## $\textbf{Algorithm 1} \ \, \text{Recursive route-counting function}$

```
 \begin{split} &\textbf{function} \  \  \, \text{countRoutes}\,(m,\ n) \\ &\textbf{if} \  \, n=0 \  \  \, \text{OR}\,\, m=0 \\ &\textbf{return} \  \, 1 \\ &\textbf{end} \\ &\textbf{return} \  \, \text{countRoutes}\,(m,\ n-1) \  \, + \  \, \text{countRoutes}\,(m-1,\ n) \\ &\textbf{end} \\ &\textbf{end} \\ \end{aligned}
```

## Algorithm 2 Recursive route-counting function

```
\label{eq:cache} \begin{split} & \textbf{cache} \leftarrow \textbf{Dict}\,(\,) \\ & \textbf{function} \ \ \textbf{count} \textbf{Routes}\,(m,n) \\ & \textbf{if} \ \ n \ = \ 0 \ \ \textbf{OR} \ m \ = \ 0 \\ & \quad \textbf{return} \ \ 1 \\ & \textbf{end} \\ & \\ & \textbf{try} \\ & \quad \textbf{return} \ \ \textbf{cache}\,[\,(m,\ n)\,] \\ & \textbf{end} \\ & \\ & \quad \textbf{cache}\,[\,(m,\ n)\,] \leftarrow \textbf{count} \textbf{Routes}\,(m,\ n-1) \ + \ \textbf{count} \textbf{Routes}\,(m-1,\ n) \\ & \quad \textbf{return} \ \ \textbf{cache}\,[\,(m,\ n)\,] \\ & \quad \textbf{end} \\ \end{split}
```