COMPGI13: Assignment 1

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Note

A one-week deadline extension was granted due to medical issues.

The error/cost learning curves for 1a-d present training values over the minibatches. An additional graph is presented at the end of P1 exemplifying error tracking over the full training set (for (1b)).

Code

The implemented programs are printed in the appendix. See the README file for more information. **Tools:** Code was developed using Python 3.6.0 and TensorFlow version 0.12.1. The following packages were utilised: matplotlib 2.0.0 (pyplot) for learning curve and confusion matrix plotting, scikit-learn 0.18.1 for confusion matrix functionality and numpy 1.12.0 for numerical operations.

• P1: exercise1a.py, exercise1b.py, exercise1c.py, exercise1d.py

Parameter optimisation

Hyperparameters are tuned for each model individually and the models are trained until convergence. Learning curves have been utilised to monitor the training process and decide on suitable hyperparameters; 'regularising' if necessary by early stopping to avoid overfitting/'plateauing'. A minibatch size of 50, a SGD step length of 0.5 and a total number of 5,000-6,000 training iterations work well to train all models in P1. Note that one 'training iteration' refers to a training epoch over a given minibatch (not over the full set of examples).

1 MNIST with TensorFlow

Training and testing rates for all models at the end of the optimisation are presented at the end of the section. The TensorFlow random seed is fixed at 555 to facilitate result reproducibility.

(1a) 1 linear layer, followed by a softmax. The weights/biases are initialised as zeros. The training and testing errors are reported via Figure 1. Figure 1 presents a plot of the errors as a function of iterations during training.

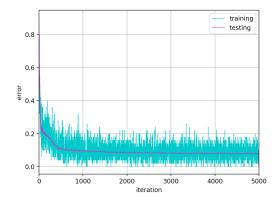


Figure 1: Plot of training/testing errors as a function of iterations during training for (1a).

The cross-entropy loss (training and testing) is also plotted as a function of training iterations. This plot is presented in Figure 2.

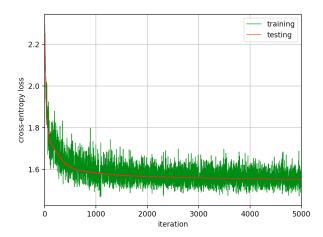


Figure 2: Plot of training/testing cross-entropy loss as a function of training iterations for (1a).

The greater variance in training values wrt testing values in Figure 1 and Figure 2 corresponds to the much smaller size of the 'minibatch' training sets wrt to the full test set. Figure 3 presents the confusion matrix for all classes over all iterations for the test set. Note that the colour map does not represent error probabilities; it represents the actual number of misclassified points. The right-hand-side bar provides a key. Also note that values have not been normalised with respect to the true labels. Correct classifications hugely outweigh errors; they have been omitted to obtain a better sense of the error spread. We can see that the most common errors involve mistaking a 5 (true) for a 3 or an 8 (predicted).

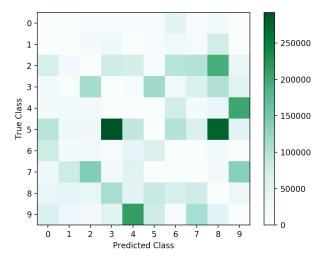


Figure 3: Confusion matrix over all iterations for the test set for (1a).

(1b) 1 hidden layer (128 units) with a ReLU non-linearity, followed by a softmax. The weights are initialised using zero-mean random values from a normal distribution with 0.1 standard deviation. For the hidden layer, the biases are initialised in the same manner but with a standard deviation of 1. The linear layer biases are initialised as zeros. The training and testing errors are reported via Figure 4. Figure 4 presents a plot of the errors as a function of training iterations. Training in this case is performed over 6,000 iterations. The cross-entropy loss (training and testing) is also plotted as a function of iterations during training. This plot is presented in Figure 5. Figure 6 presents the confusion matrix for all classes over all iterations for the test set. We can observe that the new model makes a lesser number of total mistakes (see right-hand-side bar), having learned to confuse less a 5 for an 8 for example.

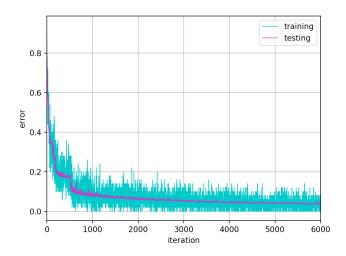


Figure 4: Plot of training/testing errors as a function of training iterations for (1b).

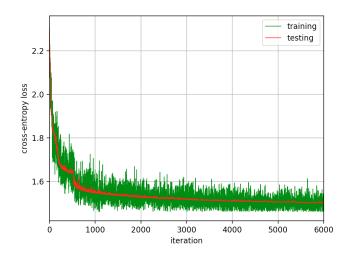


Figure 5: Plot of training/testing cross-entropy loss as a function of iterations during training for (1b).

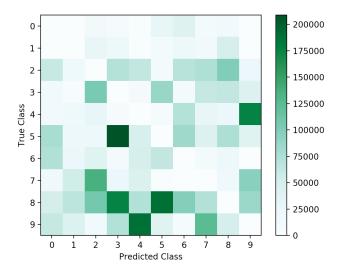


Figure 6: Confusion matrix over all iterations for the test set for (1b).

(1c) 2 hidden layers (256 units) each with a ReLU non-linearity, followed by a softmax. The weights are initialised using zero-mean random values from a normal distribution with 0.1 standard deviation. For the

hidden layers, the biases are initialised in the same manner but with a standard deviation of 1. The linear layer biases are initialised as zeros. The training and testing errors for these settings are reported via Figure 7. Figure 7 presents a plot of the errors as a function of iterations during training.

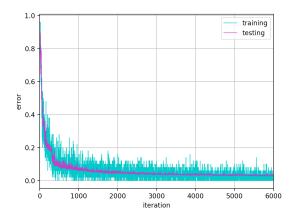


Figure 7: Plot of training/testing errors as a function of iterations during training for (1c).

The cross-entropy loss (training and testing) is also plotted as a function training iterations. This plot is presented in Figure 8.

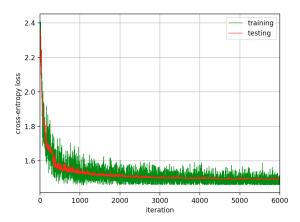


Figure 8: Plot of cross-entropy loss as a function of training iterations for (1c).

Figure 9 presents the confusion matrix for all classes over **all iterations** for the **test set**. We can observe that accuracy has improved further, with the model improving in its classification of 'fives'.

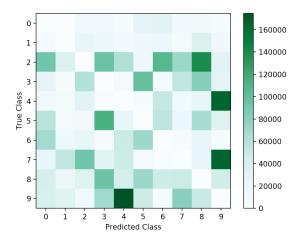


Figure 9: Confusion matrix over all iterations for the test set for (1c).

(1d) 3 layer convolutional model (2 convolutional layers followed by max pooling), followed by 1 non-linear layer (256 units), followed by softmax. The weights of the hidden ReLU layer and the linear layer are initialised using zero-mean random values from a normal distribution with 0.1 standard deviation. For the convolutional layers, the weights are initialised from a truncated zero-mean normal distribution with the same standard deviation. The biases for ReLU and linear layer are generated in the same manner as for 1b-c. For the convolutional layers, these are generated as constant 0.1s. The training and testing errors for these settings are reported via Figure 10. Figure 10 presents a plot of the errors as a function of iterations during training. In this case, accuracy measurements have been taken every 40 training iterations to make the procedure less computationally expensive.

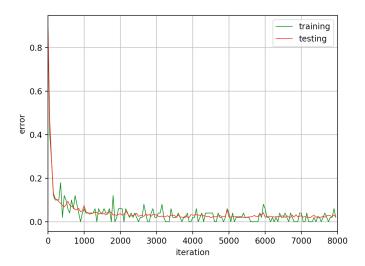


Figure 10: Plot of training/testing error as a function of training iterations for (1d).

Figure 11 presents the confusion matrix for all classes over measurements made every 40 iterations for the **test set**. Note that sampling biases may be present in this representation. The total number of errors is strongly dependent on mistakes made in the first epochs. Given the frequency at which measurements are taken, factors such as weight initialisation or the random seed play a larger role in shaping the matrix.

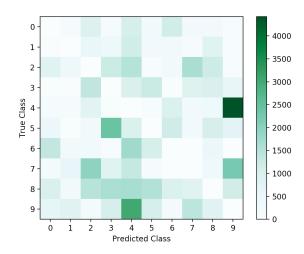


Figure 11: Confusion matrix over selected iterations for the test set for (1d)

Final error table

The training and testing errors at the end of the optimisation for P1 are recorded in Table 1.

Exercise	Training error (%)	Test error (%)
1a (5,000 epochs)	8.12	7.95
1b (6,000 epochs)	3.16	3.96
1c (6,000 epochs)	1.95	2.95
1d (8,000 epochs)	3.33	3.46

Table 1: Training/testing errors for P1 at the end of the optimisation.

As expected, it appears that the more complex models perform better. Errors for **1a-c** are reasonably close to previously recorded values. The errors for **1d** were worse than expected, being approximately 2% off from optimality. This suggests that either hyperparameter optimisation was too coarse or that alternative weight/bias initialisations should be considered. Models **1b,c** appear to overfit on the training set.

Additional Graph

Insofar, the presented graphs feature training values over each minibatch. The graph below (Figure 12) provides the training errors over the full train set and the testing errors over the full test set over all iterations for (1b).

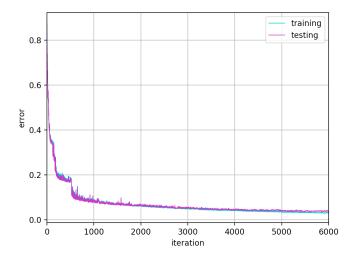


Figure 12: Plot of training (over entire set)/testing error as a function of training iterations for (1b).

2 MNIST without TensorFlow

(2ai) The derivative of the loss function with respect to the scores z, $\frac{\partial loss}{\partial z}$, is computed as follows. Firstly, consider the loss of a single example i; the softmax classifier computes this loss as,

$$loss_i = -\log\left(\frac{\exp(z_i[y_i])}{\sum_{c=1}^{10} \exp(z_i[c])}\right),\,$$

where z is the input array to the softmax layer and z[c] denotes the c-th entry of such. Note that the term in parentheses is equivalent to the softmax output, which we will denote $o_i[k]$ (for example i, array element k). Then,

$$o_i[k] = \frac{\exp(z_i[k])}{\sum_{c=1}^{10} \exp(z_i[c])},$$

and,

$$loss_i = -\log(o_i[y_i]).$$

The gradient for example i, $\frac{\partial loss}{\partial z_i[k]}$, can be decomposed by using the chain rule, with,

$$\frac{\partial loss}{\partial z_i} = \sum_{i} \frac{\partial loss}{\partial o_j} \frac{\partial o_j}{\partial z_i}.$$

having omitted the array indexing [k]. After some algebra, note that

$$\frac{\partial o_j}{\partial z_i} = o_i(1 - o_i), \qquad i = j,$$

$$\frac{\partial o_j}{\partial z_i} = -o_i o_j, \qquad i \neq j,$$

Then for the original derivative,

$$\begin{split} \frac{\partial loss}{\partial z_i} &= \sum_j \frac{\partial loss}{\partial o_j} \frac{\partial o_j}{\partial z_i} = -\sum_j \frac{\partial \log o_j}{\partial z_i} = -\sum_j \frac{1}{o_j} \frac{o_j}{z_i}, \\ &= -(1 - o_i) - \sum_{j \neq i} \frac{(-o_j o_i)}{o_j} = -(1 - o_i) + \sum_{j \neq i} o_i = o_i - \overrightarrow{1} \quad (i = j). \end{split}$$

In the original framework, provided $y_i = k, \frac{\partial loss}{\partial z_i} = o_i - \overrightarrow{1}$.

(2aii) The derivative of the loss function with respect to the inputs x, $\frac{\partial loss}{\partial x}$ for the (1a) setup, is computed via back-propagation. Note x_j are the inputs of the linear layer, we denote softmax as inputs z_k (outputs of the linear layer) and softmax outputs as o_k . Via the chain rule,

$$\frac{\partial loss}{\partial x_j} = \sum_{k} \frac{\partial loss}{\partial o_k} \frac{\partial o_k}{\partial z_k} \frac{\partial z_k}{\partial x_j}.$$

Since in the linear layer, $z_k = \sum_j w_{jk} x_j + b_{0k}$, $\frac{\partial z_k}{\partial x_j} = w_{jk}$. From the previous exercise, we know $\frac{\partial loss}{\partial z_k} = \sum_j \frac{\partial loss}{\partial o_j} \frac{\partial o_j}{\partial z_k} = o_k - \overrightarrow{1}$. Hence,

$$\frac{\partial loss}{\partial x_j} = \sum_k (o_k - \overrightarrow{1}) w_{jk}, \quad \text{i.e.} \ \, \frac{\partial loss}{\partial x_j[n]} = \sum_k (o_k[n] - \overrightarrow{1}) w_{jk} \ \, \text{for each array element } n.$$

Similarly, the derivative of the loss function wrt the weights, $\frac{\partial loss}{\partial w}$, is computed via,

$$\frac{\partial loss}{\partial w_{ik}} = \frac{\partial loss}{\partial o_k} \frac{\partial o_k}{\partial z_k} \frac{\partial z_k}{\partial w_{ik}} = (o_k - \overrightarrow{1}) x_j$$

i.e. $\frac{\partial loss}{\partial w_{jk}} = \sum_{n=1}^{N} (o_k[n] - \overrightarrow{1}) x_j[n]$, summing over N array elements. In the same manner, for the derivative of the loss function wrt the bias, $\frac{\partial loss}{\partial b}$, via,

$$\frac{\partial loss}{\partial b_{0k}} = \frac{\partial loss}{\partial o_k} \frac{\partial o_k}{\partial z_k} \frac{\partial z_k}{\partial b_{0k}} = (o_k - \overrightarrow{1}).$$

(2aiii) The derivative of a convolution layer wrt to its parameters W is computed as follows. Firstly, we denote the gradients of the convolutional layer as δ_{ij} (e.g. $\delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}$) - with respect to the output. Two updates will be performed: one on the weights W_{ij} and another on the delta gradients. Then, the derivative of the layer loss L wrt W can be computed via the chain rule, with,

$$\frac{\partial L}{\partial W_{ij}^{(l)}} = \sum_{i'} \sum_{j'} \frac{\partial L}{\partial o_{i'j'}^{(l)}} \frac{\partial o_{i'j'}^{(l)}}{\partial W_{ij}^{(l)}} = \sum_{i'} \sum_{j'} \delta_{i'j'}^{l} \frac{\partial o_{i'j'}^{(l)}}{\partial W_{ij}^{(l)}},$$

where $o_{ij}^{(l)}$ is the output vector at layer l, and by definition, $\frac{\partial L}{\partial o_{i',j'}^{(l)}} = \delta_{i'j'}^l$. Now, we know,

$$o_{i'j'}^{(l)} = W_{i'j'}^{(l)} x_{i'j'}^{(l)} + b^{(l)},$$

where $b^{(l)}$ are the specified biases for the layer and $x_{i'j'}^{(l)}$ is the reshaped input 4-d tensor. We then have,

$$\frac{\partial o_{i'j'}^{(l)}}{\partial W_{ij}^{(l)}} = \frac{\partial}{\partial W_{ij}^{(l)}} \Big(\sum_{i''} \sum_{j''} W_{i''j''}^{(l)} x_{(i'-i'')(j'-j'')}^{(l)} + b^{(l)} \Big).$$

This expression can be presented as,

$$\frac{\partial o_{i'j'}^{(l)}}{\partial W_{ij}^{(l)}} = \frac{\partial}{\partial W_{ij}^{(l)}} \Big(\sum_{i''} \sum_{j''} W_{i''j''}^{(l)} A^{(l)} (o_{(i'-i'')(j'-j''))}^{(l-1)}) + b^{(l)} \Big),$$

where A is an activation mapping intaking the output of the previous layer. Expanding and computing partial derivatives for the expression above yields zeros other than when i = i'' and j = j'' for the weights. Therefore, this implies,

$$\frac{\partial o_{i'j'}^{(l)}}{\partial W_{ij}^{(l)}} = \frac{\partial}{\partial W_{ij}^{(l)}} \left(W_{ij}^{(l)} A^{(l)} (o_{(i'-i)(j'-j)}^{(l-1)}) \right) = A^{(l)} (o_{(i'-i)(j'-j)}^{(l-1)}),$$

since i' - i'' = i' - i, j' - j" = j' - j. We can substitute the expression above into our original equation to give,

$$\frac{\partial L}{\partial W_{ij}^{(l)}} = \sum_{i'} \sum_{j'} \delta_{i'j'}^{(l)} A^{(l)}(o_{(i'-i)(j'-j)}^{(l-1)}) = \delta_{ij}^{(l)} * A^{(l)}(o_{(-i)(-j)}^{(l-1)})$$

Note that an 180 degree rotation of the kernel transforms $(o_{(-i)(-j)}^{(l-1)})$ to $(o_{(i)(j)}^{(l-1)})$. Hence,

$$\frac{\partial L}{\partial W_{ij}^{(l)}} = \delta_{ij}^{(l)} * A^{(l)}(\ rot_{180}\ o_{(ij)}^{(l-1)}).$$

The derivative of the convolution layer loss wrt to its input x (4-dim tensor) is computed as follows. Via backpropagation (chain rule), we have,

$$\frac{\partial L}{\partial x_{ij}^{(l)}} = \sum_{i'} \sum_{j'} \frac{\partial L}{\partial o_{i'j'}^{(l)}} \frac{\partial o_{i'j'}^{(l)}}{\partial W_{i'j'}^{(l)}} \frac{\partial o_{i'j'}^{(l)}}{\partial x_{ij}^{(l)}}$$

From the previous question , and since $\frac{\partial o_{i'j'}^{(l)}}{\partial x_{ij}^{(l)}} = W_{i'j'}^{(l)},$

$$\frac{\partial L}{\partial x_{ij}^{(l)}} = \left(\delta_{ij}^{(l)} * A^{(l)}(\ rot_{180}\ o_{(ij)}^{(l-1)})\right) \sum_{i'} \sum_{j'} W_{i'j'}^{(l)}.$$

2b-e not attempted due to time contraints - other assignments.

Appendix

The model restoring procedure has been commented out for all exercises.

exercise1a.py

```
model_folder = "savedmodels/"
  # question
  subdirectorv = "1a/"
   model_filename = model_folder + subdirectory + "model_1a.ckpt"
  # load and read MNIST data, import tensorflow
21
  from tensorflow.examples.tutorials.mnist import input_data
  # import dataset with one-hot class encoding
   print("Loading the data.....")
   mnist = input_data.read_data_sets(data_dir, one_hot=True)
   print("Data has been loaded.")
   import tensorflow as tf
27
  # import sklearn, matplotlib for confusion matrix generation functionality
29
  import matplotlib.pyplot as plt
  from sklearn.metrics import confusion_matrix
  # import numpy for train/test accuracy/loss tracking
33
  import numpy as np
34
  # operating system interface to save cost/accuracy history.
36
  import os
37
  # set a random seed for reproducibility
   tf.set_random_seed(555)
40
41
  # define hyper-parameter values
42
   n_attributes = 784
43
   n_{classes} = 10
44
   optimiser\_step\_size = 0.5
   tot_training_iterations = 5000
   minibatch_size = 50
   print_update_every = 200
48
49
  # one-hot encoding converted to single number class by storing
  # index of highest element.
  mnist.test.klass = np.array([lbl.argmax() for lbl in mnist.test.labels])
52
  # placeholder for data x, (784 pixels/attributes)
  # placeholder for label y
  x = tf.placeholder(tf.float32, [None, n_attributes])
  y = tf.placeholder(tf.float32, [None, n_classes])
  # placeholder for true single number class (label)
  y_klass = tf.placeholder(tf.int64, [None])
59
  # neural network model building
  # define variables - initialise weights, biases as tensors full of zeros
   weights = tf. Variable (tf. zeros ([n_attributes, n_classes]))
   biases = tf. Variable (tf. zeros ([n_classes]))
  # linear regression layer
  y_temp = tf.add(tf.matmul(x, weights), biases)
  # softmax performed
  y_pred = tf.nn.softmax(y_temp)
  # one-hot encoding -> predicted numerical label (class), index of largest
      element in row
  y_pred_klass = tf.argmax(y_pred, dimension = 1)
  # model is trained using cross-entropy loss function
```

```
cost = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(labels=y,
      logits=y_pred))
   # stochastic gradient descent for minimising objective
   optimiser = tf.train.GradientDescentOptimizer(optimiser_step_size).minimize(
      cost)
   # define the accuracy - percentage of times correct prediction
76
   correct_prediction = tf.equal(tf.argmax(y_pred, 1), tf.argmax(y, 1))
   accuracy = tf.reduce_mean(tf.cast(correct_prediction, tf.float32))
   # error is 1-accuracy
   error = 1 - accuracy
80
81
   # save weights, biases for train. Saver
82
   tf.add_to_collection('vars', weights)
   tf.add_to_collection('vars', biases)
84
   saver = tf.train.Saver()
87
   with tf. Session() as sess:
88
       # operation to initialise all variables
89
       tf.global_variables_initializer().run()
91
       print("Training started.....")
92
       # initialisation of arrays keeping track of loss/error history
       train_loss_history = np.zeros(tot_training_iterations)
       test_loss_history = np. zeros(tot_training_iterations)
95
       train_error_history = np.zeros(tot_training_iterations)
96
       test_error_history = np.zeros(tot_training_iterations)
97
       tot\_confmat = np.zeros((n\_classes, n\_classes))
       # loop over each training iteration
       for iteration in range(tot_training_iterations):
100
           # load 'minibatch_size' examples in each training iteration
           xbatch , ybatch = mnist.train.next_batch(minibatch_size)
           # error/loss for test and train calculated for each iteration
103
           train_error = error.eval(feed_dict={x: xbatch, y: ybatch})
104
           train_error_history[iteration] = train_error
           train_loss = cost.eval(feed_dict={x: xbatch, y: ybatch})
106
           train_loss_history[iteration] = train_loss
107
           test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.
108
               test.labels })
           test_error_history[iteration] = test_error
109
           test_loss = cost.eval(feed_dict={x: mnist.test.images,y: mnist.test.
110
               labels })
           test_loss_history[iteration] = test_loss
           # print error/loss values every 'print_update_every' iteration
112
               track progress
           if iteration % print_update_every = 0:
113
                print ("iteration %d, training error %g" % (iteration, train_error
114
                   ))
               print("iteration %d, training loss %g" % (iteration, train_loss))
115
               print("iteration %d, testing error %g" % (iteration, test_error))
116
                print("iteration %d, testing loss %g" % (iteration, test_loss))
           # saves error/loss history to directory
118
           np.savez(os.getcwd() + "/traininghistory1a", train_loss_history=
119
               train_loss_history,
                     test_loss_history=test_loss_history,
                     train_error_history = train_error_history,
121
                     test_error_history = test_error_history)
122
           # run optimiser for each batch - update gradients
123
           optimiser.run(feed_dict={x: xbatch, y: ybatch})
124
```

```
# groups test set images, one-hot encoding, true numerical labels of
               test set
            feed_dict_test = {x: mnist.test.images, y: mnist.test.labels, y_klass
126
               : mnist.test.klass}
           # test set true numerical classes (labels)
127
            true_klass = mnist.test.klass
128
           # test set predicted numerical classes (labels)
129
            pred_klass = sess.run(y_pred_klass, feed_dict=feed_dict_test)
           # sci-kit learn functionality to generate a confusion matrix
131
            confmat = confusion_matrix(y_true=true_klass, y_pred=pred_klass)
132
            tot_confmat = np.add(tot_confmat, confmat)
133
       saver.save(sess, model_filename)
134
       print("Training finished.")
135
       print ("=
                                               =\n")
136
       # in final confusion matrix, the diagonal is set to zeros.
137
       # correct classifications hugely outweigh errors and are ignored in the
           confusion matrix plot
       for i in range (n_classes):
139
            tot\_confmat[i,i] = 0
140
       141
       # confusion matrix plots
142
       143
       # text print of confusion matrix
144
145
       print(tot_confmat)
       # image print of confusion matrix
146
       plt.figure()
147
       plt.imshow(tot_confmat, interpolation='none', cmap=plt.cm.BuGn)
148
       # make plot nice
149
       plt.colorbar()
150
       tick_marks = np.arange(n_classes)
151
       plt.xticks(tick_marks, range(n_classes))
152
       plt.yticks(tick_marks, range(n_classes))
       plt.xlabel('Predicted Class')
154
       plt.ylabel('True Class')
155
       plt.savefig('laconfusionmatrix.png')
156
       157
       # learning curve plots for cross entropy loss
158
       159
       plt.figure()
       traininghistory = np.load(os.getcwd() + "/traininghistory1a.npz")
161
       trainingloss = traininghistory["train_loss_history"]
162
       testingloss = traininghistory["test_loss_history"]
163
       axisx = np.arange(tot_training_iterations)
164
       plt.plot\left(\left.axisx\right.,\ trainingloss\right.,\ "g-",\ linewidth=0.8,\ label="training"\right)
165
       plt.plot(axisx, testingloss, "r-", linewidth=0.8, label="testing")
166
       plt.grid()
167
       plt.legend()
       plt.xlabel("iteration")
169
       plt.ylabel("cross-entropy loss")
170
       plt.xlim(0, tot_training_iterations)
171
       plt.show()
       plt.savefig('lalosslearningcurve.png')
173
       ####
174
            learning curve for error
       ###
       ###
176
       plt.figure()
177
       traininghistory2 = np.load(os.getcwd() + "/traininghistory1a.npz")
178
       trainingerror = traininghistory2 ["train_error_history"]
179
       testingerror = traininghistory2 ["test_error_history"]
180
```

```
axisx2 = np.arange(tot_training_iterations)
                   \begin{array}{l} plt.\,plot\,(\,axisx2\,,\,\,trainingerror\,,\,\,"\,c-"\,,\,\,linewidth\,=\,0.8\,,\,\,label="\,training"\,)\\ plt.\,plot\,(\,axisx2\,,\,\,\,testingerror\,,\,\,"m\!-\!"\,,\,\,\,linewidth\,=\,0.8\,,\,\,\,label="\,testing"\,) \end{array}
182
183
                   plt.grid()
184
                   plt.legend()
185
                   plt.xlabel("iteration")
186
                   plt.ylabel("error")
187
                   plt.xlim(0, tot_training_iterations)
                   plt.show()
189
                   plt.savefig('1aerrorlearningcurve.png')
190
191
192
193
        ### RESTORE MODEL AND OBTAIN FINAL ACCURACY
194
195
        print ("Restoring and testing model")
        with tf. Session() as sess:
197
                   new_saver = tf.train.import_meta_graph(model_folder + subdirectory + "
198
                            model_1a.ckpt.meta")
                   new\_saver.restore \,(\,sess\,\,,\,\,tf.\,train.\,latest\_checkpoint \,(\,model\_folder\,\,+\,\,deltarest) + (\,sess\,\,,\,\,tf.\,train.\,latest\_checkpoint) + (\,sess\,\,,\,\,tf.\,tr
                            subdirectory + './'))
                   all_vars = tf.get_collection('vars')
200
                   weights = all_vars[0]
201
                   biases = all_vars[1]
                   test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.test.
203
                            labels })
                   print ("The final test error is %g" % (test_error))
204
205
206
        exercise1b.py
        Neural network model to classify MNIST digits consisting of 1 hidden layer
        units) with a ReLu non-linearity, followed by a softmax.
       # for compatibility
       from __future__ import absolute_import
        from __future__ import division
        from __future__ import print_function
       # data directory
 10
        data_dir = "./MNIST_data/"
 11
 12
       # for saving the model
 13
        model_folder = "savedmodels/"
       # question
        subdirectory = "1b/"
        model_filename = model_folder + subdirectory + "model_1b.ckpt"
 17
        # load and read MNIST data, import tensorflow
        from tensorflow.examples.tutorials.mnist import input_data
 20
        # import dataset with one-hot class encoding
        print ("Loading the data.....")
        mnist = input_data.read_data_sets(data_dir, one_hot=True)
        print("Data has been loaded.")
        import tensorflow as tf
 25
       # import sklearn, matplotlib for confusion matrix generation functionality
```

```
import matplotlib.pyplot as plt
  from sklearn.metrics import confusion_matrix
29
30
  # import numpy for train/test accuracy/loss tracking
  import numpy as np
32
33
  # operating system interface to save cost/accuracy history.
  import os
  # set a random seed for reproducibility
37
  tf.set_random_seed(555)
38
39
  # define hyper-parameter values
  n_{attributes} = 784
41
  n_{classes} = 10
  optimiser_step_size = 0.5
  tot_training_iterations = 6000
  minibatch_size = 50
45
  print_update_every = 200
  # number of units in hidden layer 1
  n_units_h1 = 128
49
  # one-hot encoding converted to single number class by storing
  # index of highest element.
51
  mnist.test.klass = np.array([lbl.argmax() for lbl in mnist.test.labels])
52
53
  # placeholder for data x, (784 pixels/attributes)
  # placeholder for label y
  x = tf.placeholder(tf.float32, [None, n_attributes])
  y = tf.placeholder(tf.float32, [None, n_classes])
  # placeholder for true single number class (label)
  y_klass = tf.placeholder(tf.int64, [None])
60
61
  # neural network model building
  # define variables - initialise weights, biases
  # hidden layer 1
  W_h1 = tf. Variable (0.1 * tf.random_normal([n_attributes, n_units_h1]), name="
      W_h1")
  b_h1 = tf. Variable (tf.random_normal([n_units_h1]), name="b_h1")
  # add ReLu non-linearity to hidden layer 1
  hidden1 = tf.nn.relu(tf.matmul(x, W_h1) + b_h1)
  # linear layer
  W = tf. Variable (0.1 * tf.random_normal([n_units_h1, n_classes]), name="W")
  b = tf. Variable (tf. zeros ([n_classes]), name="b")
  # put together linear layer
  z = tf.matmul(hidden1, W) + b
  # softmax performed
  y_{pred} = tf.nn.softmax(z)
  # one-hot encoding -> predicted numerical label (class), index of largest
      element in row
  y_pred_klass = tf.argmax(y_pred, dimension=1)
  # model is trained using cross-entropy loss function
  cost = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(labels=y,
      logits=y_pred))
  \# 0.61 optimal
  optimiser = tf.train.GradientDescentOptimizer(optimiser_step_size).minimize(
      cost)
  # define the accuracy - percentage of times correct prediction
```

```
correct\_prediction = tf.equal(tf.argmax(y\_pred, 1), tf.argmax(y, 1))
   accuracy = tf.reduce_mean(tf.cast(correct_prediction, tf.float32))
   # error is 1-accuracy
   error = 1 - accuracy
   # save final weights, biases for train. Saver
   tf.add_to_collection('vars', W)
   tf.add_to_collection('vars', b)
91
   saver = tf.train.Saver()
92
93
   # NEURAL NEIWORK TRAINING COMMENCES
94
   with tf. Session () as sess:
95
       # operation to initialise all variables
96
       tf.global_variables_initializer().run()
97
       print ("======
       print("Training started .....")
       # initialisation of arrays keeping track of loss/error history
100
       train_loss_history = np.zeros(tot_training_iterations)
101
       test_loss_history = np.zeros(tot_training_iterations)
       train_error_history = np.zeros(tot_training_iterations)
103
       test_error_history = np.zeros(tot_training_iterations)
104
       tot\_confmat = np.zeros((n\_classes, n\_classes))
105
       # loop over each training iteration
106
       for iteration in range (tot_training_iterations):
107
           # load 'minibatch_size' examples in each training iteration
108
           xbatch, ybatch = mnist.train.next_batch(minibatch_size)
109
           # error/loss for test and train calculated for each iteration
110
            train_error = error.eval(feed_dict={x: xbatch, y: ybatch})
111
            train_error_history[iteration] = train_error
112
            train_loss = cost.eval(feed_dict={x: xbatch, y: ybatch})
            train_loss_history[iteration] = train_loss
            test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.
115
               test.labels})
            test_error_history[iteration] = test_error
116
            test_loss = cost.eval(feed_dict={x: mnist.test.images, y: mnist.test.
117
               labels })
            test_loss_history[iteration] = test_loss
118
           # print error/loss values every 'print_update_every' iteration
               track progress
            if iteration \% print_update_every = 0:
120
                print ("iteration %d, training error %g" % (iteration, train_error
121
                   ))
                print("iteration %d, training loss %g" % (iteration, train_loss))
122
                print("iteration %d, testing error %g" % (iteration, test_error))
123
                print("iteration %d, testing loss %g" % (iteration, test_loss))
           # saves error/loss history to directory
           np.savez(os.getcwd() + "/traininghistory1b", train_loss_history=
126
               train_loss_history ,
                     test_loss_history=test_loss_history,
127
                     train_error_history=train_error_history,
                     test_error_history=test_error_history)
129
           # run optimiser for each batch - update gradients
130
           optimiser.run(feed_dict={x: xbatch, y: ybatch})
           # groups test set images, one-hot encoding, true numerical labels of
132
               test set
            feed_dict_test = {x: mnist.test.images, y: mnist.test.labels, y_klass
133
               : mnist.test.klass}
           # test set true numerical classes (labels)
134
```

```
true_klass = mnist.test.klass
135
            # test set predicted numerical classes (labels)
136
            pred_klass = sess.run(y_pred_klass, feed_dict=feed_dict_test)
137
            # sci-kit learn functionality to generate a confusion matrix
138
            confmat = confusion_matrix(y_true=true_klass, y_pred=pred_klass)
139
            tot_confmat = np.add(tot_confmat, confmat)
140
       # saving
141
        saver.save(sess, model_filename)
        print("Training finished.")
143
        print ("=
                                                  =\n")
144
        # in final confusion matrix, the diagonal is set to zeros.
145
       # correct classifications hugely outweigh errors and are ignored in the
146
           confusion matrix plot
        for i in range (n_classes):
147
            tot\_confmat[i, i] = 0
148
       # confusion matrix plots
150
       151
        # text print of confusion matrix
152
        print(tot_confmat)
        # image print of confusion matrix
154
        plt.figure()
155
        plt.imshow(tot_confmat, interpolation='none', cmap=plt.cm.BuGn)
156
       # make plot nice
        plt.colorbar()
158
        tick_marks = np.arange(n_classes)
159
        plt.xticks(tick_marks, range(n_classes))
160
        plt.yticks(tick_marks, range(n_classes))
161
        plt.xlabel('Predicted Class')
162
        plt.ylabel('True Class')
163
        plt.savefig('1bconfusionmatrix.png')
164
        165
        # learning curve plots for loss/error
166
       167
        plt.figure()
168
        training history = np.load (os.getcwd() + "/training history1b.npz")
169
        trainingloss = traininghistory ["train_loss_history"]
170
        testingloss = traininghistory["test_loss_history"]
171
        axisx = np.arange(tot_training_iterations)
        173
174
        plt.grid()
175
        plt.legend()
        plt.xlabel("iteration")
177
        plt.vlabel("cross-entropy loss")
178
        plt.xlim(0, tot_training_iterations)
        plt.show()
        plt.savefig('1blosslearningcurve.png')
181
       ####
182
            learning curve for error
        ###
183
184
       ###
        plt.figure()
185
        traininghistory2 = np.load(os.getcwd() + "/traininghistory1b.npz")
186
        trainingerror = traininghistory2["train_error_history"]
        testingerror = traininghistory2["test_error_history"]
188
        axisx2 = np.arange(tot_training_iterations)
189
        \begin{array}{l} plt.\,plot\,(\,axisx2\,\,,\,\,trainingerror\,\,,\,\,\,"c-"\,\,,\,\,linewidth\,=\!0.8\,,\,\,label="\,training"\,)\\ plt.\,plot\,(\,axisx2\,\,,\,\,\,testingerror\,\,,\,\,\,"m-"\,\,,\,\,\,linewidth\,=\!0.8\,,\,\,\,label="\,testing"\,) \end{array}
190
191
        plt.grid()
192
```

```
plt.legend()
                       plt.xlabel("iteration")
194
                       plt.ylabel("error")
195
                       plt.xlim(0, tot_training_iterations)
196
                       plt.show()
197
                       plt.savefig('1berrorlearningcurve.png')
198
199
201
         ### RESTORE MODEL AND OBTAIN FINAL ACCURACY
202
203
          print ("Restoring and testing model")
204
          with tf. Session() as sess:
205
                      new_saver = tf.train.import_meta_graph(model_folder + subdirectory + "
206
                                 model_1b.ckpt.meta")
                       new\_saver.restore \, (\,sess \;, \; tf.train.latest\_checkpoint \, (\,model\_folder \; + \; 1) \, and \,
207
                                 subdirectory + './'))
                       all_vars = tf.get_collection('vars')
208
                       weights = all_vars[0]
209
                       biases = all_vars[1]
                       train_error = error.eval(feed_dict={x: mnist.train.images, y: mnist.train
211
                                 .labels})
                       test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.test.
212
                                 labels })
                       print("The final train error is %g" % (train_error))
213
                       print("The final test error is %g" % (test_error))
214
215
216
          exercise1c.py
          Neural network model to classify MNIST digits consisting of 2 hidden layers
          units each) with a ReLu non-linearity, followed by a softmax.
   3
        # for compatibility
         from __future__ import absolute_import
          from __future__ import division
          from __future__ import print_function
         # data directory
  10
          data_dir = "./MNIST_data/"
  11
  12
         # for saving the model
          model_folder = "savedmodels/"
         # question
  15
```

import sklearn, matplotlib for confusion matrix generation functionality

 $model_filename = model_folder + subdirectory + "model_1c.ckpt"$

from tensorflow.examples.tutorials.mnist import input_data

mnist = input_data.read_data_sets(data_dir, one_hot=True)

load and read MNIST data, import tensorflow

import dataset with one-hot class encoding

print ("Loading the data.....")

print("Data has been loaded.")

import tensorflow as tf

subdirectory = "1c/"

18

19

20

24

```
import matplotlib.pyplot as plt
  from sklearn.metrics import confusion_matrix
29
30
  # import numpy for train/test accuracy/loss tracking
  import numpy as np
32
33
  # operating system interface to save cost/accuracy history.
  import os
  # set a random seed for reproducibility
37
  tf.set_random_seed(555)
38
39
  # define hyper-parameter values
  n_attributes = 784
41
  n_{classes} = 10
  optimiser_step_size = 0.5
  tot_training_iterations = 6000
  minibatch_size = 50
45
  print_update_every = 200
  # number of units in hidden layer 1
  n_units_h1 = 256
  # number of units in hidden layer 2
  n_units_h2 = 256
  # one-hot encoding converted to single number class by storing
52
  # index of highest element.
53
  mnist.test.klass = np.array([lbl.argmax() for lbl in mnist.test.labels])
  # placeholder for data x, (784 pixels/attributes)
56
  # placeholder for label y
  x = tf.placeholder(tf.float32, [None, n_attributes])
  y = tf.placeholder(tf.float32, [None, n_classes])
  # placeholder for true single number class (label)
  y_klass = tf.placeholder(tf.int64, [None])
  # neural network model building
  # define variables - initialise weights, biases
  # hidden layer 1
  W_h1 = tf.Variable(0.1 * tf.random_normal([n_attributes, n_units_h1]), name="
      W_h1")
  b_h1 = tf. Variable(tf.random_normal([n_units_h1]), name="b_h1")
  # add ReLu non-linearity to hidden layer 1
  hidden1 = tf.nn.relu(tf.matmul(x, W_h1) + b_h1)
  # hidden layer 2
  W_h2 = tf.Variable(0.1 * tf.random_normal([n_units_h1, n_units_h2]), name="
      W_h2")
  b_h2 = tf. Variable(tf.random_normal([n_units_h2]), name="b_h2")
  # add ReLu non-linearity to hidden layer 2
  hidden2 = tf.nn.relu(tf.matmul(hidden1, W_h2) + b_h2)
  # linear layer
  W = tf. Variable(0.1 * tf.random_normal([n_units_h2, n_classes]), name="W")
  b = tf. Variable (tf. zeros ([n_classes]), name="b")
  # put together linear layer
  z = tf.matmul(hidden2, W) + b
  # softmax performed
  y_{pred} = tf.nn.softmax(z)
  # one-hot encoding -> predicted numerical label (class), index of largest
      element in row
  y_pred_klass = tf.argmax(y_pred, dimension=1)
```

```
# model is trained using cross-entropy loss function
   cost = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(labels=y,
       logits=y_pred))
   \# 0.61 optimal
   optimiser = tf.train.GradientDescentOptimizer(optimiser_step_size).minimize(
   # define the accuracy - percentage of times correct prediction
88
   correct_prediction = tf.equal(tf.argmax(y_pred, 1), tf.argmax(y, 1))
   accuracy = tf.reduce_mean(tf.cast(correct_prediction, tf.float32))
   # error is 1-accuracy
91
   error = 1 - accuracy
92
93
   # save final weights, biases for train. Saver
   tf.add_to_collection('vars', W)
95
   tf.add_to_collection('vars', b)
   saver = tf.train.Saver()
98
99
   # NEURAL NEIWORK TRAINING COMMENCES
100
101
   with tf. Session() as sess:
102
       # operation to initialise all variables
103
       tf.global_variables_initializer().run()
104
       print ("==
105
       print("Training started .....")
106
       # initialisation of arrays keeping track of loss/error history
107
       train_loss_history = np.zeros(tot_training_iterations)
108
        test_loss_history = np.zeros(tot_training_iterations)
109
       train_error_history = np.zeros(tot_training_iterations)
110
       test_error_history = np.zeros(tot_training_iterations)
111
       tot_confmat = np.zeros((n_classes, n_classes))
112
       # loop over each training iteration
       for iteration in range (tot_training_iterations):
114
           # load 'minibatch_size' examples in each training iteration
115
           xbatch, ybatch = mnist.train.next_batch(minibatch_size)
           # error/loss for test and train calculated for each iteration
117
            train_error = error.eval(feed_dict={x: xbatch, y: ybatch})
118
            train_error_history[iteration] = train_error
119
            train_loss = cost.eval(feed_dict={x: xbatch, y: ybatch})
            train_loss_history [iteration] = train_loss
121
            test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.
122
               test.labels)
            test_error_history[iteration] = test_error
            test_loss = cost.eval(feed_dict={x: mnist.test.images, y: mnist.test.
124
            {\tt test\_loss\_history} \, [\, {\tt iteration} \, ] \, = \, {\tt test\_loss}
125
           # print error/loss values every 'print_update_every' iteration
126
               track progress
            if iteration % print_update_every == 0:
127
                print("iteration %d, training error %g" % (iteration, train_error
128
                    ))
                print ("iteration %d, training loss %g" % (iteration, train_loss))
129
                print("iteration %d, testing error %g" % (iteration, test_error))
130
                print("iteration %d, testing loss %g" % (iteration, test_loss))
           # saves error/loss history to directory
132
           np.savez(os.getcwd() + "/traininghistory1C", train_loss_history=
133
               train_loss_history,
                     test_loss_history=test_loss_history,
134
                     train_error_history=train_error_history,
135
```

```
test_error_history=test_error_history)
           # run optimiser for each batch - update gradients
137
           optimiser.run(feed_dict={x: xbatch, y: ybatch})
138
           # groups test set images, one-hot encoding, true numerical labels of
139
              test set
           feed_dict_test = {x: mnist.test.images, y: mnist.test.labels, y_klass
140
               : mnist.test.klass}
           # test set true numerical classes (labels)
           true_klass = mnist.test.klass
142
           # test set predicted numerical classes (labels)
143
           pred_klass = sess.run(y_pred_klass, feed_dict=feed_dict_test)
144
           # sci-kit learn functionality to generate a confusion matrix
145
           confmat = confusion_matrix(y_true=true_klass, y_pred=pred_klass)
146
           tot_confmat = np.add(tot_confmat, confmat)
147
       # saving
148
       print("Training finished.")
       print ("=
                                             —\n")
150
       saver.save(sess, model_filename)
151
       # in final confusion matrix, the diagonal is set to zeros.
152
       # correct classifications hugely outweigh errors and are ignored in the
          confusion matrix plot
       for i in range (n_classes):
154
           tot\_confmat[i, i] = 0
155
       # confusion matrix plots
157
       158
       # text print of confusion matrix
159
       print(tot_confmat)
160
       # image print of confusion matrix
161
       plt.figure()
162
       plt.imshow(tot_confmat, interpolation='none', cmap=plt.cm.BuGn)
163
       # make plot nice
       plt.colorbar()
165
       tick_marks = np.arange(n_classes)
166
       plt.xticks(tick_marks, range(n_classes))
167
       plt.yticks(tick_marks, range(n_classes))
168
       plt.xlabel('Predicted Class')
169
       plt.ylabel('True Class')
170
       plt.savefig('confmat1c.png')
       172
       # learning curve plots for loss/error
173
       174
       plt.figure()
175
       training history = np.load (os.getcwd() + "/training history1c.npz")
176
       trainingloss = traininghistory ["train_loss_history"]
177
       testingloss = traininghistory["test_loss_history"]
       axisx = np.arange(tot_training_iterations)
       180
181
       plt.grid()
182
183
       plt.legend()
       plt.xlabel("iteration")
184
       plt.ylabel("cross-entropy loss")
185
       plt.xlim(0, tot_training_iterations)
       plt.show()
187
       plt.savefig('1closslearningcurve.png')
188
       ####
189
       ####
            learning curve for error
190
       ###
191
```

```
plt.figure()
        traininghistory2 = np.load(os.getcwd() + "/traininghistory1c.npz")
193
        trainingerror = traininghistory2 ["train_error_history"]
194
        testingerror = traininghistory2 ["test_error_history"]
195
        axisx2 = np.arange(tot_training_iterations)
196
        plt.plot(axisx2, trainingerror, "c-", linewidth=0.8, label="training")
197
       plt.plot(axisx2, testingerror, "m-", linewidth=0.8, label="testing")
198
        plt.grid()
        plt.legend()
200
        plt.xlabel("iteration")
201
        plt.ylabel("error")
202
        plt.xlim(0, tot_training_iterations)
203
        plt.show()
204
        plt.savefig('1cerrorlearningcurve.png')
205
206
208
209
   ### RESTORE MODEL AND OBTAIN FINAL ACCURACY
210
211
   print ("Restoring and testing model")
212
   with tf. Session() as sess:
213
       new_saver = tf.train.import_meta_graph(model_folder + subdirectory + "
214
           model_1c.ckpt.meta")
        new_saver.restore(sess, tf.train.latest_checkpoint(model_folder +
215
           subdirectory + './'))
        all_vars = tf.get_collection('vars')
216
        weights = all_vars[0]
217
        biases = all_vars[1]
218
        train_error = error.eval(feed_dict={x: mnist.train.images, y: mnist.train
219
           .labels})
        test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.test.
           labels })
        print("The final train error is %g" % (train_error))
221
        print("The final test error is %g" % (test_error))
222
223
224
   exercise1d.py
   ""
   Neural network model to classify MNIST digits consisting of a 3 layer
       convolutional
   model (2 convolutional layers followed by max pooling), followed by one non-
       linear
   layer (256 units) followed by a softmax.
 4
  # for compatibility
   from __future__ import absolute_import
   from __future__ import division
   from __future__ import print_function
   # data directory
11
   data_dir = "./MNIST_data/"
12
13
   # for saving the model
   model_folder = "savedmodels/"
15
   # question
   subdirectory = "1d/"
   model_filename = model_folder + subdirectory + "model_1d.ckpt"
```

```
# load and read MNIST data, import tensorflow
  from tensorflow.examples.tutorials.mnist import input_data
  # import dataset with one-hot class encoding
  print ("Loading the data.....")
  mnist = input_data.read_data_sets(data_dir, one_hot=True)
  print("Data has been loaded.")
  import tensorflow as tf
  # import sklearn, matplotlib for confusion matrix generation functionality
28
  import matplotlib.pyplot as plt
29
  from sklearn.metrics import confusion_matrix
30
  # import numpy for train/test accuracy/loss tracking
32
  import numpy as np
33
  # operating system interface to save cost/accuracy history.
35
  import os
36
37
  # set a random seed for reproducibility
  tf.set_random_seed (555)
39
40
  # define hyper-parameter values
  n_attributes = 784
  n_{classes} = 10
43
  optimiser_step_size = 0.5
44
  tot_training_iterations = 8000
  minibatch_size = 50
  print_update_every = 50
  # number of units in hidden layer 1
  n_units_h1 = 256
49
  # one-hot encoding converted to single number class by storing
51
  # index of highest element.
52
  mnist.test.klass = np.array([lbl.argmax() for lbl in mnist.test.labels])
  # placeholder for data x, (784 pixels/attributes)
55
  # placeholder for label y
  x = tf.placeholder(tf.float32, [None, n_attributes])
  y = tf.placeholder(tf.float32, [None, n_classes])
  # placeholder for true single number class (label)
  y_klass = tf.placeholder(tf.int64, [None])
  # function which initialises (and returns) bias variables,
62
  # it intakes the specified shape 'config' of the variables and creates
  # a tensor of bias values populated with 0.1s (FOR LINEAR LAYER)
  def create_biases (config):
       init = tf.constant(0.1, shape=config)
66
       return tf. Variable(init)
67
  # function which initialises (and returns) weight variables.
  # it intakes the specified shape 'config' of the variables and creates
  # a tensor of weights from a truncated normal distribution (SD 0.1)
  def create_weights(config):
       init = tf.truncated_normal(config, stddev=0.1)
       return tf. Variable (init)
74
75
 # function which intakes input x and filter W and returns and computes
 # their 2D convolution, which is returned - 'SAME' padding and plain (no
```

```
strides)
   def convolution(x, weights):
78
       return tf.nn.conv2d(x, weights, strides=[1, 1, 1, 1], padding='SAME')
79
80
   \# function which intakes input x and performs max pooling on it with 2z2
       sliding
   # windows - no overlapping pixels.
82
   def pooling(x):
       return tf.nn.max_pool(x, ksize=[1, 2, 2, 1], strides=[1, 2, 2, 1],
           padding='SAME')
85
   # neural network model building
   # define variables - initialise weights, biases
  # convolution layer 1 w/b
   Wconvol1 = create_weights([3, 3, 1, 16])
   bconvol1 = create\_biases([16])
   # before applying layer 1, reshape x to 4d tensor
   reshaped1 = tf.reshape(x, [-1,28,28,1])
   # apply convolution layer 1
   hiddenconv1 = tf.add(convolution(reshaped1, Wconvol1), bconvol1)
   # apply max pooling 1
   pooling1 = pooling(hiddenconv1)
   # convolution layer 2 w/b
   Wconvol2 = create_weights([3, 3, 16, 16])
   bconvol2 = create\_biases([16])
   # apply convolution layer 2
100
   hiddenconv2 = tf.add(convolution(pooling1, Wconvol2), bconvol2)
   # apply max pooling 2
   pooling2 = pooling(hiddenconv2)
103
   # FLATTEN
   reshapeflat = tf.reshape(pooling2, [-1, 7*7*16])
   # hidden layer
   W_h1 = tf. Variable (0.1 * tf.random_normal([n_attributes, n_units_h1]), name="
107
      W_h1")
   b_h1 = tf. Variable (tf.random_normal([n_units_h1]), name="b_h1")
   # add ReLu non-linearity to hidden layer 1
   hidden1 = tf.nn.relu(tf.matmul(reshapeflat, W_h1) + b_h1)
   # linear layer
  W = tf. Variable(0.1 * tf.random_normal([n_units_h1, n_classes]), name="W")
   b = tf. Variable (tf.zeros ([n_classes]), name="b")
  # put together linear layer
  z = tf.add(tf.matmul(hidden1, W), b)
  # softmax performed
  y_pred = tf.nn.softmax(z)
  # term definitions
118
   # one-hot encoding -> predicted numerical label (class), index of largest
       element in row
   y_pred_klass = tf.argmax(y_pred, dimension=1)
120
   # model is trained using cross-entropy loss function
121
   cost = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(labels=y,
       logits=y_pred))
   # run optimiser
123
   optimiser = tf.train.GradientDescentOptimizer(optimiser_step_size).minimize(
124
       cost)
   # define the accuracy - percentage of times correct prediction
   correct_prediction = tf.equal(tf.argmax(y_pred, 1), tf.argmax(y, 1))
126
   accuracy = tf.reduce_mean(tf.cast(correct_prediction, tf.float32))
  # error is 1-accuracy
   error = 1 - accuracy
```

```
# save final weights, biases for train. Saver
131
   tf.add_to_collection('vars', W)
132
   tf.add_to_collection('vars', b)
134
   saver = tf.train.Saver()
135
136
   # NEURAL NEIWORK TRAINING
   with tf. Session() as sess:
138
       sess.run(tf.global_variables_initializer())
139
       # operation to initialise all variables
140
       tf.global_variables_initializer().run()
141
142
       print("Training started.....")
143
       # initialisation of arrays keeping track of loss/error history
144
       train\_error\_history = []
       test_error_history = []
146
       selected = []
147
       tot\_confmat = np.zeros((n\_classes, n\_classes))
148
       # loop over each training iteration
       for iteration in range (tot_training_iterations):
150
           # load 'minibatch_size' examples in each training iteration
151
           xbatch , ybatch = mnist.train.next_batch(minibatch_size)
            if iteration % print_update_every = 0:
154
                # error/loss for test and train calculated for each iteration
155
                train_error = error.eval(feed_dict={x: xbatch, y: ybatch})
156
                train_error_history.append(train_error)
157
                test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist
158
                   . test.labels })
                test_error_history.append(test_error)
159
                selected.append(iteration)
160
                print ("iteration %d, training error %g" % (iteration, train_error
161
                   ))
                print("iteration %d, testing error %g" % (iteration, test_error))
162
                # saves error/loss history to directory
163
                np.savez(os.getcwd() + "/traininghistory1d",
164
                         train_error_history=train_error_history,
165
                          test_error_history=test_error_history)
                # print error/loss values every 'print_update_every' iteration
167
                   to track progress
                # groups test set images, one-hot encoding, true numerical labels
168
                     of test set
                feed_dict_test = {x: mnist.test.images, y: mnist.test.labels,
169
                   y_klass: mnist.test.klass}
                # test set true numerical classes (labels)
170
                true_klass = mnist.test.klass
                # test set predicted numerical classes (labels)
                pred_klass = sess.run(y_pred_klass, feed_dict=feed_dict_test)
173
                # sci-kit learn functionality to generate a confusion matrix
174
                confmat = confusion_matrix(y_true=true_klass, y_pred=pred_klass)
                tot_confmat = np.add(tot_confmat, confmat)
176
           # run optimiser for each batch - update gradients
177
            optimiser.run(feed_dict={x: xbatch, y: ybatch})
       print("Training finished.")
179
       print ("=
180
       # save model
181
       saver.save(sess, model_filename)
182
       # in final confusion matrix, the diagonal is set to zeros.
183
```

```
# correct classifications hugely outweigh errors and are ignored in the
          confusion matrix plot
       for i in range (n_classes):
185
           tot\_confmat[i, i] = 0
186
       187
       # confusion matrix plots
188
       189
       # text print of confusion matrix
       print(tot_confmat)
191
       # image print of confusion matrix
192
       plt.figure()
193
       plt.imshow(tot_confmat, interpolation='none', cmap=plt.cm.BuGn)
194
       # make plot nice
195
       plt.colorbar()
196
       tick_marks = np.arange(n_classes)
197
       plt.xticks(tick_marks, range(n_classes))
       plt.yticks(tick_marks, range(n_classes))
199
       plt.xlabel('Predicted Class')
200
       plt.ylabel('True Class')
201
       plt.savefig('confmat1d.png')
202
       203
       # learning curve plots for loss/error
204
       205
       plt.figure()
       model_history = np.load(os.getcwd() + "/traininghistory1d.npz")
207
       train_error = model_history["train_error_history"]
208
       test_error = model_history["test_error_history"]
209
       x_axis = np.arange(0, tot_training_iterations, print_update_every)
210
       211
212
       plt.grid()
213
       plt.legend()
       plt.xlabel("iteration")
215
       plt.ylabel("error")
216
       plt.xlim(0, tot_training_iterations)
217
       plt.show()
218
       plt.savefig('1derrorlearningcurve.png')
219
       ####
220
       ###
       ###
222
223
224
225
226
227
   ### RESTORE MODEL AND OBTAIN FINAL ACCURACY
228
   print ("Restoring and testing model")
230
   with tf. Session() as sess:
231
       new_saver = tf.train.import_meta_graph(model_folder + subdirectory + "
232
          model_1d.ckpt.meta")
       new_saver.restore(sess, tf.train.latest_checkpoint(model_folder +
233
          subdirectory + './'))
       all_vars = tf.get_collection('vars')
234
       weights = all_vars[0]
       biases = all_vars[1]
236
       train_error = error.eval(feed_dict={x: mnist.train.images, y: mnist.train
237
           .labels})
       test_error = error.eval(feed_dict={x: mnist.test.images, y: mnist.test.
```