F1TENTH Autonomous Racing **(Due Date:)**

Lab 5: Scan Matching

Instructor: INSTRUCTOR *Name:* STUDENT NAME, *StudentID:* ID

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission. All sources of material must be cited. The University Academic Code of Conduct will be strictly enforced.

THIS IS A GROUP ASSIGNMENT. Submit one from each team.

1 Theoretical Questions

1. $M_i = \begin{pmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \end{pmatrix}$

   (a) Show that $B_i := M_i^T M_i$ is symmetric.

   (b) Demonstrate that B_i is positive semi-definite.

   Solution:

   $$M_i = \begin{bmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \end{bmatrix}$$
   $$B_i = \begin{bmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \\ p_{i0} & p_{i1} & p_{i0}^2 + p_{i1}^2 & 0 \\ -p_{i1} & p_{i0} & 0 & p_{i0}^2 + p_{i1}^2 \end{bmatrix}$$

   The eigenvalues of the matrix B_i are 0, 0, $p_{i0}^2 + p_{i1}^2 + 1$ and $p_{i0}^2 + p_{i1}^2 + 1$. All these values are for sure greater or equal to zero, therefore the matrix B_i is positive semi-definite and obviously symmetric as proven by construction.

2. The following is the optimization problem:

$$x^* = \text{argmin}_{x \in \mathbb{R}^4} \sum_{i=1}^{n} \|M_i x - \pi_i\|_2^2 \quad \text{s.t.} \quad x_3^2 + x_4^2 = 1$$

(a) Find the matrices M, W and g which give you the formulation

$$x^* = \text{argmin}_{x\in\mathbb{R}^4}\, x^T M x + g^T x \quad \text{s.t. } x^T W x = 1$$

Solution:

Answer here

(b) Show that M and W are positive semi definite. **Solution:**

Answer here