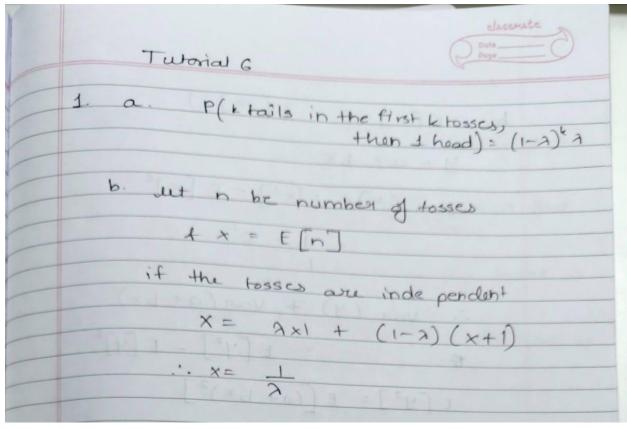
Data Science, 2022

Tut 6: Machine Learning 1

- 1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ .
- a. What is the probability of obtaining the first head at the (k + 1)-th toss?
- b. What is the expected number of tosses needed to get the first head?



- 2. [Probability] Assume X is a random variable.
- a. We define the variance of X as: $Var(X) = E[(X E[X])^2]$. Prove that $Var(X) = E[X^2] E[X]^2$.
- b. If E[X] = 0 and $E[X^2] = 1$, what is the variance of X? If Y = a + bX, what is the variance of Y?

 $x \Rightarrow x$ and om variable

a $Var(x) = E[(x - E[x])^2]$ to prove, $Var(x) = E[x^2] - E[x]$ $= E[x^2] - 2E[x = [x]] + E[x]^2$ $= E[x^2] - 2E[x] + E[x]^2$ $= E[x^2] - E[x]^2$

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b. E[x]=0 E[x2]=1
y=a+bx
Var (x)= E[x2] - E[x]2
= 1 - 0 ²
·· Var (y) 7 Var (a+bx)
$= E[y^2] - E[y]^2$
$E[y^2] = E[(a+bx)^2]$
$= E\left[a^2 + 2abX + b^2X^2\right]$
$= a^2 + 2ab E[x] + b^2 E[x^2]$
$= a^2 + b^2$ $= [Y] = [a + bX]$
= a + bE(x) $= q.$
$VOY(Y) = 2a^2 + b^2 - (a)^2$ $= b^2$
- [1] - [2] =
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- 3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5} , Aku predicts that a black beauty is a winning horse. This also means that with probability $1 10^{-5}$, Aku predicts that a black beauty will not win.
- a. Given a horse, what is the probability that it wins?
- b. What is the probability that Aku correctly predicts a black beauty is winning?

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ER 3

Let A be the event " Aku predicts that the horse is a winning horse"

Let -A be the event " how predicts that the horse is not a winning horse"

Let w be the event that the horse is a winning horse.

Let TW be the event that the horse is anot a winning horses

Given P(wjA) = 099 P (-W/-A)= 0.99999

P(A)= 10-5

a) P(w) = P(w, A) + P(w, 7A)Probability = P(w|A)P(A) + P(w|A)P(AT)gwinning = 0.99 × 10⁻⁵ + (1-0.99999)(1-10⁻⁵)

≥ 1.99 × 10-5