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THE CIRCLES OF PROPORTION AND THE HORIZONTALL INSTRVMENT.

Both invented, and the vses of both

Written in Latine by

Mr. *W. O.*

Translated into English: and set forth for the publique benefit by

William Forster.

LONDON

Printed for Elias Allen maker of these and all other Mathematical Instruments, and are to be sold at his Shop over against St Clements church without Temple-barr.

1632.

T. Cecill sculp.

THE CIRCLE OF PROPORTION, AND THE HORIZONTALL INSTRVMENT.

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BVT

Translated into English, and set forth for the publique benefit by William Forster, *louer and practizer of the Mathematicall Sciences.*

LONDON
Printed by AVG. MATHEVVES, dwelling in the Parsonage Court, neere St. BRIDES. 1632.

TO THE HONOVRABLE AND RENOWNED FOR vertue, learning, and true valour, Sir KENELME DIGBYE Knight.

SIR,

THE excellent accomplishments wherwith you are adorned both of vertue, and learning, and particularly in the Mathematicall Sciences, *together with the Honourable respect the* Author *hereof beareth vnto your Worth, and his desire to testifie the same, hath made mee presume to present vnto you, and vnder the happy auspice of your renowned name, to publish to the world this* Treatise: *the owning whereof though I may not chalenge to my selfe, yet the birth and production, whereby it hath a being to the benefit of others, is, as vnto a second parent, due vnto me.*

For being in the time of the long vacation 1630, *in the Country, at the house of the Reue-*

rend, and my most worthy friend, and Teacher, Mr. William Oughtred (*to whose instruction I owe both my initiation, and whole progresse in these Sciences.*) *I vpon occasion of speech told him of a* Ruler of Numbers, Sines, & Tangents, *which one had bespoken to be made (such as is vsually called* Mr. Gunters Ruler) *6 feet long, to be vsed with a payre of beame-compasses.* " *He answered that was a poore invention, and the performance very troublesome: But, said he, seeing you are taken with such mechanicall wayes of Instruments, I will shew you what deuises I haue had by mee these many yeares. And first, hee brought to mee* two Rulers *of that sort, to be vsed by applying one to the other, without any compasses: and after that hee shewed mee those* lines *cast into a* circle or Ring, *with another* moueable circle *vpon it. I seeing the great expeditenesse of both those wayes; but especially, of the latter, wherein it farre excelleth any other Instrument which hath bin knowne; told him, I wondered that hee could so many yeares conceale such vsefull inuentions, not onely from the world, but from my selfe, to whom in other parts and mysteries of Art, he had bin so liberall.* " *He answered,*

" *That*

"*That the true way of Art is not by Instruments,* "*but by Demonstration: and that it is a prepo-* "*sterous course of* vulgar Teachers, *to begin* "*with* Instruments, *and not with the Sciences,* "*and so in-stead of Artists, to make their Schol-* "*lers only doers of tricks, and as it were Iuglers:* "*to the despite of Art, losse of precious time, and* "*betraying of willing and industrious wits, vnto* "*ignorance, and idlenesse. That the vse of Instru-* "*ments is indeed excellent, if a man be an Artist:* "*but contemptible, being set and opposed to Art.* "*And lastly, that he meant to commend to me, the* "*skill of Instruments, but first he would haue me* "*well instructed in the Sciences. He also shewed me many notes, and* Rules *for the vse of those* circles, *and of his* Horizontall Instrument, *(which he had proiected about 30 yeares before) the most part written in Latine. All which I obtained of him leaue to translate into English, and make publique, for the vse, and benefit of such as were studious, & louers of these excellent Sciences.*

Which thing while I with mature, and diligent care (as my occasions would giue me leaue) went about to doe: another to whom the Author *in a louing confidence discouered this intent, vsing*

more

more hast then good speed, went about to preoccupate; of which vntimely birth, and preuenting (if not circamuenting) forwardnesse, I say no more: but aduise the studious Reader, onely so farre to trust, as he shal be sure doth agree to truth & Art.

And thus most noble Sir, without any brauing flourishes, or needlesse multiplying of tautologized *and erroneous præcepts, in naked truth, and in the modest simplicity, of the* Author *himselfe (whose knowne skill in the whole Systeme of Mathematicall learning, will easily free him from the suspicion of hauing the way made for him, and the subiect vnuailed, to help his sight) I haue notwithstanding vnder the protection of your courteous fauour, and learned iudgement, persisted in my long conceiued purpose, of presenting this* tractate *to the publique view, and light. Wishing withall vnto you encrease of deserued honor, and happines.*

May the 1. 1632.

By the honourer
and admirer
of your Worthines,

WILLIAM FORSTER.

THE FIRST PART OF THIS BOOKE,

Shewing the vse of the *First side* of the Instrument, for the working of *Proportions both simple and compounded*, and for the ready and easie resolving of questions both in *Arithmetique*, *Geometrie*, and *Astronomie*, by Calculation.

CHAP. I.

Of the Description, and vse of the Circles in this First side.

1 THere are two sides of this Instrument. On the one side, as it were in the *plaine of the Horizon*, is delineated the *proiection of the Sphare*. On the other side there are divers kindes of Circles, divided after many severall waies;

together with an *Index* to be opened after the manner of a paire of Compasses. And of this side we will speake in the first place.

2 The *First*, or outermost circle is of *Sines*, from 5 degrees 45 minuts almost, vntill 90. Every degree till 30 is divided into 12 parts, each part being 5 min: from thence vntill 50 deg: into sixe parts which are 10 min: a peece: from thence vntill 75 degrees into two parts which are 30 minutes a peece. After that vnto 85 deg: they are not divided.

3 The *Second circle* is of *Tangents*, from 5 degrees 45 min: almost, untill 45 degrees. Every degree being divided into 12 parts which are 5 min: a peece.

4 The *Third circle* is of *Tangents*, from 45 degrees untill 84 degrees 15 minutes. Each degree being divided into 12 parts, which are 5 min: a peece.

5 The *Sixt circle* is of *Tangents* from 84 degrees till about 89 degrees 25 minutes.

The *Seventh circle* is of *Tangents* from about 35. min: till 6 degrees.

The *Eight circle* is of *Sines*, from about 35 minutes til 6 degrees.

6 The *Fourth circle* is of *Vnequall Numbers*, which are noted with the Figures 2, 3, 4, 5, 6, 7, 8, 9, 1. Whether you vnderstand them to bee single Numbers, or Tenns, or Hundreds, or Thousands, &c. And every space of the numbers till 5, is divided into 100 parts, but after 5 till 1, into 50 parts.

The *Fourth circle* also sheweth the *true* or *naturall Sines*, and *Tangents*. For if the *Index* bee applyed to any Sine or Tangent, it will cut the *true Sine* or *Tangent* in the fourth circle. And wee are to knowe that if the *Sine* or *Tangent* be in the *First*, or *Second circle*, the figures of the *Fourth circle* doe signifie so many thousands. But if the *Sine* or *Tangent* be in the *Seventh* or *Eight circle*, the figures in the *Fourth circle* signifie so many hundreds. And if if the *Tangent* bee in the *Sixt circle*, the figures of the *Fourth circle*, signifie so many times tenne thousand, or whole *Radij*.

And by this meanes the Sine of 23°, 30′ will bee found 3987 : and the Sine of it's complement 9171. And the Tangent of 23°, 30′ will be found 4348 : and the Tangent of it's complement, 22998. And the Radius is 10000, that is the figure 1 with foure cyphers, or circles. And hereby you may finde out both the summe, and also the difference of Sines, and Tangents.

7 The *Fift circle* is of *Æquall numbers*, which are noted with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; and every space is divided into 100 æquall parts.

This *Fift circle* is scarse of any use, but onely that by helpe thereof the given distance of numbers may be multiplied, or divided, as neede shall require.

As for example, if the space betweene 1|00 and 1|0833+ bee to bee septupled. Apply the Index vnto 1|0833+ in the Fourth circle, and it will cut in the Fift circle 03476+; which multiplyed by 7 makes 24333 : then againe, apply the Index vnto this number 24333 in the Fift circle, and it will cut in the Fouth circle 1|7512+. And this is the space betweene 1|00 and 1|0833+ septupled, or the Ratio betweene 100, and 108⅓ seven times multiplied into it selfe.

And contrarily, if 1|7512 bee to bee divided by 7 : Apply the Index vnto 1|7568 in the fourth circle, and it will cut in the fit circle 24333 : which divided by 7 giveth 03476+. Then againe vnto this Number in the Fift circle apply the Index, and in the Fourth circle it wil cut vpon 1|0833+ for the Septupartion sought for.

The reason of which Operation is, because this *Fift circle* doth shew the *Logarithmes* of Numbers. *For if the Index be applyed unto any number in the* Fourth circle, *it will in the* Fift circle *cut vpon the* Logarithme *of the same number, so that to the* Logarithme *found you praf*xe a Caracteristicall (*as Master* Brigs *termes it*) *one lesse then is the number of the places of the integers proposed* (*which you may rather call the* Graduall Number). So the Logarithme of the number 2 will bee found 0.30103. And the Logarithme of the Number 43 1/6 will bee found 1.63949.

Numbers are multiplied by Addition of their Logarithmes: *and they are Divided by Substraction of their* Logarithmes.

8 In the middest among the Circles, is a *double Nocturnall* instrument, to shew the hower of the night.

9 The right line passing through the Center, through 90, and 45 I call the *Line of Vnitie*, or of the *Radius*.

10 That *Arme of the Index* which in every Operation is placed at the Antecedent, or first terme, I call the *Antecedent arme*: and that which is placed at the consequent terme, I call the *Consequent Arme*.

CHAP.

CHAP. II.

Of the Operation of the Rule of Proportion: and also of *Multiplication*, and *Division*.

1. *Theoreme.*

IF of three numbers given, the first divide the second and the quotient multiply the third; the product shall be the fourth proportionall, to the three numbers given.

Theoreme. If of three numbers given, the second divide the first, and the quotient divide the third; this later quotient shall be the fourth proportionall, to the three numbers given.

Neither is it materiall whether of the two numbers after the first be second, or third.

2 And note that in *Reciprocal proportion*, that terme by which the question is made; But in *Direct proportion* the terme that is *homogeneal* thereto, is the first terme, or the Antecedent of the first ratio.

3 And therefore out of these foundations thus layd, (if you rightly conceive the nature of the Logarithmes) doth follow *the finding out of the fourth proportionall by this Instrument:* whereof this is the Rule.

Open the Armes of the Instrument to the distance of the first, and second number: then bring the Antecedent arme, or that which stood upon the first number unto the third, and so the consequent arme, keeping the same opening, will shew the fourth number sought for.

In which operation these foure things are diligently to be considered.

First, in constituting the places of each number in the fourth circle; whether the figures written in the spaces doe signifie Vnites, Tenns, *or* Hundreds, *&c.*

Secondly, if that arme which sheweth the fourth proportionall, doe reach beyond the line of the Radius; that then you doe account the fourth in a new circle or degree.

Thirdly, whether the fourth number sought, ought to be greater, or lesser then the third. For if a fourth number bee offered greater then the third, when it should be lesse, or lesse then the third when it should be greater; it is a signe that that number doth appertaine to a circle of another degree.

Fourthly, that looke what true distance was betweene the first and second, that the same bee supposed betweene the third and the fourth, and also on the same part.

4 And for because *Multiplication* and *Division*, have a certaine implicite proportion: we will speake of them in the first place.

5 *In Multiplication, As an Vnite is to one of the factores* (or numbers to be multiplied:) *so is the other of the factores, to the product.*

And the product of two numbers shall have so many places as there be in both the factores, if the lesser of them exceede

exceede so many of the first figures of the product: But if it doe not exceede, it will have one lesse.

6 *And in Division, As the Divisor is to an Vnite; so is the Dividend, to the Quotient.*

And the Quotient shall have so many places, as the Dividend hath more then the Divisor, if the Divisor exceede so many of the first figures of the Dividend: but if it doe not exceede, it shall have one place more.

7 Wherefore let this rule bee still carefully kept in minde: that *In Multiplication the first terme of the implicite proportion is evermore* 1: And *in Division, the first terme is the Divisor.*

And thus much concerning the operation of Proportion, Multiplication, and Division, I thought meete to admonish, least hereafter in Multiplying, or Dividing, or seeking out a fourth proportional, wee be constrained to repeate the same things many times over.

8 An example of *Multiplication.* How many pence are there in $47^{li}. 9^{sh}$? For because 1 shilling containes 12 pence, and 1 pound containes 20 shillings, that is 240 pence: you shall multiply 47 by 240, and 9 by 12, and then adde together the products.

In the first *Multiplication.*

1 . 47 : : 240 . 11280 .

For set the *Armes* of the *Index* at 1 and 47 in the fourth circle; and then bring the *Antecedent arme* (which stood at 1) vnto 240, and the *Consequent arme* will shew 11280.

Againe in the second *Multiplication.*

1 : 9 : : 12 . 108 .

For set the two *Armes* of the *Index* at 1 and 9 in the fourth circle; and bring the *Antecedent arme* unto 12, and the *Consequent arme* will shew 108. Lastly, adde together 11280 and 108 and the summe 11388 will be the number of pence contained in the said summe of $47^{li}, 9^{sh}$.

9 An example of *Division.* How many pounds, and shillings are in 11388 pence? Divide 11388 by 240: the division is thus.

240 . 1 : : 11388 . $47^{\underline{5}}$ —

For set the two Armes of the Index at 240, and 1 in the fourth circle: and then bring the Antecedent arme (which stood at 240) unto 11388; and the Consequent arme will shew 47 and almost an halfe.

But how many shillings that excesse doth containe wil appeare, if first you finde by Multiplication that 11280 pence are contained in 47^{li}: which subducted from 11388 there will remaine for the excesse 108 pence. Afterwards by division you may seeke how many shillings are in 108 pence: the diuision is thus.

12 . 1 : : 108 . 9 .

For set the Armes of the Index at 12 and 1: then bring the Antecedent arme (which stood at 12) unto 108; and the Consequent arme will shew 9.

9 *Any Fraction given may bee reduced into Decimal parts, thus.*

Set the Antecedent arme of the Index at the Denominator of the Fraction given, in the fourth circle, and the Consequent arme at the Numerator, then keeping the same distance, bring the Antecedent arme unto 1, and the consequent arme will shew the *decimal parts.*

So $\frac{240}{320}$ is 0,$\underline{75}$. And $\frac{19}{48}$ is 0,3$\underline{96}$—

CHAP.

CHAP. III.

Now follow certaine examples of Proportion.

Example I.

IF 54 Elnes of Holland bee solde for *96* shillings, for how many shillings was *9* elnes sold? The termes given are

54 . *96* : : *9* .

According to the 2 Chap. 3 Sect. Set one of the armes of the Index at the Antecedent terme 54 in the fourth circle, and the other arme at the consequent terme *96*: then keeping that distance, bring the Antecedent arme vnto *9*; and the consequent arme beyond the *line of the Radius* will shew 16 for the fourth proportionall, according to the *considerations* in the 2 Chap. Sect 3.

Therefore

54 . *96* : : *9* . *16* .

are proportionals. And *16* shillings is the price of *9* Elnes.

Example II. If 108 bushels of corne be sufficient for a company of Souldiers keeping a Fort, for 3*6* dayes, How many dayes will 12 bushels suffice that same number of Souldiers? The termes giuen are

108 . 36 : : 12 .

Set one Arme of the Index at the Antecedent terme 108 in the fourth circle, and the other Arme at the consequent terme 12, (being mindfull of the considerations in the 2 Chap. 3 Sect.) then keeping that same distance,

 bring

bring the Antecedent arme vnto 36; and the consequent arme will shew 4.

Therefore

108 . 36 :: 12 . 4 .

shall bee proportionals. And 4 is the number of dayes sought for.

Example III. There is layd vp in a Fort so much corne as will suffice for 756 Souldiers which keepe that Fort, for 196 dayes: how many Souldiers will that same corne suffice for 364 dayes?

The Proportion is reciprocall, therefore the termes given are

364 : 756 :: 196 .

Set one Arme of the Index at the Antecedent terme 364, and the other Arme at the consequent 196: and keeping the same distance, bring the Antecedent arme vnto 756; and the consequent arme will shew 407 +: And therefore for so many Souldiers will the corne laid vp suffice for 364 dayes, or 13 moneths.

Example IIII. There is a Tower whose height I would measure with a Quadrant.

I take two Stations in the same right line from the Tower: and at either Station having observed the height through the sights, I finde that the perpendicular cutteth in the nearer Station 28 degrees 7 minutes almost: and in the further Station 21 degr. 58 min. almost: and betweene both the Stations, the distance was 76 feet.

The Rule of measuring heights by two Stations is contained in these Theoremes.

Theor. As the difference of the Tangents of the arches cut in either station, is to the distance betweene

tweene the ſtations; ſo is the Tangent of the leſſer arch, to the nearer diſtance from the Tower. Againe

Theor. As the Radius is to the Tangent of the greater arch; ſo is the nearer diſtance found, to the height.

And therefore becauſe by the 1 Chap. 6 Sect. the Tangents of the arches 28°,7′—, and 21°58′—are 5342, and 4032 whoſe difference is 1310; the proportions will be

First, 1310 . 76 : : tang. 21°, 58′— . 234 .

Wherefore 234 feet is the nearer diſtance.

Second Radius . tang. 28°, 7′— : : 234 . 125 .

Wherefore 125 feet is the height ſought for.

Example V. To finde the *Declination of the Sunne* the 9th day of May.

The place of the Sunne for every day, may be had nere inough out of this Table, by Adding vnto the place of the Sun in the beginning of that moneth ſo many degrees, as there are dayes paſt in that moneth: But if the number of degrees exceed 30, the exceſſe is to be accounted in the Signe next following.

Wherefore the 9th of May the place of the Sun is ♉ 20+9, that is ♉ 29: which is 59 degr. diſtant from the next Æquinoctiall point.

The place of the *Sunne, in the beginning of every Moneth.*		
Ianuary	♑	21
February	♒	22
March	♓	20
April	♈	21
May	♉	20
Iune	♊	19
Iuly	♋	18
Auguſt	♌	18
Septemb.	♍	18
October	♎	17
Novemb.	♏	18
Decemb.	♐	19

These things being knowne, the Rule is delivered in this Theoreme.

Theor. As the Radius is to the sine of the sunnes distance from the next Æquinoctiall point; so is the sine of the sunnes greatest declination, to the sine of the declination sought for.

The proportion will be

Radius . sine 59° :: sine $23^\circ, 30'$. sine $19^\circ, 59'$.

And so much is the Declination sought for.

Example VI. To finde the *Right ascension of the Sunne*, the 9^{th} day of May.

Seeke the place of the Sunne for the day proposed in the former *Table*; and the Sunnes distance from the next Æquinoctial point, as in the former example.

These things being knowne, the Rule is by one of these two Theoremes.

Theor. As the Radius, is to the sine of the complement of the sunnes greatest declination; so is the tangent of the sunnes distance from the next Æquinoctiall point, to the tangent of the distance of the right ascension of the sunne, from the same Æquinoctial-point. Or

Theor. As the tangent of the greatest declination of the Sunne, is to the Radius; so is the tangent of the declination of the sunne for the time proposed, vnto the sine of the right ascension of the sunne from the next Æquinoctial point.

The proportions will be either

Radius . sine $66^\circ, 30'$ :: tang. 59° . tang. $56^\circ, 46'$.

Or, tang. $23^\circ, 30'$. Radius :: tang. $19^\circ, 59'$. sine $56^\circ, 46'$.

CHAP.

CHAP. IIII.

Of Continuall proportion, *Or of* Progression Geometricall.

1 T*HE Ratio of a Progreſſion is the quotient of the conſequent terme divided by his antecedent.* And therefore in the Inſtrument it is *the diſtance taken betweene the termes in the fourth circle, by the opening of the Index.*

2 To *Double, Triple,* or *Multiply* how often ſoever *any Ratio given*, is nothing elſe but *ſo often to put together the ſaid ſpace or diſtance betweene the termes*, as is ſhewed in Chap 1.Sect.7.

As for example, if the *Ratio 60*, to *65* be propoſed to be *ſeptupled.*

Set the Armes of the Index at *60*, and *65*: and then with the ſame opening, bring the Antecedent arme which was at 60, vnto 1, and the conſequent arme will cut 1|0833 + in the fourth circle, and 03476 + in the fift circle: this latter number being multiplied by 7 maketh 24333; vnto which number in the fift circle applying the Index, it will in the fourth circle cut 1|7512, which is the multiplied number ſought for.

But becauſe in a little Inſtrument, the arch cut in the fift circle, cannot be eſtimated exactly: and a ſmall error in the beginning often repeated, by multiplying is made great: it is the moſt ſafe way, to take the *Logarithmes* of the termes of the *Ratio* out of the *Canon*, and to multiply them by the number given: As I have done in theſe examples.

Logarithmes taken out of the Canon.

100	2 . 0000000
104	2 . 0170333
105	2 . 0211893
106	2 . 0253059
$106\frac{2}{3}$	2 . 0280287
107	2 . 0293838
$107\frac{1}{2}$	2 . 0314085 —
108	2 . 0334238 —
$108\frac{1}{3}$	2 . 0347621
$108\frac{1}{2}$	2 . 0354297 +
$108\frac{3}{4}$	2 . 0364293 —
109	2 . 0374265
110	2 . 0313927

Or if the *Canon* be wanting; you may come neerer the marke, if that first single opening of the Index being kept, and the Antecedent arme set at 1; you transferre the Antecedent Arme, unto that place which the consequent arme doth cutt; and the consequent arme will cut the same space *duplicated.* Then holding the consequent arme in that same place, open the Antecedent arme unto 1. and afterward with that *duplicated opening,* bring the Antecedent arme to the *duplicated space,* and the consequent arme will cut the *space quadrupled.* Thirdly, bring the Anticedent arme unto the *quadrupled space* and the consequent arme, keeping that *duplicated opening,* will cut the space *sextupled.* Lastly, having againe taken a single opening, bring the Antecedent arme unto the number, or *space sextupled*; and the consequent arme will shew the *Ratio* sought for *septupled.*

And this manner of working may bee observed for as many Multiplications as you please of any *Ratio* given.

3 The

3. The *Ratio*, and *first terme being given, to continue the same unto any number of termes.*

Open the Armes of the Index, the one unto the Antecedent of the *ratio* given, and the other unto the consequent: then the same opening being kept, bring the Antecedent Arme vnto the first terme given, and the consequent arme will shewe the second terme: againe bring the Antecedent arme vnto the second terme found, and the consequent arme will shew the third. After that bring the Antecedent arme unto the third terme found, and the consequent arme wil shew the fourth. And in this manner you may proceed as farre as you please.

As for example, If a Progression in the *ratio* 2 unto 5, beginning at 8, Or if a Progression in the *ratio* 100 unto 108, beginning at 5, is to be instituted; the termes in either progression will be as followeth.

1	2 . 5 ::	8	. 20
2		20	
3		50	
4		125	
5		312|5	
6		781|25	
7		1953|125	

1	100 . 108 ::	5	. 5|4
2		5|4	
3		5|832	
4		6|29856	
5		6|8024448	
6		7|346640384	
7		7|93437161472	

4 Theo-

4 *Theor:* The *ratio* of any former terme, in a row of continuall proportionals, vnto any of the termes following, is æquall to the *ratio* of the first terme unto the second, multiplied into it selfe according to the distance of that latter terme from the former.

As for example, The *Ratio* 5,4 unto 6,8024448 which is the third terme from it, is æquall to the *ratio* of 100 vn. to 108 triplicated, Or as the Cube of 100 unto the Cube of 108. Wherefore

5 **The *Ratio*, and the *first terme of the Progression* being given, to finde out *any other terme required*.**

First, multiply the *ratio* given into it selfe, according to the distance of the terme sought from the first terme, by the 2 sect: then say

As 1 is to that multuple found; so is the first terme, to the terme sought for.

Example, What will be the amount of 26 li, in 7 yeares by *Interest upon Interest*, after the rate of 20 pence in the pound?

Because 1 pound which is 20 shillings containeth 60 groates, and 20 pence conteine 5 groates, the rate of the *Interest* will bee 60 unto 65; Or 100 unto 108,33 ×; But the first terme is given 26, unto which there are to be acquired 7 other termes in continuall proportion.

First, by the 4 Chap: and 2 Sect. Let the *ratio* given be septuplicated, that is multiplied sevenfold into it selfe, which will be 1,7512. Then set the consequent arme of the Index vnto the *septuplicate number* 1,7512, and open the Antecedent arme vnto 1; and keeping the same opening

ning, bring the Antecedent arme unto 26; and the consequent arme will shew $45\underline{|53}\,\underline{12}^{li}$ the amount sought for.

6 The *Ratio*, and *any other terme of the Progression* being given, to finde the *first terme.*

First multiply the Ratio given into it selfe according to the difference of the terme given from the first terme.

Then say

As that multuple is unto 1; so is the terme given, unto the first terme.

Example, what summe in 7 yeares did amount unto $45\underline{|53}\,\underline{12}^{li}$ by Interest upon Interest after the rate of 100 unto $108\underline{|33}$ +?

First the ratio being septuplicat, by the 4 Chap. 2 Sect. will be $1\underline{|75}\,\underline{12}$. Then setting the Antecedent arme of the Index to that septuple $1\underline{|75}\,\underline{12}$, open the consequent arme unto 1: and keeping the same opening, bring the Arme unto $45\underline{|53}\,\underline{12}^{li}$: and the consequent arme will shew 26, which was the stocke, or summe of money, from which that amount did arise.

7 The *Ratio*, the *First terme*, and *any other terme of a Progression* being given, to finde *how many places the terme given is from the first terme.*

First say, As the first terme is unto 1; so is the other terme given, unto the ratio multiplied into it selfe according to the distance of that terme from the first.

Wherefore according to 1 Chap. 7 Sect. and 3 Chap. 2 Sect. by helpe of the fift circle, see how often the ratio given, is contained in that multuple found.

Example, In how many yeares did 26^{li}, by Interest upon Intereſt after the rate of 100 unto 108|33 + increaſe unto 45|5312 li?

Firſt ſet the Antecedent arme of the Index at 26, and the conſequent arme at 1: and keeping the ſame opening bring the Antecedent arme vnto 45|5312, and the conſequent arme will ſhew 1|7512: to which in the fift circle anſwereth 24333. Then becauſe unto the conſequent terme of the ratio 108|33 + there agrees in the fift circle 03476, divide 24333 by 03476, and the quotient will be 7, the number of yeares ſought for.

8 The *Firſt terme*, and *any other terme of the Progreſsion* being given to finde the *ratio of the Progreſsion*.

Firſt ſay, *As the firſt terme is unto 1: ſo is the other terme given, unto the ratio multiplied into it ſelfe according to the diſtance of that terme from the firſt.*

Wherefore according to Chap. 1. Sect. 7, by helpe of the fift circle, Let the multuple found be divided by the diſtance of the terme from the firſt.

Example, 26 li by Intereſt upon Intereſt in 7 yeares amounted unto 45|5312: what was the ratio of the Intereſt compared unto 100?

Firſt ſet the Antecedent arme of the Index at 26, and the conſequent arme at 1: and then keeping the ſame opening, bring the Antecedent arme unto 45|5312: and the conſequent arme will ſhew 1|7512: unto which in the fift circle anſwereth 24333 and this number being divided by 7 will give 03476 +: unto which agreeth in the fourth circle 108|33 + the conſequent terme of the Intereſt ſought for.

9 *Two numbers being given* to finde *as many Midle proportionals betweene them as you will.*

Divide

Divide the distance of the greater number given from the lesser in the fourth circle iustly taken, according to Chap. 1. Sect. 7, by helpe of the fift circle into equall segments, one more then are the number of Midle proportionals sought for. All these segments added orderly to the first terme, doe distinguish the termes of the Progression which you seeke.

Example, Let there be foure *Midle proportionals*, sought out betweene 8 and 19|90656.

Apply the Index unto 8 in the fourth circle, and it will cut in the first circle 9039: also set the Index unto 19|91— in the fourth circle, and it will cut in the fift circle 2989; which number because it reaches beyond the Vnite line, is indeede 12989, according to Chap. 1. Sect. 7, and so is the distance iustly taken. Then subducting 9039 from 12989, there will remaine 3950: which divided by 4 + 1, the quotient will give 790. wherfore 9039 + 790, scil. 9829 in the fift circle doth agree with 9|6 in the fourth circle, which is the *first midle proportionall*. And 9039 + twise 790, scil. 10619 in the fift circle doth agree with 11|52 in the fourth circle, which is the *second midle proportionall*. And in this manner 13|824, and 16|5888 will be found the *third* and *fourth midle proportionals*.

10 *Theor*. If from the Ratio given, being multiplied in it selfe according to the number of termes, you subduct 1, and multiply the remainder, by the antecedent of the ratio. It will be

As the difference of the termes of the ratio, is unto the product; so is the first terme, to the sum of the Progression.

As for example if the *Ratio* of the Progression be R to S: and the *difference* of R taken out of S be D. and the *first terme of the Progression* a: and the whole *summe of the termes* Z it shall be, D . rat. mul^{ta} — 1 in R :: *a* . Z .

Which is the very Theoreme it selfe expressed in *Symboles of words*: that it may more easily be fixed in the phantasie. Which proportion also wee must consider, doth hold both *alternly*, and *conuersly*.

This Theoreme may otherwayes be expressed, by the equality which the product of the two midle termes hath to the product of the two extreames, thus

Ratio multiplicata — 1 in R in *a* = Z in D .

This manner of setting downe Theoremes, whether they be Proportions, or Equations, by Symboles or notes of words, is most excellent, artificiall, and doctrinall. Wherefore I earnestly exhort every one, that desireth though but to looke into these noble Sciences Mathematicall, to accustome themselues unto it: and indeede it is easie, being most agreeable to reason, yea even to sence. And out of this working may many singular consectaries be drawne: which without this would, it may be, for ever lye hid as in this present proportion: because it is

D . rat. mul^{ta} — 1 in *a* :: *a* . Z . wherefore

Rat. mul^{ta} — 1 in R . D :: Z . *a* . And

a . Z :: D . rat. mul^{ta} — 1 in R

These are exceeding easie: but this following is more difficult, and requireth attention.

In the former *aquation* it was

Rat. mul^{ta} — 1 in R in *a* = Z in D

Now because Rat. mul^{ta} — 1 in R in *a*,

and Rat. mul^{ta} — in R in *a* — R in *a*,

and Rat. mul^{ta} in R — R in *a*,

and

and Rat. mul^{ta} in $a - a$ in R,

are equall one to another, and also to Z in D, these æquations shall also be consectarious.

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in R in } a}{D} = Z.$$

and $\frac{\text{Z in D}}{a} =$ rat. mul^{ta} in R — R.

and $\frac{\text{Z in D}}{R} =$ rat. mul^{ta} in $a - a$.

and $\frac{\text{Z in D} + \text{R in } a}{\text{R in } a} =$ rat. mul^{ta}

And besides these many more. The practise whereof I leave to the industry of the studious Reader, especially having delivered the whole Art of such operations in my *Clavis Mathematica.*

Some of these I have occasion to use in the sections following.

11 *Therefore the* Ratio of a Progression, *and the* first terme, *and the* number of termes *being given, to finde the* summe of the whole progression.

For this operation the rule, or Theoreme last before serveth: for by it

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in R in } a}{D} = Z.$$

Example. If an *Annuity* of 5^{li}, be detained 7 yeares, what will be the *amount* thereof by interest upon interest after the *rate* of 100 unto 108?

Now because the *Amount is the summe of the Progression, whereof the first terme is the annuitie*, Multiply the ratio into it selfe according to the number of yeares, by Chap. 4. Sect. 2, and it will be $1|\underline{71382}$: from which if you subduct an vnite, there remaines $0|\underline{71382}$, which multiplied by 100 maketh $71|\underline{382}$: then set the Armes of the Index at 1 and $71|\underline{382}$; and bring the Antecedent arme which stood at 1, unto 5 the first terme: and the consequent arme will cut $356|\underline{915}$ (that is Rat. mul^{ta} — 1 in R in α) and lastly this number being divided by 108 — 100, scil. by 8, the quotient will give $44|\underline{6140}$, for the *summe of the whole progression*: and so much is the *amount* sought for.

11. The *Ratio*, the *Number of Termes*, and the *Summe of a Progreßion* being given, to find the *first terme*.

By the *converse* of the foregoing Theoreme, it is manifest, that

$$\text{Rat}: mul^{ta} - 1 \textit{ in } R \;.\; D \;::\; Z \;.\; \alpha \;.$$

The declaration in words was in the 10 Sect.

Example. If an *Annuitie* detained 7 yeares by Interest upon Interest, after the rate of 100 unto 108, did increase unto $44|\underline{6140}^{li}$. how much was that Annuitie?

Multiply the *Ratio* into it selfe according to the number of yeares (*per cap. 4. sect. 2.*) and the product will be $1|\underline{71382}$: from which if you subduct an Vnite, there remaines $0|\underline{71382}$: this being multipled by 100 doth make $71|\underline{382}$. Then say

$$71|\underline{382} \;.\; 8 \;::\; 44|\underline{6140} \;.\; 5 \;.$$

for

for the First terme : which was the *Annuitie* sought for.

13 The *Ratio*, the *First terme*, and the *Summe of the Progression* being given, to find the *Number of termes*.

By the Theoreme in the 10. Sect. it was

a . Z : : D . rat. mul^{ta} — 1 in R.

Wherefore set the Antecedent arme of the Index unto the Antecedent terme of the ratio, and the consequent arme unto the Summe of the Progression : and with that same opening, bring the Antecedent arme unto the difference of the termes of the ratio ; and the consequent arme wil shew a number (that is Rat. mul. — 1 in R;) which if you divide by the Antecedent terme of the ratio, and unto the quotient adde an Vnite, you shall have the ratio multiplied into it selfe according to the number of termes. Therefore taking the distance betweene the termes of the ratio, with the armes of the Index, measure by helpe of the fift circle (*per Cap.*4. *Sect.*7.) how often that distance may be found in the multiplied ratio : for so many are the termes of the progression.

Example. If an *Annuitie* of 5^{li} detained by Interest upon interest after the rate of 100 to 108, increased unto 44,6140^{li}. How many yeares was the *Annuitie* detained?

Set the Antecedent arme of the Index at 5, and the consequent arme at 44,6140, and with the same opening bring the Antecedent arme unto 8, and the consequent arme will shew 71,382 (that is Rat. mu^{ta} — 1 in R:) this number being divided by 100, will be 0,71382 : and if unto the quotient you adde 1, you shall have 1,71382 (the ratio multiplied into it selfe according to the number of yeares :) unto which in the fift circle answereth 2338 : but

but unto 108 in the fift circle there anſwereth 0334. divide therefore 2338 by 0334; and the quotient will be 7, for the number of yeares ſought for.

Or ſuch queſtions may be more eaſily performed by this Theoreme, which the induſtrious Reader may by himſelfe practiſe.

$$\frac{Z \text{ in } D + R \text{ in } \alpha}{R \text{ in } \alpha} = \text{rat. mul}^{ta}$$

14 *Theor.* If the ſumme of the whole progreſſion be divided by the ratio multiplied into it ſelfe according to the number of termes, the quotient will be the firſt terme; and that ſumme given will be the laſt terme, of another progreſſion, having the ſame ratio but one terme more.

15 And becauſe a ſumme of mony the amount whereof in any number of yeares given, by Intereſt upon intereſt, doth equall an Annuitie ſo long detained, is equivalent to the ſame Annuitie; and the amount of an Annuitie is the ſumme of a Progreſſion continued from that Annuitie. If therefore an Annuitie for any number of yeares be divided by the ratio multiplied into it ſelfe according to the number of yeares; the quotient will be the juſt price of an Annuitie to endure for ſo long. And becauſe by the 10 and 11 Sect. it hath beene ſhewed that

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in } R \text{ in } \alpha}{D} = \text{to the amount.}$$

Therefore by the 14 Sect. $\frac{\text{Rat. mul}^{ta} - 1}{\text{Rat. mul}^{ta}} \frac{\text{in } R \text{ in}}{\text{in } D}$

will be equall to the price of an Annuitie in ready money, which ſhall be the Rule for the operation following.

Where-

Wherefore also

$$\text{Rat.mul}^{ta} - 1 \text{ in R in } \alpha = \text{Rat.mu.}^{ta} \text{ in D in Pret.}$$

which proportion is thus enuntiated in words.

Theor. If from the Ratio multiplied into it selfe according to the Number of yeares you subduct an Vnite, and the remainder be multiplied continually by the Antecedent of the Ratio, and the Annuitie it selfe: *And againe,* If the Ratio multiplied into it selfe, according to the number of yeares be multiplied continually by the difference of the termes of the Ratio, and by the Price: both those products will be equall.

Example. An Annuitie of 5li, to endure for 7 yeares, is to be sold: what is it worth in ready money, after the Rate of 100 unto 108?

The Ratio multiplied into it selfe according to the number of yeares (*per Cap.* 4 *Sect.* 2) is 1,7138: subduct 1, and there will remaine 0,71382: which multiplied by 100 maketh 71,382. Then set the Antecedent arme of the Index at 1, and the consequent arme 71,382: and keeping that same opening, bring the Antecedent arme unto 5; and the consequent arme will shew 356,915; keepe this (for it is $\frac{\text{rat mul.} - 1 \text{ in R in } \alpha}{\text{D}}$). After that set the Antecedent arme of the Index at 1, and the consequent arme unto the multuple ratio 1,7138; and with the same opening bring the Antecedent arme unto 8; and the consequent arme will cut 13,7106, keepe this number also (for it is Rat.mul.in D). Lastly, place the Antecedent arme of the Index at 13,7106, and the consequent arme at 1; and with the same opening bring the Antecedent arme vnto 356,915: and the consequent arme will shew 26,022li.

which is the iuſt Price of an Annuitie of 5^{li} in readie money.

16 And by the laſt præcedent Theoreme or Rule, alſo

$$\frac{\text{Rat mul. in D in Pret}}{\text{Rat.mul.}-1\text{ in R}} = \alpha .$$

which Theoreme may bee enuntiated in words as was there ſhewed.

Example. An Annuitie for 7 yeares is bought for 26,032li. after the rate of 100 unto 108, by Intereſt upon intereſt: how much was that Annuitie?

The Ratio multiplied into it ſelfe for the number of 7 yeares (*per Cap*: 4, *Sect.* 2) is 1,7138: which multiplied continually by 8, and by the price, doth make 356,915. Divide this number found, by 71,382, (which is the *multuple ratio*) it ſelfe leſſe by an Vnite, and multiplied by 100: and the quotient will be 5^{li}, the Annuitie ſought for.

17 Alſo by *Rationation* from that præcedent rule will follow this propoſition

$$\frac{\text{R in }\alpha}{\text{R in }\alpha - \text{Pret. in D}} = \text{Rat. mul}^{ta}.$$

which is thus enuntiated in words.

Theor. If the product of the Antecedent of the ratio multiplied by the Annuitie be divided by it ſelfe, being diminiſhed by the product of the difference of the termes of the ratio multiplied by the Price: the quotient will bee equall to the ratio multiplied into it ſelfe according to the number of yeares. As

If the ratio be 100 unto 108; the Annuitie 5^{li}: and the price thereof 26,032li. Set the Antecedent arme of the

the Index at 1, and the conſequent arme at 8, the diffe-rence of the termes of the ratio: and with the ſame ope-ning bring the Antecedent arme unto $26,\underline{032}$ —: and the conſequent arme will ſhew $208,\underline{256}$: which ſubducted from 500, there will remaine $291,\underline{744}$, for the *diviſor.* Set therefore the Antecedent arme of the Index at the *diviſor* $291,\underline{744}$, and the conſequent arme at 1: and with the ſame opening, bring the Antecedent arme unto the *dividend* 500: and the conſequent arme will ſhew $1,\underline{7138}$, which is the *ratio* multiplied into it ſelfe according to the number of yeares. And by this number ſo found it will be eaſie (by helpe of the fift circle) the *ratio* of the Inte-reſt being given, to finde the continuance of the Annuitie.

Example. An Annuitie of 5^{li}, was bought for $26,\underline{032}$li—, after the rate of 100 unto 108: how many yeares is it to laſt?

Firſt ſeeke out the ratio multiplied into it ſelfe accord-ing to the number of yeares, which will be $1,\underline{7138}$, accord-ing as was even now ſhewed in this Sect. to this in the fift circle there anſwereth 2338: but unto 108 there an-ſwereth in the fift circle 0334. Divide therefore 2338 by 0334; and the quotient will be 7 for the number of yeares ſought for.

CHAP. V.

Of the Quadrating, *and* Cubing of Numbers, Sides or Rootes: *and of the* Extraction of the Quadrate, *and* Cubic side, *or* roote, *out of* Numbers, *or* Powers *given.*

1 *IF a number, side, or roote be multiplied into it selfe; the product will be a Quadrat.* And *if a quadrate bee multiplied into his owne side, or roote, the product will be a Cube.*

Wherefore 1 . N :: N . Q .

and 1 . N :: Q . C .

2 If therefore a number be given to be Quadrated *Set the Antecedent arme of the Index at* 1, *and the consequent arme at the number given: then with the same opening bring the Antecedent arme to the number given; and the consequent arme will shew the Quadrat thereof.*

And the number of figures in a Quadrat of a single root (or which doth not exceede 9) is easily found out by those Rules, that have beene before delivered concerning Multiplication. But if a side or root consist of more figures then one; for each figure after the first it acquireth two more places of figures. And if any of the figures of the root given be decimal parts, cut off from the Quadrat found, twice so many of the last figures for decimals.

Example I. The Quadrat of the side 7 is required. Say

1 . 7 :: 7 . 49

Set therefore the Antecedent arme of the Index at 1, and the consequent arme at 7; and with that opening bring

bring the Antecedent arme unto 7; and the consequent arme will shew 49 which is the Quadrat sought for.

Example II. The Quadrat of the side 57 is required.

Set the Antecedent arme of the Index at 1; and the consequent arme at 57: then with that same opening bring the antecedent arme unto 57; and the consequent arme will shew 3249, which is the Quadrat sought for, consisting of foure places.

Example III. The Quadrat of the side, or root 570 is required.

Having found as before, 3249 for the quadrat of the side 57: put thereunto two circles: and it will bee 324900, the quadrat sought for.

Example IIII. The quadrat of the side 574 is required.

Set the Antecedent arme of the Index at 1, and the consequent arme at 574; and with the same opening bring the Antecedent arme unto 574; and the consequent arme will shew 329476, the quadrat sought for consisting of sixe figures: but the two last figures cannot at all be discerned by the Instrument.

3 If a number be given to be *Cubed.*

Set the Antecedent arme of the Index at 1, *and the consequent arme at the number given; and with that same opening, bring the Antecedent arme unto the number given; and the consequent arme will shew the Quadrat; then bring the Antecedent arme unto the Quadrat, and the consequent arme with that same opening will shew the Cube of that side given.*

The number figures in a *Cube* of a single side, or root which doth not exceede 9, is easily found by that which hath beene before delivered concerning Multiplication:

 But

But if the ſide, or root conſiſt of more figures then one; for each figure after the firſt it obtaineth three more places of figures. And if any of the figures of the root given be *decimal parts*, cut off from the *Cube* found thrice ſo many of the laſt figures for *decimal parts*.

Example. The Cube of the ſide, or root 7 is required.

Say, 1 . 7 :: 7 . 49 .

againe 1 . 7 :: 49 . 343 .

Set therefore the Antecedent arme of the Index at 1, and the conſequent arme at 7; and with that opening bring the Antecedent arme unto 7, and the conſequent arme will ſhew 49 the quadrat thereof: Then ſet the Antecedent arme at 49; and the conſequent arme (with that firſt opening) will ſhew 343, which is the deſired Cube of the ſide propoſed.

Another example, The Cube of the ſide, or root 57 is required.

Set the Antecedent arme of the Index at 1, and the conſequent arme at 57; and with that ſame opening bring the Antecedent arme unto 57; and the conſequent arme will ſhew the quadrat 3249. Then ſet the Antecedent arme at 3249; and the conſequent arme keeping the former opening will ſhew 185193, which is the required Cube of that ſide propoſed, conſiſting of ſix places: but the two laſt figures cannot be known by the Inſtrument.

Example III. The Cube of the ſide 570 is required.

Hauing found as before the Cube of the ſide, or root 57 to be 185193: put thereunto three circles; and it will be 185 193000 the Cube ſought for.

Examples of greater Cubes, it will be needleſſe to ſet downe.

4 The

4 The *Extraction of the Square, or Quadrat root, or side,* is done by helpe of the fift circle, after this manner.

Set the Index at the Quadrat proposed; and of that number which it cuts in the fift circle, take halfe: then set the Index at that halfe; and it will shew in the fourth circle, the side, or root sought for.

But you must know that if the number which is the Quadrat proposed, have onely two places of Integers, the side, or root consisteth of one figure. But if it have more places of Integers, dividing them by 2, the quotient will give the true number of figures in the root, if it measure it exactly; or one lesse then the true number if any thing remaine.

Example I. The side, or root of the Quadrat 49 is required.

Set the Index at 49 in the fourth circle, and it will cut in the fift circle 6902; indeede 1. 6902 having the *graduall number* 1 præfixed, because 1 in the fourth circle signifieth 10, one *circuition* thereof being finished: the halfe whereof is 0. 8451. Then set the Index at 0. 8451, in the fift circle; and it will cut in the fourth circle 7, the side, or root sought for.

Example II. The side, or root of the quadrat 3249 is required.

Set the Index at 3249 in the fourth circle, and it will cut in the fift circle 5118; indeede 3.5118 prefixing the *graduall number* 3, because 1 in the fourth circle signifieth 1000, three *circuitions* thereof being finished: the halfe whereof is 1.7559. Then set the Index at 7559, omitting the prefixed *graduall number* 1; and it will shew in the fourth circle 57 the side sought for, consisting of two figures, because 1 in the fourth circle signifieth 10.

Example

Example III. The side of the quadrat 329476 is required.

Set the Index at 329476 in the fourth circle, and it will cut in the fift circle 5178; indeede 5.5178 prefixing the *graduall number* 5, because 1 in the fourth circle signifieth 100000, five *circuitions* thereof being past over; the halfe whereof is 2.7589; Then set the Index at 7589, omitting the *graduall number* 2 prefixed thereto; and it will shew in the fourth circle 574 the side, or root sought for, consisting of three figures, because 1 in the fourth circle doth signifie 100.

5 The *Extraction of the Cubic roote, or side* is done by helpe of the fift circle after this manner.

Set the Index at the Cube proposed; and that number which it cuts in the fift circle divide by 3. Then set the Index at that third part, and it will shew in the fourth circle the side, or roote sought for.

And you must know that if the Cube proposed have onely three places of Integers, the side, or roote thereof consisteth of one figure: But if it have more places of Integers, divide them by 3, the quotient will give the true number of the figures of the Root, if it measure it exactly; or one lesse then the true number if any thing remaine.

Example I. The side, or roote of the Cube 343 is required.

Set the Index at 343 in the fourth circle, and it will cut in the fift circle 5353; indeede 2.5353 prefixing the *graduall number* 2, because 1 in the fourth circle doth signifie 100, two *circuitions* therof being past over: the third part whereof is 8451. Then set the Index at 8451, and it will shew in the fourth circle 7, the side sought for.

Example

Example II. The side, or roote of the Cube 185193 is required.

Set the Index at 185193 in the fourth circle; and it will cut in the fift circle 2677; indeede 5.2677 prefixing the *graduall number* 5, because 1 in the fourth circle doth signifie 100000, five *circuitions* thereof being past over: the third part whereof is 1.7599. Then set the Index at 7599, omitting the prefixed *graduall number* 1; and in the fourth circle it will shew 57, the side, or root sought for, consisting of two figures, because 1 in the fourth circle doth signifie 10.

Examples of greater Cubes it will be needlesse to set downe.

F CHAP.

CHAP. VI.

Of Duplicated, *and* Triplicated proportion.

And first of Duplicated proportion.

Theoreme.

L*Ike Plaines are in a Duplicated ratio,* that is, *As the Quadrats of their homologal sides.* And therefore questions in the which the *sides of like planes are compared*, doe appertaine vnto this place.

And it is to be noted, that if three numbers be given, in which *As the quadrat of the first, is unto the quadrat of the second; so ought the third to be unto a number sought for.* Let it be thus done, *As the first number, is to the second; so is the third to a fourth;* And againe *As the first number, is to the second; so is the fourth now found, to the number sought for.*

Example I. There are two like rectangle *Area*, or *plaines*, the length of the greater, is 40 feete, the length of the lesser 24 feet: each of them paved with paving tiles; the greater hath 1200 tiles: how many shall the lesser have?

The *Area*, or *plaines* are one to the other, as the quadrats of the longitudes given. And the proportion is direct. Say therefore

1600 (Q: 40) . 576 (Q: 24) :: 1200 . 432 .

which is the number of tiles contained in the pavement of the lesser Areæ, or plaine.

Example II. How many Acres of woodland measured with a Perch, of 18 feete, are there in 73 Acres of champane land, measured with a Perch of $16\frac{1}{2}$ feete?

The-

The meaſures given $(18, 16\frac{1}{2})$ being reduced into their leſt termes, are as 12 unto 11 : and the quadrats of theſe numbers, are, 144, and 121. And the Proportion is Reciprocall. Say therefore

$$144\ (Q{:}\ 12)\ .\ 121\ (Q{:}\ 11)\ ::\ 73\ .\ 6\tfrac{49}{144}\ .$$

and ſo many are the Acres of Wood land.

Of *Triplicated proportion.*

3 *Theor.* Like Solids are in a Triplicated ratio, *that is,* As the Cubes of their homologal ſides. *And therefore queſtions in which the ſides* of like ſolids are compared, *doe appertaine unto this place.*

4 *If three numbers be given in the which*, As the Cube of the firſt is to the Cube of the ſecond ; ſo is the third number to a number ſought for. *Let it be thus done*, As the firſt number is to the ſecond ; ſo is the third to a fourth: *Againe*, As the firſt is to the ſecond ; ſo is the fourth now found unto a fift. *And thirdly*, As the firſt is to the ſecond ; ſo is that fift to the number ſought for.

Example. If $\frac{43}{100}$ lib. of gunpowder , ſuffice to charge a Gun, whereof the concave diameter is inch $1\frac{1}{2}$. How many pounds of powder will ſuffice to charge a Gunne, whoſe concave diameter is 7 inches ?

The capacities are one to another, as the Cubes of the diameters. And the proportion is direct. Say therefore

$$3|\underline{375}\ (C{:}\ 1|\underline{5})\ .\ 343\ (C{:}\ 7)\ ::\ 0|\underline{43}\ .\ 43|\underline{7}\ +$$

wherefore $43|\underline{7}$ lib. of Gunpowder, will bee needfull to be had.

Another example, $43|\underline{7}^{\text{lib}}$. of Gunpowder are ſufficient to charge a Gun, whoſe diameter in the concave is 7 inches: now there is another ſort of Gunpowder, much more ſtrong and forcible, that is in ſtrength unto the former, as 5 vnto 2: How much of this ſtronger powder, will ſuffice to charge a Gun of 4 inches diameter?

Here are two operations: the firſt ſeekes out, how much of that ſtronger powder ſufficeth to charge a Gun of 7 inches diameter: and the proportion is reciprocall, that is

$$5 \cdot 2 :: 43|\underline{7} \cdot 15|\underline{48}$$

The ſecond operation is like that in the former example.

$$343\ (C:7)\ 64\ (C:4) :: 15|\underline{48} \cdot 2|\underline{8884}$$

CHAP.

CHAP. VII.

Concerning the Meaſuring *of* Circles, Cones, Cylinders, *and* Sphæres.

1 ARchimedes in a peculiar Treatiſe found the proportion of the Diameter of a circle to the Circumference to bee a very ſmale deale greater then of 7 unto 22: And of late *Ludolph Van Ceulen* inſiſting in the ſame ſteps of *Archimedes*, hath more preciſely found it to be of 1 vnto 3,14159265358979 3 but for our Inſtrument it will be ſufficient to take the ratio of 1 vnto 3,1416, Or of 0,3183 + unto 1: leaving the diligent practizer, to more exactneſſe, if he pleaſe to uſe his Pen.

And note that the Rules following are ſet downe in proportions, to be wrought as hath been taught in Chap. 2, Sect. 3. Wherein

D, or Diam. ſignifieth the Diameter.

Dq, or Q. Diam. the Quadrat of the Diameter.

Dc, or C. Diam. the Cube of the Diameter.

R, or Rad. the Radius, or Semidiameter.

P, or Perif. the Periferie, or Circumference.

Pq, the Quadrat of the Periferie.

Long. the length.

L the ſide, or latus

Alt. the altitude.

*, ſheweth that the two magnitudes betweene which it is ſet, are to be multiplied together.

In a Circle.

2. *The* Diameter *of a circle being given, to finde the* Periferia. *Say,*

7 . 22, Or 1 . 3,1416 :: Diam . Perif .

Example. A circle is given, the Diameter whereof is 12, I would know the circumference, or Periferia of it. Say,

1 . 3,1416 :: 12 . 37,6992 .

3 *The* Periferia *of a circle being giuen, to finde the* Diameter. *Say,*

22 . 7, Or 1 . 0,3183 + :: Perif . Diam .

4 *The* Diameter *of a circle being giuen, to finde the* Area. *Say,*

7 * 4 . 22, Or 1 . 0,7854 :: Q: Diam. Area .

Or els 1 . 3,1416 :: Q: Rad . Area .

Example. A circle is giuen, the diameter whereof is 12, I would know the content, or Area of it. Say,

1 . 0,7854 :: 144 (Q: 12) . 113,0976 .

Or 1 . 3,1416 :: 36 (Q: 6) . 113,0976

5 *The* Area *of a circle being giuen, to finde the* Diameter. *Say,*

22 . 7*4, Or 1 . 1,27324 :: Area . Q: Diam .

Example. A circle is giuen, the content whereof is 113,0976, I would know the Diameter of it. Say,

1 . 1,27324 :: 113,0976 . 144 .

6 *The*

6 The Periferia *of a circle being giuen, to finde the* Area. *Say,*

22*4 . 7, Or 1 . 0|0795775 :: Q: Perif . Area .

7 The Area *of a circle being giuen, to finde the* Periferia. *Say,*

7 . 22*4, Or 1 . 12|56637 :: Area . Q: Perif .

In a Cone.

8 *The* side of a right Cone, *and the* Diameter of the base *being giuen, to find the* Superficies. *Say,*

7 . 22, Or 1 . 3|1416 :: ½ D*L . Superf .

Example. A Cone is giuen, whereof the side is 18, and the diameter of the base 12, I would know the superficies of it. Say,

1 : 3|1416 :: 108 (½ D*L) . 33|02928 .

9 The Axis, *or* height of a right Cone, *and the* Diameter of the base, *being giuen, to find the* Soliditie. *Say,*

7*4 . 22, Or 1 . 0|7854 :: Dq in ⅓ Axis . Soliditie .

Example. A Cone is giuen, whereof the Axis is 18, and the Diameter of the base 12, I would know the Solidity. Say,

1 . 0|7854 :: 864 (Dq in ⅓ axis) . 678|2856 .

In

In a Cylinder.

10 *The* side of a right Cylender, *and the* Diameter *being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : Diam : × axem . Superfic.

11 *The* Side of a right Cilinder, *and the* Diameter *being given, to finde the* Soliditie. *Say,*

7 × 4 . 22, Or 1 . 0,7854 : : Dq × L . Soliditie.

12 *The* Side of a right Cylinder, *and the* Circumference P, *being given, to finde the* Soliditie. *Say,*

22 × 4 . 7, Or 1 . 0,0795775 : : Pq × L . solidit.

In a Sphære.

13 *The* Axis, *or* Diameter *of a Sphære being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : Dq . Superficies.

14 *The* Superficies *of a Sphære being given, to finde the* Axis. *Say,*

22 . 7, Or 1 . 0,31831 : : Superfic . Dq.

15 *The* Segment *of a Sphære being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : Q : chord of ½ Segm. . Superfic.

16 *The*

16 *The* Axis, *or* Diameter *of a Sphære being giuen, to finde the* Solíditie.
Say,

7×6 . 22, Or 1 . 0|5236 :: Dc . Soliditie .

Example. A Sphære is giuen, whereof the Axis is 12, I would know the solidity of it. Say,

1 . 0|5236 :: 1728 (Dc) . 904|6208 .

17 *The* Soliditie *of a Sphere being giuen, to finde the* Axis. *Say,*

22 . 7×6, Or 1 . 1|90986 :: Soliditie . Dc .

Example. A Sphære is giuen, the Solidity whereof is 904|6208, I would know the Axis thereof. Say,

1 . 1|90986 :: 904|6208 . 1728 (Dc) .

18 *A* Segment *of a Sphere being giuen to finde the* Soliditie. *Say,*

First. *As the altitude of the other Segment, is to the altitude of the Segment giuen: so is that altitude of the other Segment increased by halfe the Axis, vnto a fourth.* Then againe say, *As* 7×3 *is to* 22, *Or as* 1 *is to* 10472 : *so is the product of the quadrat of halfe the chord of the Persferia of that Segment, multiplied by that fourth, to the Soliditie.* Viz.

7×3 . 22, Or 1 . 1|0472 :: $\left.\begin{matrix} \textit{Q: } \tfrac{1}{2} \textit{ chorda} \\ \textit{in quartam} \end{matrix}\right\}$. Solidit.

19 For note that a Sphære, is equall to two Cones, having their height and the diameter of their base, the same with the Axis of the Sphære. Or which is all one, A Sphære is two third parts, of a Cylinder, having the height and the diameter of the base the same with the Axis of the Sphære.

CHAP. VIII.

Concerning *Plaine*, and *Solide Measures*.

1 THe diuiding of the *Carpenters ruler* into Inches, and halfe, and quarters, and halfe quarters of Inches, that is of every Inch into eight parts, is most inartificiall, and vnfit for measuring, by reason of the manifold denominations, which must be brought into one, and is hard to bee done of them that are vnskilfull. I would wish therefore that euery Inch were diuided into 10 parts, or rather that the foot were diuided into 100 parts, which is best of all: for then there will neede no reduction. And all other diuisions must bee reduced vnto this, by these Rules following.

2. If the measures be taken vpon a Ruler diuided into Inches and tenth parts of an Inch, first take out all the whole feet, and then diuide the Inches remaining, with their decimall parts if there be any by 12.

Example. How many feet and decimall parts of a foot, are in Inches 17|3 ?

First take out the whole foot which is 12 Inches, and there will remaine Inches 5|3 : which being diuided by 12, you shall haue 442 thousand parts almost: wherefore Inches 17|3, is feet 1|442 ——. And contrariwise, feet 1|442 —, shall be reduced into Inches 17|3, being multiplied by 12.

3. If the measure bee taken vpon a Ruler diuided into inches and halfe quarter, that is each inch into 8 parts, First you must reduce the eight parts into decimals of Inches, by diuiding the number of parts given by 8 the Denominator thereof: and afterward by the former Rule,

Rule, reduce the inches, and decimall parts, into decimall parts of a foot.

Example. How many feet, and decimall parts of a foot, are in 7 inches, and 5 eight parts?

Firſt diuide the 5 eight parts by 8, and you ſhall haue 625 thouſand parts: which being put to 7 inches, will make inches 7|625: Again diuide theſe by 12, as was ſhewed in the former rule: and the whole meaſure will be feete 0|635. And contrariwiſe feet 0|635, will be reduced into inches 7|625, being multiplied by 12.

4. I muſt aduiſe all thoſe that haue occaſion, to meaſure Plaines, or Solids, to make themſelues very perfect in this kind of Reduction (becauſe moſt Rulers they ſhall ordinarily meet withall, are diuided into inches, and halfe quarters) which will be very eaſie to them, if they doe but remember, that *In diuiſion the firſt terme of the proportion implyed, is the Diuiſor it ſelfe: but in Multiplication, the firſt terme is euermore* 1. as hath beene ſhewed in Chap: 2, Sect: 7. And therefore, preſuming on the diligence of the Practiſer herein, I ſhall not neede in this kind of meaſuring, to ſpeake any more of inches, but of feet and decimall parts of feet, as if the Ruler were ſo diuided.

Of *Plaine meaſures.*

5. *A Parallelogram*, or foure ſided rectangle Superficies, being propoſed, to find the *length of a Superficiall foote.*

Take with your Ruler the breadth thereof in feet, and decimals of a foot: and by the breadth ſo taken diuide 1. the quotient ſhall be the length of a ſuperficiall foot.

Example. A boord is feet 1|17 broad, how much thereof will make a foot?

Diuide 1 by 1|17, the quotient will bee 0|855 almoſt: ſo much ſhall the length of a foot be, which multiplying the parts by 12, will giue inches 10|26. And againe thoſe parts multiplied by 8, will giue 2 eight parts of an inch.

Example, II. In tileing, or healing they vſe to reckon by the Square, which is 10 foot euery way, in all 100 feet. There is roofe, feet 16|25 broad, how much thereof maketh a Square?

Diuide 100 by 16|25 the quotient will bee 6|154 almoſt: ſo much ſhall be the length of one ſquare; which multiplying the parts by 12, will be 6 feet, and inches 1|848 — almoſt. And againe thoſe parts multiplied by 8, will giue ſomewhat more then 6 eight parts of an inch.

Example, III: In pauing they vſe to reckon by the yard, which is 3 feet euery way, in all 9 feet. There is a roome to bee paued, which is feet 17|25 broad; how much thereof maketh a yard?

Diuide 9 by 17|35, the quotient will be 0|519 almoſt, the length of one yard.

6. ***A fower ſided rectangle Superficies,*** with all the ***oppoſite ſides paralleſt*** being propoſed, to find the ***content.***

Take with your Ruler both the breadth and length of, and multiply the one number into the other.

Example, A board is feet 1|17 broad, and feet 16|32 long: how many feet doth it containe in all?

Multiply 16|32 by 1|17 the product will be feete 19|1 almoſt: the whole quantity of that board.

Example. II. A certaine barne tiled, hath the breadth of of the roofe feet, 16|25, and the length of the barne is feet 47, how many squares of tiling hath it?

Double the length (that you may haue both sides of the roofe) and it will be 94, which being multiplied by 16|25, will giue feet 1527|5. Againe diuide those feete by 100, so shall you haue squares 15|275.

Example. III. A certaine hall paued hath the breadth feet 17|35, and the length feet 30|5, how many yards doth it containe?

Multiply 17|35 by 30|5, the product will be feet 229|175. Diuide these by 9 and the quotient will bee yards 58|797.

7. *A fower sided Superficies* with the *two sides of length onely parallel* being proposed, to find the *content* thereof.

Take with your Ruler the length of the two parallell sides thereof: adde both those numbers together, multiply halfe that summe, by the breadth of the Superficies taken the neareſt way over, and the product will bee the content thereof.

Example. A *Trapezium*, or fower sided figure is proposed, hauing two sides thereof parallell, the length of the longer parallel side is feete 18|75 and the length of the shorter side is feete 14|45, the breadth thereof being taken the nearest way over is feete 12|5, I would know how many feete are contained in the whole Superficies?

The length of the parallell sides are feet 18|75, and 14|45, which added together make 33|2, halfe whereof is 16|6, which multiplied by the breadth 12|5, the product will be 207|5, so many feet are contained in the superficies of the Trapezium proposed.

8. A fower sided Superficies which hath none of the sides parallel, as also euery plaine figure of more sides the fower being proposed, must with *Diagoniall* lines bee diuided into triangles. And note that euery such figure containeth so many triangles as it hath sides, abating two out of the number. Then those triangles are to be measured seuerally as followeth.

9 To find the *content, or Area of a Triangle.*

Take the perpendicular height, or neerest distance betweene the base or knowne side, and the angle opposite: and by that height multiply halfe the base, or multiply the whole base by halfe that perpendicular height: and the product shall be the content, of that Triangle.

But if it be an *Æquilater triangle*: say,

1000 . 433$\underline{0127}$ :: the side of the triangle . Area.

10. To find the *content of a segment of a Circle*, whereof the *Periferia* is giuen in *degrees and decimall parts.*

First say, As 100000, is to 1745$\underline{329253}$: so is the Arch in degrees, to the Arch in the diuisions of the Radius. keepe this number found.

Againe by the 6 Sect: 1 chap: find out the *true sinus* of the Arch given. Then take the difference of these two numbers found, by subducting the *Sinus* out of the Arch. And lastly multiply halfe that difference by the Radius 100000, the product shall be the content of that segment.

11. The

11 *The* chord of any arch, *together with the* Radius, *or semidiameter of the whole circle being giuen, to finde out the* Arch it selfe. *Say,*

As the Radius giuen, is to halfe the chord (reckoned in the fourth circle) *so is* 100000, *to the sinus of halfe the arch* (to be reckoned in the first, or eighth circle). Wherfore double the arch found, and so haue you the arch of the chord proposed.

12 *To finde* a Quadrat, *or* Square equall to a superficies *giuen.*

First, seeke out (as hath beene taught) the content of that superficies: then take the quadrat roote thereof by Chap. 5, Sect. 4.

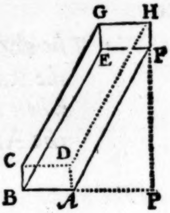
Of Solid measures.

13 *In* a Columne, or Cylinder, *hauing the* base, *to finde* how much of it maketh a foot solid.

By a *Columne* I meane a solid body arising from a plaine base, the angular lines whereof are paraliell, and equall: and if the angular lines make right angles with the base, it is *a right Columne,* and the length is the height thereof: but if they make oblique angles, it is an *Oblique Columne,* and the length is not the height, but the height is a perpendicular line let downe from the top of the Columne vnto the base, extended if neede bee: as in the

Diagrame,

Diagrame, the Solid ABCDEFGH, is an *Oblique Columne*, because the angular line EB, standeth obliquely vpon the side of the base BA, and indeede vpon the base it selfe. Wherefore the *height* of it shall be equall to the line FP, let fall from the top vnto the base extended.



And after this manner also a *Cylinder*, and a *Pyramid*, and a *Cone* is esteemed either *right*, or *oblique*, and the *height* taken accordingly.

First therefore the base is to be found, of what fashion soeuer it is, as hath euen now beene shewed, either in this Chapter, or in the last before: and then diuide 1 by that same base: the quotient shall bee, the height of a Section thereof, which is equall to one foot solid.

Example. A Columne, or peece of tymber, whose sides are all parallel, hath the breadth feete 1,75, and the thicknesse thereof is feet 1,25: which multiplied together the product will be 2,1875. Diuide therefore 1, by 2,1875: and the quotient shall be 0,457143 almost. And so much is the height of a solid foot, of that peece of timber.

14 *Hauing the* Base, *and the* height of a Columne, *or* Cylinder, *to finde the* whole content.

Multiply the base into the height, and the product shall be the content.

Example. A Columne hath the base feet 2,1875, and the height thereof is feet 17,34, how many feet are contained in the whole?

Multiply

Multiply the baſe 2,1875, by 17,24 and the product will be 36,086875, ſo many ſolid feet are contained in that Columne.

And in this very manner may you finde the content of a Cylinder, hauing either the diameter, or circumference giuen, together with the height.

15 To meaſure tapering timber, *the* baſe, *or* baſes thereof, *together with the* height *being giuen.*

A Tapering peece of timber, according as the baſe thereof is right lined, or circular, is either a *Pyramide* or a *Cone*, or elſe a *ſegment* of one of theſe two: If it be a compleat Pyramide, or Cone, it hath but one baſe, Multiply that baſe by $\frac{1}{3}$ of the height, and the product ſhall be the content.

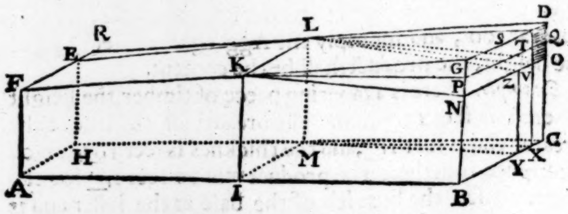
But if it be the ſegment of a Pyramide, or Cone, Firſt finde out the baſes at both the ends, and multiply the one by the other, and out of the product thereof extract the Quadrat root: then adde together both the baſes, and that quadrat root, and multiply the Aggregate thereof by $\frac{1}{3}$ of the height, the product ſhall be the content.

Example. There is a tapering peece of timber, the height whereof is feet 12,6, and the breadth of the baſe at the greater end is feet 1,75, and the thicknes is feet 1,32, which multiplied together, the product will be feet 2,31 for the greater baſe: the breadth of the baſe at the leſſer end is feet 1,2, and the thickneſſe there, is feete 0,91, which multiplied together, the product will be feet 1,092 for the leſſer baſe. Multiply the baſes together, the product will be 2,52252, the quadrat root whereof is 1,588 almoſt, to which if you adde the ſummes of both the baſes, the aggregate will be 4,99, which being multiplied by $\frac{1}{3}$ of the height, *ſcil.* 4,2, the product will be feet 20,958, the content of the peece of timber.

16 Wherfore that vulgar manner which *Carpenters* vse in measuring of tapering timber, is not true: for if a peece of timber be tapering, they measure it in the very midle, and take the base, or Section there, multiplying it by the whole length. Which their manner of working, I say, is erroneous.

For first by practise a content will be giuen, euer lesse then the true content found according to the former Sect: which way of working is infallibly true, as is *Analitically* demonstrated, in my *Clauis mathematica*, Cha. 20, Sect. 15.

And secondly I say, that the product of that middle base, or Section, multiplied by the length, shall bee lesse then the true content, by foure Pyramides, hauing for their bases, a rectangle vnder halfe the difference of the thicknesse at the ends, and a quarter of the difference of the breadths: and are as long as halfe the peece of timber: Or which is all one, by a Parallelepipedon, vnder halfe the differences of the breadthes, and thicknesses, at both ends, and a third part of the whole length.



Which that I may shew, suppose one quarter of a tapering peece of timber giuen, sawed in sunder, at halfe the breadth, and halfe the thicknesse be *ABCDGFEH*: the midle Section is *IKLM*. Measure vpon the greater base *BN*, and *CO*, equall to *AF*, and *BY*. and *NZ*, equall to *FE*. Diuide *CY*, and *DO* in the midst. in the points *X*, and *Q*, and draw the lines *PQ*, *NO* paralell to *BC*,

and

and *TZ*, *SX* parallel to *BG*. Measure also *FER*, equall to *GS*, or *KL*. And so the Paralelegram *BPTX*, shall be equall to the middle Section *IKLM*. Lastly, draw the lines *LO*, *LV*, *LZ*, and *KP*, *KN*, and *MX*, *MY*.

I say that in this one quarter of the peece of timber, A solid hauing the bases equall to *IKLM*, and the length *AB* (which is the vsuall measuring of Carpenters) is lesse then the true content, by the Pyramide *DSVOL*, in which *DO*, or *SV*, is halfe the difference of thicknes, and *DS*, or *VO*, one quarter of the difference of breadths in both ends: and the height of it equall to halfe the length.

For the two solides *AFEHIKLM*, *BNZYIKLM*, are apparantly equall, againe the two solide wedges *GSTPKL*, *PTVNKL* are equall, and also the two solide wedges *OCXVLM*, *VXYZLM* are equall. Now if you turne ouer the wedge *GSTPKL*, vnto the lesser part *AFEHIKLM*, it will ouer-reach in thicknesse, at the lesser end the quantity of *ER*, and if you also turne ouer the wedge *OCXVLM*, vnto the lesser part, you shall find it to fill vp the former ouer-reaching, and to make an exact Parallelepipedon, the bases or ends whereof are equall to the middle Section *IKLM*. but ouer and aboue those two wedges turned ouer, you shall haue left the Pyramide *DSVOL*, which was to be proued.

And in like manner if a round tapering solide, or peece of timber, be measured by the middle Section, or circular base, I say, That the product thereof multiplied by the length, shall be lesse then the true content, by a Cylinder, the diameter of whose base, is equall to halfe the difference of the diameters of the two bases: and the length is one third part of the whole length.

Chap. IX.

Concerning the Measuring, *or* Gauging of Vessells.

1 A Wine, or Beere vessell, whether Pipe, Hogshead, Barrell, Kilderkin, or Firkin, and such like, is in forme of a *Sphæroides*, hauing the two ends equally cut off: and accordingly may be measured thus.

Measure the two diameters of the Vessell, in inches, or else in tenth parts of a foote, the one at the bung hole, the other at the head, and also the length within. And by the diameters found, finde out the circles; then adde together two third parts, of the greater circle, and one third part of the lesse: Lastly, multiply the aggregate by the length: so shall you haue the content of the Vessell, either in Cubic inches, or cubic tenth parts of a foot.

Example. Suppose a vessell, having the Diameter at the bung 32 inches, and at the head 18 inches, and the length 40 inches. The quadrat of 32 is 1024. and the quadrat of 18 is 324: Say then euermore,

1 . 0,5236 :: 1024 . 536,166, $\frac{2}{3}$ the circle at the bung.

1 . 0,2618 :: 324 . 84,823 , $\frac{1}{3}$ circle at the head.

Or else by Chap. 7, Sect. 4.

The aggregate of those two circles is *620,989*, which being multiplied by 40, the length giueth 24839,56 cubic inches for the whole content of that vessell.

M. *Edm.*

2 Mr. ***Edm: Gunter*** in his second booke of the Crosse Staffe, ***Chap.* 4.** pretending to shew the manner of gauging Wine vessels, beginneth with these words.

" *The Vessels which are heere measured, are supposed to bee* Cylin-
" ders,*or reduced into* Cylinders *by taking the meane, betweene the Di-*
" *ameter at the head, and the Diameter at the boungue, after the usuall*
"*manner.*

And according to this supposition, teacheth to finde a *Gauge point*, for a gallon of wine, in that his imagined ***Cylindriacall vessell.***

Because his words are cautelous, and subterfugious, wee must a little examine them; for if his way bee true, my Rule before set downe, though grounded vpon demonstration, cannot stand.

Well then, that reduction of a wine Vessell into a Cylinder, is either true, or false; if it bee true what neede those ambiguities of *Vessels which are here measured:* and *are supposed to be, &c.* and *after the vsuall manner?* if false why is it not noted, but deliuered as a Rule to confirme an error. And what meaneth, ***the meane betweene the Diameter at the head, and the Diameter at the boungue?*** is it the ***meane Geometricall***, or ***Arithmeticall***, that is the ***meane proportionall***, or ***that which equally differeth*** from both? such shifting is vnworthy an Artist.

First therefore, Let it be ***the meane in respect of difference***, which is equall to ***halfe the summe of the two Diameters:*** I say that the Vessell cannot truely be reduced to a ***Cylinder*** by such ***a meane Diameter.*** For seeing it is most apparent that such a Vessell, is greater in the middle then at the ends, the boords, or sides thereof, shall from the middle to the ends, goe either streight, and so the Vessel shall be as it were, two equall segments of Cones, set base to base: Or else arching, and so the Vessel shall (as before I sayd, and is commonly taken for a truth) bee a ***Sphæroides***, hauing the two ends equally cut of.

If it be considered as two segments of Cones: the measure by that *meane Diameter*, or middle Section is quite false, as hath beene demonstrated in the former Chapter, Sect: 16, 17. and will be giuen lesse then the true content, although the sides goe straight: much more then, if the sides goe arching; for that conuexnesse, must needs yeeld a greater capacity. And therefore in neither can that manner of Gauging be true.

Againe, if the *meane Diameter* be vnderstood to be the *meane proportionall betweene the two Diameters*, it is much more false, for betweene any two numbers, the *meane Geometricall* is lesse then the *meane Arithmeticall.*

Thus much I haue thought good in this place ingenuously to signifie to the inexpert Learner, that hee might not beguile himselfe with a prejudged opinion.

3. I haue shewed *the measuring of vessels by the cubic inch:* but our vsuall reckoning is by the *Gallon*, and parts thereof. Wee must therefore doe the best we can, to inquire the *true quantitie of a Gallon in inch measure*, which will bee difficult to doe exactly, both because the *Standards* vsually are not streight sided, but a little arching, neither doe they agree perfectly one with another: but what partly by experience, both mine owne, and others, which hath come to my sight; and partly by reasoning shall seeme to me most probable, I will not refuse to set downe.

4 Our *English Gallon* is vnderstood to bee either in *Ale measure*, or *Wine measure*: and these two measures not a little differing. And first we will inquire about our *Ale measure.*

I my selfe haue measured Bushels, and Pecks, which haue exactly beene fitted to the *Standards*, and haue still in my account found *a Gallon to containe better then* 270

cubic

cubic inches, indeed much about 272, or 273 as preciſely as I could meaſure in a veſſell not truely regular. Alſo my worthy friend *Maſter William Twine*, who hath vndergone great paines and charge, in finding out the *true content of our Engliſh meaſures*, gaue vnto me two ſeverall meaſures of an *Ale-gallon*, and thoſe in due conſideration but little differing. The one was found out by a braſſen veſſell made in manner of a Parallelepipedon, the baſe whereof was exactly ſixe inches ſquare, and the ſides diuided into inches and twentieth parts: into which veſſell he powring out a *ſtandard Gallon of Queene Elizabeth*, filled with water, found it therein to ariſe vnto 7 inches, and 6 tenth parts: which being computed maketh cubic inches $273|\underline{6}$. The other was found by taking the dimenſions of that *ſtandard Gallon*, which was made in forme of *a ſegment of a Cone*, but that the ſides were a little arching: the dimenſions were thus; the Diameter of the top was inches $6|\underline{5}$: the Diameter of the bottome was inches $5|\underline{3}$: and the height of it was inches $9|\underline{8}$: which being caſt vp by Chap: 8, Sect: 15, will be found to containe cubic inches $268|\underline{85}$: differing from the former, only cubic inches $4\frac{3}{4}$: which difference might well ariſe through the curuitie of the ſides. Theſe meaſures he did not only take himſelfe, but to giue me ſatiſfaction, ſhewed me the experience in the ſaid *Veſſell and Standard*: but the truth is, I obſerued the *Standard*, beſides the arching of the ſides, to bee not exactly circular within, nor the brimme of an even height, nor the bottome plaine: and in taking the height of the water in the *Veſſell*, our ſight was not able to eſtimate the aſcent thereof ſo preciſely, that a ſpoonefull of water, more or leſſe, could breed any ſenſible difference.

What therefore ſhall wee doe in this difficultie? indeede looke to the firſt ground, and principle of our Engliſh meaſuring, from Barley cornes. For the length of

3 Barley

3 Barley cornes taken out of the middle of the eare is an *Inch*, or *Uncia*, that is a twelfth part of a foot. 3 feete make a *Yard*: and 16 feet and an halfe, that is 5 yards and an halfe, a *Perch*, with which wee measure our land; for 40 perches is a *Furlong*, and 8 furlongs an *English mile*: and againe 40 square perches is a *Roodland*, 4 roodland make an *Acre*. So then a Perch which is feet $16\frac{1}{2}$, or yards $5\frac{1}{2}$ is as it were the beginning of all land measure in length: and a square Perch which is feete $272\frac{1}{4}$, is as it were the beginning of all land measure in the superficiall content.

Now therefore seeing in *Vessels a gallon is as it were the beginning of Vessell measure* (for a *Pottell* is but a diminatiue of it, and *a quart*, the quarter) it is not vnlikely that our wise Ancestors had such a consideration also in solide measures, that as a square Perch (the beginning of Superficiall land measure) did containe 272 square feete and a quarter; so a Gallon (the beginning of vessell measure) should containe 272 Cubic inches and a quarter. And the rather seeing that the ancient *Geographers*, diuide a foote into 4 *Palmes*, a palme being 3 of our inches, as 3 feete are a yard. So that as the side of a square Perch consisteth of yards $5\frac{1}{2}$, a Gallon also should consist, of a number of Cubic inches the square side whereof is palmes $5\frac{1}{2}$.

Wherefore sauing the exact truth when it shall appeare, and in the meane time the more probable reasons of other men, I make bold to tender this my coniecture, to the censures of more diligent Inquirers, *That the measure of an English Ale gallon should be a square Vessell of inch $16\frac{1}{2}$, or Palmes $5\frac{1}{2}$ every way, and 1 inch deepe*: that is $272\frac{25}{100}$ Cubic inches.

5. And this my opinion may peraduenture receiue some confirmation by the inquiry of an *English Wine Gallon*.

Master

M. *Henry Briggs* that learned Geometrician, and my very louing friend, made an experiment, with a Cubicall vessell, which was 12 inches euery way, which hauing filled with water carefully measured, found it to containe 7 gallons and an halfe wanting *a moment*, as hee himselfe long since, being then of *Gresham Colledge*, signified to me. Now if it had contained exactly 7 gallons, and an halfe, a *Wine gallon* should haue beene $230\underline{,4}$ cubic inches, but because it wanted a little, the gallon must be somewhat bigger, for which *moment* therefore if you will put 6 hundred parts of an inch, the *Wine gallon shall containe* 231 *cubic inches.*

Againe M. *Gunter* in the place before mentioned sheweth, that the common opinion is that at *London*, a *Cylindriacall vessell*, whose diameter is 38 inches, and length 66 inches, doth containe 324 gallons: wherefore by this account a gallon should bee 231 cubic inches almost exactly, which in both so neerely agreeing, wee may well conclude, *That an English Wine gallon doth containe* 231 *cubic inches.*

It is also commonly receiued, that the reason of the greatnesse of an *Ale gallon* aboue the *Wine gallon* is, that because of the frothing of the Ale or Beere, the quantity becommeth lesse, and therefore such liquors that did not so yeeld froth, as Wine, Oyle, and the like, should in reason haue a lesser measure. If then we compare these two gallons together, we shall finde that

$$272\underline{,25} \;.\; 231 \;::\; 16,5 \;.\; 14\,.$$

which abatement might to our Ancestors, in apportioning those measures, seeme to be reasonable.

6 *To finde* how many Cubic tenth parts of a foot are in a gallon, *both of* Beere, *and* Wine : *or also in* any number of Cubic inches.

Because there be in a Cubic foote, 1000 Cubic tenth parts, and 1728 Cubic inches, say,

1728 (C: 12) . 1000 (C: 10) : : $272|\underline{25}$. $157|\underline{5521}$ — cubic tenth parts of a Beere gallon.

And 1728 (C:12) . 1000 (C: 10) : : 231 . $133|\underline{6803}$— cubic tenth parts in a Wine gallon.

Also 1728 . 1000 : : $24839|\underline{56}$. $14374|\underline{746}$, cubic tenth parts, are in the vessell measured in Sect. 1.

7 And according to these measures so found, you may easily finde the content of a *pint*, or *quart*, or *peck*, or *bushell*, which two last are to be reckoned in *Ale measure*.

8 *The* Content of a Vessell, *being giuen* in cubic inches, *or in* cubic tenth parts of a foot, *to finde* how many gallons it containeth.

This is easily done if you diuide the content giuen in inches, by $272|\underline{25}$ for *Ale measure :* and by 231, for *Wine measure*. But if the content be giuen in decimall parts of a foote diuide it by $157|\underline{5521}$ — for Ale measure; and by $133|\underline{6803}$ — for Wine measure.

Example. How many wine gallons are in a vessell containing $24839|\underline{56}$ cubic inches, or $14374|\underline{746}$ cubic tenth parts of a foote. Diuide $24839|\underline{56}$ by 231, or diuide $14374|\underline{746}$ by $133|\underline{68}$, and the quotient shall be $107|\underline{53}$ wine gallons.

CHAP.

CHAP. X.

Concerning the Comparison *of sundry* Metals, *in* quantity *and* weight.

1 IF foure pieces of Metalls, whereof the third is of the same kinde with the first, and the fourth of the same kind with the second, are proportionall, their grauities also, or weights, shall be proportionall.

2. If there bee foure pieces of metall, whereof the third is of the same kind with the first, and the fourth of the same kinde with the second: and the first and second be of equall greatnesse, and the third and fourth of equall weight; the weights of the first and second, shall be reciprocall to the magnitudes of the third and fourth.

3. Two Sphæres of the same matter are in weight, as the cubes of their Diameters, are in magnitude. *Et contra.*

4. Pieces of Metall if they bee of equall magnitude, haue their weights in direct proportion, as is here set downe: but if they be of equall weight, they haue their magnitudes in proportion reciprocall: According to the experiments of *Marinus Ghetaldi*, in his tractate called *Archimedes promotus.*

Gold.	3990	Brasse.	1890
Arg. Viu.	2850	Iron.	1680
Lead.	2415	Tinne.	1554
Siluer.	2030		

5. To finde the *Weight of a Sphære of Tinne,* hauing the *Diameter* 1 *Inch, or else* 1 *tenth part of a foot.*

Take a piece of Tinne, and turne it exactly in a Lathe, into a Cylinder, hauing both the Diameters of its base, and also the length equall, to the Diameter of the Sphere giuen. Then weigh that Cylinder, that you may haue the weight thereof in graines: And laſtly, take two third parts of the whole number of graines: for the weight of the Sphære.

After this manner *Marinus Ghetaldi* found a Cylinder of 1 inch, or twelfth part of a foot thick, and long to weigh 1824 graines: whereof $\frac{2}{3}$ is 1216, the weight of a Sphære of that thickneſſe.

Againe if you ſay,

1000 (C:10) . 1728 (C:12) :: 1216 . 2101,$\underline{248}$

You ſhall haue the weight of a Sphære whoſe Diameter is one tenth part of a foot.

Wherefore alſo *a Cubed inch of Tinne weigheth* 2322,$\underline{4}$ —, and *a Cubed tenth part of a foote weigheth* 4013,$\underline{1}$ —

And note that *Mar: Ghetaldi* vſeth the ancient *Roman foot*, which by the meaſure ſet downe in his booke ſeemeth to be very little leſſe, then our vſuall *English foot*, if not exactly the ſame.

Note alſo that he diuideth *one pound*, into 12 *ounces*, and euery *ounce* into 24 *Scruples*, and euery *Scruple* into 24 *graines*: So that *an ounce* with him weigheth 576 *graines*: and *a pound* 6912. Whereas our *English pound of Troy weight by Aſſize*, or *Goldſmiths weight*, is but 5760 graines, and our *ounce* 480. But whether the *Raguzean graine*, be the ſame with our *English*, I leaue to be tryed by the diligent Practizer.

6 To finde the *Weight of a Sphære of Tinne, at any other Diameter aſsigned.*

Multiply the Cube of the Diameter giuen by 1216, if it be in inch meaſure; or by 2101,$\underline{248}$, if the meaſure be by decimall parts of a foot: and the product will be the weight of that Sphære.

And contrariwiſe to find the Diameter of a Sphære of Tinne, by the weight giuen in graines. Diuide the weight giuen in graines by 1216 if you would haue inch meaſure:

measure: or by $2101{,}\underline{248}$ if you would measure by decimall parts of a foot, and the quotient shall be the Cube of the Diameter.

7. To finde the *Weight of a Sphare of any Mettall, at any Diameter* giuen, either *in Inch measure,* or *in decimall parts of a foot.*

First by Sect. 6, seeke the weight of a Sphære of Tinne, at that Diameter: then by Sect. 4, say, *As the proportionall number of Tinne, is to the proportionall number of that other Metall: so is the weight of the Sphare of Tinne now found, to the weight of the Sphare proposed.*

Example. Suppose a Sphære of Iron, whose Diameter is 3 inches; what shall be the weight thereof?

First, the weight of a Sphære of Tinne, of 3 inches Diameter, will be found to be 32832 graines. Then say,

1554 . 1680 :: 32832 . $35494{,}\underline{054}$ graines the weight of the Sphære proposed.

8 To finde the *Diameter of a Sphare of any Metall in inch measure, or decimall parts of a foot,* the *weight* thereof being giuen.

First by the contrary of Sect. 6, seeke the Cube of the Diameter of a Sphere of Tinne of that weight. Then by Sect. 4, say reciprocally. *As the proportionall number of that other Metall, is to the proportionall number of Tinne: so is the Cube of the Diameter now found, to the Cube of the Diameter of the Sphare proposed.*

Example. A Sphære of Iron weigheth $35494{,}\underline{054}$ grains: how many inches is the Diameter thereof?

First, the Cube of the Diameter of a Sphere of Tinne of $35494{,}\underline{054}$ graines weight, will be $29{,}\underline{18910695}$, then say, reciprocally,

1680 . 1554 :: $29{,}\underline{18910695}$. 27—

The Cubic root whereof is 3, the Diameter of a Sphære of Iron of that weight proposed.

CHAP. XI.

Concerning the *Ordering of Soldiers, in any kinde of rectangular forme of battaile.*

1 BAttailes are considered either in respect of the number of men, or in respect of the forme of ground. *As a square battaile of men* is that which hath an equall number of men, both in *Rank and File*, though the ground on which they stand, bee longer on the *File*, then on the *Ranke*. And *a square battaile of ground* is that which hath the *Ranke* as long as the *File*, though the men in *Ranke* be more then in *File*.

2. In respect of the number of men, it is called either *a square battaile*, or *a double battaile*, or *a battaile of the grand front*, which is quadruple, or *a battaile of any proportion, of the number in Ranke, to the number in File.*

3. If it bee *a square battaile of men:* Extract the quadrat root out of the whole number of men, and the same shall be the number of Souldiers, to be set in a Ranke.

Example. 576 Souldiers are to bee martialled in a square battaile, that so many may be in Ranke, as in File.

Take the quadrat root of 576, which is 24: the same shall be the number to be placed in a Ranke.

4. If it be *a double battaile of men:* Extract the quadrat root out of halfe the number of men, and the same doubled shall bee the number of Souldiers to bee set in a Ranke.

Example. 1458 Souldiers, are to be placed in a double battaile; so that twise so many may be in Rank as in File.

Take

Take halfe the given number 1458, which is 729, the quadrat root whereof is 27: double it and you ſhall haue 54 men to be placed in a ranke.

5. If it be *a quadruple battaile*, which is called *of the great front*: Extract the quadrat root out of one quarter of the number of men, and the ſame quadrupled ſhall be the number of Souldiers to be ſet in a ranke.

Example. 1024 Souldiers are to bee martialled into a *battaile of the grand front*, ſo that fower times ſo many be in *ranke* as in *file*.

Take one quarter of 1024 the number given, which is 256, the quadrat root whereof is 16: quadruple it, and you ſhall haue 64 men to be placed in a ranke.

6. If a battaile bee required of any other forme, that is, if a Ratio be given, according to which the number of men in Ranke, ſhall be to the number in file. Multiply the two termes of the Ratio given: Then ſay *As the product is to the quadrat of the terme which is for the ranke*, or *As the terme which is for the file, is to the terme which is for the ranke: ſo is the whole number of Souldiers, to the quadrat of the number of men to be placed in a ranke.*

Example. 1944 Souldiers, are to be martialled ſo, that the number of the ranke, be to the number of the file, as 8 vnto 3, that is for 8 men in ranke, 3 are to be ſet in file.

First multiply the two termes of the Ratio 8 and 3, the product whereof is 24, alſo quadrat 8, the terme of the ranke, which will be 64. Then ſay,

3 . 8 :: . 1944 . 5184 .

out of which extract the quadrat root 72, and it will giue you the true number of the ranke.

7. In reſpect of the forme of ground, the battaile is either

either *a square of ground*, or longer one way then the other. For the distance, or order of Souldiers martialled in array, is distinguished either into *Open order*, or *Order*.

Open order is when the very centers of their places, are distant 7 feet asunder, both in ranke and file.

Order is when the centers of their places, are distant 3 feet and a halfe in *ranke*, and so much in *file*. Or else 3 feet and a halfe in *ranke*, and 7 feet in *file*: which last *order*, and whatsoeuer *order* else there is, in which the distance of the *rankes* one from another is greater, then the distance of *files*, causeth that *a square of men*, maketh not *a square of ground*, but the ground is longer on the *file* then on the *ranke*.

8 If it be *a square battaile of ground*, the centers of the distances being feet $3\frac{1}{2}$ in *ranke*, and 7 feet in *file*. Because $3\frac{1}{2}$ is halfe of 7, the ratio of the distances, is as 1 unto 2. And seeing the number in ranke, to the number in file, is reciprocall to the distances, the ratio of the number of men in ranke, to the number of men in file, shall be as 2 unto 1. And so the Rule shall be the same with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke; so is the whole number of Souldiers, to the true number of the ranke.*

Example, 1352 Souldiers, are to bee set in *a square of ground*, that their distances may be feet $3\frac{1}{2}$ in ranke, and 7 feet in file.

The Ratio of the *ranke* to the *file*, shall reciprocally be, as 7 to $3\frac{1}{2}$, that is as 2 to 1. Say therefore

$$1 \,.\, 2 :: 1352 \,.\, 2704,$$

the quadrat roote whereof 52 is the number of men to be set in a ranke.

9 If a battaile wherein the distance in ranke is vnequall to that in file, be longer one way then the other, accor-

ding

ding to any Ratio giuen : there is to be consi·dered a double ratio, one reciprocall in respect of the distances, the other according to the forme of the ground. Wherefore to finde the Ratio of men in rank, to the men in file, Multiply the two termes of the ranke, for the ranke, and the two termes of the file, for the file. And then the Rule shall bee the same with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke : so is the number of Souldiers, to the quadrat of the true number of the ranke.*

Example. 10290 souldiers, are to be set in a battaile, so that they may stand onely 3 feet asunder in ranke, and 7 feet in file, and the length of the ground for the ranke, to the length of the ground for the file, shall haue the ratio of 5 vnto 2.

First in respect of the distances, the Ratio of Rank to file, reciprocally is as 7 vnto 3. Secondly, in respect of the ground, the ratio of rank, to file, is as 5 to 2. Wherefore by multiplication of like termes, the true ratio of rank to file shall be 7×5 to 3×2, that is as 35 to 6. Say therefore

$$6 \ . \ 35 \ :: \ 10290 \ . \ 60025$$

the quadrat root whereof is 245, the number of men to be set in ranke.

10 If 1000 Souldiers, may be lodged in a square, of 300 feete, how many feete must the side of a square be, which will serue to lodge 5000? Say,

$$1000 \ . \ 5000 \ :: \ 300 \times 300 \ . \ 450000,$$

the quadrat roote whereof 671 — is the square side sought for.

And this is the order for resolution of all other questions of this sort.

CHAP. XII.

A collection of the most necessarie Astronomicall operations,

1 BEfore wee deliuer the Rules of such operations, it will not be inconuenient, to set downe certaine *Reductions*, wherof we may haue frequent vse.

To reduce sexagesime parts into decimals. Diuide the sexagesimes giuen by 60.

Example. How many decimals are 34′, 12″?

Here are required two reductions, first of the seconds into decimals of minutes: then of the minutes with their decimals, into decimals of degrees, Thus

$$60 \;.\; 1 \;::\; 12'' \;.\; 0|\underline{2}$$

Againe

$$60 \;.\; 1 \;::\; 34'|\underline{2} \;.\; 0|\underline{57}^{\circ}$$

Wherefore 34′, 12″ are equall to $0|\underline{57}$ of a degree.

And contrariwise *to reduce decimall parts of degrees in sexagesimes.* Multiply the decimall part giuen by 60.

Example. How many sexagesime parts are $0|\underline{57}^{\circ}$?

$$1 \;.\; 60 \;::\; 0|\underline{57}^{\circ} \;.\; 34|\underline{2}'$$

Againe

$$1 \;.\; 60 \;::\; 0|\underline{2} \;.\; 12''$$

To *reduce houres into degrees.* Multiply the houres with their decimall parts by 15.

Example

Example. How many degrees are $8^{Ho}, 34', 12''$; that is by the former reduction $8|\underline{57}^{Ho}$? thus

$$1 \;.\; 15 \;::\; 8|\underline{57} \;.\; 128|\underline{55}$$

Wherefore Houres 8, 34′, 12″ doe containe $128|\underline{55}$ degrees.

And contrariwise *to reduce degrees into houres.* Diuide the degrees with their decimall parts by 15.

Example. How many houres are in degrees $128|\underline{55}$?

$$15 \;.\; 1 \;::\; 128|\underline{55} \;.\; 8|\underline{57}$$

2 It is to be vnderstood, that if foure numbers are proportionall, their *Order* may be so transposed, that each of those termes, may bee the last in proportion. In this manner,

I. As the first is to the second; so is the third to the fourth.

II. As the third is to the fourth; so is the first to the second.

III. As the second is to the first; so is the fourth to the third.

IIII. As the fourth is to the third; so is the second to the first.

Wherfore euery proportion doth implicitly containe foure *Orders*, two descending, and two ascending, as may be seene by their combinations: By one of which *orders*, if of foure proportionall numbers, any three be giuen, that other which is vnknowne, may be found out.

Example. To finde out any of these,

termes
1 As the Sine of the complement of the *suns declination,*
2 is to the Sine of the compl. of his *altitude* ;
3 So is the Sine of the *Sunnes Azumith* from the meridian,
4 to the Sine of *the horary distance from the meridian.*

If the first, second, and third termes be giuen, the fourth shall be found out by the *I order.*

If the first, third, and fourth termes be giuen, the second shall be found out by the *II order.*

If the first, second, and fourth termes bee giuen, the third shall be found out by the *III order.*

If the second, third, and fourth termes be giuen, the first shall be found out by the *IIII order.*

3. To finde out any one of these.

termes
1 As the *Radius,* or totall Sine
2 is to the Sine of the *distance, or longitude of the Sunne* in the Ecliptic, from the next Æquinoctial point :
3 So is the Sine of the *Sunnes greatest declination*(which is the angle of the Ecliptic with the Æquinoctial),
4 To the Sine of the *Sunnes declination* in that longitude.

4. To

4. To finde out any one of these.

termes
1 As the *Radius,*
2 is to the Sine of the *Sunnes right ascension,* from the next æquinoctiall point:
3 So is the tangent of the *Sunnes greatest declination,*
4 to the tangent of the *Sunnes declination* in that place:

5. To finde out any one of these.

termes
1 As the *Radius*
2 is to the Sine of the compl. of the *Sunnes greatest declination:*
3 So is the tangent of the *longitude of the Sunne* from the next æquinoctiall point.
4 to the tangent of the *Right ascension of the Sunne,* from the same æquinoctial point.

6. To finde out any one of these.

termes
1 As the *Radius*
2 is to the Sine of the compl. of the *longitude of the Sunne* from the next æquinoctiall point;
3 So is the tangent of the *Sunnes greatest declination,*
4 to the tangent of the compl. of the *angle of the Ecliptic with the Meridian.*

7. To finde out any one of these.

termes
1 As the *Radius*,
2 is to the Sine of the *Sunnes greatest declination:*
3 So is the Sine of the compl. *of the Sunnes right ascension* from the next æquinoctial point,
4 to the Sine of the compl. of the *Angle of the Ecliptic with the Meridian.*

8. To finde out any one of these.

termes
1 As the Sine of the compl. of the *Poles height*,
2 is to the *Radius*;
3 So is the Sine of the *Sunnes declination*,
4 to the Sine of the *Sunnes Amplitude ortive*, that is the arch of the horizon from the place of the Sunnes rising or setting to the true East, or West point.

9. To find out any one of these.

termes
1 As the *Radius*,
2 is to the Sine of the *Sunnes greatest amplitude ortive*, which is in the Tropicks:
3 So is the Sine of the *longitude of the Sunne* from the next æquinoctial point,
4 to the Sine of the *Sunnes amplitude ortive.*

Or alſo of theſe.

termes

1 As the Sine of the compl. of the *Poles height,*
2 is to the Sine of the compl. of the *Sunnes greateſt declination:*
3 So is the Sine of the *Sunnes longitude* from the next æquinoctial point,
4 to the Sine of the *Sunnes amplitude ortiue.*

10. To finde out any one of theſe.

termes

1 As the *Radius,*
2 is to the tangent of the *height of the Pole.*
3 So is the tangent of the *Sunnes declination,*
4 to the Sine of the *Sunnes Aſcenſional difference.*

11. To finde out any one of theſe.

termes

1 As the *Radius*
2 is to the Sine of the *height of the Pole:*
3 So is the tangent of the *Sunnes amplitude ortiue,*
4 to the tangent of the *Sunnes aſcenſionall difference,*

12. To finde out any one of these.

termes

1 As the Sine of the compl. of the *Sunnes declination,*
2 is to the *Radius:*
3 So is the Sine of the compl. of the *Sunnes amplitude ortive,*
4 to the Sine of the compl. of the *Sunnes ascensionall difference.*

13. To finde out any one of these.

termes

1 As the tangent of the *height of the pole,*
2 is to the *Radius:*
3 So is the tangent of the *Sunnes declination,*
4 to the Sine of the *Suns horary distance from the Meridian, being due East or West.*

14. To finde out any one of these.

termes

1 As the Sine of the *height of the pole*
2 is to the *Radius:*
3 So is the Sine of the *Sunnes declination,*
4 to the Sine of the *Sunnes altitude being due East, or West.*

15. To find out any one of these.

termes
1. As the *Radius,*
2 is to the Sine of the *height of the Pole:*
3 So is the Sine of the *Sunnes declination,*
4 to the Sine of the *Sunnes altitude aboue the Horizon at sixe of the Clocke.*

16. To find out any one of these.

termes
1 As the *Radius,*
2 is to the Sine of the complement of the *Poles height.*
3 So is the tangent of the *Sunnes declination.*
4 to the tangent of the *Sunnes Azumith from the North meridian, at 6 of the Clocke.*

17 The *hower of the Sunnes Rising, and setting* is found out by the *Ascensionall difference.* For if you reduce the degrees of the Ascensionall difference, into howers, it will shew you how much the Sunne riseth, or setteth before, or after 6 a Clock.

18. The *Oblique ascension* also of the Sunne is found out by the *Ascensionall difference.* For if you subduct the Sunnes Ascensionall difference, out of the right ascension of the Sunne, from the beginning of *Aries,* for the sixe Northerne signes which are ♈, ♉, ♊, ♋, ♌, ♍, or if you adde it thereto, for the sixe Southerne Signes, which are ♎, ♏, ♐, ♑, ♒, ♓, you shall have the Sunnes oblique ascension.

19. The *declination of the Sunne,* and his *Altitude* aboue the Horizon at any time, together with the *height of the Pole* being giuen, to find the *hower of the day.* Say,

As the *Radius*,
is to the Sine of the complement of the *Sunnes declination*.
So is the Sine of the compl. of the *Poles height*,
to a fourth number. Keepe it,

Then out of the Sunnes distance from the North pole, subduct the complement of the Pole; and of that remaines, and the complement of the Sunnes altitude, take both the *Summe*, and also *the difference*. And say againe,

As the *fourth before kept*,
is to the Sine of *halfe that summe*:
so is the Sine of *halfe the difference*,
unto a number which being multiplied
by the Radius, is equall to the quadrat,
of the Sine of halfe the Angle of the
Sunnes horary distance from the Meridian.

20. The *declination of the Sunne*, and his *altitude* aboue the Horizon at any time together with the *height of the Pole* being given, to find the *Sunnes Azumith*. Say,

As the *Radius*
is to the Sine of the compl. of the *Suns altitude*.
So the Sine of the compl. of the *Poles height*,
is to *a fourth*, Keepe it,

Then out of the complement of the Sunnes altitude, subduct the complement of the Pole; and of that remaines, and the distance of the Sunne from the North Pole, take both the *Summe*, and also the *difference*: and say againe,

As

As the *fourth before kept,*
is to the Sine of *halfe the summe:*
So is the Sine of *hal_e the difference,*
vnto *a number* which being multiplied
by the Radius, is equall to the quadrat of
the Sine of halfe the angle of the *Sunnes*
horiz_ntall distance from the Meridian.

• 21. To find the *length of the Crepuscu-lum,* or *Twilight.*

Betweene the light of the day, and the darkenesse of the night, the Twilight is set by the wise Creator; that wee here vpon the earth might not in an instant, passe from one extreame into another, but by successiue degrees. *The Twilight is nothing else but the refraction of the Sunnes beames, in the densitie of the aire.* And *Pet. Nonnius* to find the length of the Twilight, watched the time after Sunne set, when the twilight in the West was shut in, so that no more light appeared there, then in any other part of the sky neere the Horizon: then by one of the knowen fixed Stars, having taken the true hower of the night, found by many obseruations, that at the time of shutting in the Twilight, the Sunne was vnder the Horizon 18 degrees, and vntill the Sunne was gone so low, the Twilight continued. Say therefore

As the *Radius*
is to the Sine of the compl. of the *Sunnes declination:*
So the Sine of the compl. of the *heigth of the Pole,*
is to a fourth, Keepe it.

Then out of the Sunnes distance from the South Pole, subduct the complement of the Pole; and of that remaines, and degrees 62, take both the *Summe* and also the *difference*; and say againe,

As the *fourth kept*,
is to the Sine of *halfe the summe*:
so is the Sine of *halfe the difference*,
to *a number* which being multiplied
by the Radius is equall to the quadrat
of the Sine of *halfe the angle of the Sunnes distance at the ending of the Twilight, from the high Noone next to it.*

Wherefore if out of the whole angle conuerted into howers, you subduct halfe the diurnall arch, or the hower of the Sunnes setting, you shall haue the true length of the *Crepusculum*, or *Twilight*.

22. To find the *length of the least Crepusculum in the yeare.*

The Sunne being in the winter *Tropic* maketh the longest Crepusculum, of the whole winter halfe yeare, and from thence, as the dayes increase, the *Crepuscula* doe decrease vntill they come to bee shortest, which is in a certaine Parallel, betweene that Tropic, and the Æquinoctiall: the declination whereof is thus found out.

As the tangent of the *complement of the Pole*,
is to the *Sine thereof*:
So is the *tangent of 9 degrees*,
to the Sine of the *declination of the Parallell, in which the Sunne maketh the shortest Crepusculum of the whole yeare.*

23 But before the *Crepusculum* come to bee shortest, there is another Parallel, in which the Crepusculum is equall to that in the Æquinoctiall: the declination wherof is thus found out.

As the *Radius*
is to the Sine of the *altitude of the Pole*:
So is the *Sine of* 18 *degrees*
to the Sine *of declination of the Parallel in which the Sunne maketh the Twilight equall to that in the Æquinoctiall.*

24. *If an Arch of the Ecliptic, be equall to his Right ascension, one end thereof* beeing knowne, to find out the *other end.* Say,

As the Sine of the Compl. *of the declination of the arch giuen.*
is to the *Radius*:
so is the Sine of the compl. of the *greatest declination,*
to the Sine of the compl. of the *other and.*

25. To find the *poynt of one quadrant of the Ecliptic,* wherein the *difference of longitudes cease to be greater, then the differences of the right ascensions.*

Multiply the Sine of the complement of the greatest declination, by the Radius, and out of the product extract the quadrat root: the same shall bee the Sine of the complement of the declination sought for.

26. To find the *quantitie of the angles, which the circles of the 12 Howses make with the Meridian.* Say

As the *Radius*
is to the Tang. of 60 degr. for the 11 th, 9th, 5th, and third howers, or to the Tang. of 30 deg. for the 12th, 8th, 6th, and second howses;
so is the Sine of the *complement of the Pole,*
to the tang. of the *compl. of any howse with the Meridian.*

And note that on the Easterne part of the vpper hemisphære, there are *three circles of Howses,* the *Horoscope,* which is also the Horizon, and next to that is the *circle of the* 12th *Howse*, then the *circle of the* 11th *Howse.* On the Westerne part also, are *three circles of Howses,* the *circle of the* 7th *Howse,* which also is the Horizon, and next thereto the *circle of the* 8th *Howse*, then the *circle of the* 9th *Howse.* But the *circle of the* 10th *Howse, is the very vpper Meridian it selfe. Contrary Howses* are 1 and 7: 2, and 8: 3 and 9: 4 and 10: 5 and 11: 6 and 12.

27. Resolue the whole time from the Noone last past into degrees (by multiplying the howers with their decimall parts by 15, according to Sect: 1) which adde vnto the right Ascension of the Sunne: and you shall haue the right ascension of the point of the Æquator in the vpper Meridian, which is called the *Right ascension of Medium cœli.*

28. Adde 90 degrees to the Right ascension of *Med. Cœli:* and it shall be the *degree of the Æquator then rising vpon the East Horizon.*

29. If the *first quadrant of the Æquator doe arise*, the beginning of ♈ is distant from the meridian Eastward, so much as is the distance of the *Right ascension of Med. cœli*, from 360. But if the *second quadrant of the Æquator doe arise*, the beginning of ♈, is distant from the Meridian Westward, so much as is the distance of ☉, from the *Right ascension of Med. cœli*.

And in both of them the lower angles of the Ecliptick with the Meridian, on the East side is obtuse, and on the West side acute : and the 90th degree of the Ecliptick, commonly called *nonagesimus gradus*, is on the East part.

30. If the *third quadrant of the Æquator doe arise*, the beginning of ♎ is distant from the Meridian Eastward, so much as is the distance of the *Right ascension of Med. cœli* from 180. But if the *fourth quadrant of the Æquator doe arise*, the beginning of ♎, is distant from the Meridian Westward, so much as is the distance of 180, from the *Right ascension of Med. cœli*.

And in both of them the lower angle of the Ecliptick with the Meridian, on the East side is acute, and on the West side obtuse : and on the 90th degree is on the West part.

31. The *point of the Ecliptick culminant in the Meridian*, which is called *Medium cœli*, or *Cor cœli*, and is the *cuspis of the* 10th *house*, may be found by Sect. 5.

32. The *declination* of the said *culminant point*, may be found by Sect. 3. VVherefore also by adding or subducting that declination, to, or from the elevation of the

Æquator,

Æquator, (which is the complement of the Pole) the *Altitude of Med. cœli* may be had.

33. The *Angle of the Ecliptick with the Meridian*, may be found by Sect. 7.

34. To finde the *Altitude of the 90 degr.* Or the *Angle of the Ecliptick with the horizon.*

As the *Radius*
is to the Sine of the compl. of the *altitude of Med: cœli.*
So is the Sine of the *angle of the Ecliptick with the Meridian*,
to the Sine of the compl. of *the angle sought for.*

35. To finde the *Azumith of 90 degr.* which is also the *Amplitude ortive of the Ascendent, or Horoscopus.*

As the *Radius*,
is to the Sine of the *Altitude of Med. cœli.*
So is the tang. of the *Angle of the Ecliptick with the Meridian*,
to the tang. of the compl. of the *distance of that Azumith from the Meridian.*

36. To finde the *Horoscopus*, or *Ascendent degree of the Ecliptick*, Or the *Cuspis of the first house.*

The Distance of the Azumith of 90 degrees from the Meridian, is equall to the Amplitude ortive of the Ascendent degree. Wherefore the Ascendent degree of of the Ecliptic, may thence bee found, by Sect: 8, or 9. Or else thus

As the *Radius*
is to the Sine of the complement *of the angle of the Ecliptic with the Meridian:*
So is the tang. of the complement of the *altitude of Med. Cæli,*
to the tangent of the *distance of Med. cœli from the Ascendent degree.*

37. To find the *parts of the angle of the Ecliptic with the Meridian, cut with an arch perpendiculer to the Circle of any of the Howses.* Say

As the *Radius*
is to the Sine of the compl. of the *altitude of Med. cœli:*
so is the tangent of the *circle of any Howse with the Meridian,*
to the tang: of the compl: of the *part of that angle, which is next the Meridian*

Then subduct that part found out of the whole Angle for the remaining or latter part.

38. To find the *Distance of the cuspis of any howse, from Med: cæli.* Say

As the Sine of the compl. of the *later part of the angle of the Ecliptic with the Meridian,*
is to the Sine of the compl: of the *former part of that angle:*
So is the tang. of the *altitude of Med: cœli,*
to the tang: of the *distance of the cuspis of that Howse sought for.*

39. To find the *Altitude of the Pole aboue any of the circles of the Howses.*

First find out the Angle which the circle of the Howse proposed maketh with the Meridian, by Sect: 23: And then say.

As the *Radius*
is to the Sine of the *angle of the circle of the Howse with the Meridian:*
So is the Sine of the *height of the Pole aboue the Horizon of the place,*
to the Sine of the *height of the Pole aboue that circle of position.*

40. The *longitude*, and *latitude of any fixed Starre* being given, to find out the *Right ascension*, and *Declination* thereof.

The angle which the Circle of the Sunnes longitude maketh with the Meridian, at the Pole of the Ecliptic, I call the *Angle of longitude.*

And the angle which the Circle of the Sunnes Right ascension, maketh with the Meridian at the Pole of the world, I call the *Angle of right Ascension.* The condition and quantitie of which two angles, is thus found out.

In

In *Starres of the Northerne latitude*

If the longitude be in the I quadrant of the Ecliptic: subduct it out of 90: the remaines will bee the *angle of longitude*, acute. And the Angle of Right ascension, being found, must be added vnto 270.

If the longitude bee in the II quadrant: subduct 90 out of it: the remaines will bee the angle of *longitude*, acute. And the Angle of right ascension being found must be taken out of 270.

If the longitude bee in the III quadrant: subduct 90 out of it, the remaines will be the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be taken out of 270.

If the longitude bee in the IIII quadrant, subduct it out of 90+360: the remaines will bee the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be added vnto 270.

In *Starres of the Southerne latitude*

If the longitude be in the I quadrant, subduct 270 out of it + 360: the remaines will be the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be taken out of 90.

If the longitude bee in the II *Quadrant*, subduct it out of 270: the remaines will bee the *angle of longitude*, obtuse. And the Angle of right ascension being found, must bee added to 90.

If the longitude be in the *III* quadrant: subduct it out of 270; the remaines will be the *angle of longitude*, acute. And the Angle of right ascension being found, must bee added to 90.

If the longitude be in the *IIII* quadrant: Subduct 270 out of it: the remaines wil be the *angle of longitude*, acute. And the Angle of right ascension being found, must bee subducted out of 90 * 360.

Then say,

As the *Radius*, or totall Sine,
is to the Sine of the complement, or excesse of
the *angle of longitude*:
So is the tang. of the compl. of the *latitude*,
to the tang. of the *first base*.

If the angle of longitude bee obtuse; vnto the first base found, adde the greatest declination deg. $23\frac{1}{2}$: and the summe shall be the *second base*; and the angle of right ascension shall be acute.

But if the angle of longitude be acute; out of the *first base* subduct the greatest declination: and the remaines shall be the *second base*. And the angle of right ascension shall be obtuse.

Or else out of the greatest declination of the Sun, subduct the *first base*; and the remaines shall be the *second base*: and the angle of right ascension shall be acute.

Say againe,

As the Sine of the *second base*,
is to the Sine of the *first base*:
So is the tang. of the *angle of longitude*,
to the tang. of the *angle of right ascension*.

VVhence

VVhence by adding or ſubducting as was before deliuered in the conditions of thoſe angles, ſhall be giuen the Right aſcenſion of that Starre ſought for.

Laſtly ſay

As the tang. of the *ſecond baſe*,
is to the *Radius*:
So is the Sine of the compl. or exceſſe of the angle *of right aſcenſion*
to the tang. of *Declination.*

VVhere note that if the *ſecond baſe* exceede 90 degr: the declination found, ſhall not be of the ſame kinde, that the latitude is, but in the contrary Hemiſphære.

A Table of the Right Aſcenſions, and Declinations, of 40 of the cheifeſt fixed Starres, Calculated for the yeare of our Lord. 1650.

Names of Starres	Right Aſcenſion		Declination			mag.
The Polar Starre	7°	47′	80°	27′	N	2
Andromedaes girdle	12	32	33	48	N	2
the former horne of ♈	23	38	17	37	N	4
the bright ſtarre in the head of ♈	26	56	21	48	N	3
the iawe of the Whale	41	03	2	42	N	2
Meduſaes head	41	27	39	35	N	3
the eye of the Bull	64	00	15	46	N	1
the Goat ſtarre	72	44	45	35	N	1
the former ſhoulder of Orion	76	38	4	59	N	2
the latter ſhoulder of Orion	84	07	7	18	N	2
the great dogge ſtarre	97	27	16	13	S	1
the higher head of ♊	108	01	32	35	N	2
the leſſer Dogge ſtarre	110	17	6	06	N	2
the lower head of ♊	111	00	28	49	N	2
the Cribb, or Manger	125	4	20	52	N	*Neb.*
the heart of Hydra	137	39	7	10	S	2
the heart of the Lion	147	27	13	39	N	1
In the loynes of the Lion	163	54	22	26	N	2
In the tayle of the Lion	172	49	16	32	N	1
In the girdle of Virgo	189	32	5	20	N	3

Names of Starres	R. Ascen.		Declination			mag.
Aliot	189°	36	57	53	N	2
Vindemiatrix	191	15	15	51	N	3
Spica Virginis	196	44	9	17	S	1
Arcturus	209	56	21	34	N	1
the Southerne ballance	217	56	14	32	S	2
the Northerne ballance	224	31	8	2	S	2
the bright ſtar in the ſerpents neck	231	49	7	35	N	3
the heart of Scorpius	242	4	25	34	S	1
the head of Hercules	254	40	14	51	N	3
the head of Ophiucus	259	41	12	52	N	3
bright ſtarre in the Harp	276	17	38	30	N	1
bright ſtarre in the Vultur	293	27	8	1	N	2
vpper horne of ♑	299	30	3	32	S	3
the left hand of ♒	307	10	10	43	S	4
the left ſhoulder of ♒	318	18	7	2	S	3
the mouth of Pegaſus	321	49	8	18	N	3
the right ſhoulder of ♒	326	59	1	58	S	3
Fomahant	339	29	31	23	S	1
In the vpper wing of Pegaſus	341	53	13	21	N	2
In the tip of the wing of Pegaſus	358	52	13	15	N	2

41. The *longitude*, and *latitude of any two Starres* being giuen, to finde their *distances*.

If the Starres haue both the same longitude, differing onely in latitude; the difference of latitude, is the distance of the starres.

And if they differ onely in longitude hauing the same latitude; Say

As the *Radius*,
is to the Sine of *halfe the difference of longitude:*
So is the Sine of the compl. of the latitude giuen,
to the Sine of *halfe the distance of the Starres.*

But if they differ both in longitude, and latitude, whether the latitudes be both of the same kinde, or one Northerne, and the other Southerne. Take the difference of both the starres, from the pole of the Ecliptick, toward which the starre hauing the greater latitude is. And say,

As the tang. of the compl. of the *lesser distance from the pole,*
is to the *Radius:*
So is the Sine of the compl. or *excesse of the difference of longitudes,*
to the tang. *of the first base.*

Take this *first base* out of the greater distance from the pole, and the remaines shal be the *second base*, Then say,

As the Sine of the compl. of the *first base*,
is to the Sine of the compl. of the *second base:*
So is the Sine of the compl. of the *lesser distance from the pole*,
to the Sine of the compl. of the *distance of the two Starres.*

If

If any man will take paines to calculate (by this last Rule) the distances of some noted starrs of the first, second, and third magnitudes, round about the heauens, which are not aboue 5, or 6 degrees, at the most, one from the other: and shall keepe them written in his booke: they may serue as a Rule, or Instrument, whereby he may reasonably estimate with his eye, the distance of any Planet, or Comet, or other apparition from a knowen fixed starre, not very farre remote: by comparing the distance which hee would know with some of those knowen distances which he shall find, either to be equall, or else to haue some proportion thereto.

42. The *longitude*, and *latitude of any two Cities* being giuen, to find their *distance*.

The manner of the operation is the very same with the former, vnto which therefore I referre the Reader: onely I will note, that in the heauens, the longitude and latitude is taken in respect of the Ecliptic, which being the way of the Sunne, all the starrs in their proper motion, haue reference vnto it, as vnto their measure and rule. But in the Earth the principall Circle is the Equinoctiall, diuiding it into the Northerne, and Southerne hemisphæres. And therefore in the earth, the longitude, and latitude is reckoned by the Æquinoctiall.

The distance of two places vpon the Earth, being found in degrees, may bee conuerted into English miles, by taking 60 miles for euery degree, and one mile for euery minute.

43. To find *at what hower a fixed starre commeth into the Meridian any day.*

Seeke the Right ascension of the Sunne, for that day, by Sect: 5; and subduct it out of the Right ascension of the Starre. And reduce the degrees remaining into howers, by Sect: 1. The same shall shew how long time from the Noone before, the same starre shall come into the Meridian.

Wherefore if at any time of the night, a Starr whose Right ascension is knowne, be in the Meridian, the hower of the night is easily found.

44. The *height of any knowne Starre* aboue the Horizon, being by any means given, to find the *hower of the night.*

First seeke out the hower of that starrs comming into the Meridian the same day, by Sect. 43. Againe seeke out the horary distance of that starr from the Meridian, according to Sect: 19. And then if the starr bee on the East side, not yet come to the Meridian, take the difference of those two numbers; but if the starre bee past the Meridian, take the Summe of them, for the houre of the night.

45. The *height of the Pole* being given to find the *comming of any fixed Starre, in the due East, or West.* Say

As the *Radius*
is to the tang: of the *starres declination:*
So is the tang: of the *compl: of the Pole,*
to the Sine of the compl. of the *Starres horary distance from the Meridian.*

46. The *height of the Pole* being giuen, to find the *Altitude of any fixed starr a-boue the Horizon, being due East or West.* Say,

As

As the Sine of the *height of the Pole,*
is to the *Radius*:
so is the Sine of the *Starrs declination,*
to the Sine of the *Altitude, at due East or West.*

47. By the *Altitudes of any two knowne fixed starrs taken when they are both in the same Azimith, to find the height of the Pole.*

First say,

As the Sine of the *difference of the starrs altitudes,*
is to the Sine of the *difference of their Right ascensions*;
so is the sine of the *neerer starrs distance from the apparent Pole,*
to the Sine of *an angle to be kept.*

Againe compare the furthest starrs distance from the Pole with the distance from the *Zenith,* and say

As the *Radius*
is to the Sine of the compl. of the *Angle kept*:
so is the tang: of the *lesser of the compared arches,*
to the tang: of the *first base.*

Subduct the *first base* out of the greater of the two compared arches; and the remaines shall bee the *second base.*

Then lastly say,

As the Sine of the complement *of the first base,*
is to the Sine of the compl. *of the second base*:
so is the Sine of the compl. of the *lesser of the two compared arches,*
to the Sine of the *height of the Pole.*

48. To find out the *horizontall Parallax of the Moone.*

First the distance of the Moone from the Center of the earth must be knowne in Semidiameters of the earth: which vnto them that are acquainted with the Theorie of the Planets, is not very difficult. And whereof peraduenture, I may hereafter teach the practise, by most easie and exact instruments, which I haue long since framed.

Say,

As the *distance of the Moone, from the center of the earth,*
is to the *Semidiameter of the earth:*
So is the *Radius,*
to the Sine of the *Moones horizontall parallax in that distance.*

49. The *horizontall Parallax of the Moone* being given, to find *her Parallax in any apparent altitude.*

As the *Radius,*
is to the Sine of the *altitude of the Moone:*
so is the Sine of the *horizontall Parallax,*
to the Sine of the *Parallax in that altitude.*

50. The *place of the Moone in the Ecliptic* hauing little or no latitude (as in the Eclips of the Sunne) together with *her Parallax of altitude* being giuen, to find the *Parallaxes of her longitude,* and *latitude.*

If the Moone bee in the 90th degree of the Ecliptic: shee hath no Parallax of longitude, and the Parallax of latitude, is the very Parallax in that altitude.

But

But if the Moone be not in the 90^{th} deg. say,

As the *Radius*
is to the tang. of the *angle of the Ecliptick with the horizon*:
So is the Sine of the compl. of the *distance of the Moone, from the Ascendent, or descendent degree of the Ecliptick*,
to the tang of the compl. of the *angle of the Ecliptick with the Azumith of the Moone.*

Againe say,

As the *Radius*
is to the Sine of that *angle*:
So is the *Parallax of the Moones altitude*,
to the *Parallax of her latitude.*

Lastly say,

As the *Radius*
is to the Sine of the compl. of that *former angle*:
So is the *Parallax of the Moones altitude*,
to the *Parallax of her longitude.*

which is to bee added to the true motion of the Moone, if she be on the East part of the 90^{th} degree of the Ecliptick: or to be subducted out of it, if she be on the West part.

Many other *Astronomicall* and *Geographicall problemes* might be added. But because it is impossible to set downe all, which may be of vse, at some time or other: I haue in the next Chapter deliuered briefly the *doctrine of triangles* fitted vnto practise: with all the seuerall cases belonging thereto.

CHAP. XIII.

Of Trigonometria, *or the manner of calculating both* Plaine, *and* Sphæricall triangles. *And first concerning certaine generall notions, and rules necessary thereto.*

IN euery triangle both Plaine, and Sphærical, the greater side subtendeth the greater angle. And the greater angle hath the greater side opposite vnto it. Also the greater angle lyeth to the lesser side, and the greater side hath the lesser angle lying vnto it.

In euery plaine triangle, any two angles being giuen, the third is also giuen: and one of the angles being giuen, the summe of both the other two is giuen. For all the three angles together, are equall to two right angles, that is to 180 degrees.

In a plaine rectangle triangle, one of the acute angles is the complement of the other. Where note that when the complement is named without any other addition, it is meant of the arch, which is wanting of a quadrant of that circle, or 90 degrees. In like manner the excesse is meant of the arch, which is aboue a quadrant. But when it is said the complement to a semicircle, it is vnderstood of so many degrees as will make vp 180.

But in a Sphæricall rectangle triangle, one of the oblique angles is alwayes greater then the compl: of the other.

If two arches together make vp a Semicircle, the excesse of the greater arch, is equall to the complement of the lesser.

The

The ſame Right Sine, and the ſame Tangent, and Secant, doth belong both to the arch it ſelfe, and alſo to the complement of it to a Semicircle. But their verſed Sines differ: For the verſed Sine of an arch leſſe then a quadrant, is equall to the difference of the *Radius*, and the Sine of the complement of that arch: and the verſed Sine of an arch greater then a quadrant, is equall to the ſumme of them. And the verſed Sine is thus found out, As the *Radius* is to the Sine of halfe the arch; ſo is the Sine of that halfe arch, to halfe the verſed Sine of the whole arch.

In a right angled triangle both Plaine, and Sphæricall, one of the ſides containing the right angles, is called the Baſe, and the other the Cathetus: and the ſide ſubtending the right angle is the Hypotenuſa. And know that euery rectangled triangle, is moſt fitly noted with the letters *ABC*; ſo that *BA* may be the Baſe, and *CA* the Cathetus, and *BC* the Hypotenuſa: and *B* the angle at the baſe, and *C* the angle at the Cathetus, and *A* the right angle. Likewiſe euery oblique-angled triangle with the letters *BCD*; ſo that out of the angle *C* a perpendicular *CA*, being let downe, it may in the baſe *BD* diſtinguiſh the two caſes *BA* and *DA*, which are the baſes of the two particular triangles into which it is cut. And in noting the triangles with letters, obſerue diligently, that if any angle be giuen together with one of the ſides including it, the ſame angle be noted with *B*; and the ſide with *BC*.

If both the angles at the Baſe *BD* be acute, the perpendicular *CA* ſhal fall within the triangle: And $BD = BA + DA$ that is *BD* is equall to the ſumme of *BA* and *DA*. And if the angle *B* be obtuſe, the perpendicular *CA* ſhall fall without the triangle, beyond the obtuſe angle *B*: And $BD = DA - BA$, that is *BD* is equall to the exceſſe of *DA* aboue *BA*: or if the angle *B*. be obtuſe, the perpendicular *CA*: ſhall fall without the triangle beyond the

obtuſe

obtuse angle D: And $BD = BA - DA$, that is BD is equall to the excesse of BA aboue DA. The lesser case being still taken from the greater angle.

And note that this signe $\times$, or *pl* (that is *plus*) sheweth that the magnitude before which it is set, is affirmed and positiue in nature; and therefore to bee added. And that this signe — or *mi* (that is *minus*) sheweth the magnitude before which it is placed, is denied, and priuatiue in nature; and therefore to be substracted, as you may see in those former examples.

Againe, some magnitudes are taken seuerally and apart: as $s\ BA$, that is the Sine of the Base; $s\ co\ BC$, that is the Sine of the complement of the Hypotenusa; $t\ B$, that is the tangent of the angle ABC at the base; $t\ co\ C$, or $t\ co\ ABC$, the tangent of the complement of the angle at the Cathetus: So also $\sqrt{q}Z$, that is the quadrat side of the plaine Z. And some magnitudes are taken vniuersally, and then they are included in pricks: as $s : \frac{DC + BD - BC}{2} :$ that is the Sine of halfe the arch, which is composed of the summe of the two arches DC, and BD, abating thereout the arch BC. So also $\sqrt{q} : Z \times X :$ that is the quadrat side of the two plaines Z and X put together: also $\sqrt{q} : Q$ in R: Or $\sqrt{q} : Q \times R$: that is the quadrat side of a rectangular plaine, the two sides whereof are the lines Q and R, or some fourth proportionall already found, and the *Radius*, or Semidiameter, which is the totall Sine. For by the signe in, or $\times$, I vse to expresse multiplication.

When any triangle is giuen to be resolued by Trigonometrie, note the parts thereof (either sides or angles) which are giuen and knowne, with a little line drawne crosse each such part: and note the vnknowne part which is sought for with a little circle.

And

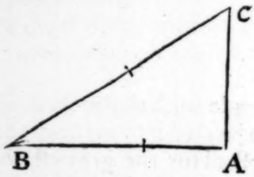
And if a triangle Sphæricall (bee it right angled or oblique-angled) proposed hath two sides each of them seuerally greater then a quadrant: you shall in resoluing thereof, keepe the least side with the least angle opposed to it: and for the two other both sides, and angles, take the complements of them to a semicircle.

Lastly, if a triangle with all the three angles giuen, be required to be conuerted into a triangle hauing the three sides giuen. You shall for the greatest angle of the triangle proposed, & for the greatest side subtending it, take the complements to a semicircle; keeping the other two lesser angles, with their subtendent sides as they are.

O THE

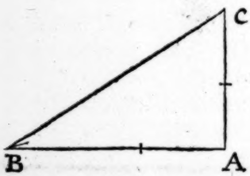
THE CALCVLATION OF PLAINE *right-angled-triangles.*

I.



BC . BA :: R . *s co* B (*s* C) .

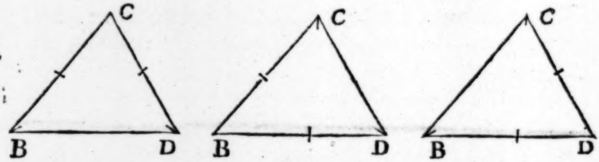
II.



BA . CA :: R . *t* B (*t co* C) :

Of plaine Oblique-angled triangles.

III.

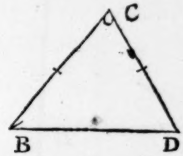


s B . DC :: *s* D . BC :: *s* C . BD .

and here it is necessary to be knowne, whether the angle sought for be greater, or less then a right angle, or 90 deg.

First

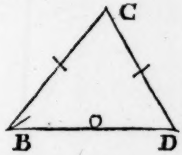
IIII.



Firſt ſeek the angle D, by the III; then both the angles B and D being ſubducted out of 180, you ſhall haue

$$180 - B - D = C .$$

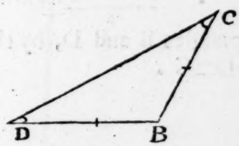
V.



Firſt ſeeke the angle D, by the III; then both the angles B and D being ſubducted out of 180, Say

$$s\,B \,.\, DC :: s : 180 - B - D \,.\, BD .$$

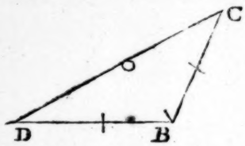
VI.



Let the ſide BD be greater then the ſide BC:
First, $BD + BC \,.\, BD - BC :: t\,\frac{180-B}{2} \,.\, t\,Q$.
then for the other two angles:
$\frac{180-B}{2} + Q =$ the greater. $\frac{180-B}{2} - Q =$ the leſſer.

Let

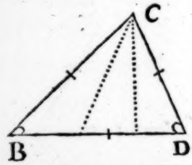
VII.



Let the side BD be greater then the side BC:

First, the angles C and D are to be sought, by the VI. and the side DC, by the III.

VIII.



Take the greatest side BD for the base: and let the side BC, be greater then the side DC. First say,

$BD . BC+DC :: BC-DC . Q$ (viz $BD-2DA$).

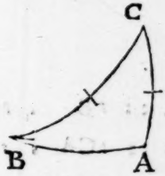
then $\frac{BD+Q}{2} = BA$. $\frac{BD-Q}{2} = DA$.

Nextly seeke the angles B and D, by the III.

Lastly $180-B-D=C$.

THE

THE CALCVLATION OF Sphæricall right-angled, and quadrantall triangles.

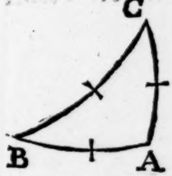


I.

R . *s*B :: *s*BC . *s*CA .

II.

R . *s*B :: *sco*BA . *sco*CA .

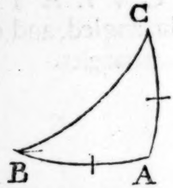


III.

R . *sco*BA :: *sco*CA . *sco*BC .

 R . *s*BA

IIII.

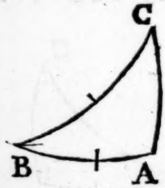


R . sBA :: tB . tCA .

R . tcoB :: tCA . sBA .

R . sBA :: tcoCA . tcoB .

V.



R . scoB :: tBC . tBA .

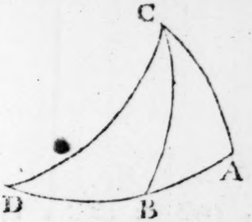
R . tcoBC :: tBA . scoB .

R . scoB :: tcoBA . tcoBC .

VI.

R . scoBC :: tB . tcoC .

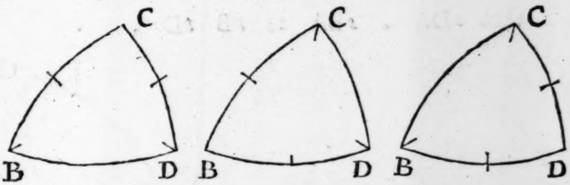
If



VII.

If a triangle BCD be quadrantall, hauing one ſide BC equall to a quadrant; vpon the pole D deſcribe an arch of a great circle CA, cutting the ſide DB extended in A: and ſo making a right-angled triangle ABC without the other. This outward right-angled triangle ſhall be reſolued in ſteed of the quadrantall propoſed.

Of Sphæricall Oblique-angled triangles.



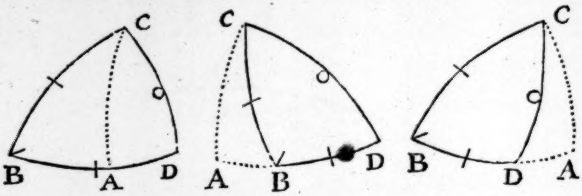
VIII.

sB . sDC :: sD . sBC :: sC . sBD .

and in theſe it is neceſſary to bee knowne whether the terme ſought for be greater then a quadrant, or not. The ſame alſo is to be knowne in the tenne rules next following, if the ſides BC and DC are both giuen.

First,

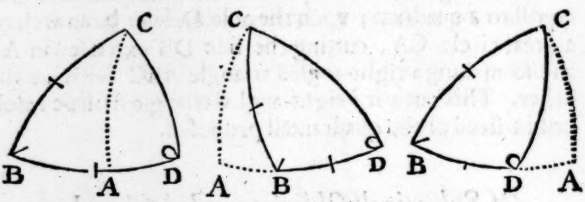
IX.



First, R . *sco*B :: *s*BC . *s*BA .

then, *sco*BA . *sco*DA :: *sco*BC . *sco*DC .

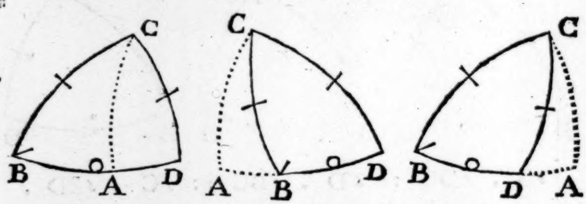
X.



First, R . *sco*B :: *s*BC . *s*BA .

then *s*DA . *s*BA :: *s*B *s*D .

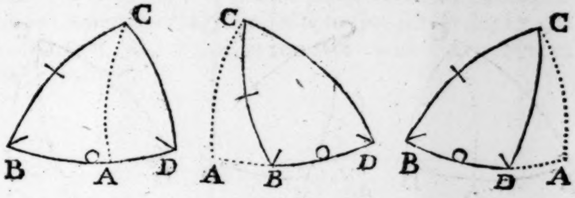
XI.



First, R . *sco*B :: *s*BC . *s*BA

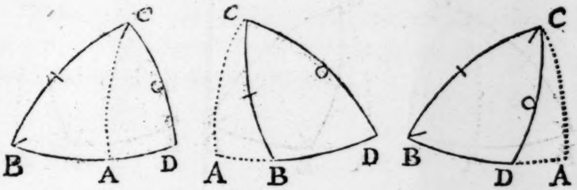
then, *sco*BC . *sco*DC :: *sco*BA . *sco*DA .

First,



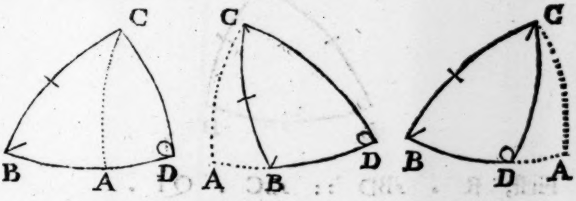
XII.

First, R . *sco*B :: *t*BC . *t*BA .
then, *t*D . *t*B :: *s*BA . *s*DA .



XIII.

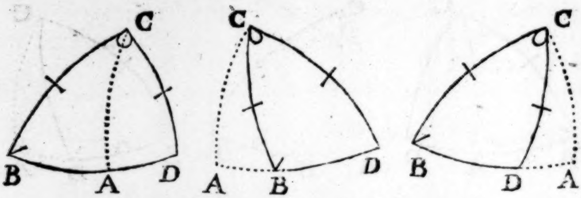
First, R . *sco*BC :: *t*B . *tco*BCA .
then, *sco*DCA . *sco*BCA :: *t*BC . *t*DC .



XIIII.

First, R . *sco*BC :: *t*B . *tco*BCA .
then, *s*BCA . *s*DCA :: *sco*B . *sco*D .

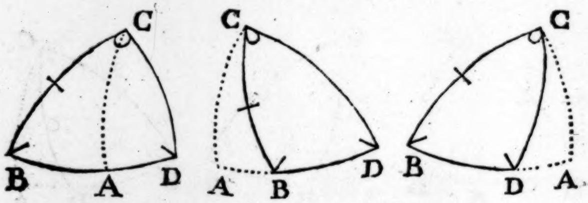
XV.



First, R . *sco*BC :: *t*B . *tco*BCA .

then, *t*DC . *t*BC :: *sco*BCA . *sco*DCA .

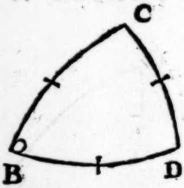
XVI.



First, R . *sco*BC :: *t*B . *tco*BCA .

then, *sco*B . *sco*D :: *s*BCA . *s*DCA .

XVII.

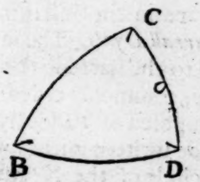


First, R . *s*BD :: *s*BC . QI :

then, QI . $s{:}\,\frac{DC+BD-BC}{2}$ :: $s{:}\,\frac{DC+BC-BD}{2}$. QII :

See what QII cutteth in the fift circle, which is of equall diuisions: and thereto adde the *Radius*, by setting 1 before

1 before that number. Diuide the whole into two equall parts: and reckoning one halfe in that fift circle, set the Index to it, and it shall in the first circle cut the Sine of halfe the angle B.



If all the three angles be giuen: conuert the triangle into another hauing all the three sides giuen: and resolue the same for the triangle proposed. XVIII.

P 2 THE

Chap. XIIII.

Of the Nocturnall Dials.

THere are in the Instrument, two seuerall *Nocturnall Dyals.* The innermost of them is fitted to the starre in the rump of the great Beare, commonly called *Aliot.* The other is composed of 12 seuerall starres: whose names you shall finde written within, neere to the center.

The outermost circle of the Nocturnall Dyall is diuided into twise 12 houres: each houre being subdiuided into quarters, and are noted with figures belonging to the houres, as may be seene in the Instrument.

The middlemost circle of the Nocturnall is diuided into 12 moneths, hauing their names written: each moneth being distinguished into tenth dayes with longer lines; and into fift dayes with shorter lines. And if the Instrument be large enough, each day of the monethes throughout the yeare, may be noted with pricks.

In the innermost circle are the diuisions and names of 12 fixed starres: which are these.

The bright star in the head of ♈.	Spica Virginis
the Bulls eye	the North ballance
the latter shoulder of Orion	the head of Ophiuchus
the little dogge	the heart of the Vultur
the heart of the Lyon	the mouth of Pegasus
the tayle of the Lyon	the tip of the wing of Pegasus.

To finde out the houre of the night by Aliot.

Seeke the day of the moneth in the annuall circle of the Nocturnall: and apply the Index thereto: marke what houre it cutteth, in the houre circle. Remember this

houre

houre for all that day: then at night when you would finde out the houre, hold vp your Instrument by the handle, and moue it vp and downe, till you see the pole starre through the middle hole, and the starre called *Aliot* by the limbe: Set the Index or labell to *Aliot*, and marke what houre the labell cutteth, for if vnto this houre you adde the houre kept in minde for that day, the summe of both shall giue you the true houre of the night: so that you cast out 12 houres, from the said summe if it shall chance to be more.

Example. If on the 15 th of Nouember you would find the houre of the night by the starre *Aliot.* Apply the index to the day of the moneth, and it will cut in the houre circle 8 and an halfe: then suppose the Index being set to the starre *Aliot*, as aboue taught, doth cut in the houre circle 10. these two numbers being added together, the summe will be $18\frac{1}{2}$, out of which subduct 12; and the remaines $6\frac{1}{2}$, will be the true houre of the night.

To finde out the houre of the night by the Inner Nocturnall Dyall.

To performe this it is necessary that you know the true Meridian of the place wherein you are, and can finde it out by night, which you may thus doe. Hauing a Meridian line drawne in some window, or other conuenient place (as is shewed in the Second part of this booke, *Vse* 19) stick vp therein a long needle perpendicularly, and watch till the Sunne casteth the shadow of the needle, vpon the Meridian line. Or else in a true Sunne Dyall obserue when the shadow falls iust on 12 a clock, for then is the Sunne in the Meridian. Wherefore goe instantly into some place about your house where you may see some marke, either a chimney, or the corner of an house, or else some tree, or such like, directly betweene you and the Sunne: then haue you the true Meridian.

Or otherwise you may in a cleare night goe into some plaine place neere your house, and setting vp a straight pole perpendicularly on the ground, goe a good distance from it Southwards; and then moue vp and downe, till you see the top of your pole, directly betweene your eye, and the North polar starre: then set vp another pole perpendicularly betweene your feet, so that both your poles, and the Polar starre, may be in one right line. And then going backe againe to your first pole, looke what knowne starre is directly ouer your last pole, for that starre is in the Meridian. You may therefore instantly goe to some conuenient place, and take a marke whereby you may at all times know the Meridian as is afore taught.

When therefore at any time of the night you would know what a clocke it is, goe to that place where you stood, and looking directly ouer your marke, see if any of the 12 fixed starres, bee in the Meridian; or if none of them be therein, obserue which two of them are on either side thereof, and what part of that space is in the Meridian. Then goe into the light, and take your instrument, and set the Index to that starre, or point which you saw in the Meridian: marke what houre it cutteth, for that same houre being added to the houre, which the day of the moneth sheweth, shall giue you the true houre of the night: so that you cast out 12 houres, from the said summe, if it shall chance to be more.

Example. Suppose the fifth of December, that the middle point of the space betweene the bright starre in the head of *Aries*, and the *Bulls eye*, bee in the Meridian. Set the Index to the middle point of the space betweene those two starres in the Instrument: and it will cut in the houre circle 2 and an halfe: then againe set the Index to the fifth of December, and in the houre circle it wil cut 7: which

which added vnto 2 and an halfe, giueth 9 and an halfe, for the true houre of the night.

Another example. Suppose the 19th of December, that one third part of the ſpace betweene the *Bulls eye*, and the *right ſhoulder of Orion*, be in the Meridian. Set the Index to one third part of that ſpace in the Inſtrument, and it will cut in the houre circle 4 and halfe a quarter almoſt: againe, ſet the Index to the 19th of December, and in the houre circle it will cut 6, which being added vnto 4, and halfe a quarter almoſt, giueth 10 and almoſt halfe a quarter for the houre of the night.

THE

THE SECOND PART OF THIS BOOKE.

Shewing the vse of the Second side *of the Instrument, for the working of most questions, which may be performed by the* Globe: *And the* declination of Dyals, *vpon any kinde of Plaine.*

VPon the *second side* of the Instrument, is delineated the *proiection of the vpper Hemisphere vpon the plaine of the Horizon*: The *Horizon* it selfe is vnderstood to bee the innermost circle of the limbe: and is diuided on both sides, from the points of *East*, and *West* into degrees, noted with 10, 20,30, &c. vnto 90. And the center of the Instrument is the *Zenith*, or *Verticall point*.

Within the *Horizon*, the middle straight line, or Diameter pointing North and South, is the *Meridian*, or 12 a clock line: and the other short arching lines, on both sides of it are the *houre lines*, distinguished accordingly by their figures.

figures. Theſe houre lines ſhould indeede bee drawne through the whole plaine, croſſing one another in the Pole of the world: but that the Inſtrument may be more faire, they are onely drawne ſhort.

And becauſe diuers excellent vſes, doe require the totall delineation of the houre circles, I haue in a ſeuerall paper, inſcribed intirely, both the houre lines, and alſo two other circles betweene them, containing euery one fiue degrees. (But if the Inſtrument were large enough to receiue them, it were beſt if euery degree had his circle: and ſo euery 15 circle ſhould bee an houre line.) And of the parallels, there needes no more, but the Æquinoctial, and both the Tropics.

For as much as there will be great vſe of this paper Inſtrument; I haue in the 24 Vſe ſhewed the manner of making it: ſo that any that is ingenuous, and ready handed may himſelfe delineate one ſufficient enough to ſerue his turne, for any eleuation.

The two arches which croſſe the houre lines meeting on both ſides in the points of interſection of the ſixe a clocke lines with the Horizon, are the *two Semicircles of the Ecliptick*, or Annuall circle of the Sunne: the vpper of which arches ſerueth for the *Summer halfe yeare*, and the lower for the *Winter halfe yeare*: and are therefore diuided in 365 dayes: which are alſo diſtinguiſhed into 12 moneths with longer lines, hauing their names ſet downe: and into tenthes, and fifthes with ſhorter lines: and the reſt of the dayes with pricks: as may plainly bee ſeene in the Inſtrument.

And this is for the ready finding out of the *place of the Sunne* euery day: and alſo for ſhewing of the *Sunnes yearely motion*: becauſe by this motion the Sunne goeth round about the heauens in the compaſſe of a yeare, making

ing the *foure parts*, or *seasons* thereof. Namely the *Spring*, in that quarter of the Ecliptick which beginneth at the intersection on the West side of the Instrument, and is therefore called the *Vernall intersection*. Then the *Summer* in that quarter of the Ecliptick which beginneth with the intersection of the Meridian in the highest point next the Zenith. And after that *Autumne* in that quarter of the Ecliptick, which beginneth at the intersection on the East side of the Instrument, and is therefore called the *Autumnall intersection*. And lastly, the *Winter* in that quarter of the Ecliptick, which beginneth at the intersection with the Meridian, in the lowest point next the Horizon.

But besides this yearely motion, the Sunne hath a *Diurnall* or *dayly motion*, whereby it maketh day and night with all the diuersities, and inequalities thereof: which is expressed by those other circles drawne crosse the houre lines: the middlemost whereof being grosser then the rest, meeting with the Ecliptick in the points of the *Vernall*, and *Autumnall intersections*, is the *Æquinoctiall*: and the rest on both sides of it, are called the *Parallels*, or *Diurnall arches of the Sunne*: the two outermost whereof are the *Tropics*, because in them the Sunne hath his furthest *digression*, or *Declination* from the Æquinoctiall, which is degrees 23½: and thence beginneth againe to returne to the Æquinoctiall. The vpper of the two Tropics next the center (in this our Northerne Hemisphære) is the *Tropick of Cancer*: and the Sunne being in it, is highest into the North, making the longest day of Summer. And the lower next the Horizon, is called the *Tropick of Capricorne*; and the Sunne being in it, is lowest into the South, making the shortest day of Winter.

Betweene the two Tropics, and the Æquinoctiall infinite such *parallel circles* are vnderstoode to be contained: for the Sunne is what point soeuer of the Ecliptick it is caried, describeth by his lation, a circle parallel to the Æquinoctial. Yet those parallels which are in the Instrument, though drawne but to euery second degree of Declination, may be sufficient to direct the eye, in imagining and tracing out, through euery day of the whole yeare in the Ecliptick, a proper circle, which may be the *Diurnall arch of the Sunne* for that day. For vpon the right estimation of that imaginary parallel, doth the manifold vse of this Instrument especially rely: because the true place of the Sunne, all that day, is in some part or point of that circle. Wherefore for the better conceiuing, and bearing in minde thereof, euery fift parallel, is herein made a little grosser then the rest.

I Vse. *And thus by the eye, and view only, to behold and comprehend* the course of the Sunne, *both for his* Annuall *and* Diurnall motion, *may be the first vse of this Instrument.*

II Vse. *To take the* height of the Sun *aboue the Horizon.*

Set vp the pinne, (which is therefore made fit for the hole at the center) perpendicular in the center: and put the Indices on both sides, downe vpon the Meridian, that they with their waight, may not sway the Instrument any way as it hangeth: then with a threed put into the hole aboue in the handle, hang it perpendicularly, bearing the edge toward the Sunne, that the pinne may cast a shadow, vpon the degrees in the limbe: for that degree which the shadow of the pinne cutteth in the limbe, is the height of the Sunne aboue the Horizon, at that present.

III Vse.

III. Vse. *To find the* Declination of the Sunne *euery day.*

Looke the day of the moneth proposed in the Ecliptic, and marke how many degrees the prick shewing that day, is distant from the Equinoctiall, either on the Summer, or Winter side, *viz.* North, or South.

Example. I. What will the Declination of the Sunne be, vpon the 11th day of *August*? Looke the 11th day of *August*: and you shall find it in the sixt Circle aboue the Æquinoctiall: now because each Parallel, standeth (as hath beene said before) for 2 degrees, the Sunne shall that day decline North-wards 12 degrees.

Example. II. What Declination hath the Sunne, vpon the 24th day of *March*? Looke the 24th day of *March*, and you shall find it, betweene the second, and third Northerne parallels, as it were an halfe and one fift part more of that distance from the second: reckon therefore 4 degrees for the two Circles, and one degree for the halfe space: so shall the Sunnes declination bee 5 degrees, and about one fift part of a degree Northward, that same day.

Example. III. What Declination hath the Sunne vpon the 13th day of *Nouember*? Looke the 13th day of *Nouember*, and you shall find it below the Equinoctiall, tenne parallels and about one quarter, which is 20 degrees, and an halfe South-wards. So much is the Declination. And according to these examples iudge of all the rest.

IIII. Vse. *To find the* Right ascension of the Sunne *euery day.*

Imagine an hower line through the day of the moneth giuen, and marke in what point it will crosse the

Æquinoctiall: then lay a Ruler, or a streight Scroule of paper, to the Pole of the world (noted in the Instrument with *P W*) and that same point. For the Ruler shall in the innermost Circle of the limbe, of the South side, cut the Right ascension of the Sunne for that day, to be reckoned from the West, to the point of intersection, for the first, or vpper Semicircle of the Ecliptic: or from the East together with 180, for the second, or lower Semicircle of the Ecliptic.

V. Vse. *To find the* longitude of the Sunne, *or in what* degree of the Signe *he is euery day.*

The Pole of the first Semicircle of the Ecliptic is noted *P I.* and the Pole of the second Semicircle is noted *P I I.* Lay a Ruler, or a streight Scroule of paper, to the day of the moneth, and the proper Pole of the Semicircle of the Ecliptic, in which it is: for the Ruler shall in the innermost Circle of the limbe, on the South side, cut the degree of the Sunnes place in the Ecliptic, reckoning it in the same manner as you did in finding the Sunnes Right ascension: and the Arch thus found is called the longitude of the Sunne. which may bee expansed into signes, by reckoning on the limbe, from the West to South ♈, ♉, ♊, and from South to East ♋, ♌, ♍: then backe againe from East to South ♎, ♏, ♐; and lastly from South to West ♑ ♒, ♓, allowing 30 degrees, for each of those twelue signes.

VI. Vse. *To find the* Diurnall Arch, *or* Circle of the Sunnes course *euery day.*

The Sunne euery day by his motion (as hath beene said) describeth a Circle parallell to the Æquinoctiall, which is either one of the Circles in the Instrument, or some

ſome-where betweene two of them. Firſt then ſeeke the day of the moneth; and if it fall vpon one of thoſe Parallels, that is the Circle of the Sunnes courſe that ſame day: But if it fall betweene any two of thoſe Parallels, imagine in your minde, and eſtimate with your eye, another Parallel through that point, betweene thoſe two Parallels, keeping ſtill the ſame diſtance from each of them.

As in the firſt of the three former Examples, The circle of the Sunnes courſe, vpon the 11th day of *August*, ſhall be the very ſixt Parallel aboue the Æquinoctiall towards the Center.

In Example II. The Circle of the Sunnes courſe vpon the 24th day of *March*, ſhall bee an imaginary Circle betweene the ſecond, and third Parallels, ſtill keeping an halfe of that ſpace, and one fift part more of the reſt from the ſecond.

In Example III. The Circle of the Sunnes courſe vpon the 13th day of *Nouember*, ſhall be an imaginary Circle, betweene the tenth, and eleauenth Parallels, below the Æquinoctiall, ſtill keeping one quarter of that ſpace from the tenth.

VII. Vſe. *To find the* Riſing *and* Setting of the Sunne *euery day*.

Seeke out (as was laſt ſhewed) the imaginary Circle or Parallel of the Sunnes courſe, for that day, and marke the point where it meeteth with the Horizon, both on the Eaſt and Weſt ſides thereof, for that is the very point of the Sunnes riſing, and ſetting that ſame day, and the hower lines which are on both ſides of it, by proportioning the diſtance reaſonably, according to 15 minuts, for the quarter of the hower, will ſhew the hower of the Sunnes riſing, on the Eaſt ſide, and the Sunnes ſetting on the Weſt ſide.

VIII. Vſe

VIII. Vse. *To know the reason, and manner, of the* Increasing, *and* Decreasing of the dayes and nights *throughout the whole yeare.*

When the Sunne is in the Æquinoctiall, it riseth, and setteth at *6* a Clock, for in the instrument, the intersection of the Æquinoctiall, and the Ecliptic with the Horizon, is in the *6* a clock Circle on both sides. But if the Sunne bee out of the Æquinoctiall, declining toward the North, the intersections of the Parallel of the Sunne with the Horizon, is before *6* in the Morning, and after *6* in the Euening: and the diurnall Arch of the Sunne, greater then 12 howers; and so much more great, the greater the Northerne Declination is. Againe if the Sun be declining toward the South, the intersections of the Parallel of the Sunne with the Horizon, is after *6* in the Morning, and before *6* in the Euening; and the diurnall Arch lesser then 12 howers; and by so much lesser, the greater the Southerne Declination is.

And in those places of the Ecliptic in which the Sun most speedily changeth his Declination, the length also of the day is most altered, and where the Ecliptic goeth most parallel to the Æquinoctiall changing the Declination but little, the length of the day also is but little altered.

As for example, when the Sunne is neere vnto the Æquinoctiall, on both sides, the dayes increase, and also decrease suddenly and apace: because in those places, the Ecliptic inclineth to the Æquinoctiall in a manner like a streight line, making sensible declination. Againe when the Sunne is neere his greatest Declination, as in the height of the Summer, and the depth of Winter, the

dayes

dayes keepe for a good time, as it were at one stay, because in those places the Ecliptic is in a manner parallel to the Æquinoctiall, scarce altering the declination: and because in those two times of the yeare, the Sunne standeth as it were still, at one declination; they are called the *Summer Solstice*, and the *Winter Solstice*. And in the meane spaces, the neerer every place is to the Æquinoctiall, the greater is the diversitie of dayes.

Wherefore we may hereby plainely see, that the common receiued opinion, that in every moneth, the dayes doe equally increase, is erroneous.

Also wee may see that in Parallels equally distant from the Æquinoctiall, the day on the one side, is equall to the night on the other side.

IX. Vse. *To find the* Ascensionall difference of the Sunne *every day.*

Seeke out the time of the Suns Rising, or Setting that same day (by the VII Vse) and see how much it differeth from sixe a clocke, then conuert the same difference into degrees (as was taught in 1 Part. Chap. 12. Sect. 1.) by multiplying the howers with their decimall parts, by 15. And so haue you the Ascensionall difference for that day.

X. Vse. *To find out the* Oblique ascension of the Sunne *every day.*

Seeke out the Sunnes Right ascension (by the IIII Vse) and the Ascensionall difference (by the IX Vse.) And if the Sunne be in the first Semicircle of the Ecliptic, Subduct the Ascensionall difference, out of the Right ascension: But if the Sunne be in the second Semicircle of the Ecliptic, adde the Ascensionall difference to the Right ascension: and you shall haue the oblique ascension.

XI. Vse. *To find how farre the Sunne riseth, and setteth, from the true East and West points, which is called the* Sunnes Amplitude ortiue, *and* occasiue.

Seeke out (as was shewed in the VI. Vse) the imaginary Circle, or Parallel of the Sunnes course, and the points of that Circle in the Horizon, on the East, and West sides, cutteth the degree of the Amplitude Ortiue, and occasiue.

XII. Vse. *To find the* length of every day and night.

Double the hower of the Sunnes setting, and you shall haue the length of the day; and double the hower of the Sunnes rising, and you shall haue the length of the night.

XIII. Vse *To find the* true place of the Sunne, vpon the Instrument, *which answereth to the point, wherein the Sunne is in the heauens: and is the ground of all the questions following.*

Take with your Instrument the height of the Sunne, and reckon it on the moueable Index, or Labell: and then moue the said Labell, till you find the height of the Sunne, exactly to fall vpon the Parallel of the Sunne for that day, on the East side if it bee in the Fore-noone, and on the West side, if it bee in the After-noone; the point of intersection, where the Index, or Labell crosseth the Parallel

rallel, in that point of the Sunnes altitude, shall bee the true place of the Sunne on the Instrument.

XIIII. Vse. *To find the* Hower of the day.

The true place of the Sunne on the Instrument (found out as was last shewed) sheweth among the hower lines the true hower of the day.

XV. Vse. *To find out the* Azumith *or* verticall Circle *in which the Sunne is, or the Horizontall distance of the Sunne from the Meridian.*

The Index or Labell fastned at the Center, is a moueable Azumith: apply therefore the edge thereof, vnto the true place of the Sunne on the instrument (found out as was shewed by XIII. Vse.) And marke what point of the Horizon, or Limbe, the same edge of the Labell cutteth; reckon how many degrees of the Horizon, are intercepted betweene that point, and the Meridian line, or South point, either on the East, or West side: and that Arch shall be the Horizontall distance sought for, whereby is shewed the Azumith of the Sunne at that instant: and consequently the Angle which the verticall Circle, or Azumith of the Sunne maketh with the Meridian.

XVI. Vse. *The* Azumith of the Sunne *being knowne, to find out the* Altitude of the Sunne, *and* the Hower of the day.

Set the edge of the Labell to the Azumith given, and marke in what point the same edge crosseth the Parallel of the Sunne for that day: that point of intersection sheweth the height of the Sunne aboue the Horizon, vpon the Labell: and also it sheweth the hower of the day, among the hower lines.

XVII. Vse. *To find at what* hower the Sunne commeth to be *full* East, *or* West *every day in Summer.*

Apply the edge of the Labell, vnto the East, or West points of the Limbe, and marke in what point, the said edge cutteth the Parallel of the Sunne for that day, for that same point among the hower lines, shall shew the time of the Sunnes comming to be full East, or West in that day, and likewise of what altitude the Sunne will be aboue the Horizon, at that time of his being full East, or West.

XVIII. Vse. *To find the* height of the Sunne at high Noone every day, *and likewise at every other hower.*

Marke in what point the Parallel of the Sunne for that day, cutteth the line of that hower, for which you would know the Sunnes altitude: And vnto that point of intersection, apply the edge of the moueable Labell, or Index: and thereon shall you find, the very degree of the Sunnes altitude, at that hower.

By this XVIII Vse, and by the XVI, are made the Quadrants, described by *Gemma Frizius*, *Munster*, *Clauius*, Mr. *Gunter*, and others: and also all manner of *Rings*, *Cylinders*, & innumerable other *topicall Instruments*, for the finding out of the hower, and other like conclusions. And likewise the reason, of finding the hower of the day, by a mans shadow, or by the shadow of any Gnomon, set vp perpendicular to the Horizon, or else parallel to it.

XIX. Vse. *To find out the* Meridian line, and the points of the compasse *without a Magneticall needle, yea more exactly then with a needle.*

Take the height of the Sunne, by the shadow of the pinne: and apply the same height, reckoned on the Index, or Labell to the parallel of the Sunne for that day, whereby you haue the true place of the Sunne, in the instrument, as hath beene shewed in the XIII. Vse. Then keeping both the Labell at that point, and the pinne vpright in the Center, hold, or set your instrument parallel to the plaine of the Horizon, with the pinne toward the Sunne, and moue it gently, till the shadow of the pinne shall fall, exactly vpon, the fiduciall edge of the Labell. For then the Meridian line of the instrument, shall be in the true Meridian of the place: and the foure quarters of the instrument, shall looke into the foure cardinall points, of East, West, North, and South. Wherefore if with a bodkin, you make a prick at each end of the Meridian of your instrument where it standeth: and with a Ruler draw a line through them: the same shall bee the Meridian of that place.

This is a most excellent practise, for finding out the Meridian in any place, and is in an instant performed, and that easily. And hereby you may examine the *Variation of the Compasse.* And also exactly place any Sunne Dyall.

XX. Vse. *Considerations for the* vse of the instrument in the night.

In such questions as concerne the night, or the time before Sun rising, and after Sunne setting, the instrument representeth the lower Hemisphære, wherin the Southerne

Pole is eleuated. And therefore the Parallels, which are aboue the Æquinoctiall, shall bee for the Southerne, or Winter Parallels, and those beneath the Æquinoctiall, for the Northerne, or Summer parallels. And the East shall be accounted for West, and the West for East: and the North shall bee accounted for South, and the South for North: contrary to that which was before, when the Instrument represented the vpper Hemisphære.

XXI. Vse. *To find* how many degrees the Sunne is vnder the Horizon, at any time of the night.

Seeke the declination of the Sunne for the day proposed: and at the same declination, on the contrary side, imagine a Parallel for the Sunne that night: and marke what point of it *is* in the very hower and minute proposed: then set the Index, or Labell to that point of the Parallel, and it will shew you thereon the degree of the Sunnes depression vnder the Horizon.

XXII. Vse. *To find out the* length of the Crepusculum, or Twilight *euery day*.

Because the question concerneth the night time, you must seeke out the Sunnes Parallel, for the night, on the other side of the Æquinoctiall, hauing the same declination with that which the day of the moneth sheweth: then moue about the Labell, vntill the said Parallel cutteth the edge thereof in the 18th deg: on the West side for the Morning Twilight, and on the East side for the Evening Twilight, of the same day.

And note that in the height of Summer, the Twilight in our Horizon, continueth all night long: because the

the same goeth not vnder the Horizon, full 18 degrees

XXIII. Vse. *To find the* Declination of any Wall, or Plaine.

Take a board hauing one streight edge, and a line drawne perpendicular vnto that edge: apply the streight edge vnto the Wall, at what time the Sunne shineth theron, holding the board parallel to the plaine of the Horizon: and hang vp a thread with a plummet, so that the shadow of the thread may fall on the board, crossing that perpendicular line. Then take with your Instrument the height of the Sunne, and instantly make two pricks, in the shadow of the thread on the board, a good way distant one from the other: and laying a Ruler to those two pricks, draw a line, which line shall be the Azumith of the Sunne, on the board: againe with the height of the Sunne lastly taken, find out on your instrument, the Azumith of the Sunne; or the Angle which the Sunnes Azumith maketh with the Meridian, (by the XV. Vse.) And on the board taking the intersection of the shadow line with the perpendicular for the Center, describe a Circle equall to the innermost Circle of the Limbe: (which you may easily doe, if you set one foot of your compasses vpon the East, or West point, and extend the other foot vnto 60 degrees, on the same innermost Circle, for this distance is equall to the *Radius* thereof.) Againe with your compasses, take of the Arch betweene the Azumith of your Instrument, and the Meridian, and set that on the Circle of the board, that way that the true South is: and through the end of that Arch measured on the board, draw a streight line for the Meridian. Lastly take with your compasses, the Arch intercepted between the Meridian on the board, and the perpendicular line, and by applying it to the inmost Circle of the limbe, from the East, or West points see how many degrees it

con-

containeth: for that is the declination of the Wall Or else you may find the Meridian vpon the board, by XIX Vse.

If the Angle of the Meridian with the perpendicular, on the board, be a right Angle, the Wall is direct East, or West.

But if the Meridian fall vpon the perpendicular, or be parallel thereto making no Angle with it, the Wall is direct North, or South.

XXIIII. Vse. *The* Art of Dyalling. *And first how to make the Instrument in paper, promised in the beginning of this second part.*

For the Delineation of this instrument in paper, it will bee necessary first to shew the manner how the Semidiameter is to bee graduated, or diuided into degrees: and how the Centers, and Semidiameters, of the seuerall kinds of Arches are to be found.

Vpon halfe a sheet of strong large Dutch paper, (the larger, the better) draw two streight lines, making a right Angle neere one of the corners, the one through the length, and the other through the breadth of the paper; which two lines I therefore call the longer, and the shorter perpendicular.

Vpon the right Angle point, being the Center, with a Semidiameter equall to that by which you intend to delineate your instrument, describe a quadrant of a Circle: and on the point where it meeteth with the shorter perpendicular, draw a long tangent line parallel to the longer pependicular.

Diuide the Quadrant into 90 degrees, among which from the beginning at the shorter perpendicular, reckon the eleuation of the Pole, for which you will make your in-

instrument, and applying a Ruler to the end thereof, and to the Center, where the Ruler cutteth the tangent line make a prick. And taking with your compasses the distance from the Center to that prick, measure it vpon the shorter perpendicular: this shall be the *Semidiameter of the sixt hower Circle.* At the end thereof draw another long line parallel also to the longer perpendicular.

Then out of the Center vnto the second parallel through every degree of the quadrant, draw fine streight lines, cutting also the first Parallel. The intersection of those lines with the first Parallel, shall be *The scale of centers of Arches.* And their intersection with the second Parallel shall be *The scale of centers of hower Circles.* And the segments of those lines, intercepted betweene the Center, and the first Parallel, shall be the *Semidiameters of Arches*: and the whole lines betweene the Center and the second Parallel, shall bee *The Semidiameters of hower Circles.* And that you may know for what Circle, every Center, and Semidiameter serueth, you shall note every fift line from the beginning, with the figures 5, 10, 15, 20, &c. Set vnder the second Parallel, vnto 90 which will fall vpon the longer perpendicular: that so you may readily find the Center, and Semidiameter of any Circle required.

Againe divide the first 45 degrees of the Quadrant in the middest: and applying your Ruler to the Center, and to every one of those halfe divisions, where in each place the Ruler cutteth the first Parallel, or tangent line, make a prick. So shall you haue vpon the tangent line betweene the shorter perpendicular and the midlemost line 45, a third scale, which is, *The scale of 90 degrees*, for the graduating of the Semidiameter of your instrument on the paper: In which you shall also distinguish every fift degree, with figures set vnder the tangent line.

Having thus prepared your paper of scales with lines neatly and exactly drawne, keepe it by you to haue it still in a readinesse for the making, and vsing of the Instrument in paper. The making whereof is thus.

Take with your compasses the Semidiameter of the Quadrant in your paper of scales: and therewith vpon a peice of strong Dutch paper, Describe the Horizontall Circle: which you shall cut into two Semicircles with a Meridian line drawne through the Center: divide them into Quadrants in the points of East, and West: and each Quadrant into 90 degrees to be marked with figures iust as is done in the Instrument.

Then with your compasses take the eleuation of the pole vpon the scale of degrees in your paper: & set it vpon the Meridian line from the Center which way you please: that shall bee the intersection of the Æquinoctiall with the Meridian. Also reckon the complement of the height of the Pole, vpon the scale of Centers of Arches, and with your compasses take the distance from the end thereof to the Center: the same shall bee the Semidiameter of the Æquinoctiall, to bee drawne from the East point of the Horizon through the point of intersection with the Meridian vnto the West point.

Againe take with your compasses vpon the scale of degrees in your paper the complement of the height of the Pole: and set it vpon the Meridian on the other side of the Center from the Æquinoctiall: there shall bee the Pole of the Æquinoctiall, or of the World, in which all the houre lines shall crosse one another.

Nextly vnto the height of the Pole both adde, and also subduct 23½ degrees: and with your compasses take both those Arches in the scale of degrees: and set them in the Meridian from the Center, one falling beyond the Æquinoctiall, and the other short of it: those shall be the intersections of the two Tropics with the Meridian. Seek also

also by Sect: 8, Chapt: 12 of the first part, the Amplitude ortiue of the Sun, having 23½ degrees declination: and reckon it being found, in the Horizontall Circle from the East and West points both wayes: those are the points of intersection of the Tropics with the Horizon on both sides: and so having three points in each Tropic, you may easily through them drawe the Circles.

Moreover with your compasses, take the distance betweene the Center and the second Parallel in your paper, which is the Semidiameter of the sixt houre Circle: and set it on the Meridian from the Pole, beyond the Æquinoctiall: that shall be the Center of the sixt houre Circle: vpon which you may draw the same Circle, from the East point of the Horizon through the Pole to the West point. Then through the center of the sixt houre Circle erect a line perpendicular to the Meridian, extending it infinitly on both sides of the Meridian: and in that line both wayes, pricke downe the Centers of the horary Circles, out of the scale in paper: And lastly opening your compasses from every one of those Centers vnto the Pole severally, describe all the horary Circles, or at least every fift of them, and so is your paper instrument perfectly finished.

The vse of this instrument on paper is, that lines, and arches may bee designed vpon it with a fine pennicell of blacke lead, and afterward be wiped out againe. Wherefore it will bee needfull for him that will vse this instrument, to all the purposes thereof, to get a good paire of large compasses with three points, one sharpe, another for Inke, a third for blacke Lead. And I suppose it would doe well to fasten over your instrument a peice of thinne oyled paper, through which the lineaments may be conspicuous: and vpon it to trace such lines, and arches as you haue occasion to vse: that so your instrument may be kept cleane, and last longer.

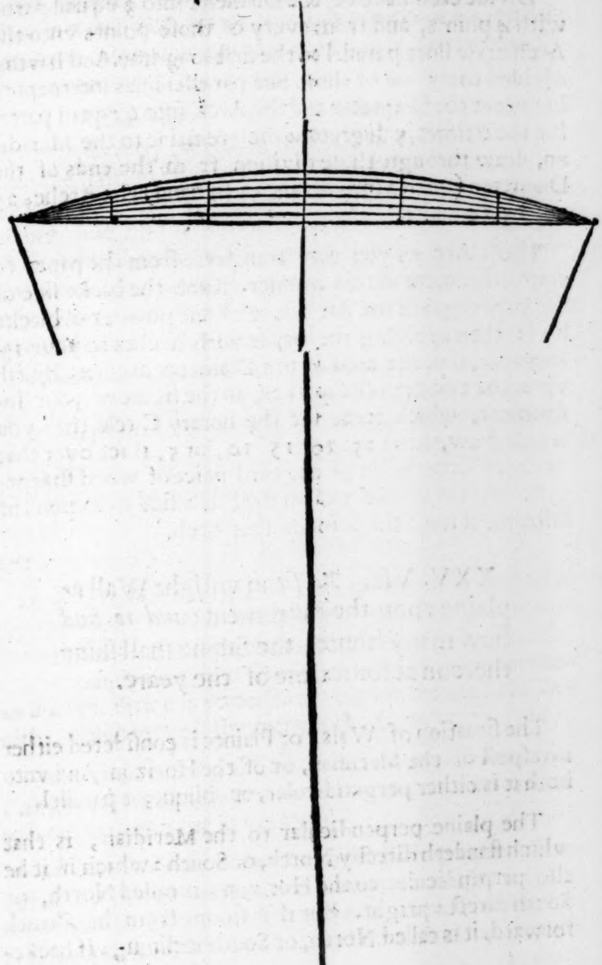
For as much as in delineating the horary Circles, which are within 30 degrees of the Meridian, the Semidiameters will be too long for your compasses: you may in that streight thus helpe your selfe. First say,

> As the *Radius*,
> is to the Sine of the *Elevation of the Pole*;
> So is the tang: of the *distance of any Horary Circle from the Meridian*, suppose 25, or 20, or 15, or 10, or 5 degrees,
> to the *arch of the Horizon betweene the Meridian, and that horary Circle*.

Reckon this distance on the limbe of your Instrument from both ends of the Meridian, and marke it. Thus doe for the 25th, 20th, 15th, 10th, and 5th horary Circle on both sides of each end of the Meridian.

Then in any peice of cleane paper, through the middest of the longer way, draw a line: and toward one end which (I call the upper ende) crosse the same with a perpendicular line exactly equall to the Diameter of your Instrument, the point of Intersection being the center.

Take with your Compasses out of the paper of scales, the semidiameter of 60. degr: (which you may well doe for an ordinary instrument): and setting one foot on either end of the Diameter, that point wherein the other foot shall cutt the first long line, make your Center, and thereon draw an Arch through both ends of the Diameter, and cutting the vpper part of the first long line: this Arch is equall to that horary Circle, which is distant from the Meridian 30 degrees the complement of 60.



Divide each halfe of the Diameter into 3 equall parts, with 4 points, and from every of those points vnto the Arch draw lines parallel to the first long line. And having divided euery one of those fiue parallel lines intercepted betweene the Diameter and the Arch into 6 equall parts, for the 6 times 5 degrees which remaine to the Meridian, draw through those divisions from the ends of the Diameter (with a smooth and even hand) the Arches 25 20, 15, 10 and 5.

Those Arches you may transferre from the paper to your instrument in this manner. Rubb the backe side of the paper against the Arches, with fine powder of blacke lead: then applying the paper with Arches to your instrument, that the ends of the Diameter may exactly fall vpon the two opposite markes, in the limbe of your instrument, which serue for the horary Circle that you would draw, either 25, 20 15, 10, or 5, trace over that Arch with the point of any hard peice of wood sharpened: and the blacke lead on the backe side will vpon the instrument leaue the print of that Arch.

XXV. Vse. *To set* an vpright Wall *or* plaine vpon the instrument: *and to find* how many houres the Sunne shall shine thereon at some time of the yeare.

The situation of Wals, or Plaines is considered either in respect of the Meridian, or of the Horizon. And vnto both it is either perpendicular, or oblique, or parallel.

The plaine perpendicular to the Meridian, is that which standeth directly North, or South: which if it be also perpendicular to the Horizon, is called North, or South direct vpright. But if it stoope from the *Zenith* forward, it is called North, or South inclining: If backe-ward

ward, it is called North or South reclining. And note that in a ſtooping Plaine that ſide which is toward the Horizon, is inclining, and that which is toward the Zenith is reclining.

The *Plaine oblique to the Meridian* is that which ſtandeth not directly North, or South, but declineth one ſide into the Eaſt, and the other into the Weſt: and is therefore called *Declining Eaſtward, or Weſtward,* according as either ſide of the Plaine looketh: As if an vpright Wall being Southerne, declineth from the South into the Eaſt, it is called *South declining Eaſtwards vpright.* But if it be not vpright, it is called *South declining Eaſtward,* and *inclining, or reclining.*

The *Plaine parallel to the Meridian,* is that which looketh directly Eaſt, or Weſt; and accordingly, hath his denomination, whether it bee *Vpright, Inclining, or Reclining.*

The *Plaine Parallel to the Horizon,* is called Horizontall: and is repreſented by the inſtrument it ſelfe, or at leaſt by the innermoſt Circle of the limbe thereof.

And note that the Arch of Declination, is reckoned from the next Eaſt, or Weſt point. And that the Arch of Inclination, or Reclination is reckoned from the Zenith, or the complement of it from the Horizon. So that euery vpright Plaine is vnderſtood to paſſe through the Zenith, which in the inſtrument is the Center.

And thus having ſhewed the ſeuerall affections of Plaines, wee will now proceed to ſhew the manner how to ſet them vpon the Inſtrument.

A Direct North, or South vpright Plaine, is repreſented in the Inſtrument by a line drawne through the Center from the Eaſt point to the Weſt, which is alſo the Horizontall interſection of the Plaine. And by it you ſhall

ſhall ſee that the Southerne ſide or face of the plaine is open to all the houres betweene ſixe in the morning and ſixe in the evening. And that about *London*, the Northerne ſide, onely in the Summer enioyeth the Sunne from his riſing till after ſeven in the morning: and from before 5 a clocke in the afternoone, till his ſetting.

A direct East, or West upright plaine, is repreſented in the Inſtrument by the Meridian, which is alſo the Horizontall interſection of the plaine. And in it you ſhall ſee that all the forenoone houres are open to the Eaſt ſide: and all the afternoone houres to the Weſt ſide.

A Declining Plaine is thus ſet upon the Inſtrument, reckon on the Horizon the arch of Declination, from the Eaſt, or Weſt point: and at the end draw a line through the Center vnto the oppoſite point of the Horizon: So that each ſide thereof may be open to that point, either Eaſt, or Weſt, into which the Declination is ſuppoſed. That line ſo drawne throught the center is the Horizontall interſection of the plaine, and repreſenteth the plaine it ſelfe, if it bee vpright. For example, there is about *London* an vpright Wall declining Eaſtwards 35 degrees: which I would ſet vpon the Inſtrument. Hold the Southerne part of the Inſtrument to you, and reckon from the Eaſt backward into the North vpon the Horizon 35 degrees: there draw a line through the Center: this line ſhall not onely vpon the South ſide repreſent a Southerne Plaine declining Eaſtward 35 degrees. But alſo vpon the North ſide ſhall repreſent a Northerne Plaine declining Weſtward 35 deg. And moreover it will appeare that on the Southerne ſide ſhall bee drawne the houres from almoſt 4 a clocke in the morning, till 3 in the afternoone. And that on the Northerne ſide ſhall bee drawne vpon one ſide 4 a clocke in the morning onely: and vpon the other ſide all the houres from 3 in the afternoone till Sunne ſet.

And so consequently the declination of an vpright wall, or Window being given, it may be found at what houre the Sunne vpon any day in the yeare will come to that Wall, or Window, and when it will goe from it. As in the former example, There is about *London* a Northerne wall declining Westward 35 deg. I would know at what time of the day the Sunne will begin to shine vpon it on the 24th day of *March.* Set the Index at 35 deg: from West toward South: and because that day the Sunnes Declination is 6 degrees Northward; Looke at what houre the sixt Parallel aboue the Æquinoctiall toward the Center meeteth with the Index so placed: and you shall find it at 3 a clock in the Afternoone. Wherefore at that time the Sunne will begin to shine vpon the Wall that same day.

The Poles of every vpright Wall are in the Horizon 90 deg: that is a quarter of a Circle, distant from the line representing the Plaine. Wherefore if vpon that line in the Center you erect a perpendicular, the ends therof in the Horizon shall be the poles of that Plaine: and are so farre distant from the North and South points, as the Plaine it selfe is from the East, and West.

XXVI. Vse. *To set an* Inclining, *and* Reclining Wall, *or* Plaine *vpon the Instrument: and to find* how many houres the Sunne shall shine thereon, *at some time of the yeare.*

When you haue *an Inclining,* or *Reclining Plaine* to be described on the Instrument. First the Horizontall intersection is to be set thereon, as if it were vpright; together with the line perpendicular thereto, in which are the Poles of the Plaine: according as was taught in the XXV Vse.

Then vpon the ſcale of degrees in your paper, reckon the arch of Inclination, or Reclination; and with your compaſſes take, & ſet it in your Inſtrument vpon the line perpendicular to the Horizontall interſection of your Plaine, from the Center that way into which the Inclination, or Reclination tendeth: the ſame ſhall bee the vppermoſt point of your Plaine.

Againe, with your Compaſſes take the Complement of inclination, or reclination, both upon the ſcale of degrees, and alſo upon the ſcale of centers of arches in your paper: and ſet both ſpaces upon the ſame perpendicular line, but on the other ſide of the center (extended if need be): At the ſhorter of thoſe ſpaces ſhal be the pole of your plaine: and at the longer of them ſhal be the Center of it.

Laſtly, ſetting one foot of your Compaſſes in the center of your Plaine, and extending the other foot to the vppermoſt point, deſcribe in your Inſtrument an Arch of a Circle: which if you haue done well, will exactly fall vpon the ends of the Horizontal interſection of your Plaine. That Arch ſhall repreſent your Plaine, inclining vpon the lower ſide, which is toward the Horizon, or Limbe: but reclining vpon the vpper-ſide, which is toward the Zenith, or Center. And ſo either ſide ſhall ſhew in what hower lines the Sunne, at ſome time of the yeare, will ſhine vpon it: that in delineating a Dyall thereon, it may not be combered with vnneceſſary houre lines. For Example, ſuppoſe that the former Plaine, which with the South declined Eaſtward 35 deg: doe alſo incline 41 deg: 30 min. Wherefore alſo with the North ſide it ſhall decline Weſtward 35 deg: and Recline 41 deg: 30 min. Deſcribe this plaine vpon your Inſtrument with an Arch of a Circle, found out as was taught laſt before. And it will appeare that vpon the Inclining ſide ſhall be drawne all the houre lines from almoſt 4 in the morning, to 4 in the afternoone: And vpon the Reclining ſide ſhall bee drawne firſt 4 and 5 in the morning: and then beginning at 8 a clocke, all houres to Sunne ſet.

And

And note that in all Northerne plaines, the North Pole is eleuated: and in all Southerne plaines the South Pole is eleuated. Except such North inclining, and South reclining Plaines, that in the Instrument fall below the North Pole, betweene it and the Horizon: For in them the contrary Pole is eleuated. And also that a direct East and West plaine, if it Recline, hath the North Pole eleuated: and if it Incline, the South Pole.

XXVII. Vse: *The Plaine being set vpon the Instrument, to find the* distances of the houre lines, *and* Substile from the Horizontall Intersection. *And also the* height of the Stile aboue the Substile.

Euery Dyall either hath a Center in which all the houre lines, together with the Substile, and Stile doe meet: or else it hath no center, & so they are al parallel one to another.

If the Plaine being set vpon the Instrument, cutteth the Pole of the Æquinoctiall (that is the point in which all the horary Circles crosse the Meridian) the Dyal to be drawne vpon that Plaine shall haue no Center. But if it cutteth not the Pole, the Dyal shall haue a Center. And of these Dyals with Centers wee will first intreate: as being most proper for the vse of the Instrument.

Behold therefore the Pole of your plaine heedily what horary Circle it falleth vpon: Or if it fall betweene any two, the distance of each being reasonably apportioned, imagine a horary Circle passing through it. Marke in what point that horary Circle, either real, or imagined, doth cut the Plaine, that same point shall bee the place of the Substile in your Plaine: and the height of the Stile aboue it, is the Arch of that horary Circle intercepted betwixt the Pole of the Æquinoctial, and the point of the Substile noted in the Plaine.

Therefore applying a Ruler to the Pole of your Plaine, carry it about vnto all the intersections of the Plaine with the houre Circles, and the substile seuerally: and where in euery place the Ruler shall cut the innermost Circle of the Limbe, there make a visible marke: For the arches of the Limbe intercepted betweene the Horizontall points of your Plaine, and euery one of those marks, shall be the distance intended to be sought.

But for the Horizontall plaine, the ends of the houre Circles in the Limbe of your instrument, doe giue the distance without any more adoe.

Concerning the height of the Stile aboue the Substile: It is apparent by the instrument, that in a Horizontal dyal, the *6* a clocke line lyeth directly East and West: and the Meridian perpendicular to it, directly North and Scuth. And that the Meridian is the Substile. And that the height of the Stile aboue the Substile, is equall to the height of the Pole in that place.

It is also apparent, that in all direct North and South Dyals, the *6* a clocke Line is drawne parallel to the Horizon, and the Meridian perpendicular to it: And that the Meridian is the substile. And that the height of the Stile aboue the substile, if the Plaine bee vpright, is equall to the complement of the height of the Pole. But if it be North inclining, or South Reclining, it is equall to the difference, of the height of the Pole, and the Arch of Inclination, or Reclination. And if the Plaine bee North Reclining, or South Inclining, it is equall to the Summe of the complement of the height of the Pole and of the Arch of Inclination, or Reclination. And if the Plaine fall vpon the Æquinoctiall, the stile shall stand vp perpendicular vpon it in the Center: and the hower lines shall be drawne all at equall Angles, *viz.* 15 degrees one from another.

In

In ſuch Dyals as haue not the Meridian for the ſubſtile, the height of the Stile aboue the Subſtile is thus found by the Inſtrument. It was ſhewed before that the height of the ſtile aboue the ſubſtile is the Arch of the horary Circle through the Pole of the Plaine, intercepted betweene the pole of the Æquinoctiall, and the point noted in the Plaine for the ſubſtile. Therefore from the Horizontall points, or interſections of that horary Circle reckon 90 degrees both wayes: and thereto through the Center draw an obſcure line: in which line ſhall be both the Inclination of that horary Circle, which is the diſtance of the interſection, or vppermoſt point thereof from the Center: and alſo the Pole. Then with your compaſſes take that diſtance, or Inclination: and ſetting it vpon the ſcale of degrees in your paper, ſee how many degrees it containeth vpon that ſcale. Againe vpon the ſame ſcale take with your compaſſes the complement of that inclination or diſtance, which being ſet vpon the obſcure line on the other ſide of the Center, ſhall ſhew the Pole of that horary Circle. Laſtly applying a Ruler to the Pole of that horary Circle, and both to the Pole of the Æquinoctiall, and to the point of the ſubſtile in the Plaine ſeuerally: marke where in both places the Ruler cutteth the innermoſt limbe of your inſtrument: For the degrees of the limbe intercepted betwixt thoſe markes, ſhall be the height of the ſtile, aboue the ſubſtile, which was ſought for.

And by this which hath beene taught, you ſhall find that in an vpright Dyall declining, as before 35 deg: from the South into the Eaſt, or from the North into the Weſt, the ſubſtile ſhall fall vpon that horary Circle, which is about 3 deg: after 9 a clock in the morning: and the ſtile eleuated aboue the ſubſtile about 31 deg: And alſo that in a South Dyall declining Eaſtward 35 deg: and inclining 41 deg: 30 min: Or in a North declining Weſtward 35 deg: and reclining 41 deg: 30 min: the ſubſtile ſhall fall vpon that horary Circle which is about 8 degrees after *6* a clock: and the ſtile eleuated aboue the ſubſtile about *63* deg. 30 minutes.

XXVIII. Vſe. *The making of all manner of plaine* Dyals with Centers.

I haue already ſhewed how to find out in our Inſtrument the diſtances of all the houre lines, and the ſubſtile, from the Horizontal interſection. Now the delineating of a Dyal is nothing elſe but to transferre thoſe diſtances out of your Inſtrument into the Dyal plaine, euery one in his due ſituation: and then through them, out of the Center, to draw ſuch houre lines as ſhall be of vſe, together with the ſubſtile.

The due ſituation of thoſe diſtances vpon the Dyal plaine, dependeth on the true placing of the Meridian, or 12 a clocke line: for that being truely deſcribed, all the reſt will be eaſie enough.

Firſt therefore I will ſhew the manner how the Meridian, or 12 a clocke line, is to be deſcribed.

Take in your Dyal ſome point for the Center, where you ſhall thinke fit: and through it draw a line parallel to the plaine of the Horizon. Croſſe it in the Center with a perpendicular line. And hauing opened your compaſſes to the length of the Semidiameter of your paper Inſtrument, deſcribe on the Center a Circle equall to the innermoſt Limbe thereof. In which Circle the line parallel to the Horizon is for the Horizontal interſection: and the other for the line perpendicular to it: and the Circle it ſelfe repreſenteth the plaine: Marke therein the Eaſt and Weſt ſides of the Plaine with *E* and *W*.

In the Horizontal, and in al North and South direct Plaines, both vpright, and ſtooping; and in all vpright

dec-

declining plaines, the Meridian is perpendicular to the Line parallel to the Horizon.

In North inclining, and South reclining plaines, the Meridian is to bee drawne on that side of the Dyal plaine either East, or West, into which the declination is: But in North inclining and South reclining, on the contrary side. And if the plaine bee Northerne, the Meridian shall be aboue the Line parallel to the Horizon: and if the plaine be Southerne, it shal be vnder it. And if the contrary Pole be eleuated, it shall be drawne through the Center into the opposite Quadrant of the Circle in your Dyal plaine.

Lastly in a direct East and West plaine, both inclining and reclining, the Meridian is the same with the line parallel to the Horizon.

Wherefore with your compasses take the distance in the limbe of your Instrument, from the next Horizontall point, vnto the marke of the Meridian; and measure it vpon the Circle of the Dyal plaine, in that part, and on that side, according as in consideration of the eleuated Pole, and of the qualitie of the Plaine, was shewed to be agreeable. And at the end of that arch, through the Center, draw a line for the Meridian.

Againe with your compasses take the distances in the limbe of your Instrument, betweene the marke of the Meridian, and the markes of all the houre Lines seuerally: and setting them vpon the Circle of the Dyal plaine orderly from the Meridian, the Forenoone houres on the West side of it, and the Afternoone houres on the East side: at the end of euery one of those arches draw the houre Lines: and distinguish them with their proper figures accordingly.

Lastly fasten the stile in the Center, so that it may hang perpendicular vnto the plaine in the Substile, at the iust height. And because the stile in every Dyal is vnderstood to be a segment of the Axis of the world which

which is a line imagined to passe from the North to the South Pole through the Center of the earth; the stile being rightly placed shal still with the end point towards the eleuated Pole, that is vpward from the Center, if the North Pole be eleuated; or downeward from the Center if the South Pole be eleuated.

XXIX. Vse. *The making of all manner of plaine* Dyals not hauing Centers.

If the plaine represented on the Instrument (as was taught before in the XXV and XXVI Vses) cut the Pole of the Æquinoctiall, it is an horary Circle, either one of them which are drawne in the Instrument, or falling betweene some two of them: and the Dyall plaine it selfe shall not crosse the axis of the world, but lye parallel to it, without any Angle of eleuation. And therefore such a Dyal can haue no Center: But the stile, the substile, and all the houre lines shall be parallel one to another.

Every such Plaine represented on the Instrument,

Either, *First* it is the Meridian of the place, the Horizontall intersection whereof is the 12 a clocke Line drawne from North to South: and the Dyall made thereon, is a direct East, or West vpright Dyal: In which the substile is distant from the Line, in the Circle of the Dyall plaine parallel to the Horizon, with an Arch equall to the eleuation of the Pole, and vpward toward the Pole. And is also the 6 a clocke line in your Dyal.

The rest of the houre lines are thus described. Draw through the substile, in any point, a long Line at right Angles: that line shall bee the Æquinoctial intersection vsually called the *Contingent line:* And taking a conuenient distance for the stile to hang parallel over the substile (according to the greatnesse of your Dyall plaine) measure

meaſure it vpon the ſubſtile from the Æquinoctiall interſection: and vpon the end of that meaſure, deſcribe halfe a Circle for the Æquinoctiall it ſelfe. Diuide each Quadrant thereof from the ſubſtile, into 6 equall parts, or houres. Then applying a Ruler to the Center, and to euery one of thoſe diuiſions ſeuerally, where in euery place the Ruler ſhall cut the long line of Æquinoctiall interſection, make pricks: and through thoſe pricks draw the houre lines, parallel to the Subſtile, or 6 a clocke line: diſtinguiſhing ſo many of them as bee needfull, with their figures: that is all the Forenoone houres on the Eaſt plaine, and all the Afternoone houres on the Weſt plaine. But in theſe Dyals there is no 12 a clock line, it being infinitly diſtant from the Subſtile. Laſtly hang the ſtile directly over the Subſtile, and parallel to it, at the diſtance formerly taken. And thus are your Eaſt, and Weſt Dyals finiſhed.

Or *Secondly*, it is the ſixt houre Circle, the Horizontal interſection whereof is the line of Eaſt, and Weſt: and the Dyal made thereon is direct North inclining, or South reclining, with an Arch equal to the complement of the height of the Pole. And the parallel to the Horizon is the Æquinoctial interſection: and the line perpendicular to it is the 12 a clocke line, and alſo the Subſtile.

The reſt of the houre Lines, from 7 a clocke in the morning, to 5 in the euening, are thus deſcribed. Take a conuenient diſtance for the Stile from the Subſtile, meaſuring it vpon the Subſtile from the Æquinoctial interſection: and on the end of that ſpace deſcribe the Semicircle of the Æquinoctial, to bee diuided on both ſides of the Subſtile into 6 houres: through euery one of which out of the Center, a Ruler being applyed; at the points of the ſeuerall interſections of the Ruler with the Æquinoctial interſection, draw the houre Lines parallel to the Subſtile, or 12 a clocke Line: diſtinguiſhing them with their figures, namely 11, 10, 9, 8, 7, on the Weſt ſide:

and 1, 2, 3, 4, 5, on the East side: but in these Dyals there is no sixe a clocke Line, it being infinitly distant from the Substile. Lastly hang the Stile directly over the Substile, and parallel to it, at the distance formerly taken.

Or *Thirdly*, it is North inclining, or South reclining, and also declining: in which.

As the tangent of the *Elevation of the Pole*,
is to the *Radius*;
So is the Sine of the compl: of *Declination*,
to the tang: of the compl: of *Inclination* or *Reclination*.

The Plaine being set vpon the Instrument by the Arches of Declination and stooping thereof (as hath beene taught in XXVIV se) shall cut the pole of the Æquinoctiall. Apply therefore a Ruler to the Pole of the plaine, and to the Pole of the Æquinoctial; and the point in which it cutteth the Limbe, marke for the substile: which is to bee transferred vnto the Circle of the Dyal plaine, by taking the distance betweene that point, and the next Horizontal intersection, and setting it on that Circle from the line parallel to the Horizon, vpward if the plaine be North: or downe-ward if the plaine bee South: and on that side which is contrary to the Declination. The substile being thus found, draw a long line perpendicular to it, for the Æquinoctial intersection. And taking a conuenient distance for the stile from the substile, measure it vpon the substile from the Æquinoctiall intersection: and on the end of that space describe the Semicircle of the Æquinoctial. Then looke in your Instrument how many degrees of the Æquinoctial are intercepted betweene the Meridian, and the Arch representing your Plaine: and reckoning the same number of degrees vpon the Æquinoctial of the Dyal plaine, from the substile towards the side of Declination, there make a marke for the Meridian point thereof: in which

you

you must begin to diuide the Æquinoctial semicircle into houres both wayes: And that being diuided, apply a ruler to the center, and to euery one of the diuisions: and at the points of the seuerall intersections of the ruler with the Æquinoctial intersection, draw the houre lines parallel to the substile. Set 12 at that houre line which was drawne at the intersection through the Meridian point of the Equinoctial: and 11, 10, 9, 8, &c. on the West side: and 1, 2, 3, 4, &c, on the East side. Lastly, hang the stile directly ouer the substile, and parallel to it, at the distance formerly taken.

XXX Vse. *How by* Sines and tangents *to calculate the* places of the Meridian, *and* Substile, *and the* height of the Stile *aboue it: and the* distance of the Meridian of the Æquinoctiall from the Substile; *together with the* places of houre lines, *both by calculation, and also Geometrically.*

I haue already taught the making of all manner of plaine Dyals most easily by the Instrument, for the same height of the Pole. But if any man either want an Instrument, or else desireth greater exactnesse, I will also here shew how to performe the same by calculation, on the other side of the Instrument.

In a plaine erect Dyall declining.

As the *Radius*
is to the Sine of *declination*;
So is the tang. of the compl. of the *Poles height.*
to the tang. of the *distance of the*
substile from the Meridian.

Againe

As the *Radius*

is to the Sine of the compl. *of Declination*;

So is the Sine of the compl. of the Poles *height*,

to the Sine of the *height of the stile*

aboue the substile.

Thirdly

As the Sine of the *Poles height*

is to the *Radius*;

So is the tang. of *Declination*,

to the tang. of the *Distance of the Meridian of the Æquinoctiall from the substile.* And this distance is euer lesse then 90 degrees.

In a plaine East and West Inclining *and* Reclining Dyall.

As the *Radius*

is to the Sine of the compl. of { *inclination* / *reclination*

So is the tang. of the *height of the Pole*,

to the tang. of the *distance of the substile from the Meridian.*

Againe

As the *Radius*

is to the Sine of { *inclination* / *reclination*

So is the Sine of the *height of the Pole*,

to the Sine of the *height of the stile aboue the substile.*

Thirdly,

As the Sine of the compl. of the *Poles height*,

is to the *Radius*;

So is the tang. of the compl. of { *inclination* / *reclination*

to the tang. of the *distance of the Meridian of the Æquinoctiall from the substile.* And this distance is euer greater then 90 degrees.

In-

In plaine Dyalls *both* declining *and also* inclining, *and* reclining.

As the *Radius*
is to the Sine of $\left\{ \begin{array}{l} \textit{inclination} \\ \textit{reclination} \end{array} \right.$
So is the tang: of *Declination*,
to the tang. of the compl. of *the distance of the Meridian, from the line parallel to the horizon.*

Againe
As the *Radius*
is to the Sine of the compl. of *Declination*;
So is the tang. of the compl. of the *Poles height*,
to the tang. of *Base I*.

If the Dyall be South reclining, or North inclining, the *summe of Base I*, and of the complement of Inclination or reclination shall be *Base II*. But if the Plaine be South inclining, or North reclining, the *difference of Base I*, and of the complement of inclination, or reclination shall bee *Base II*.

Then say thirdly,
As the Sine of the compl. *of Base I*,
is to the Sine of the *compl. or excesse of Base II*;
So is the Sine of the *height of the Pole*,
to the Sine of the *height of the stile above the substile.*

Fourthly,
As the Sine of the compl. of *the height of the stile above the substile*,
is to the Sine of *Declination*;
So is the Sine of the compl. of the *height of the Pole*,
to the Sine of the compl. of the *distance of the substile from the line parallel to the horizon.*

Fiftly

As the Sine of the compl. of the *height of the stile aboue the substile,*
is to the Sine of *Declination ;*
So is the Sine of the compl. of { *inclination* / *reclination*
to the Sine of the *distance of the Meridian of the Æquinoctiall from the substile.*

And note that in South reclining, and North inclining Plaines, if *Base II* be lesse then a quadrant, the contrary pole is eleuated aboue the Plaine: And if *Base II* be equall to a quadrant, the Plaine doth cut the Pole of the Æquinoctiall.

Now concerning the placing of the substile vpon the Dyall plaine (as I haue already in the *XXVIII Vse* shewed for the Meridian) Wee are to know, First that the substile is to be drawne vpward from the line parallel to the horizon, if the Plaine be Northerne; or downeward from it, if it bee Southerne. Except in North reclining, and South inclining Dyalls, in which the *Base I* exceedeth the complement of inclination, and reclination: for in them it is quite contrary. And secondly that the substile is to be drawne in the contrary side from the Declination. But in North inclining, and South reclining Dyalls, in which the contrary Pole is eleuated, the substile must be drawne through the center into the opposite quadrant of your Dyall circle.

Lastly, the *houre lines* in all manner of plaine Dyalls, are thus to be found.

If the substile and houre bee both on the same side of the Meridian: the arch of the Æquinoctiall betweene the substile, and the houre line, shall bee equall to the difference of the two distances, namely of the houre line from noone, and of the Meridian of the Æquinoctiall from the substile. But if the substile bee vpon one side of the Meridian,

ridian, and the houre on the other side: it shall be equall to the summe thereof. Then say

As the *Radius*
is to the Sine of the *height of the stile aboue the substile*;
So is the tang. of the *arch of the æquinoctiall, betweene the substile and the houre line*,
to the tang. of the *arch of the circle of your Dyall plaine, betweene the substile and that lower line.*

Or else you may without calculation Geometrically inscribe the houre lines in Dyals hauing centers (for how to doe it in Dyals not hauing centers, I haue already shewed in the *XIX Vse*) thus.

Describe in your Dyall plaine a line for the stile, at the same height or distance from the substile, that the true stile ought to haue. Take also in the substile (as in reason you shall see fit) a point, and through it draw at right angles a long line, for the contingent, or Æquinoctiall intersection. Againe from the same point let fall a perpendicular vnto the stile: the length of this perpendicular is the nearest distance betweene that point and the stile: and it is also the distance of the center of the Æquinoctiall from that point: measure it therefore vpon the substile, the contrary way from the center of the Dyall: and hauing thus the center of the Æquinoctiall, describe thereupon toward the contingent line one halfe of the Æquinoctiall circle: which if the substile be the Meridian, or 12 a clock line of your Dyall, you must begin to diuide into houres at the substile: But if the substile and Meridian of your Dyall be seuerall lines, apply a ruler to the center of the Æquinoctial, and to the intersection of the 12 a clock line with the contingent, and there draw a line: this line shal bee the Meridian of the Æquinoctial: at which you must begin to diuide the Æquinoctiall circle into houres, both wayes. Then applying a ruler vnto the center of the Æquinoctial & euery one of those diuisions, where the

the ruler in euery place shal cut the contingent line, there make a marke: and lastly, through euery one of those marks from the center of the Dyall, draw the houre lines themselues.

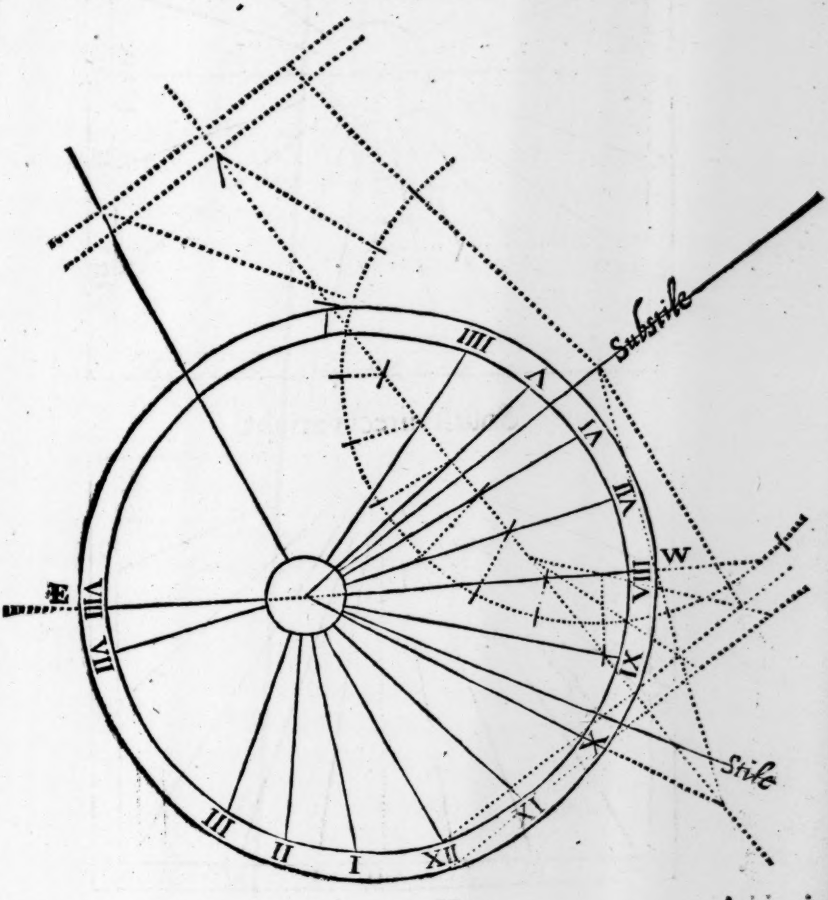
And if in any houre line it shall happen, that the ruler so applied will not reach to intersecat the contingent line: you may thus help your selfe. Which rule also may serue you to find the Meridian of the Æquinoctiall, as often as the intersection of the Meridian of the Dyall with the contingent, falleth without your paper or plaine.

Draw the houre line as farre as it will goe. And take with your Compasses the distance of the intersection point of the contingent with the substile, both from the center of the Dyal, & frō the center of the Æquinoctial. And taking at all aduenture a point in the contingent line, on that side in which the houre line is, measure from that point on the contingent, both those distances: and at the ends of them both draw two lines parallel to the substile, crossing the contingent. Then applying a ruler to the point, which you tooke at all aduenture, and to the intersection of the parallel, which hath the distance of that center, whence the houre line giuen proceedeth, with that houre line: where the ruler shall cut the other parallel, make a prick: and measure the distance betweene that prick and the contingent, vpon the former parallel, on the other side of the contingent. Lastly, out of the proper center through the end of that measure, draw a line: which shall be that you desire.

An example of this Geometricall way of delineating the houre lines you shall finde in the description of a South vpright Dyall declining 35 degr. and reclining degr. 41 min. 30. by considering whereof these rules will be found exceeding plainely set downe: As also all the other rules and obseruations here deliuered, to one that is any whit pregnant and ingenious, will neede no other exemplification, then the inspection of the instrument it selfe, and of these seuerall Dyalls following

FINIS.

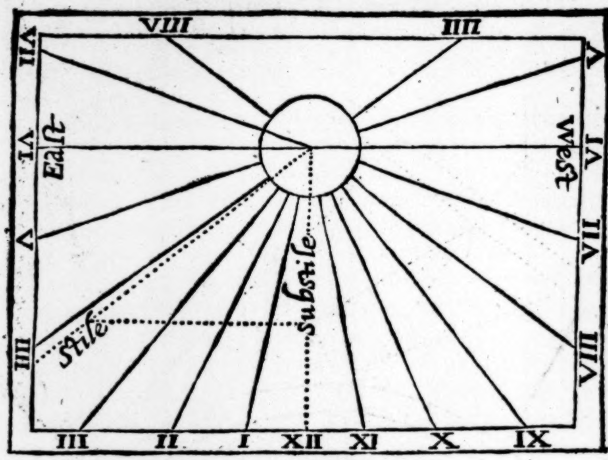
A North Dyall declining Eastwards 15 deg. reclining 41 deg. 30 min. Latitude 51 deg. 30 min.



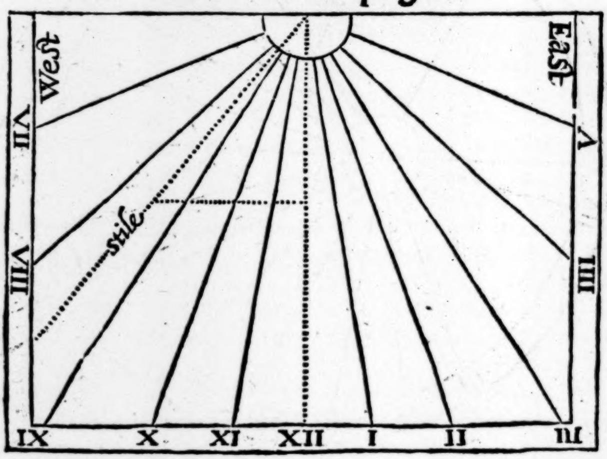
X

A Hori-

A Horizontall Dyall.

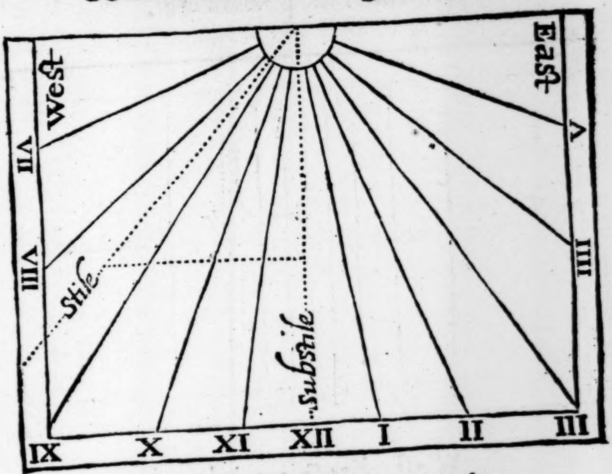


South direct vpright.

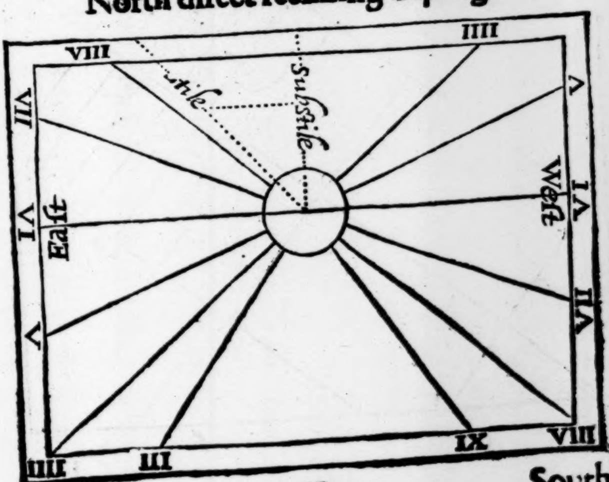


South

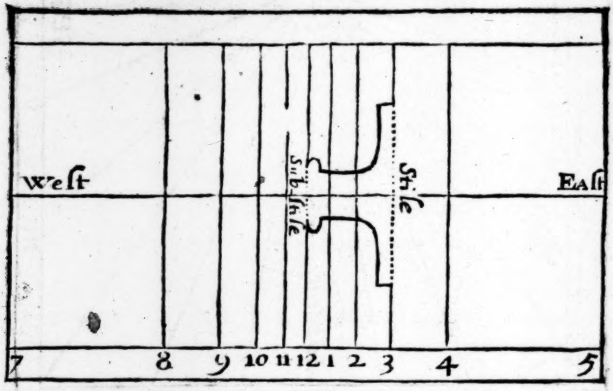
South direct inclining 24 deg.



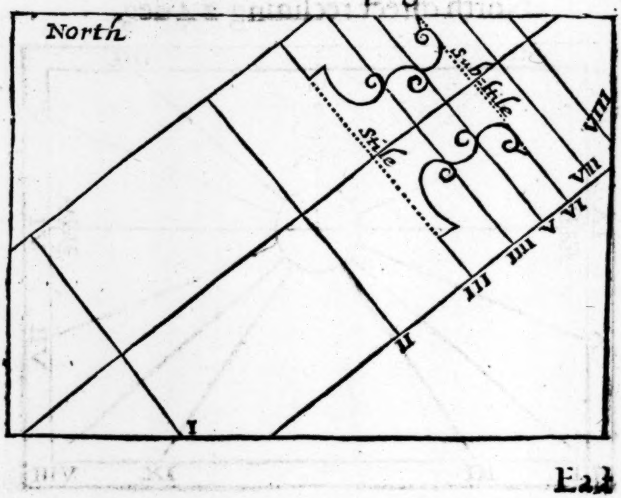
North direct reclining 24 deg.



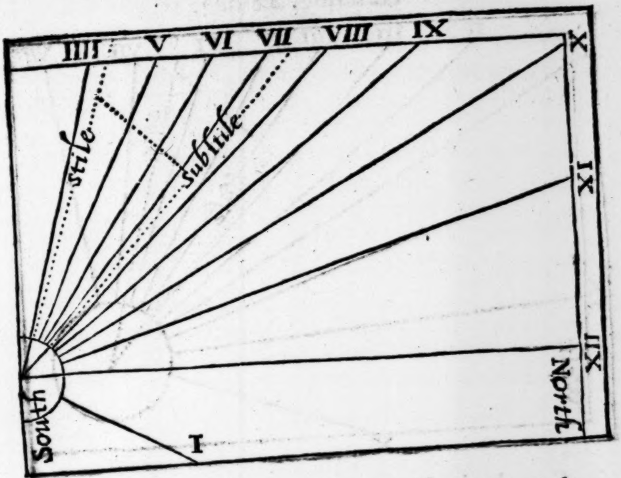
South direct reclining æquall to the complement of the poles height.



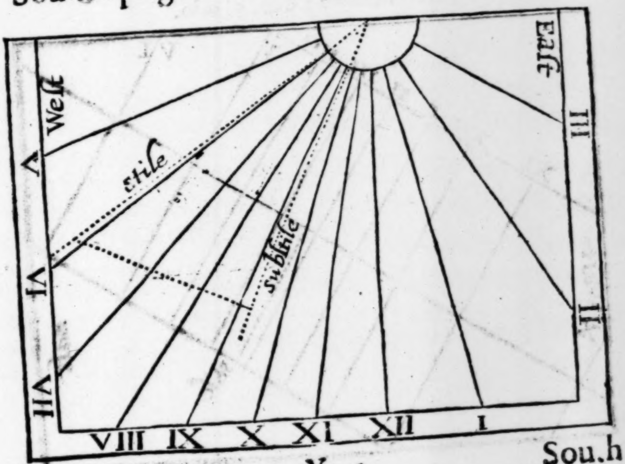
West direct vpright.



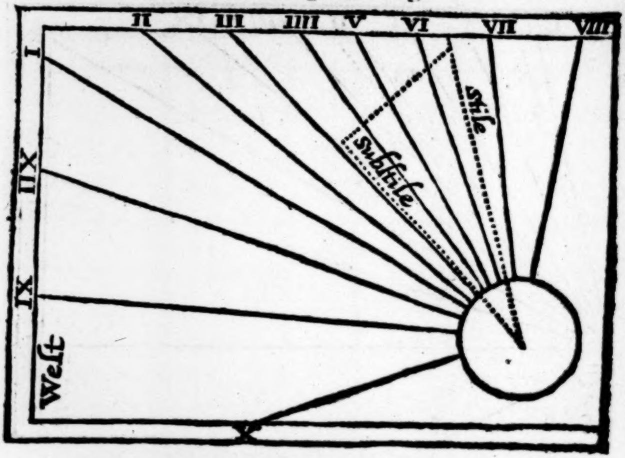
East direct reclining 32 deg.



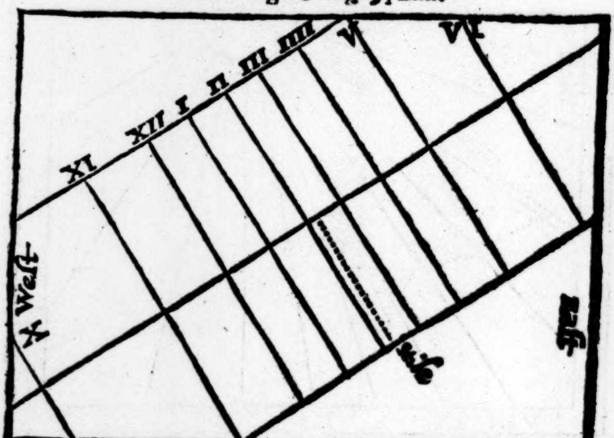
South vpright declining Eastward 35 deg.



South declining Weſtward 76 degr.
reclining. 48 deg.



South declining Weſtward 61 degr.
Reclining 21 deg. 5½ min.



The Tranſlator to the Readers.

Courteous Readers, by reaſon of my abſence whileſt this worke was in the Preſſe, ſome faults haue eſcaped, which otherwiſe might happily haue bin avoyded: for the Compozer being vnacquainted with that manner of *Æquations expreſſed by letters*, where any *fraction* was on the one ſide, he miſplaced them a little out of order, which ought to haue bin ſet thus

$$\frac{\text{Rat. multa} - 1 \text{ in R in } a}{\text{D}} = \text{z}$$

Other ſmall faults, either letters miſtaken, or points miſplaced, I requeſt you to amend, where you find them amiſſe: the chiefeſt that I haue yet noted are theſe which follow; and are thus to be corrected.

Pag. 11, in the table for, ♌ make Ω. *pag.* 16, *lin* 34 vnto 108 $,\underline{33}$ +; *pag.* 35, *lin.* 5. 61 $\frac{49}{144}$ *p* 36, *lin.* 11 17$,\underline{48}$ *p* 36, *lin.* 14 17$,\underline{48}$. 31$\underline{26}$+. *p.* 41, *lin.* 19, 1$\underline{0}$4$\underline{72}$

pag. 53, *lin.* { 10 cylindrical
18 it falſe, why
20 to confirme an error?

pag. 56, *lin.* 6, 4 roodlands & *lin* 14 diminutiue *pag.* 57, *l.* 14, opinion is, that at London a Cylindrical veſſell *pag.* 94, *lin.* 21, a circle, or 90 degrees.

pag. 99, In the triangle *VI*, the ſmall line in the angle *B*, ſhewing it to be giuen, is wanting. *pag.* 100. After the triangle *VII line* 3, and then the ſide *DC*, by the *III*.

And in the triangle *VIII*, ſet *A* at the end of the pricked perpendicular line. *Pag* 132, *line* 10, to the tangent of the arch of *pag* 143, *line* 3, In North reclining, and South inclining plaines *p.* 152, *line* 30, North Dyall declining Eaſtwards 35 degr.