NUI Maynooth | CS355
Computer Science Department | Week No. 2

EXERCISE SHEET NO. 1

Note: to draw all machines studied in CS355-370 (DFA, PDA, CFG, TM etc) you can use JFLAP (should be installed on lab machines). It also allows simulations, playing with pumping lemmas, conversions etc.

Math Preliminaries

Exercise 1. Let $A = \{a, b, \emptyset, \{a, b\}\}$. Answer with True or False.

1. $\emptyset \in A$
2. $\{a\} \subset A$
3. $\emptyset \in 2^A$
4. $\{\emptyset\} \in 2^A$
5. $\emptyset \subset 2^A$
6. $\{\{a, b\}\} \subset 2^A$

Exercise 2.
Let $A = \{a, b, c\}$ and $B = \{b, c\}$. For each of the following sets do: If the set is finite write all elements in the set and two elements not in the set. If the set is infinite write two elements in the set and two not in the set.

1. A^*
2. $(A \cap B)^*$
3. $B^* \cap \{\epsilon\}$
4. $B \cap \{\epsilon\}$
5. $A^* \cap B$
6. $\{x \in B^* : |x| \leq 2\}$
7. $B \times B^*$

DFAs

Exercise 3. Give the formal description of automaton in Figure 1 (below).

Exercise 4. Draw the state diagram of the automaton $M = (Q, \Sigma, \delta, q_0, F)$ where

1. $\Sigma = \{a, b, c\}$
2. $Q = \{q_0, q_1, q_2\}$
3. δ is given by the table:

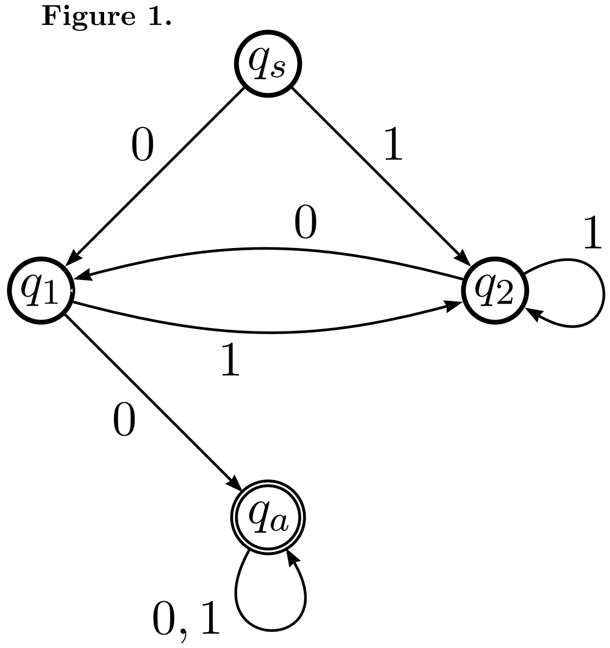
	a	b	c
q_0	q_0	q_2	q_1
q_1	q_1	q_1	q_0
q_2	q_1	q_2	q_0

4. $F = \{q_1\}$

Exercise 5. For each of the following languages, draw (if it exists) the state diagram of an automaton recognizing it. Test each automaton on two strings in the language, and two strings not in the language, plus the empty string.

1. $\{w \in \{0,1\}^* | \ w$ contains an even number of zeroes$\}$
2. $\{w \in \{a,b\}^* | \ w$ has exactly three b's $\}$
3. $\{w \in \{a,b\}^* | \ w$ has at least three a's and at least two b's$\}$
4. $\{w \in \{0,1\}^* | \ w$ contains 110 as a substring$\}$
5. $\{w \in \{0,1\}^* | \ |w| = 3\}$
6. $\{w \in \{0,1\}^* | \ |w|$ is even $\}$
7. $\{w \in \{0,1\}^* | \ w$ contains an equal number of '1' and '0'$\}$

Figure 1.



Exercise 6. Let $\Sigma = \{a, b\}$ and $A = \{w \in \Sigma^* |\ w \text{ has an odd number of } b's\}$, and $B = \{w \in \Sigma^* |\ \text{ each } b \text{ is followed by at least one } a\}$.

1. Construct DFAs M_A recognizing A, and M_B recognizing B.
2. From M_A and M_B construct DFA M for $A \cup B$ (use the product construction seen in class).
3. Suppose you want to construct M' for $A \cap B$. Fill in the blanks: For any string x, $M'(x) = \text{accept iff } M_A(x) = \_\_,\ \_\_\ M_B(x) = \_\_$.
4. From this explain what parts of M need to be modified to obtain M' [1 sentence].
5. Construct M' applying the changes you listed above.