1. 符号约定：

大写：X、Y、Z 表示随机变量.

小写：x_i，y_i，z_i 表示随机事件.

2. 概率空间：

(1) 离散随机变量 X 的概率空间为：

$$\begin{bmatrix} X \\ p(X) \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ p(x_1) & p(x_2) & \cdots & p(x_n) \end{bmatrix} \quad 0 \le p(x_i) \le 1,\ \sum_{i=1}^{n} p(x_i) = 1$$

$$\left.\begin{aligned} P_X &= [\,p(x_1)\ \ p(x_2)\ \cdots\ p(x_n)\,] \\ &= [\,P_{x_1}\ \ P_{x_2}\ \cdots\ P_{x_n}\,] \\ &= [\,p_1\ \ p_2\ \cdots\ p_n\,] \end{aligned}\right\} \text{记法}$$

(2) X 与 Y 的联合概率空间

$$\begin{bmatrix} XY \\ p(XY) \end{bmatrix} = \begin{bmatrix} x_1y_1 & \cdots & x_iy_1 & \cdots & x_ny_m \\ p(x_1y_1) & \cdots & p(x_iy_1) & \cdots & p(x_ny_m) \end{bmatrix} \quad \begin{aligned} &0 \le p(x_iy_j) \le 1 \\ &\sum_{i=1}^{m}\sum_{j=1}^{n} p(x_iy_j) = 1 \end{aligned}$$

$$P_{XY} = \begin{array}{c|cccc} & y_1 & y_2 & \cdots & y_m \\ \hline x_1 & p(x_1y_1) & p(x_1y_2) & \cdots & p(x_1y_m) \\ x_2 & p(x_2y_1) & p(x_2y_2) & \cdots & p(x_2y_m) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & p(x_ny_1) & p(x_ny_2) & \cdots & p(x_ny_m) \end{array}$$

联合概率，如XY，X为行，Y为列

边缘概率

(3) 全概率公式：

$$p(x_i) = \sum_{j=1}^{m} p(x_i|y_j)\,p(y_j) = \sum_{j=1}^{m} p(x_iy_j)$$

$$p(y_j) = \sum_{i=1}^{n} p(y_j|x_i)\,p(x_i) = \sum_{i=1}^{n} p(x_iy_j)$$

$p(x_i)$：先验概率

$p(x_i|y_j)$：后验概率.

(4) 贝叶斯公式

$$p(x_i|y_j) = \frac{p(y_j|x_i)\cdot p(x_i)}{p(y_j)} = \frac{p(x_iy_j)}{\sum_{i=1}^{n} p(x_i)\cdot p(y_j|x_i)}$$

(5) $\left.\begin{aligned} &\sum_{i=1}^{n} p(x_i \mid y_j) = 1 \\ &\sum_{j=1}^{m} p(y_j \mid x_i) = 1 \end{aligned}\right\}$

(6)

$$P_{X|Y} = \begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_m \end{array} \begin{bmatrix} p(x_1 \mid y_1) & p(x_2 \mid y_1) & \cdots & p(x_n \mid y_1) \\ p(x_1 \mid y_2) & p(x_2 \mid y_2) & \cdots & p(x_n \mid y_2) \\ \vdots & \vdots & \ddots & \vdots \\ p(x_1 \mid y_m) & p(x_2 \mid y_m) & \cdots & p(x_n \mid y_m) \end{bmatrix}$$

{ 行求和 = 1
前提/条件 作行标

(7) $P_Y = P_X \cdot P_{Y|X}$ (矩阵乘法)

(8) $p(x_i y_j) = p(x_i \mid y_j) \cdot p(y_j)$
$= p(y_j \mid x_i)\, p(x_i)$

(9) 解题:

① 已知 P_X $P_{Y|X}$.

可推出 $P_Y = P_X P_{Y|X}$、P_{XY}

从而由贝叶斯 $P(x_i \mid y_j) = \dfrac{p(x_i y_j)}{p(y_j)}$

② 已知 P_{XY}.

行、列求和: P_X P_Y

由贝叶斯: $P_{X|Y} = \dfrac{P_{XY}}{P_Y}$, $P_{Y|X} = \dfrac{P_{XY}}{P_X}$

(10). 三个随机变量 X、Y、Z.

$$P(z_k \mid x_i) = \sum_{j=1}^{m} P(z_k \mid x_i y_j) \cdot P(y_j \mid x_i)$$