

We start by defining a recursive-inductive type `bsPlay` that will encode a Brussels Sprouts position. The attributes that we keep track of are

- The set of crosses in the position.
- The set of edges in the position.
- The source and target functions that identify the beginning and ending crosses of each edge (This introduces a direction to our graph which is not part of Brussels Sprouts, but we shall ignore this extra information).
- The set of faces in the position.
- For each cross, the faces that each crossbar extends into.

This datatype is constructed inductively according to the following rule:

- Given any face of the position and any pair of crossbars that extend into that face, we may draw an edge joining the two crossbars and then add a new cross on that edge (so we have actually created two new edges).

```

\\
\\>\AgdaKeyword{module} \AgdaModule{brussels\_sprouts} \AgdaKeyword{where}\<%
\\
%
\\
\\>\AgdaKeyword{open} \AgdaKeyword{import} \AgdaModule{Data.Empty}\<%
\\
\\>[0]\AgdaIndent{2}{}\<[2]%
\\>[2]\AgdaKeyword{renaming} \AgdaSymbol{()}\AgdaDatatype{\perp} \AgdaSymbol{to} \AgdaDatatype
\\
\\>\AgdaKeyword{open} \AgdaKeyword{import} \AgdaModule{Data.Sum}\<%
\\
\\>[0]\AgdaIndent{2}{}\<[2]%
\\>[2]\AgdaKeyword{renaming} \AgdaSymbol{()}\AgdaDatatype{\_\\_} \AgdaSymbol{to} \AgdaData
\\
\\>\AgdaKeyword{open} \AgdaKeyword{import} \AgdaModule{Data.Product}\<%
\\
%
\\
\\>\AgdaKeyword{data} \AgdaDatatype{[U+2736]} \AgdaSymbol{:} \AgdaPrimitiveType{Set} \Agda
\\
\\>[0]\AgdaIndent{2}{}\<[2]%
\\>[2]\AgdaInductiveConstructor{*} \AgdaSymbol{:} \AgdaDatatype{[U+2736]}\<%
\\
%
\\
\\>\AgdaKeyword{data} \AgdaDatatype{bsPlay} \AgdaSymbol{:} \AgdaPrimitiveType{Set}\<%
\\

```

```

\>\AgdaFunction{vertices} \AgdaSymbol{:} \AgdaDatatype{bsPlay} \AgdaSymbol{→} \AgdaPrimitive
\\
\>\AgdaFunction{edges} \AgdaSymbol{:} \AgdaDatatype{bsPlay} \AgdaSymbol{→} \AgdaPrimitive
\\
\>\AgdaFunction{source} \AgdaSymbol{:} \AgdaSymbol{()}\AgdaBound{p} \AgdaSymbol{:} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{target} \AgdaSymbol{:} \AgdaSymbol{()}\AgdaBound{p} \AgdaSymbol{:} \AgdaDatatype{bsPlay}
\\
%
\\
\>\AgdaKeyword{data} \AgdaDatatype{bsPlay} \AgdaKeyword{where}\<%
\\
\>[0]\AgdaIndent{2}{}\<[2]%
\>[2]\AgdaInductiveConstructor{emptyPlay} \AgdaSymbol{:} \AgdaDatatype{bsPlay}\<%
\\
\>[0]\AgdaIndent{2}{}\<[2]%
\>[2]\AgdaInductiveConstructor{addVertex} \AgdaSymbol{:} \AgdaDatatype{bsPlay} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
\>[0]\AgdaIndent{2}{}\<[2]%
\>[2]\AgdaInductiveConstructor{addEdge} \AgdaSymbol{:} \AgdaSymbol{()}\AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
%
\\
\>\AgdaFunction{vertices} \AgdaInductiveConstructor{emptyPlay} \AgdaSymbol{=} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{vertices} \AgdaSymbol{()}\AgdaInductiveConstructor{addVertex} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{vertices} \AgdaSymbol{()}\AgdaInductiveConstructor{addEdge} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
%
\\
\>\AgdaFunction{edges} \AgdaInductiveConstructor{emptyPlay} \AgdaSymbol{=} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{edges} \AgdaSymbol{()}\AgdaInductiveConstructor{addVertex} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{edges} \AgdaSymbol{()}\AgdaInductiveConstructor{addEdge} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
%
\\
\>\AgdaFunction{source} \AgdaInductiveConstructor{emptyPlay} \AgdaSymbol{()}\<%
\\
\>\AgdaFunction{source} \AgdaSymbol{()}\AgdaInductiveConstructor{addVertex} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
\>\AgdaFunction{source} \AgdaSymbol{()}\AgdaInductiveConstructor{addEdge} \AgdaBound{p} \AgdaSymbol{→} \AgdaDatatype{bsPlay}
\\
%

```

```

\\
\>\AgdaFunction{target} \AgdaInductiveConstructor{emptyPlay} \AgdaSymbol{()}\<%
\\
\>\AgdaFunction{target} \AgdaSymbol{()}\AgdaInductiveConstructor{addVertex} \AgdaBound{p}
\\
\>\AgdaFunction{target} \AgdaSymbol{()}\AgdaInductiveConstructor{addEdge} \AgdaBound{p} \
\\
\>\<%

```