John Gowers' thesis

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1 Report

This thesis provides an in-depth analysis of the game semantics of Idealized Algol (IA), and nondeterministic and probabilistic extensions. It begins by describing the model and uses sequoidal structure to prove adequacy, and also "intensional full abstraction". It then develops two approaches to forming intensional models of extended versions of IA: via Kleisli categories and via parametric monads. Each one is presented via various universal properties and explained from an abstract perspective. The Kleisli construction is used for nondeterminism and the parametric monads for probability.

Along the way many interesting topics come up; the thesis displays an impressive facility to marshall concepts from different areas, and includes detailed proofs using diverse techniques. For example, I liked the treatment of multicategories and enrichment, and I greatly appreciated the result that must-testing observational equivalence for countable nondeterminism is sensitive to non-computable functions.

The thesis is a challenging read at times, and more high level explanation would sometimes be helpful. (Admittedly it is extremely hard to tell such a complex story in a way that makes it easy to follow.) I should add that the introduction is very well written.

There are also some technical points that need to be discussed in the viva, as listed below. In particular, the tensoring of random variables, the categorical size issues, and the benefit of the parametric monad approach.

Overall, this is clearly an impressive thesis, and a clear contribution to research into game semantics from a categorical viewpoint.

2 Viva discussion points

- Downwards closure condition in the definition of game (p32).
- Lemma 2.4.3, alternating sequences or general sequences?
- Definition 2.6.1 and 2.9.1, examples illustrating subset inclusion and tree immersion

- Start of Section 2.9.1, you say that tree immersion generalizes subset inclusion, but Proposition 2.9.8 says monomorphism whereas Proposition 2.6.6 says epimorphism.
- Proposition 2.11.8, second J intended?
- Proposition 2.13.2, commuting with tensor product?
- p70, "Suppose C is a monoidal category", are you assuming the unit is terminal.
- Theorem 2.12.3, A is fixed, there's no adunction.
- Proposition 2.13.6, arbitrary B? Not just well-opened?
- p75 "right adjoint on to its image"—what does this mean?
- p75, mid-page, there's no such adjunction, surely?
- Corollary 2.14.4, for $J(A \oslash -)$ on $\mathcal{G}$.
- Section 2.17, what's $!n$? How about using $I \cong I \oslash I$?
- Inititialization at 0 in Proposition 2.17.2 and introduction to language.
- Chapter 3, can you give a high level description of adequacy proof?
- p129, surely Jf is a morphism from a to b in the Kleisli category.
- Example 4.2.5, explain the appeal to the enriched Yoneda lemma.
- Definition 4.5.1, maybe introduce the `let` notation when you introduce the language, rather than after Theorem 4.11.11? Then use it repeatedly.
- Explain Lemma 4.5.4.
- Section 4.8, I don't think Proposition 4.8.1 is true according to your definition of strategy with costs, as you allow the costs to be nondeterministic.
- Corollary 4.9.3, what is $t_{|u|}$? Maybe t should be $\mathtt{tr}$? i
- Definition 4.9.4, missing "for all α"?
- p155, midpage, I don't think the alternative definition of must-convergence works.
- p158 midpage, "what it means" do you mean for given $w \in X^\omega$?
- Definition 4.10.9, I don't think the "i.e." is correct here.
- p162, In N, why is $f(\mathsf{ask}_{\mathbb{N}})$ needed? To prove observational equivalence using denotational equality? You could use CIU.
- p163 $[\![N]\!];\sigma$ maybe should be $\sigma;[\![N]\!]$.

- “is an instance of”?
- Proposition 4.12.1 what’s the type of α?
- Proposition 4.12.1, can you characterize intrinsic equivalence wrt $\approx$?
- Definition 5.1.1, do you mean that a Melliès morphism is a pair (x, f)? For otherwise composition is not well-defined.
- Definition 5.1.3, the coend requires $\mathcal{X}$ to be small. But your main example isn’t small, because (I think) an object is a pair (X, V) where X is a set.
- **Cat** consists of moderate categories, I think.
- Define delooping, i.e. is the diagrammatic-order composition given by $\otimes$, or the antidiagrammatic order?
- p178, “coherent isomorphism”. I would like to see this in more detail, to be sure that your condition is stronger.
- Proposition 5.4.4, “lax functor”, surely this is a map of oplax cocones.
- Definition 5.5.3, You need to assume that $\mathcal{C}$ is moderate (which is true in your example), otherwise $\pi_*\mathcal{C}$ won’t be moderate.
- Corollary 5.6.1, spell out J and ϕ.
- Corollary 5.7.2, seems to assume that any two lax limits are uniquely isomorphic. If that’s true, it should be stated.
- Spell out Definition 5.8.1.
- p189, “symmetric monoidal lax action”, what does symmetric mean here?
- Proposition 6.1.1, seems to assume that $\mathcal{X}$ is small. And Proposition 6.2.1 assumes that $\mathcal{Y}$ is. And Proposition 6.3.1 constructs non-small $\mathcal{X}$, but seems to require it to be small.
- Example 6.3.5, on the face of it the argument that $\langle V, W\rangle$ is a random variable doesn’t seem to work, and therefore RV_Ω isn’t monoidal.
- Bottom of p224, assumes $\mathcal{C}$ is small.
- p229, “we can identity” but this is a profunctor from $\mathcal{D}$ to $\mathcal{C}$ according to your definition.
- Proposition 7.16.2, for $[\mathcal{X}, \mathbf{Set}]$ to be self-enriched, $\mathcal{X}$ needs to be small. (Even though $\mathcal{X}$ isn’t small in your main example, it’s still interesting to see these results in the case that $\mathcal{X}$ is small.)
- Proposition 7.12.1, should these hats be tildes?

- Start of Chapter 8 is slightly confusing, especially whether $\mathcal{X}$ or $\mathcal{X}^{\mathsf{op}}$ is intended.
- p239 $jp \to a$ shouldn't this be $\underline{jp} \to a$?
- p239 "is the object $J[\![T]\!]_{\mathcal{G}}$", isn't J identity-on-objects?
- p239, ω doesn't go from 1 but from I as stated on the next line.
- Is it important that your version of Idealized Algol has no empty datatype?
- p242, and if your version of Idealized Algol had only boolean and command datatypes, what would go wrong?
- Definition 8.4.2, X has to be jp, right?
- p249, 4 lines down, shouldn't u be an element of $(jN)^*$, according to Definition 8.3.1?
- Proposition 8.5.7, but x is an object of $\mathcal{X}$, not a set.
- A random variable is a pair (X, V), right?
- Explain bottom of p252.
- Section 8.7, how does the first sentence follow from Proposition 8.7.1?
- Section 8.8, maybe considering $IA_{(X_i)_{i\in I}}$ would give you the best of both worlds?
- Proposition 8.9.3, B isn't mentioned.
- Definition 8.9.4, is this representation-independent?