## 1 Lexer

The Lexer reads the proof code and produces the list of tokens that represents it.

```
module Lexer where
import Data.Char
import Data.List.Split
```

These tokens are represented as LexerTokens, and are as follows

```
data Token = BOF
              ID String
              Natural Int
              LParen
              RParen
              LBrack
              RBrack
              SQuote
              DQuote
              Arrow
              Lambda
              Exists
              ForAll
              OpAdd
              OpSub
              OpMul
              OpDiv
              OpMod
              OpEqual
              0pLT
              OpGT
              {\tt OpAnd}
              0p0r
              Negation
              Bottom
              Comma
              Colon
              Equiv
              Туре
              True
              False
              Null
              Undefined
```

```
| TBoolean
| TNatural
| TSymbol
| TList
| TChar
| TypeOf
| Let
| Native
| EOF
| deriving (Show)
```

The string itself is chopped up by the munch function, which uses the simplified maximal munch algorithm to produce tokens.

```
munch :: String → (Token, String)
munch = extractTokenStr Start ""
```

As you may have noticed, the actual munching is performed by extract-TokenStr, while munch simply serves as an entry point. Before defining that, however, we require a few helper definitions.

First are the states which the state machine used for munching can be in:

```
data State = Start | Identifier | Number | Single | PossibleArrow |
PossibleEquiv
```

Then, we have a few helper functions which can identify classes of characters. isIdent checks that a character is a valid character for an identifier, i.e. alphanumeric, or an underscore.

isSingle checks that a character is one of the characters that makes up a whole token on its own.

```
\begin{split} & \text{isIdent} \, :: \, \text{Char} \, \to \, \text{Bool} \\ & \text{isIdent} \, \, \text{c} \, = \, \text{isAlphaNum} \, \, \text{c} \, \mid \mid \text{c} \, = \, \text{`\_'} \\ & \text{isSingle} \, :: \, \text{Char} \, \to \, \text{Bool} \\ & \text{isSingle} \, \, \text{c} \, = \, \text{c} \, \, \text{`elem'} \, \, \text{"()[]} \!\! \hookrightarrow \!\! + \, , \, = \!\! \slash \! / \!\! \cdot \, \text{\slash} \! / \!\! : \, \mid \& \, \, \slash \, \
```

```
extractTokenStr state token code = case state of
  Start

ightarrow case code of
     '-' : rest

ightarrow extractTokenStr PossibleArrow "-" rest
     ':' : rest

ightarrow extractTokenStr PossibleEquiv ":" rest
     1 : rest | isAlpha 1 || 1 == '_'

ightarrow extractTokenStr Identifier [1] rest
     n: rest \mid isDigit n \rightarrow extractTokenStr Number [n] rest
     o : rest \mid isSingle o \rightarrow extractTokenStr Single [o] rest
     \texttt{w} \, : \, \texttt{rest} \, \mid \, \texttt{isSpace} \, \, \texttt{w} \, \rightarrow \, \texttt{extractTokenStr} \, \, \texttt{Start} \, \, \texttt{[]} \, \, \, \texttt{rest}
                                   \rightarrow error $ "Lexer could not process character
            sequence " ++ code -- TODO: LexerError
   \  \  \, \hbox{Identifier} \quad \to \, \hbox{case code of} \\
     l : rest | isIdent l 
ightarrow extractTokenStr Identifier (l : token)
                                    \rightarrow (convertToToken token, code)
  Number

ightarrow case code of
     l : rest | isDigit l 
ightarrow extractTokenStr Number (l : token) rest

ightarrow (Natural $ read $ reverse token, code)
  {\tt PossibleArrow} \, \to \, {\tt case} \, \, {\tt code} \, \, {\tt of} \, \,
     '>' : rest

ightarrow extractTokenStr Single ">-" rest
                                   → (convertToToken token, code)
  {\tt PossibleEquiv} \, \to \, {\tt case} \, \, {\tt code} \, \, {\tt of} \, \,
                                   \rightarrow extractTokenStr Single "=:" rest
     '=' : rest
                                    \rightarrow (convertToToken token, code)
  Single
                     \rightarrow (convertToToken token, code)
```

The strings extracted by extractTokenStr, other than the numeric ones, are converted to actual tokens by convertToToken. This function expects that the token be written backwards because that's how extractTokenStr makes them.

Is that a stupid design for this function? Probably, but I think it will be ok.

```
\mathtt{convertToToken} \; :: \; \underline{\mathtt{String}} \; \to \; \underline{\mathtt{Token}}
convertToToken ""
                               = Arrow
convertToToken ">-"
                                = Arrow
convertToToken "("
                                = LParen
convertToToken ")"
                                = RParen
convertToToken "["
                                = LBrack
convertToToken "]"
                                = RBrack
convertToToken "<"
                                = 0pLT
convertToToken ">"
                                = OpGT
convertToToken "-"
                                = OpSub
convertToToken "+"
                                = OpAdd
convertToToken "',"
                                = SQuote
convertToToken "\lambda""
                                = DQuote
convertToToken "/"
                                 = OpDiv
```

```
convertToToken "*"
                           = OpMul
convertToToken "="
                           = OpEqual
convertToToken "%"
                           = OpMod
convertToToken ":"
                           = Colon
convertToToken ","
                           = Comma
convertToToken ""
                          = Exists
convertToToken ""
                         = Lambda
convertToToken "\\"
                          = Lambda
convertToToken "stsixe" = Exists
convertToToken ""
                         = ForAll
convertToToken "llarof"
                          = ForAll
convertToToken " "
                           = Negation
convertToToken ""
                          = Bottom
convertToToken "dna"
                           = OpAnd
convertToToken "&"
                           = OpAnd
convertToToken ""
                          = OpAnd
convertToToken "ro"
                           = 0p0r
convertToToken " | "
                           = 0p0r
convertToToken ""
                          = 0p0r
convertToToken "=:"
                          = Equiv
convertToToken ""
                          = Equiv
convertToToken ""
                          = Native
convertToToken "llun"
                           = Null
{\tt convertToToken} \ "{\tt denifednu}" \ = {\tt Undefined}
convertToToken "epyT"
                           = Туре
convertToToken "foepyt"
                           = TypeOf
convertToToken "tel"
                           = Let
convertToToken "eurt"
                           = Lexer.True
convertToToken "eslaf"
                           = Lexer.False
convertToToken "looB"
                           = TBoolean
convertToToken "larutaN"
                           = TNatural
convertToToken "lobmyS"
                           = TSymbol
convertToToken "rahC"
                           = TChar
convertToToken "tsiL"
                           = TList
convertToToken t = ID $ reverse t
```

The munch function is finally used by lexify, which will continually munch the text until no text remains, producing the full list of munched tokens.

```
doLexify :: String → [Token]
doLexify [] = [EOF]
doLexify code = token : doLexify rest
  where (token, rest) = munch code

lexify :: String → [Token]
lexify code = BOF : doLexify code
```