NUI Maynooth | CS355
Computer Science Department | Week No. 3

EXERCISE SHEET NO. 2

DFAs

Exercise 1. Let $\Sigma = \{a, b\}$. Show the following sets are regular (draw a DFA).

1. $\emptyset$.
2. $\{a^m a^n | \ m > n > 0\}$
3. $\{w | \ w$ begins with a and ends in $b\}$
4. $\{w | \$ the third symbol of w is $a\}$
5. $\{w | \ w$ starts with a and has odd length or starts with b and has even length$\}$
6. $\{w | \$ every odd position of w is an $a\}$
7. $\{w | \ w$ contains an even number of a or exactly two $b\}$
8. $\{w | \ w$ is any string except aa and $aaa\}$
9. $\{w | \ w$ contains neither the substrings ab nor $ba\}$.
10. $\{w | \ w$ does not end in $aab\}$.
11. $\{xy | \ x, y \in \Sigma^*$ and x contains more a's than $y\}$

Exercise 2. Let $\Sigma = \{a, b\}$ and $A = \{w \in \Sigma^* | \ w$ contains $abab\}$.

1. Show that $A \in \text{REG}$ by giving the state diagram of a DFA M recognizing A. Test your DFA on $aababb$, $bbaababb$.
2. Given any set $L \subseteq \Sigma^*$, define its complement by $\bar{L} = \{x \in \Sigma^* | \ x \notin L\}$. Describe the set $\bar{A}$.
3. Using your DFA M for A, construct a DFA for $\bar{A}$.
4. Using the same idea, prove the following: REG is closed under complement i.e., for any $A \in \text{REG}$, $\bar{A} \in \text{REG}$.

Exercise 3.

1. Using the DFA M from the previous exercise, show using the formal definition seen in class that M accepts string $aababb$.
2. Write a formal definition for "N rejects string w", where N is a DFA.
3. Write a formal definition for "N rejects string w", where N is an NFA.

NFAs

Exercise 4. Let $\Sigma = \{0, 1\}$. Give the state diagrams for the following sets.

1. $\{w|\ w$ ends with $01\}$ with 3 states
2. $\{w|\ w$ contains the substring $1101\}$ with 5 states
3. $\{w|\ w$ contains an odd number of 0, or exactly two $1\}$ with 6 states
4. $\{0\}$ with 2 states
5. $\{\epsilon\}$ with 1 state
6. $\{0\}^*$ with 1 state

Exercise 5.

1. Draw an NFA recognising the set of binary strings ending in 00. Your NFA should have at most three states.
2. Modify your NFA into a DFA using the procedure seen in class. Simplify your machine by removing useless states (i.e. with no incoming arrows).