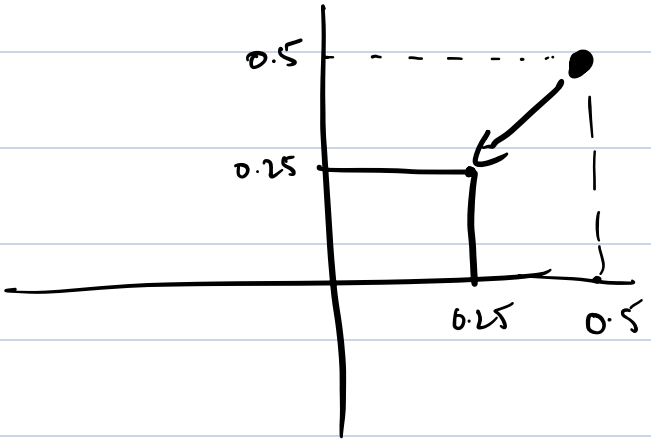
HW1

1) $0.5 \times 0.5 \longrightarrow 0.25 \times 0.25$

Scaling $\begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$



$C_x = C_y = \frac{1}{2}$

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$0.5 \times 0.5 \longrightarrow 0.25 \times 0.5 \quad C_x = \frac{1}{2} \quad C_y = 1$

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2)

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \quad \leftarrow U_{12}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x' \\ 0 & 1 & t_y' \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \quad \leftarrow U_{23}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & t_x' \\ 0 & 1 & t_y' \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}}_{U_{13}} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & t_x + t_x' \\ 0 & 1 & t_y + t_y' \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{12} \cdot U_{23} = U_{13}$$

3) matlab → graph

$$\begin{pmatrix} x_g \\ y_g \\ 1 \end{pmatrix} = \begin{pmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_m \\ y_m \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_g' \\ y_g' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_g \\ y_g \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_m \\ y_m \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_x & 0 & t_x \\ 0 & c_y & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_m \\ y_m \\ 1 \end{pmatrix}$$

(x_g, y_g)	$\longrightarrow$	(x_m, y_m)
0, 635	$\longrightarrow$	(565, 1518)
5, 635	$\longrightarrow$	(646, 1518)

4) n control pts

$x_1, y_1 \longrightarrow x_1', y_1'$

$\vdots$

$x_n, y_n \longrightarrow x_n', y_n'$

$P_x = [a\ b\ c\ d\ e\ f]$

$P_y = [A\ B\ C\ D\ E\ F]$

$$X' = H P_x$$

$$\begin{bmatrix} x_1' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} x_1^2 & y_1^2 & x_1 y_1 & x_1 & y_1 & 1 \\ & & & \vdots & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} =$$

$n \times 1$ $\quad$ $n \times 6 \cdot 6 \times 1$

$$y' = H\, p_y$$

$$\Rightarrow \quad p_x = \frac{H^{-1} x'}{H^{-1} y'}$$

$$p_y = \frac{H^{-1} y'}{}$$