1 Monads and Lawvere Theories

One of the oldest ideas relating to the treatment of effects in computer science is the idea that an effect should be captured by a suitable monad (or comonad) on our category, and that the effectful category should then be given by the Kleisli category of that monad. Typically the base category is taken to be the category of sets, and it is not always obvious how to relate the construction to other categories – for instance, finite nondeterminism is often regarded as being captured by some finite powerdomain monad on **Set**. However, some particular examples are clearly more general. For example, the *state monad* $S_W X = W \multimap (X \otimes W)$ exists in any monoidal closed category, while the *reader monad* $R_W X = W \Longrightarrow X$ exists in any Cartesian closed category (and the equivalent *reader comonad* $R^W X = X \times W$ exists in any Cartesian category).

Example 1.1. Dan Ghica's category of *slot games* is equivalent to the Kleisli category for the reader monad $R_{\mathbb{C}}$ on a more traditional category of games, where $\mathbb{C}$ is the singleton or command-type game. Slot games are used to measure time complexity; we can think of the Kleisli morphism $\mathrm{ask} \colon (\mathbb{C} \Longrightarrow X) \to X$ (given, in the base category, by the identity on $(\mathbb{C} \Longrightarrow X)$) as being analagous to composing with a term $\mathrm{wait}_T \colon \mathbb{C}$ that pauses for some fixed length of time.