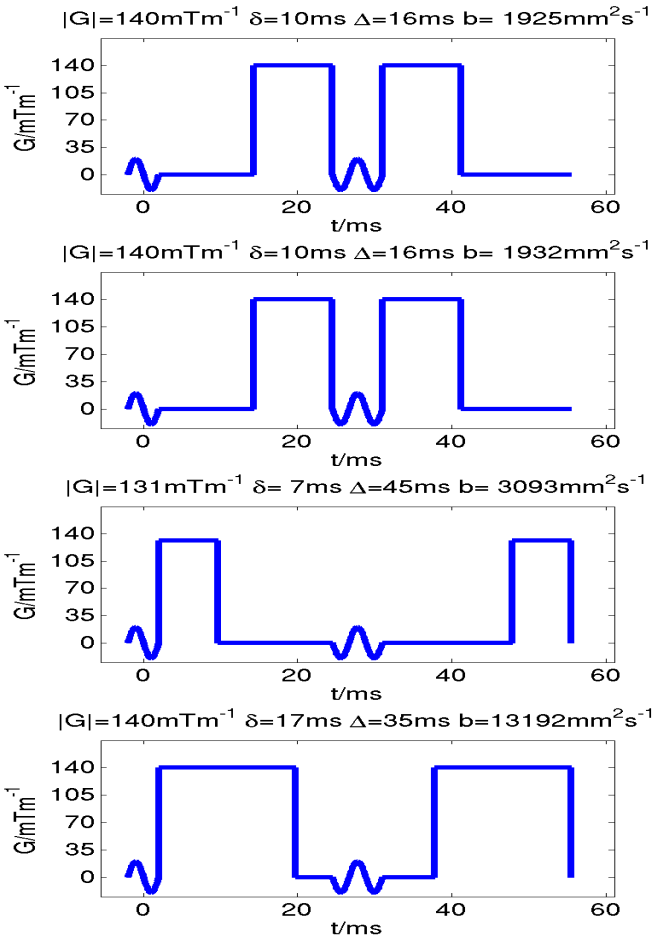
Protocol & nb of scans (% of param. variation)	CV (Max/Mean)	Error (Max/Mean)
Standard 70 scans (0%)	17.8 %	2.3 %
Standard 70 scans (10%)	19.2% / 17.7%	2.8% / 2.3%
Standard 70 scans (20%)	21.3% / 17.9%	3.4% / 2.5%
Bisdas 70 scans (0%)	14.3%	1.8%
Bisdas 70 scans (10%)	14.7% / 13.8%	2.2% / 1.7%
Bisdas 70 scans (20%)	15.5% / 14%	2.0% / 1.7%
Optimized 70 scans (0%)	10.0%	0.8%
Optimized 70 scans (10%)	11.7% / 10.5%	1.1% / 0.9%
Optimized 70 scans (20%)	12.6% / 10.9%	1.6% / 1.0%
Optimized 55 scans (0%)	10.6%	0.8%
Optimized 55 scans (10%)	12.4% / 11.6%	1.0% / 1.0%
Optimized 55 scans (20%)	13.9% / 12.0%	1.6% / 1.1%
Optimized 35 scans (0%)	14.1%	1.3%
Optimized 35 scans (10%)	17.1% / 15.1%	1.4% / 1.1%
Optimized 35 scans (20%)	17.3% / 14.9%	2.0% / 1.4%

Optimised designs

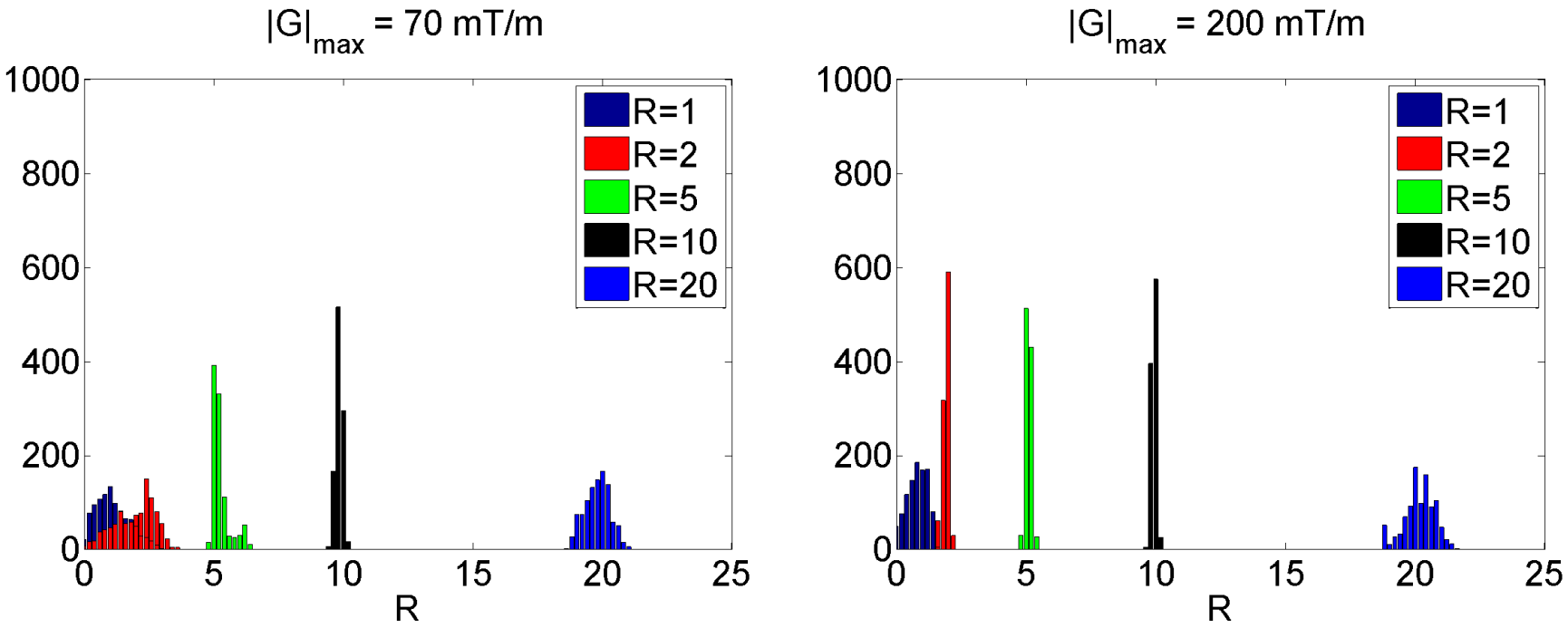
In-vivo: Gmax = 60mT/m

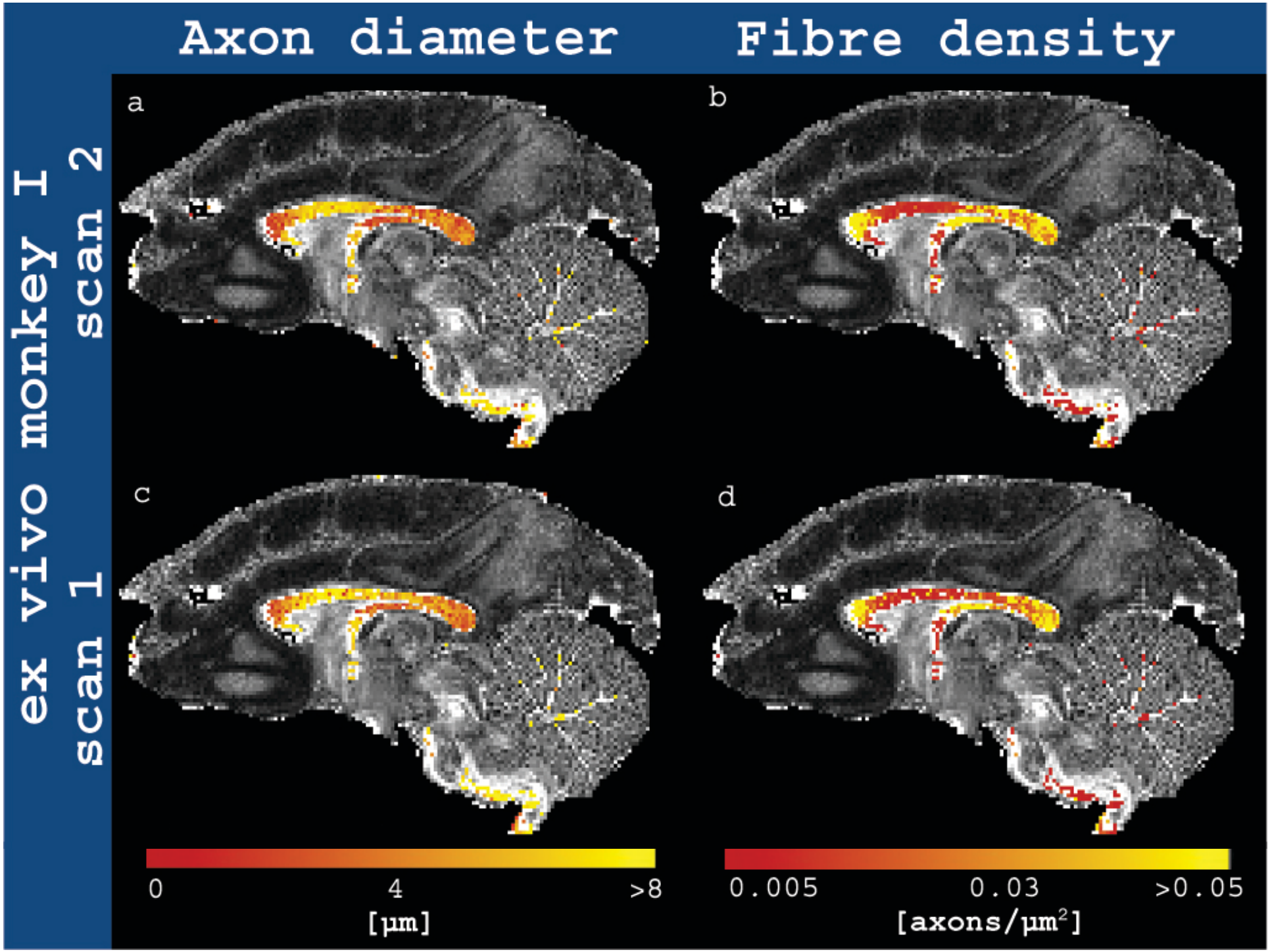


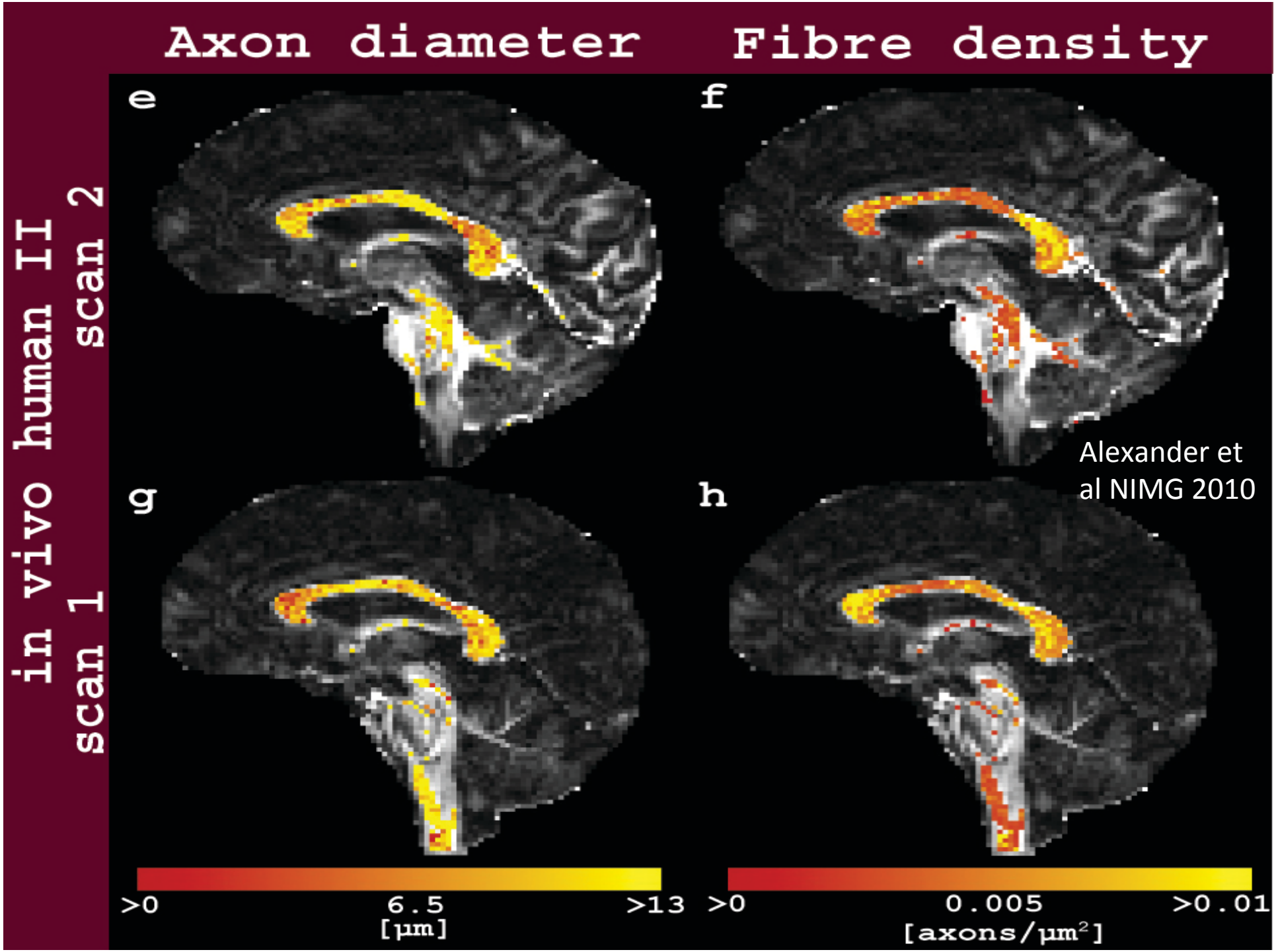
Fixed: Gmax = 140mT/m



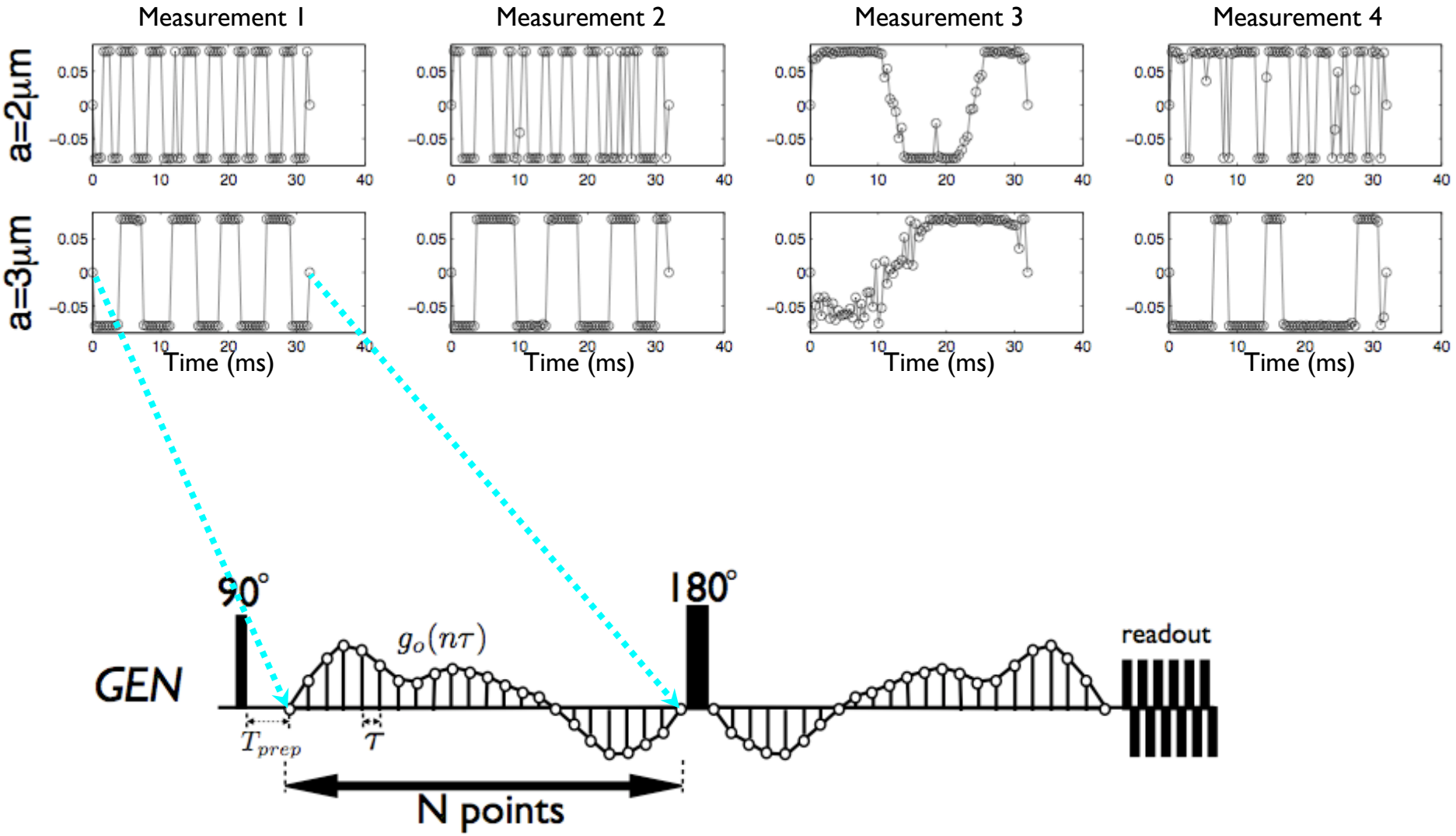
Posterior distributions on *R*




Axon diameter
Fibre density
ex vivo monkey I
scan 1
scan 2
a
b
c
d
0
4
>8
[µm]
0.005
0.03
>0.05
[axons/µm²]


Axon diameter
Fibre density
in vivo human II
scan 1
scan 2
e
f
g
h
Alexander et
al NIMG 2010
>0
6.5
[µm]
>13
>0
0.005
[axons/µm²]
>0.01

Drobnjak JMR 2010



Drobnjak JMR 2010

MCMC posterior distributions on axon diameter. Gmax = 40mT/m; N=4.