

Lab 5: Scan Matching

Instructor: INSTRUCTOR

Name: STUDENT NAME, StudentID: ID

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission. All sources of material must be cited. The University Academic Code of Conduct will be strictly enforced.

THIS IS A GROUP ASSIGNMENT. Submit one from each team.

1 Theoretical Questions

$$1. M_i = \begin{pmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \end{pmatrix}$$

- (a) Show that $B_i := M_i^T M_i$ is symmetric.
- (b) Demonstrate that B_i is positive semi-definite.

Solution:

$$M_i = \begin{bmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \\ p_{i0} & p_{i1} & p_{i0}^2 + p_{i1}^2 & 0 \\ -p_{i1} & p_{i0} & 0 & p_{i0}^2 + p_{i1}^2 \end{bmatrix}$$

The eigenvalues of the matrix B_i are 0, 0, $p_{i0}^2 + p_{i1}^2 + 1$ and $p_{i0}^2 + p_{i1}^2 + 1$. All these values are for sure greater or equal to zero, therefore the matrix B_i is positive semi-definite and obviously symmetric as proven by construction.

2. The following is the optimization problem:

$$x^* = \operatorname{argmin}_{x \in \mathbb{R}^4} \sum_{i=1}^n \|M_i x - \pi_i\|_2^2 \quad \text{s.t.} \quad x_3^2 + x_4^2 = 1$$

- (a) Find the matrices M , W and g which give you the formulation

$$x^* = \operatorname{argmin}_{x \in \mathbb{R}^4} x^T M x + g^T x \quad \text{s.t. } x^T W x = 1$$

Solution:

Answer here

- (b) Show that M and W are positive semi definite. **Solution:**

Answer here