KØBENHAVNS UNIVERSITET
DET NATUR- OG VIDENSKABELIGE FAKULTET

Data-Driven Financial Models

Final Assignment

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Group members' contributions:

It is noted that despite us working together on every question, Ahad is most competent in questions 1a, 1b, 1c, 1d, for William questions 1e, 1f, 1g, 1h, for Firat questions 2a, 2b, 2c, and for Noa question 3.

(1) Portfolio optimization:

a) **Diversification:** Choose financial stock indices (at least 7) for different markets or sectors. Time interval 20 years, monthly data. When necessary, convert all values in one domestic currency (e.g. USD).

Solution a).

We collect monthly data from December 2002 to December 2022 from Yahoo Finance for the following financial stock indices (in this order), similar to what we did in the previous assignments:

- S&P 500 (ˆGSPC)
- Dow Jones Transportation Average (ˆDJT)
- Russell 2000 (ˆRUT)
- iShares US Financials ETF (IYF)
- Health Care Select Sector SPDR ETF (XLV)
- Energy Select Sector SPDR ETF (XLE)
- NASDAQ Composite (ˆIXIC)

and get following output:

Index	Date	ˆGSPC	ˆDJT	ˆRUT	IYF	XLV	XLE	ˆIXIC
1	2003-01-01	-0.0278	-0.0610	-0.0289	-0.0121	0.0003	-0.0203	-0.0110
2	2003-02-01	-0.0171	-0.0589	-0.0318	-0.0292	-0.0225	0.0290	0.0125
3	2003-03-01	0.0083	0.0393	0.0111	-0.0028	0.0323	-0.0040	0.0027
4	2003-04-01	0.0779	0.1225	0.0895	0.1136	0.0351	-0.0033	0.0878
5	2003-05-01	0.0496	0.0317	0.1009	0.0523	0.0204	0.1009	0.0861
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
236	2022-08-01	-0.0434	-0.0526	-0.0220	-0.0183	-0.0594	0.0262	-0.0475
237	2022-09-01	-0.0980	-0.1393	-0.1023	-0.0870	-0.0297	-0.1113	-0.1109
238	2022-10-01	0.0768	0.1191	0.1038	0.1162	0.0957	0.2337	0.0383
239	2022-11-01	0.0524	0.0755	0.0213	0.0629	0.0462	0.0127	0.0427
240	2022-12-01	-0.0608	-0.0897	-0.0687	-0.0630	-0.0231	-0.0412	-0.0914

Instead of using the asset prices we use logarithmic returns as estimates of the net returns. In doing so we get exactly 20 years of monthly data (January 2003 to December 2022) of the log returns for the seven the stock indices. S&P 500 is a broad index that tracks the stock prices for the largest companies in the US and thus includes many different sectors. Russell 2000 tracks the stock prices for companies in the US with a smaller market value. The rest of the stock indices each represent different sectors of the US economy ranging from healthcare to energy.

b) **Estimate:** expected yearly returns and covariance matrix with a rolling window of 10 years (in annual steps).

Solution b).

For each rolling window of 10 years with annual steps (e.g. 2003 to 2012, 2004 to 2013 and so on) we estimate the yearly returns and covariance matrix by using the *mean()* and *cov()* functions respectively. We then multiply these by 12 to annualize the results. The yearly return and covariance matrix for the first window are the following:

```
mu =

    0.0483
    0.0832
    0.0796
    0.0090
    0.0572
    0.1319
    0.0816


C =

    0.0225    0.0259    0.0283    0.0306    0.0146    0.0236    0.0256
    0.0259    0.0439    0.0359    0.0375    0.0140    0.0237    0.0301
    0.0283    0.0359    0.0413    0.0392    0.0167    0.0293    0.0349
    0.0306    0.0375    0.0392    0.0530    0.0191    0.0232    0.0332
    0.0146    0.0140    0.0167    0.0191    0.0164    0.0126    0.0153
    0.0236    0.0237    0.0293    0.0232    0.0126    0.0531    0.0250
    0.0256    0.0301    0.0349    0.0332    0.0153    0.0250    0.0335
```

For the rest of the expected returns and covariances we refer to the attached Matlab code.

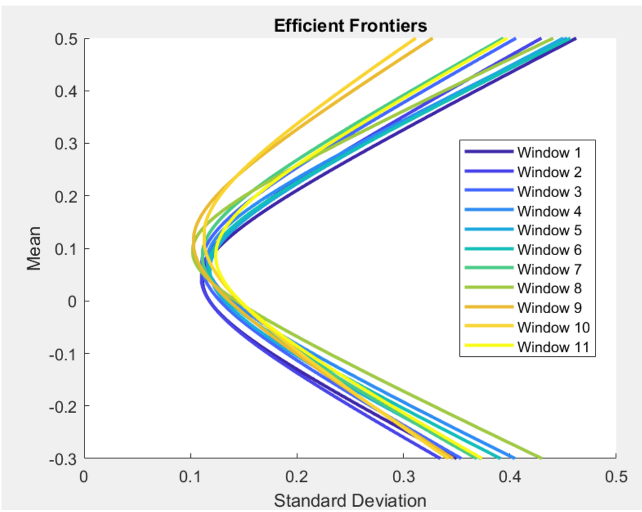
c) **Efficient frontier:** show in one or more figures the 11 different efficient frontiers (one for each rolling window) and explain in your report the diversification benefit.

***Solution c)**.*

For each of the 11 windows we choose to find the tangent portfolio for when the return of the risk-free asset is 0,01 and 0,05 (these numbers are chosen arbitrarily). To do this, we use the code from slide 7 in *ch06_OR*. For each window i we get the optimal portfolio weights $x^*_{0.01,i}$ and $x^*_{0.05,i}$ and expected returns $x^*_{0.01,i} \cdot \mu_i = \mu^*_{0.01,i}$ and $x^*_{0.05,i} \cdot \mu_i = \mu^*_{0.05,i}$, where μ_i is the expected return of the stock indices in window i. Since these portfolios are on the efficient frontier, the rest of the frontier can then be found by combining the two portfolios, i.e. the portfolio weights, expected return and standard deviation of a portfolio P on the efficient frontier can be calculated as:

$$
\begin{aligned}
x_{P,i} &= X x^*_{0.01,i} + (1-X) x^*_{0.05,i} \\
\mu_{P,i} &= X \mu^*_{0.01,i} + (1-X) \mu^*_{0.05,i} \\
\sigma_{P,i} &= \sqrt{x_{P,i}^\top C_i x_{P,i}},
\end{aligned}
$$

where $X \in \mathbb{R}$ and C_i is the covariance matrix for window i. We plot the 11 efficient frontiers in the following:



Where we have color coded the windows such that the first window is dark blue and as each window is added, they become more and more yellow. We see that the efficient frontiers are very similar except for perhaps windows 9 and 10, where they are shifted slightly upwards compared to the rest. This could be caused by the fact that windows 9 and 10 both only include the end of the 2007-2008 crisis and thus the beginning of the recovery phase as well as only the beginning of the Covid crisis. Therefore these windows include mostly periods of economic stability. It is well known (p. 263, EGBG) that crises increase correlation efficients, thereby reducing the benefits of diversification. For the same amount of risk in these two windows, people are therefore able to get more return hence the upward shift. While the beginning of window 11 shares similar characteristics as windows 9 and 10, it is different from these two since it covers yet another shock to the American economy, the war in Ukraine, that started an energy crisis which affected most markets negatively.

But generally, with our diverse stock indices, the curvature of the plots suggest that by diversifying and thus avoiding perfect correlations, combining assets reduces risk.

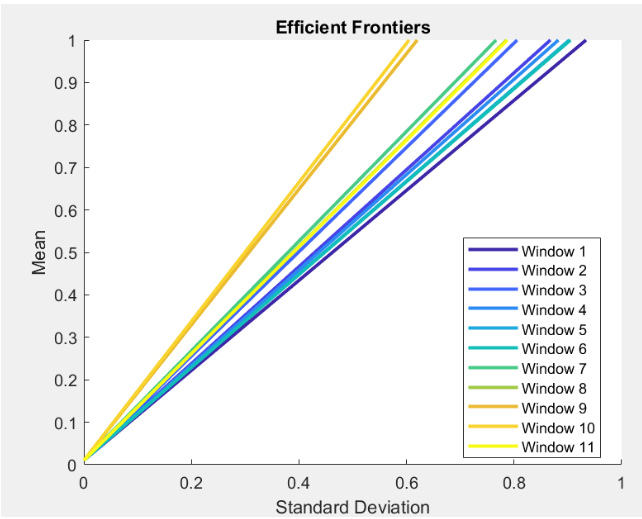
d) **Tobin separation:** repeat the assignment c), now including a risk-free rate of 1%. What are the consequences for the portfolio management in such a world?

Solution d).

We now include a risk-free rate of 1%. We can use the same method as in c) to find the tangent portfolio for each window when the risk-free rate is $R_F = 0{,}01$. For each window i we then get the portfolio weights of the tangent portfolio x_i^*, and we can calculate its expected return and standard deviation as $\mu_i^* = x_i^* \cdot \mu_i$ and $\sigma_i^* = \sqrt{(x_i^*)^\top C_i x_i^*}$ respectively, where μ_i and C_i are the expected return and covariance matrix for window i. Every portfolio on the efficient frontier P can then be found by combining the tangent portfolio with the risk free asset with different choices of lending and borrowing. The expected return and standard deviation of such a portfolio P can be calclulated as:

$$\mu_{P,i} = X\mu_i^* + (1-X)R_F$$
$$\sigma_{P,i} = X\sigma_i^*,$$

for window i, according to slide 22 in *ch05_OR*. We get the following plot:



We now see that instead of getting "sideways parabolas", we get straight lines (capital market line, CML). The reason for only combining the risk-free asset with the tangent portfolio is because according to the separation theorem (p. 100, EGBG), all investors would hold the same portfolio of risky assets, the tangent portfolio. Therefore every other portfolio on the frontier is just a linear combination of the tangent portfolio and the risk-free asset where the weights sum to 1, and it is therefore also possible to avoid all risk by only investing in the risk-free asset. We also see that again, all windows except windows 9 and 10 are fairly similar, where these two are shifted noticeably upwards. This is due to the same reasons as described in c).

e) **Asset allocation:** choose a constant required return and calculate the optimal asset allocation for the case of the Tobin separation efficient frontier in the different years. How high is the portfolio turnover on average each year?

Solution e).

We choose our required expected return to be 8% (chosen arbitrarily). For every window i, let $\mu^*_{P,i} = 0{,}08$ denote the expected return of the optimal portfolio, which must be equal to 0,08. We also define the excess returns $\mu^e_i = \mu_i - R_F 1$ and $(\mu^*_{P,i})^e = \mu^*_{P,i} - R_F$. According to David Lando and Rolf Poulsen's *Finance 1 and Beyond (October, 2023)* (LP) p. 55, for each window i the portfolio weights for the stock indices can be calculated as

$$\hat{x}^*_{P,i} = \frac{(\mu^*_{P,i})^e}{(\mu^e_i)^\top C_i^{-1} \mu^e_i} C_i^{-1} \mu^e_i.$$

The weight for the risk free asset is thus $x^*_{R_F} = 1 - \hat{x}^*_{P,i} 1$. We then get the optimal asset allocation for the Tobin separation efficient frontier for window i as $(x^*_{P,i})^\top = ((\hat{x}^*_{P,i})^\top, x^*_{R_F})$. For the first window we get:

```
pfw =

   -1.8271
    0.3042
   -0.1206
    0.0712
    0.5962
    0.3449
    0.7826
    0.8486
```

where the first seven are the portfolio weights for the stock indices and the last one is the portfolio weight for the risk-free asset. For the remaining 10 portfolio weights, we refer to the attached Matlab code.

We now wish to calculate the portfolio turnover on average each year. To do this, we use the following formula from Nikolaus Hautsch and Stefan Voigt's "Large-scale portfolio allocation under transaction costs and model uncertainty" p. 16:

$$TO = \frac{1}{10} \sum_{i=1}^{10} \left\| x^*_{P,i+1} - \frac{x^*_{P,i} \circ (1 + \mu_{i+1})}{1 + x^*_{P,i} \mu_{i+1}} \right\|_1,$$

where the operator $\circ$ is the element-wise multiplication and $\| \cdot \|_1$ is the 1-norm. We get the following results:

```
turnover = 0.9572
```

i.e. we get a turnover of 95,72%. This is quite high and suggests that we have to change our portfolio often in order to get an expected return of 8%. This is however not surprising given the many crises from 2003 to 2023 and therefore the need to change the portfolio weights of the stock indices.

f) **Backtest:** conduct a backtest (ex-post) for the optimized portfolio

(with annual re-balancing, see point e). Calculate the average return and the standard deviation for your ex-post portfolio out of sample? (i.e. your portfolio is created by calculating the mean and covariance in-sample to find your optimal portfolio, BUT then you hold your portfolio for one year out-of-sample and calculate the return you have had during this period. Then you repeat the same process again and again for each period.)

Solution f).

For each of the $i = 1, \ldots, 11$ rolling windows of 10 years, we calculated the weights of the portfolios that would have given an expected return of 8% in that window, $x^*_{P,i}$. We follow the strategy as described in the question where at the end of each window we buy the portfolio that the past 10 years suggest that we should buy in order to get an expected return of 8%. We then hold it and sell it exactly at the end of the next year. In doing so we get exactly 10 realized returns for the years 2013 to 2022. Letting $\mu_{2013}, \ldots, \mu_{2022}$ denote the expected returns during years 2013 to 2022 including the return of the risk-free asset, we calculate the realized returns of our portfolio with annual rebalancing as:

$$R_{strat,2013} = x^*_{P,1} \cdot \mu_{2013}, \ldots, R_{strat,2022} = x^*_{P,10} \cdot \mu_{2022}.$$

We then use the *mean()* and *std()* functions in Matlab to estimate the expected return and standard deviation for our portfolio and we get:

```
mean_P = 0.0580
std_P = 0.0954
```

Thus, the expected return of this strategy is 5,8% and standard deviation is 9,54%. This expected return is lower than 8% and the standard deviation is also relatively high in comparison, suggesting that this strategy is very risky, as expected given the many shocks in the timespan our data covers.

g) **Beta:** use a broad stock index to test, whether our portfolio is in line with the

$$\bar{R}_P = R_F + \left(\bar{R}_M - R_F\right)\beta_P$$

CAPM prediction:

Has your portfolio created "alpha"? Is the "alpha" significant? (see Jensen's alpha)

Solution g).

S&P 500 is a broad index that is often used as a benchmark. We thus let S&P 500 represent the market portfolio. According to slide 4 in *Empirical tests of CAPM OR*, we can test whether our portfolio is in line with CAPM by fitting a linear model of the form:

$$R_{strat,t} - R_F = \alpha_{strat} + \beta_{strat}(R_{M,t} - R_F) + e_{strat,t}$$

to find "alpha" and estimate β for our portfolio. We already have data for the $R_{strat,t}$-s for $t = 2013, \ldots, 2022$. The corresponding returns for our benchmark portfolio, the $R_{M,t}$-s, can simply be computed by finding the average monthly return for S&P 500 during year t and annualize by multiplying by 12. Using the *fitlm()* function to fit a linear model with "$R_{strat,t} - R_F$" as response variable and "$R_{M,t} - R_F$" as covariate, we get the following results for our coefficients:

```
alpha = 0.0103
p_value = 0.7143
beta = 0.4236
p_value = 0.0284
```

We see, that we estimate β for our portfolio β_{strat} to be 0,4236. We also see, that we have found an "alpha" with the value $\alpha_{strat} = 0{,}0103$. From the second output above, we have conducted a t-test and with a significance level of 5%, we see that the intercept "alpha" is statistically insignificant, and we can therefore not reject the null-hypothesis that it might be 0. The insignificance of "alpha" could suggest that CAPM holds, but it could also be caused by the fact that we only have 10 datapoints in our regression.

h) **Timing:** calculate the *Treynor-Mazuy* measure and give an interpretation to these results (portfolio returns)

Solution h).

To calculate the Treynor-Mazuy measure, we fit a linear model of the form

$$R_{strat,t} - R_F = a_{strat} + b_{strat}(R_{M,t} - R_F) + c_{strat}(R_{M,t} - R_F)^2 + e_{strat,t},$$

as described on slide 12 in *Chp 25 and more*. We fit the model in Matlab using the *fitlm()* function. We let "$R_{strat,t} - R_F$" be the response variable and "$(R_{M,t} - R_F)$" and "$(R_{M,t} - R_F)^2$" be covariates. We get the following output for a, b and c:

```
a = 0.0275
p_value = 0.5068
b = 0.4649
p_value = 0.0345
c = -0.7071
p_value = 0.5488
```

We see that the intercept $a_{strat} = 0{,}0275$ and the α_{strat} from g) are of the same order of magnitude, but still quite different. However, with a p-value of 0,5068, it still seems the intercept is statistically insignificant. The $b_{strat} = 0{,}4649$ is similar to β_{strat} in their value as to be expected. We estimate c in the Treynor-Mazuy model to $c_{strat} = -0{,}7071$ suggesting that the fund's timing is very poor and that it does not anticipate the market movements very well, where beta is not changed properly at the right moments. This makes sense since, as we have stated many times, the years from 2003 to 2022 had many crises making it difficult to predict the market. It is however also statistically insignificant meaning we cannot reject the null-hypothesis that c might be 0. But even so, c being 0 still suggets that the fund's timing is poor.

(2) Bonds:

a) Use five different "real" bonds and calculate for these bonds the yield to maturity, duration and convexity.

Solution a).

In this question, we began by researching five different bonds, which we found on Nasdaq Copenhagen, selecting them specifically from corporate bonds. The reason for choosing Nasdaq Copenhagen compared to other pages is that it provides more detailed information regarding the type of bond, which facilitates the calculations required for this part of the assignment. We choose the following bonds in the same order as used in the MATLAB code:

- Nordic Investment Bank (EMTN 1196): Coupon rate: 0,125%, clean price: $7{,}45 \cdot 84{,}5$ EUR, maturity: 6 periods, payment frequency: 1, months since last coupon payment: 11,5.
- Nordic Investment Bank (EMTN 1089): Coupon rate: 0,375%, clean price: $7{,}45 \cdot 91{,}481$ EUR, maturity: 15 periods, payment frequency: 1, months since last coupon payment: 6.
- ALM. Brand A/S (Alm Brand Tier 2): Coupon rate: 4,63%, clean price: $7{,}45 \cdot 98{,}75$ EUR, maturity: 28 periods, payment frequency: 4, months since last coupon payment: 3.
- Arbejdernes Landsbank A/S (ALSNP2027V): Coupon rate: 4,7567%, clean price: $7{,}45 \cdot 100{,}914$ EUR, maturity: 11 periods, payment frequency: 4, months since last coupon payment: 1.
- DLR Kredit A/S (SNP FRN July 2027): Coupon rate: 5,0733%, clean price: $7{,}45 \cdot 102{,}5$ EUR, maturity: 10 periods, payment frequency: 4, months since last coupon payment: 0,3.

All necessary information about the bonds, including coupon rates, current prices, frequency, and so forth, was implemented based on data obtained from Nasdaq and can be found in the MATLAB code under section 2.a of the assignment. We note that all the bonds we chose are traded in DKK.

We begin by calculating the Yield to maturity using *OR_chp20*, slide 3 and textbook *EGBG* p.520 and 521. The calculation is based on the following formula:

$$P_{dirty} = \sum_t \frac{C(t)}{(1+y)^t}$$

Where P_{dirty} represents the bond price (Dirty Price), $C(t)$ denotes the cash flow at time t, t is the time until maturity for the bond, and y is the variable to be calculated, representing the yield to maturity. But now, we need to explain how to calculate the Dirty Price and Cash Flows to continue. The cash flows can be calculated as follows: We will use the following for each periodic cash flow for a bullet bond:

$$\text{Cash Flow (coupon)} = \frac{\text{Coupon Rate (CPRs)}}{\text{Payment Frequency}} \times \text{Face Value (100)}$$

Note that for a bullet bond, at the maturity the bondholder receives both the final coupon payment and the face value (100). Therefore, the final cash flow is instead calculated as:

$$\text{Final Cash Flow} = \text{Face Value (100)} \times \left(1 + \frac{\text{Coupon Rate (CPRs)}}{\text{Payment Frequency}}\right).$$

Next, we have to calculate the dirty price. We know that Dirty Price = Market Price (Clean Price)+Accrued Interest. But what is accrued interest? We know if the bond is traded in the middle of a coupon payment period, the buyer must compensate the seller for the interest accrued since the last payment. Therefor the formula for this is:

$$\text{Accrued Interest} = \text{Face Value (100)} \times \text{Coupon Rate (CPRs)} \times \frac{\text{Months Since Last Payment}}{12}$$

With these formulas, we have all the necessary components. We implemented them in MATLAB with using solver function *fzero()* and obtained the following YTM values for each bonds in same order: (Note that in all parts of Task 2 in the Matlab code where the value 7.45 appears, it simply indicates the conversion from DKK to EUR)

```
Yield_to_maturity =0.0296
Yield_to_maturity =0.0097
Yield_to_maturity =0.0465
Yield_to_maturity =0.0425
Yield_to_maturity =0.0396
```

For two of our bonds, we obtained YTM values that are very close to the effective interest rate in Nasdaq, which indicates the accuracy of our results. For the three other bonds, Nasdaq has not calculated the yield to maturity and we can thus not compare our results with theirs. A lower YTM, as seen with Bond 2 (0,97%), indicates a bond priced at a significant premium or offering minimal returns due to its low coupon payments and short maturity. Such bonds are typically favored by investors seeking safety and stability over income. On the other hand, higher YTMs, such as Bond 3 (4,65%) and Bond 4 (4,25%), suggest these bonds offer more attractive effective returns. These could be the result of higher coupon rates, longer maturities, appealing to investors looking for higher yields, albeit with potentially more sensitivity to market interest rates.

For duration, we use formula 22.3 from the textbook EGBG, which is expressed as follows:

$$D = \frac{1}{P_{dirty}} \sum_{t=1}^{T} t \frac{C(t)}{(1+y)^t}$$

We have calculated all the necessary information before; now we only need to implement the formula in MATLAB, resulting in the following values:

```
Mac_Duration =5.9793
Mac_Duration =14.5919
Mac_Duration =24.0723
Mac_Duration =10.3809
Mac_Duration =9.4635
```

We know that Macaulay-Duration represent the weighted average time until the bond's cash flows are received, considering the present value of those cash flows. Bonds 2 and 3, with durations of **14,59** years and **24,07** years, are much more sensitive to interest rate changes than Bonds 1, 4, and 5, which have shorter durations. Bonds with longer maturities (like Bond 3) naturally have higher durations, as most of their cash flows occur far into the future, increasing the weighted average time to recoup the investment and we can say that investors seeking lower interest rate risk may prefer shorter-duration bonds (like Bond 1), while those seeking higher yields and are comfortable with interest rate risks might opt for longer-duration bonds (like Bond 3).

For convexity, we used formula 22.4 on p. 563 from the textbook EGBG, which is expressed as follows:

$$C = \left(\frac{1}{2}\right) \frac{\sum_{t=1}^{T} \frac{t(t+1)C(t)}{(1+y)^t}}{P_{dirty}}$$

We have calculated all the necessary information before; now we only need to implement the formula in MATLAB, resulting in the following values:

```
Convexity =20.9038
Convexity =115.6696
Convexity =330.9945
Convexity =61.0669
Convexity =51.0784
```

We know that Convexity complements duration by explaining the curvature of the price-yield relationship. Together, duration and convexity provide a comprehensive picture of a bond's interest rate risk and we can see that convexity is highest for Bond 3 (330,9945), which correlates with its high duration. Bonds with higher convexity will experience greater price increases for a given decline in interest rates, and smaller price decreases for a rise in rates and Bond 1 has the lowest convexity, implying a more linear price-yield relationship compared to others. Thus, we have achieved the desired results for this question.

b) Calculate the duration and convexity of a portfolio of these bonds, if EUR 100.000, – is invested in each of them.

Solution b).

We will calculate the portfolio duration, which is the weighted average of the individual bond durations, with the weights being the proportion of the portfolio value invested in each bond. Similarly, we will calculate the portfolio convexity, which is the weighted average of the individual bond convexities, providing a measure of the portfolio's curvature in the price-yield relationship. That is, we will use the following two formulas from page 568 (EGBG) for these calculations:

$$D_{\text{portfolio}} = \sum_{i=1}^{n} w_i \cdot D_i$$

and

$$C_{\text{portfolio}} = \sum_{i=1}^{n} w_i \cdot C_i$$

where $w_i = \frac{\text{Value Invested in Bond } i}{\text{Total Value Invested in Portfolio}}$, C_i is convexity of bond i and D_i is duration of bond i and $n = 5$ in our case for the five bonds. We now have all the necessary information. We simply need to use our results from the previous question and implement the formulas in Matlab and we get following values:

```
pfDur =12.8976
pfConv =115.9426
```

The portfolio duration of **12,90 years** indicates that, on average, it will take this amount of time for the portfolio's cash flows to be received (in present value terms). This duration reflects the interest rate sensitivity of the portfolio. This means the portfolio will experience a 12% price change for a 1% change in interest rates (approximately). The portfolio convexity of **115,9426** represents the portfolio's sensitivity to large changes in interest rates. Convexity captures the curvature in the price-yield relationship and improves the accuracy of bond price estimates under larger rate changes and this high convexity indicates the portfolio's price sensitivity to yield changes has significant curvature, providing additional price increases for yield drops.

c) Estimate the potential decline in the market value of your portfolio, if the yield increases by 150 basis points.

Solution c).

We will now estimate the potential decline in the market value of a portfolio if the yield increases by 150 basis points, equivalent to 1,5%. To perform this calculation, we will use a formula that incorporates our knowledge of duration and convexity from the previous question. Specifically, we will refer to the textbook EGBG footnotes on page 564, which are expressed as follows

$$\frac{\Delta P_{dirty}}{P_{dirty}} = -D\Delta y + C\left(\Delta y\right)^2.$$

where $\frac{\Delta P_{dirty}}{P_{dirty}}$ is the percentage price change and Δy is change in yield (here, $\Delta y = \frac{0{,}015}{(1+y)}$ representing change with 150 basis points), and we already have knowledge of the remaining variables. We now have all the necessary information. We just need to use our knowledege from the previous questions and implement the formulas in MATLAB and we get following values:

```
pricechange =
   -0.0827
   -0.1912
   -0.2770
   -0.1367
   -0.1259

Total Portfolio Price Change (in EUR) =
  -8.1359e+04

Portfolio Price Change as a Percentage =
   -0.1627
```

The elements of the first vector represent the percentage price change for each bond in the portfolio due to the 150 basis point yield increase. And we can say that Bond 3 ($-27{,}70\%$) shows the largest percentage decline, likely due to its high duration and convexity, indicating that it is more sensitive to yield changes. Bond 1 ($-8{,}27\%$) shows the smallest percentage decline, suggesting it has the lowest sensitivity to interest rate changes, likely due to a shorter duration or lower convexity. The absolute decline in portfolio value is approximately 81359 EUR. This represents the monetary loss across the entire portfolio due to the yield increase of 150 basis points. The portfolio percentage decline is approximately 16,27%. This reflects the weighted average sensitivity of the portfolio, taking into account the individual bond durations, convexities, and their equal investment weights.

(3) Limit of Arbitrage:

Please find an example of a case where we see the limit of arbitrage in real life and discuss it in detail. Explain the case and the arbitrage opportunity. Can you take advantage of such an opportunity? What are the risks?

Solution.

The example we choose is the case of mispricing of the Treasury Inflation Protected Securities (TIPS) relative to Treasury bonds. Our study of this case is based on the research papers *Why Does the Treasury Issue TIPS? The TIPS - Treasury Bond Puzzle* by Matthias Fleckenstein, Francis A. Longstaff and Hanno Lustig as well as *Limits to Arbitrage and Mispricing in TIPS* by Lorenzo Bretscher. TIPS are mostly the same as a normal treasury bond, it is a type of U.S. government bond issued by the U.S. Treasury Department. The big difference between the two is the fact that TIPS are adjusted for inflation, therefore the interest payments can increase with inflation. The reason this is interesting to talk about, is the fact that this is one of, if not the largest case of arbitrage ever recorded. TIPS principal and coupon payments are adjusted for inflation based on the Consumer Price Index (CPI). In contrast, nominal Treasury bonds have fixed payments unaffected by inflation. Although TIPS and nominal Treasuries are issued by the same government and are backed by the same credit, research demonstrates that nominal Treasury bonds are often systematically overpriced relative to their cash-flow-matched TIPS equivalents when adjusted through inflation swaps. This mispricing reflects inefficiencies and arbitrage limits in financial markets.

Quantifying the Mispricing

The price gap has frequently exceeded 10$ per 100$ notional and reached over 20$ during the 2008-2009 financial crisis. These anomalies, representing arbitrage opportunities, persisted even in the presence of active participants seeking to exploit them. The total estimated mispricing during the period exceeded 56$ billion, highlighting significant inefficiencies in this major fixed-income market.

Strategy Outline

Theoretically, we can exploit this mispricing this way:

Step 1: Buy undervalued TIPS.

Step 2: Use an inflation swap to transform TIPS' inflation-adjusted cash flows into fixed payments that mirror a nominal Treasury bond's cash flows.

Step 3: Short overvalued nominal Treasury bonds, realizing a profit equal to the price differential.

Case Study: Financial Crisis of 2008-2009

During the financial crisis, the mispricing between nominal Treasuries and TIPS reached record levels. At its peak, the arbitrage profit exceeded 20$ per 100$ notional, but the market constraints discussed above limited the ability of arbitrageurs to close the gap. The TIPS market became particularly illiquid, making it harder to execute trades at favorable prices. Financial institutions faced balance sheet pressures, reducing their capacity to deploy capital into arbitrage trades.

Limits to Arbitrage

While the above strategy seems straightforward in theory, significant real-world constraints prevent arbitrageurs from fully exploiting these inefficiencies. TIPS markets are less liquid than nominal Treasury markets, especially during crises. This can widen bid-ask spreads, increasing transaction costs and reducing the attractiveness of the trade. Mispricing can persist for extended periods, particularly during market crises when investors "flee to liquidity" and prefer nominal Treasuries. The strategy's profitability diminishes as funding and opportunity costs rise over time. Also, arbitrage often requires leveraging positions, such as shorting nominal Treasuries.

Market volatility or persistent mispricing can trigger margin calls or increase borrowing costs, making the trade untenable. Also the problem with actually producing the trades, since the arbitrage relies on precision across three markets: TIPS, nominal Treasuries, and inflation swaps. Delayed execution or mismatched trades may result in losses. There's also the regulatory and institutional constraints, such as short-selling restrictions and regulations impose barriers for certain market participants. Small investors may lack access to inflation swap markets or sufficient capital to enter such trades.