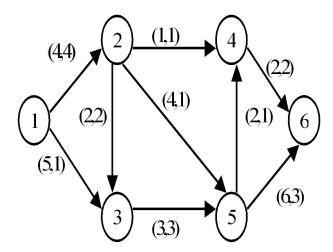
## MP305 Practical 2020/2021 - Network Flows I

The Python notebook Network\_Flows\_I that contains the maximal network flow algorithm can be accessed via any web browser. See the MP305 Blackboard web page for details and instructions.

Solutions to all questions with (\*) have to be submitted as a pdf document through Blackboard. This practical is worth 3% of your final grade.

1. Find the maximal flow for minimal capacity for the network below where the capacity c(i,j) and the flow  $\phi(i,j)$  is shown on each arc (i,j):

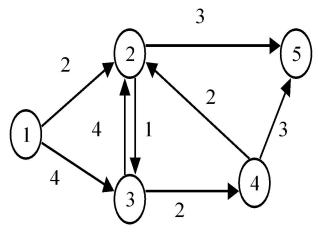


This is the example discussed in the lecture notes. You may read in the data for this example from the Python notebook Network\_Flows\_I. See the MP305 Blackboard web page for details and instructions.

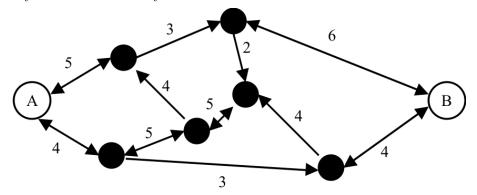
- (a) Find the incremental network and capacities at each iteration of the Ford Fulkerson algorithm.
- (b) In the **last** iteration when the maximal flow is found, identify which arcs are normal and which are inverted in the incremental network.
- (c) Hence find the minimal capacity cut of this network flow model.

In the following two problems, first define the network and its capacities following the template of problem 1 and then run the Python code.

2. (\*) Find the maximal flow through the following network with the given capacities:



- (a) Set the initial flow to 0 at each arc and find the incremental network and capacities at each iteration of the Ford-Fulkerson algorithm.
- (b) In the **last** iteration when the maximal flow is found, identify which arcs are normal and which are inverted in the incremental network.
- (c) Hence find the minimal capacity cut of this network flow model.
- 3. (\*) A road network is shown below with the capacity on each road indicated. Notice that many roads are two way.



- (a) Find the maximal flow through the network from A to B.
- (b) Compare this to maximal flow from B to A.