Heuristic Search

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Dept. Computer Science, Maynooth University, Co. Kildare, Ireland.

Search - Overview

Different Search strategies are used in many areas of AI

1. Blind Search
2. Heuristic Search
   - A*
3. Adversary Search (Game playing)
   - MiniMax
   - α–β Pruning
- Also Graph Search

Why **Search**?

- **Solve** problems
  - Oxo, Farmer-Fox-Goose, combinatoric problems
  - Chess, Checkers & competitive games
  - Shortest path to exit a maze
  - Find the best path for an NPC to attack a player character in a Computer Games…
- Create **predictive** models of
  - war, economics & other domains

Problem Solving

- Many AI problems can be solved by **Searching**
- Different search techniques are employed, each with their own characteristics
- **Generate and Test** **(G-n-T)**
  - **Problem Spaces** are **Generated** representing (all/many) possible solutions
  - **Test** different spaces to identify successful solutions

Sliding Tile Puzzles

- Such as 8 and 15 tile puzzles
- **Goal** state and **Start** State :

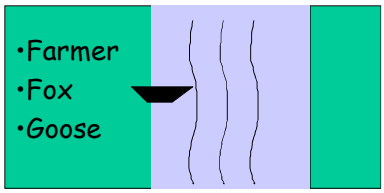
1	2	3
4	5	6
7	8	

8	5	3
1	2	7
6	8	

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

http://www.jaapsch.net/puzzles/fifteen.htm
http://www.tilepuzzles.com/default.asp?p=13

Farmer, Fox and Goose (1)



- Get all three across the river safely
- Constraints
  - The boat can only fit two "people" at a time
  - Fox and Goose can never be alone together on either bank, or in the boat.

Blind Search

- For small problem spaces we can generate every possible solution, using:
  - **DFS** - Depth First Search order
  - **BFS** - Breadth First Search order
- However, blind search strategies can only solve toy problems
  - Chess has 10^{140} different possible board states and all positions could never be generated
    - >> number of nanosecond since the big bang!

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Backtracking

- **Backtracking** occurs when the part of the search space is found to be unproductive
- By a roll-back to a previous recursive position, we continue searching alternate branches
  - backtracking occurs very frequently during problem solving
  - Prolog executes programs as a search and backtrack process

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Lookahead

- Determines the "depth" of the search space to be explored
- Its really just a simple integer parameter that's decremented every time the recursive function calls itself
  - Limited by a zero lower bound
  - Depth Limited Search

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DFS - Depth First Search

- Recursive blind-search mechanism
- **Pre-Order** search examines the root before visiting either sub-tree
    - yielding a program employing **tail-recursion**, with a very low memory overhead
    - + 2 3
  - **Post-order** examines the root after subtrees
    - 2 3 +
  - **In-order** visits one subtree, then the root and then the other subtree
    - 1 + 2

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BFS - Breadth First Search

- Recursive blind-search mechanism
- **Post-Order** search; stores the root node while it examines both sub-trees
    - yielding a program employing **head-recursion**, with a very high memory overhead
      - storing the entire search tree
  - Post-order examines the root after subtrees
  - In-order visits one subtree, then the root and then the other subtree
  - Sigma(n) = Sigma(n-1) + n
  - Sigma(n) = n + Sigma(n-1)

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Iterative Deepening (ID)

- Breadth FS searches to a predetermined depth.
- ID tries all depths, 1, then 2 then 3 etc.

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The 8/15 Tile Puzzles

- In the case of the 8-tile puzzle:
- **Goal State** might be:


1	2	3
4	5	6
7	8	

- but **Start State** might be

8	5	3
1	2	7
6	8	

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

http://www.jaapsch.net/puzzles/fifteen.htm

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Search Spaces

- Consider the following search space(s)

2	8	3
1	6	4
7		5

Start state

Generate each 3 alternate next state from the initial state.
-generate(Board) -> board

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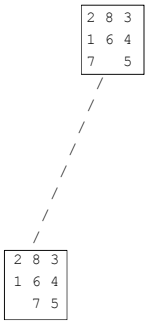
Search Spaces

- Consider the following search space(s)



Start state

Generate each 3 alternate next state from the initial state.
-generate(Board) -> board

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Search Spaces

- Consider the following search space(s)

Start state

Generate each 3 alternate next state from the initial state.
-generate(Board) -> board

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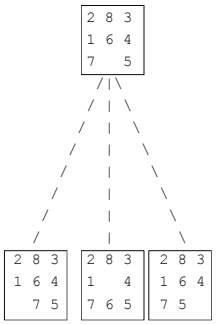
Search Spaces

- Consider the following search space(s)



Start state

Generate each 3 alternate next state from the initial state.
-generate(Board) -> board

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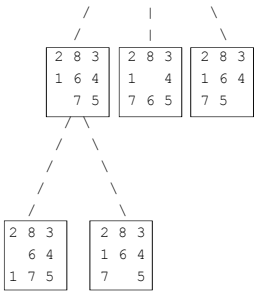
Search Spaces ctd..

- expanding one branch further, we get



2 subsequent states for one of those states

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Complexity of Search

- **Branching Factor** determines the extent (size) of the search space - B
- Path length - L
- Total Search space - T
- $T=B(B^L - 1)/(B - 1)$
  - $T=B + B^2 + B^3 + + B^L$

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Farmer, Fox, Goose & Beans (2)

- Once upon a time a **farmer** went to market and purchased a **fox**, a **goose**, and a **bag of beans**. On his way home, the farmer came to the bank of a river and hired a boat. But in crossing the river by boat, the farmer could carry only himself and a single one of his purchases - the fox, the goose, or the bag of the beans.
- If left alone, the fox would eat the goose, and the goose would eat the beans.
- The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact. How did he do it?
- **Solution:** Bring goose over, Return, Bring fox or beans over, Bring goose back, Bring beans or fox over, Return, Bring goose over
- Thus there are seven crossings, four forward and three back.

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Heuristic Function - eg

- Heuristics are **Rules-of-Thumb** which help identify potentially fruitful portions of the problem space
  1. *"capturing pieces helps win at chess"*
  2. *"advancing up the board is a good idea"*
  3. *"loosing a piece damages your chances"*
  4. *"try to occupy the centre of the board"*
- **Static Evaluation Function** yields a numeric score for each solution space
  - *problem space* maps onto a *number*

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Heuristic Search

- The heuristic function can be used to guide the search process
  - focus on the states with a high heuristic value
  - eliminate some search states with very low heuristic values
    - and their sub-states, their sub-sub-states *etc.*

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Greedy Best First Search

- Heuristic search mechanism
- Like depth-first search, but guided by the heuristic value

1. Generate all *next* states (look ahead one "move")
   - We might be able to take a short-cut here
2. Select the (1) best of these states – adopting this as our current state
   - Using our heuristic function
3. Go to 1.

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Best First Search

- Use the heuristic to identify (beforehand) the best squares on the board
- Use this ordering to "guide" Best First Search
  - 5, 1, 3, 7, 9, 2 ,4 ,6 8,

1	**2**	**3**
4	**5**	**6**
7	**8**	**9**

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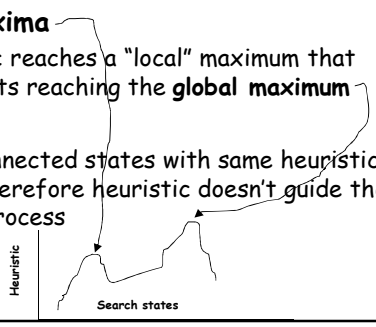
Local Maxima & Plateau

- 2 Main problems that plague Heuristic search
- **Local Maxima**
  - Heuristic reaches a "local" maximum that inhibits its reaching the **global maximum**
- **Plateau**
  - Many connected states with same heuristic value. Therefore heuristic doesn't guide the search process



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A* Search

Best First Search Algorithm

See also http://en.wikipedia.org/wiki/A*_search_algorithm

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"Repeated States" Anomaly

- Of course, there are many ways to reach many game states.
- In chess, different placement order produce the same state
  - but by different paths
- We don't want to duplicate path information
- Path A says its 5 move but path B says its 7 moves
  - only store the "shortest" path

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DAG

- Trees – DAG – Graph
- DAG - Directed Acyclic Graph

- A* ensure the search stays as a tree
  - Never turns into a DAG

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A* Fundamentals

- **Generalised Best-First Search**
- Extends one node per **iteration**
- **f'** approximation of current node's worth
- **g** cost of getting to current node (actual)
- **h'** additional cost of getting to solution

f' = g + h'

- **Closed:** (*internal*) set of nodes whose successors have all been examined *(internal nodes?)*
- **Open**: (*leaf*) set of nodes whose successors have not yet been examined *(leaf nodes?)*

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A* = Best F. S.

- **f'** approximation of nodes worth
- **g** cost of getting to current node (actual)
- **h'** additional cost of getting to solution
  - BFS => h'=0
- **f' = g + 0**

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A* Intro

1. Pick **Bestnode** with lowest f'
2. Compute Successor (Bestnode)
   - g(Bestnode) + new-cost
3. Set Successor to point back to Bestnode, so we can recover moves later
4. If Successor ε Open, then link it (*Successor(Bestnode)*) to the cheaper of Bestnode-path or Oldnode-path, by comparing g values

Note: Each node points to the best parent node

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A*

5. If Successor ε Closed, call it Old

If newpath to old is less than old-path to Old then set g and f' values of Old appropriately

BUT must propagate f' to Successors

DFS Successors(Old) altering g until find a node with equivalent or better g value

But: If in propagating g down, the path may become better than nodes current parent,

then Stop else reset parent and continue propagating

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A*

6. If Successor not in Open or Closed, put in Open and add to Bestnodes successors

f'(Successor) = g(Successor) + h'(Successor)

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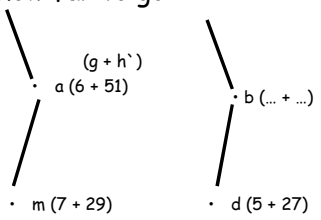
A* Example

g = actual cost of getting to state

measured in number of moves?

h' = estimate of how far to go



•4 states depicted a, b, m, d g and h` values are illustrated

•a and b are in closed m and d are leaf nodes – in open

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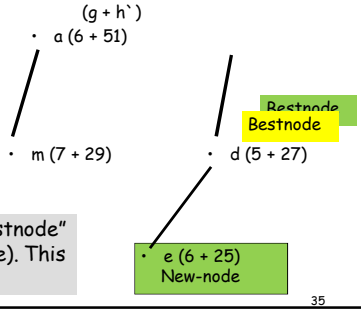
Situation 1 A*

The Simplest case is when we develop a totally new successor of d(Bestnode) e.



Select and Develop "Bestnode" d to create New-Node(e). This is a successor of d.

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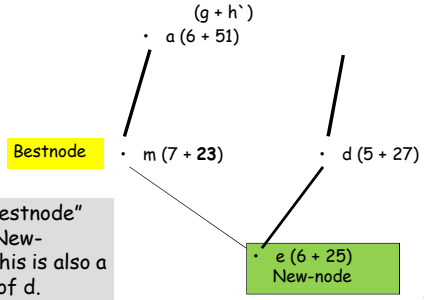
Situation 2 A*

M is Bestnode, but its successor e already exists and has a better path ...



Develop "Bestnode" to create New-Node(e). This is also a successor of d.

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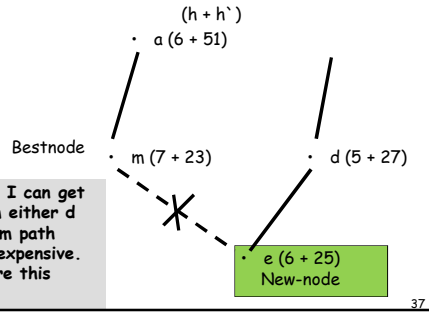
Situation 2 A*

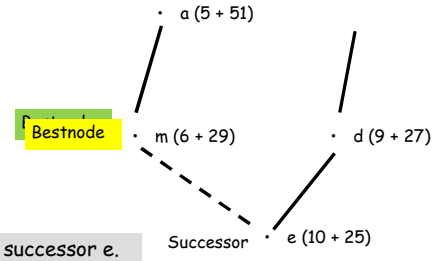
.. Then we don't maintain the newer but less-efficient path.



In this example I can get to state e from either d or m. The new m path would be more expensive. So, I *don't* store this possibility!

Situation 3 A*

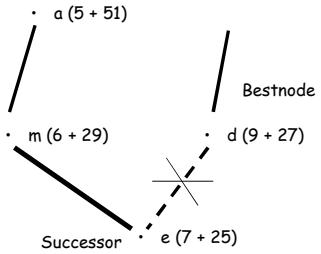
M is Bestnode. Its successor e already exists, but we now have a better path to e



D generated its successor e. But e is also a successor of m. We only keep one path to e

Situation 3 A*

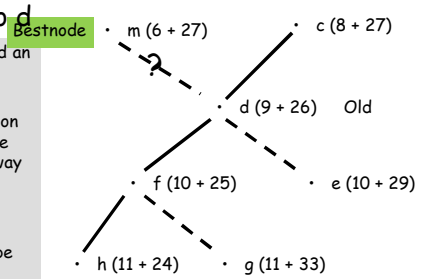
... so we must update that successor. So we must replace the d-b branch with m-e.



We keep the new "short cut"!

Situation 4 A*

M is Bestnode, its successor d already exists and had "children" and we found a shortcut to d



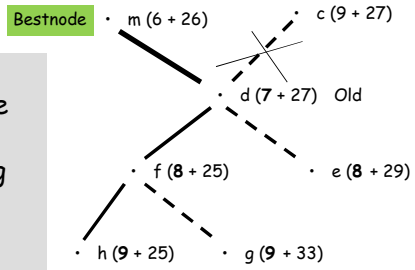
We have generated an existing node – d.

d has many follow-on states, but have we found a cheap**ER** way of getting to d?

If we're luck, the existing path will be cheapest…

Situation 4 A*

Re-parent d (from c to m) and propagate the changes to the successors of d (f,e,h,g...)



Updating a Closed node might mean propagating **new** values down the tree.

- Other heuristically founded search techniques

Tabu Search

- As stated, Hill-Climbing (best F. S.) often works dreadfully
  - Getting stuck in local maxima at the very beginning
- Tabu Search overcomes this problem for almost all domains (sub-optimal)
- Maintain a list of "recent" nodes & Never revisit a recent node
  - even if you have to reduce the heuristic value to avoid this.

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BLAST Algorithm

- Blast is a **heuristic** based search used by DNA researchers
- They often look for a "target" of 1000 nucleotides in a GenBank database of billions of nucleotides
  - …AGTTAC… target query
  - | | | |
  - …ACTTAG… GenBank database
- 50 times faster than optimal, but less accurate

1. Look for a partial "**seed**" match
2. **Extend** seed match in both directions

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Blast Heuristic

- Basic Local Alignment Search Tool
- Only **high-scoring** matches are passed to 3rd & final stage
  - heuristic requires = 11 nucleic seeds

3. Blast then looks for **gaps** in alignment between squences
   - …AGTTAC… target
   - ||| \\
   - …AGT…TAG… GenBank database

Blast paper is highly cited. See Document Ranking notes nearer end of course.

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Adversary Search
Game Playing Search

Search in the presence of an opponent

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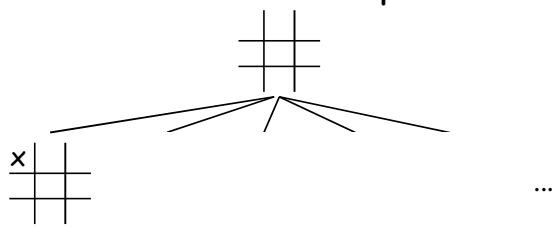
OXO Search Space



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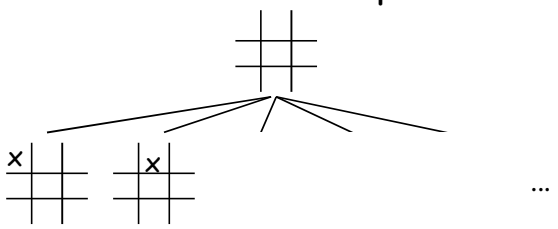
OXO Search Space



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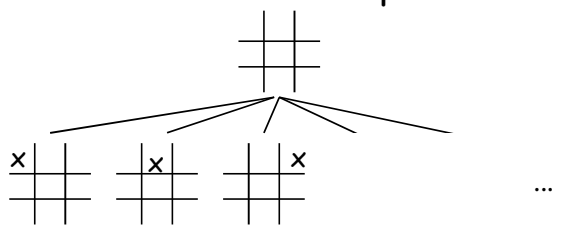
OXO Search Space



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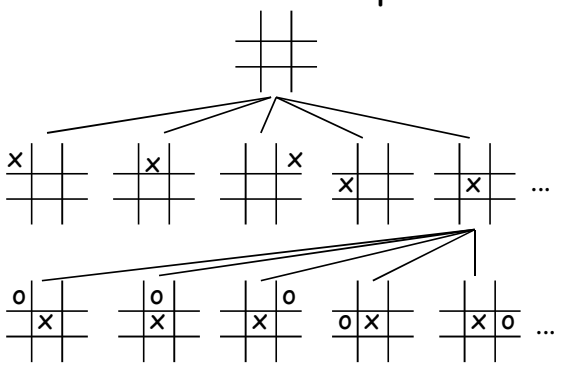
OXO Search Space



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Adversary Search

- Many **games** involve competition between opponents
- When generating the problem space we must assume the opponent will counteract our moves
  - and not willingly try to loose
- Our search procedure must know whose move it is in order to know how to apply the heuristic rule

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Game Theory

- **Zero-sum** states that one player's gain is the opponent's loss
  - these are the simplest games, covers the games we will look at *eg chess, tic-tac-toe*
- **Zero-sum payoff rule**
- As opposed to Mutual loss and Mutual gain (cooperative) games
  - Consensus decision making (eg contract law)

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Game Theory 2

- **Equilibrium** is when both player are unlikely to change their strategies
- **Equilibrium** can be achieved when no change in strategy can improve the players position
  - regardless of what the other player does
- Game theory is also used in the study of (competivive) Economic (Nash, 1/3 Nobel in Economics '94), Political and Warfare systems
  - See "Nash Equilibrium".

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Game Theory 2

- **Minimax Theorem**
  - Fundamental theorem of Game Theory
- **Every Finite, Zero-Sum, two-person Game has optimal Mixed Strategies.**
  - Mixed Strategy: A collection of moves together with a corresponding set of weights which are followed probabilistically in the playing of a game.

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MiniMax

- A depth-first depth-limited search which examines all states
- **2 phase** algorithm
  - descend to the search ply limit and apply the heuristic function
  - propagate these values up the tree
- My move: I **maximises** the heuristic value
- Opponents move: Opponents **minimises** the heuristic value function(h)
  - Alternate between max. and min. levels

http://en.wikipedia.org/wiki/File:Plminmax.gif

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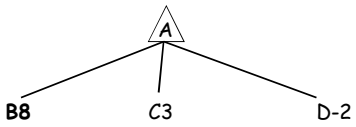
Minimax Eg



•Generate and evaluate future game positions

• We *would* select B as it gives us the best (maximum) heuristic score, but…

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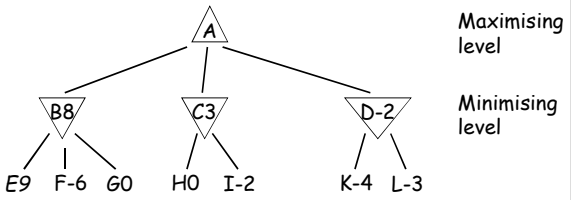
Minimax Eg



- Two-ply moves generated
- At the limit of search space, Leaf values get "backed up" to previous levels
- Which **move** should we select?

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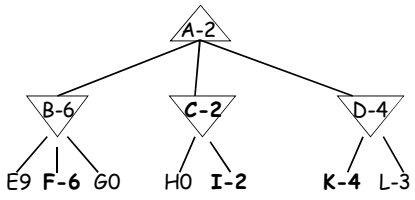
Minimax Eg



- Selecting B lets the opponent "clobber" us - will end up in F(-6)
- So, select C as even a clever opponent can't hurt us more than I(-2)

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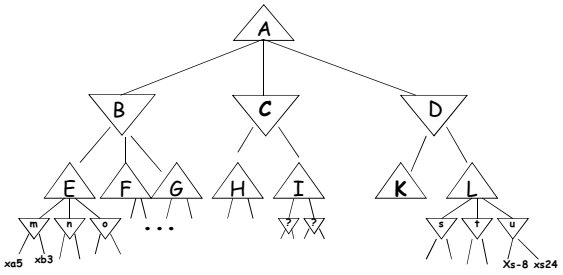
4-ply search



- What a 4-ply search might look like

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Minimax Problems

- Exhaustive
- Explores unnecessary search spaces
  - slow, inefficient etc.
- A more clever algorithm could do better
  - see α–β pruning which can double the explored space

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α–β Pruning

- Remove **(prune)** *irrelevant* states from MiniMax search tree
  - depth first expansion of Minimax tree
- Two threshold values (global variables) required, one for each player
- α **alpha** - represents the **lower** bound of the **maximising** level
  - *alpha at least*
- β **beta** - represents the **upper** bound of the **minimising** level
  - *beta below*

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α–β Pruning

- α–β pruning allows MiniMax to search much "deeper" within the same amount of time
  - removes many irrelevant states
- So, we can now look 6 moves ahead instead of just 4
  - Or maybe even 8 moves ahead!!

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α–β Pruning

- Terminate a search
  1 below a MAX node with α >= β of *any* of its MIN ancestors
  2 below a MIN node with β <= α of *any* of its MAX ancestors
- Depth-first search means we only ever consider nodes along one path at a time
- Pruning for the Maximising player, means cutoff at the minimising level
  - and vice versa

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- Check out the Wikipedia entry
  - http://en.wikipedia.org/wiki/Alpha-beta_pruning
  - View the animated graphic half way down the page
  - It will take about 10 minutes to view the full playable-gif.

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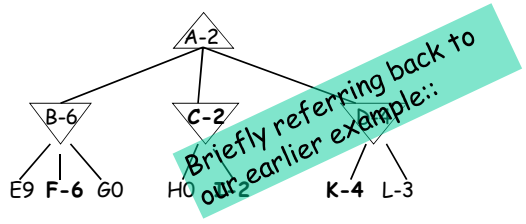
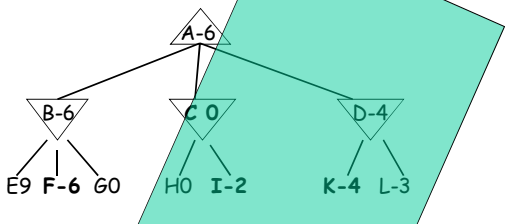
Minimax Eg



- Selecting B lets the opponent "clobber" us - will end up in F(-6)
- So, select C as even a clever opponent can't hurt us more than I(-2)

65

Minimax Eg



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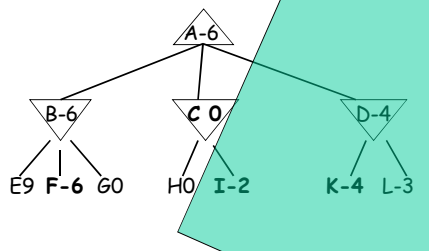
66

Minimax Eg



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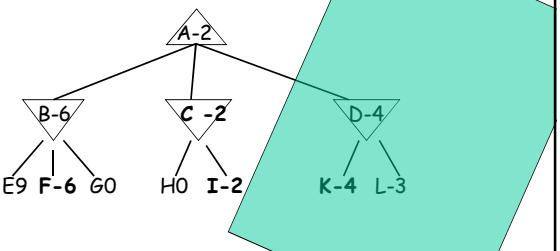

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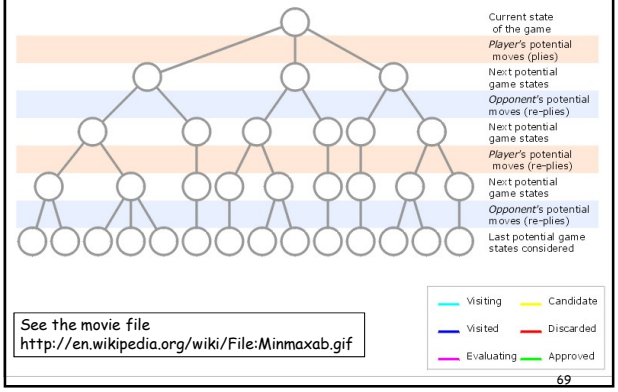
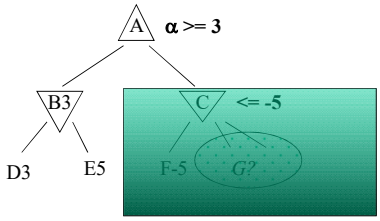
Minimax with alpha-beta pruning on a two-person game tree of 4 plies



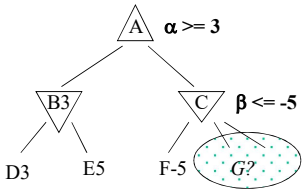
See the movie file
http://en.wikipedia.org/wiki/File:Minmaxab.gif

Simple α–β Pruning



- Regardless of what G is worth, we are guaranteed a score of -5 or less at C
- This is worse than α=3, so we don't continue with the rest of the C sub-tree

Simple α–β Pruning



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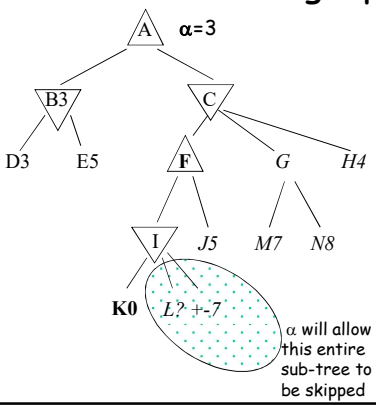
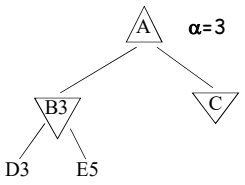
α based Pruning - preview



α–β Eg



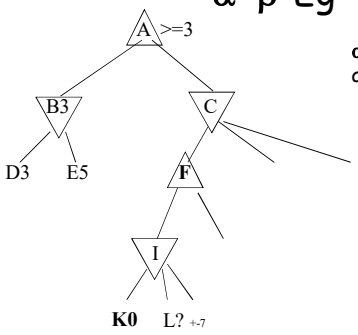
Maxmising level

Minimising level

Maxmising level

Minimising level

α–β Eg

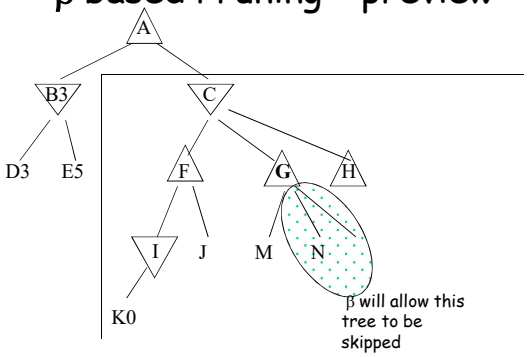


After examining K0, so *I* <= 0 (β=0), thus F >= 0 (which would α=0) - so abandon the rest of the C sub-tree

Using alpha

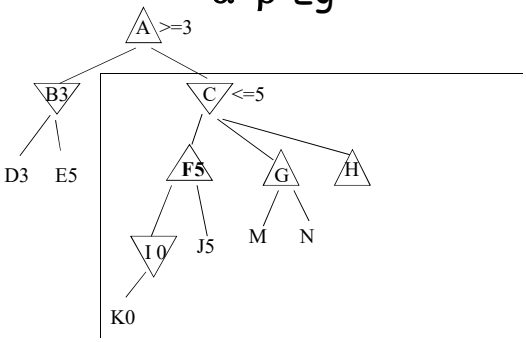
- D3 gets backed up so A>3 and α gets 3.
- Then K<-0, so I is guaranteed a max of 0, (β=0) which means F is guaranteed a minimum of 0
- This is less than the existing α=3, so we don't need to explore the rest of I!
  - so skip nodes: L and its siblings

β based Pruning - preview



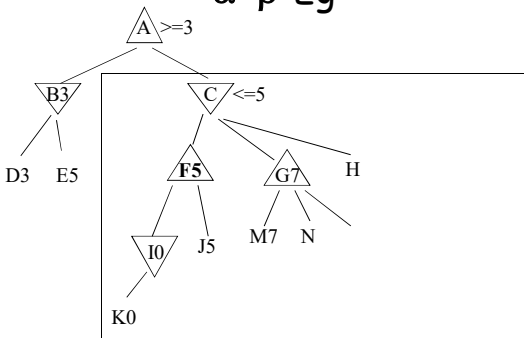
Moving on to the opponents move...

α–β Eg



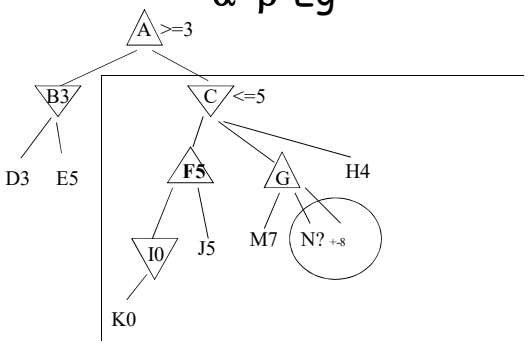
after examining J5, this is passed to F becoming α(*C*)<=5. Now...

α–β Eg



Expanding G to M we get 7, which is passed up to G. But 7>β so this player will not choose G.

α–β Eg



so the rest of the G sub-tree can be "pruned"!

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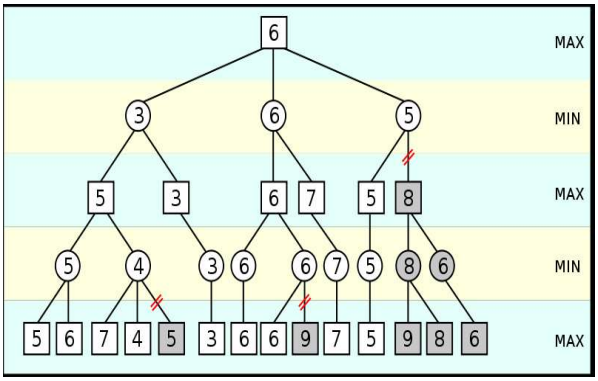
α-β Pruning Example



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Using beta

- β prunes branches of a Maximising node
- after finding that Fβ=5
  - C is guaranteed 5 or less
- Now expand G, giving M7 which is passed back to G
- but 7 > 5(beta), my opponent trying to minimise the Heuristic value
  - so player will not choose G as this would lead to a score of at least 7
  - so skip the rest of G

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Quiescent Search

- A further optimisation strategy utilized by the Deep Blue Chess program
- *Don't cut off the search in the middle of an exchange of pieces*
- If there is a large variation in the heuristic value across "sister" states, then extend the search depth for that sub-stree

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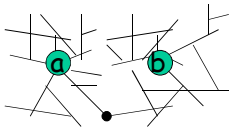
Bi-Directional Graph Search

- **Graph Searching** algorithm
  - labeled, interleaved iterative deepening



http://en.wikipedia.org/wiki/Six_Degrees_of_Kevin_Bacon

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6 degrees of Kevin Bacon

- If you use Kevin Bacon (the actor) as an end point, you can link him in six degrees or less to almost any other performer.
- Here's how we connect Kevin Bacon to the Alien in the Alien1 movie:
  - Kevin Bacon was in Footloose with Dianne Weist,
  - who was in The Birdcage with Gene Hackman,
  - who was in The Firm with Holly Hunter,
  - who was in Copycat with Sigourney Weaver,
  - who was in Alien with the Alien.

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Big Bushy Networks

- N neighbours for each node
- X look-ahead
- X^N -> 10^8 = 100,000,000
- 2 * $X^{(n/2)}$ -> 2 * 10^4 = 20,000

- **Small World Phenomenon**
- Choosing any 2 random people on earth, and looking at friends and friends of friends, how many "links" separate those people?

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Graph: Shortest Path

- Searching in graphs is central to many disciplines
- Epidemiology simulations and understanding
- Estimates of similarity between words
  - ***visuwords.com/***
- What is average number of links separating 2 pages on the internet?

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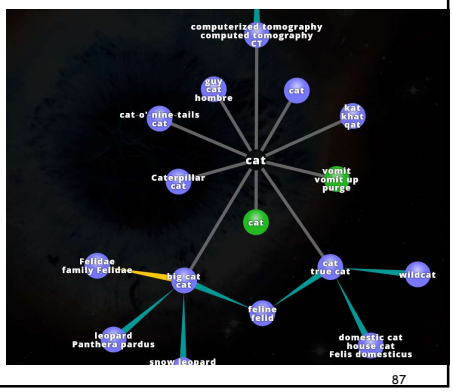
Compute similarity between words in a Thesaurus style graph



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Conclusion

- Search can solve problems
- Heuristic search can solve (some) very complex problems

1. A* Algorithm eliminates duplicate paths to search states
2. MiniMax used for adversary-based search
3. α-β pruning optimises adversary-based search

- Graph searching is broadly applicable

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