Transfinite games and the sequoidal exponential

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Abstract

Laird [Lai02] introduces the concept of a *sequoidal category*, an augmentation of a monoidal category by a new operation $\oslash$ (the sequoid) that has a natural model constructed using game semantics. The sequoid operator $\oslash$ in this game-semantic model allows us to present the exponential connective $A \mapsto !A$ of [Hyl97] and [AJM00] as a final coalgebra. Under further assumptions, one can prove coalgebraically that this final coalgebra $!A$ is the carrier for a cofree commutative comonoid on A, and is therefore a suitable model for the exponential from linear logic. The authors of [CLM13] note that it does not seem to be possible to construct cofree commutative comonoids using the sequoidal category axioms alone. I shall show that this is indeed impossible by presenting a sequoidal category of transfinite games in which the final coalgebra $!A$ admits no comonoid structure. I shall discuss ways in which this transfinite model can inform our study of the usual finitary model.

References

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