# Data Science, 2022
# Tut 6: Machine Learning 1

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ.
a. What is the probability of obtaining the first head at the (k + 1)-th toss?
b. What is the expected number of tosses needed to get the first head?

Tutorial 6

1. a. $P(k \text{ tails in the first } k \text{ tosses, then } 1 \text{ head}) = (1-\lambda)^k \lambda$

b. let $n$ be number of tosses

& $x = E[n]$

if the tosses are independent

$$x = \lambda \times 1 + (1-\lambda)(x+1)$$

$$\therefore x = \frac{1}{\lambda}$$

2. [Probability] Assume X is a random variable.
a. We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.
b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?

2. $x \rightarrow$ random variable

a. $Var(x) = E[(x - E[x])^2]$

to prove, $Var(x) = E[x^2] - E[x]^2$

$\therefore Var(x) = E[x^2 - 2x E[x] + E[x]^2]$

$= E[x^2] - 2E[x E[x]] + E[x]^2$

$= E[x^2] - 2E[x]^2 + E[x]^2$

$= E[x^2] - E[x]^2$

H.P.

b. $E[x] = 0 \quad E[x^2] = 1$

$y = a + bX$

$Var(x) = E[x^2] - E[x]^2$

$= 1 - 0^2$

$= 1$

$\therefore \; Var(y) = Var(a + bx)$

$= E[y^2] - E[y]^2$

$E[y^2] = E[(a + bX)^2]$

$= E[a^2 + 2abX + b^2X^2]$

$= a^2 + 2ab\,E[x] + b^2\,E[x^2]$

$= a^2 + b^2$

$E[y] = E[a + bX]$

$= a + b\,E[x]$

$= a.$

$\therefore \; Var(y) = a^2 + b^2 - (a)^2$

$= b^2$

3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability $10^{-5}$, Aku predicts that a black beauty is a winning horse. This also means that with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.

a. Given a horse, what is the probability that it wins?

b. What is the probability that Aku correctly predicts a black beauty is winning ?

Ex 3

Let A be the event "Aku predicts that the horse is a winning horse"

Let $\neg A$ be the event "Aku predicts that the horse is not a winning horse"

Let $W$ be the event that the horse is a winning horse.

Let $\neg W$ be the event that the horse is not a winning horse.

Given $P(W|A) = 0.99$
$P(\neg W|\neg A) = 0.99999$

$P(A) = 10^{-5}$

a) $P(W) = P(W, A) + P(W, \neg A)$

Probability of winning $= P(W|A)\,P(A) + P(W|\neg A)\,P(\neg T)$

$= 0.99 \times 10^{-5} + (1-0.99999)(1-10^{-5})$

$\approx 1.99 \times 10^{-5}$

b. Prob. that Akee predicts wining correctly.

$$P(A|W) = \frac{P(A,W)}{P(W)}$$

$$= \frac{P(W|A)\,P(A)}{P(W)}$$

$$= \frac{0.99 \times 10^{-5}}{\cancel{0.99\times}\ 1.99 \times 10^{-5}}$$

$$\simeq 0.4975$$