



Filter Design Assignment-I

EE338 - 2023 Filter Review Report

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1 Student Details

- Name : Ojas Karanjkar
- Roll No: 210070040
- Filter number assigned: 103

2 Butterworth Bandstop Filter Details

2.1 Un-normalized Discrete time specifications

- 1. $m = 23$
- 2. $q(m) = [2.3] = 2$
- 3. $r(m) = 23 - 10*2 = 3$
- 4. $BL(m) = 20 + 3*2 + 11*3 = 59$
- 5. $BH(m) = 59 + 40 = 99$
- Stopband = 59 kHz to 99 kHz.
- Transition Band Width = 5 kHz.
- Passband = 0 to 54 kHz and 104 to 212.5 kHz.
- Tolerance = 0.15.
- Nature of Passband and Stopband : Both monotonic.

2.2 Normalized discrete filter specifications

Sampling frequency = $425kHz$

$$\omega = \frac{\Omega * 2\pi}{\Omega_{sampling}}$$

- Stopband = 0.27π to 0.46π .
- Transition Band Width = 0.0235π .
- Passband = 0 to 0.25 and 0.48π to π .
- Tolerance = 0.15.
- Nature of Passband and Stopband : Both monotonic.

2.3 Converting to Analog Low-pass filter

We have the following transformation for converting into analog low-pass filter:

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

Using the above Bilinear Transformation, the passband and stopband specifications are updated as follows:

- Stopband = 0.451 to 0.88 .
- Transition Band = 0.414 to 0.4509 and 0.88 to 0.939 .
- Passband = 0 to 0.414 and 0.939 to ∞ .
- Tolerance = 0.15.
- Nature of Passband and Stopband : Both monotonic.

2.4 Frequency Transformation for Band-Stop Filter

The frequency transformation for converting a bandstop filter to low pass filter are as follows:

$$\begin{aligned}\Omega_L &= \frac{B\Omega}{\Omega_0^2 - \Omega^2} \\ B &= \Omega_{p2} - \Omega_{p1} \\ \Omega_0 &= \sqrt{\Omega_{p1}\Omega_{p2}}\end{aligned}$$

According to the above transformation, we can tabulate the updated value of stopband and passband edges:

Ω	Ω_L
0^+	0^+
$0.414(\Omega_{p1})$	1.00
$0.451(\Omega_{s1})$	1.28
$0.623^-(\Omega_0)$	$-\infty$
$0.623^+(\Omega_0)$	$+\infty$
$0.88(\Omega_{s2})$	-1.19
$0.939(\Omega_{p2})$	-0.99
∞	0^-

- Passband edge = 1 (Ω_{lp})
- Stopband edge = 1.19 (Ω_{ls})
- Tolerance = 0.15.
- Nature of Passband and Stopband : Both monotonic.

2.5 Analog Lowpass Transfer Function

Tolerance = 0.15

$$\begin{aligned}
 D_1 &= \frac{1}{(1-\delta)^2} - 1 = 0.3844 \\
 D_2 &= \frac{1}{\delta^2} - 1 = 43.44 \\
 N &\geq \left\lceil \frac{\log(\frac{D_2}{D_1})}{2\log(\frac{\Omega_s}{\Omega_p})} \right\rceil \\
 N &= 14 \\
 \frac{\Omega_p}{D_1^{\frac{1}{2N}}} &\leq \Omega_c \leq \frac{\Omega_s}{D_2^{\frac{1}{2N}}} \\
 1.03467 &\leq \Omega_c \leq 1.040042 \\
 \Omega_c &= 1.037
 \end{aligned}$$

2.5.1 Finding Poles of the transfer function

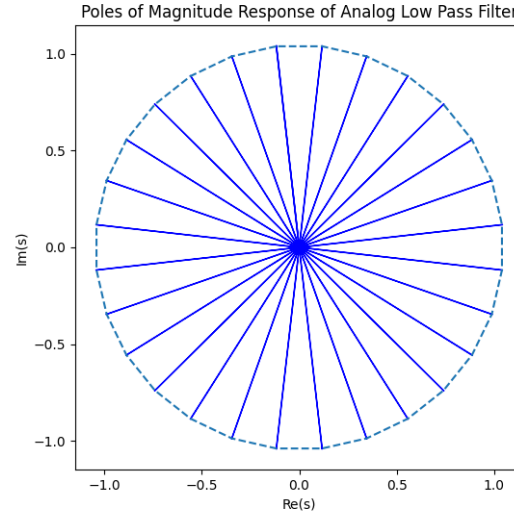
Poles can be found using the following expression:

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 0$$

Upon solving the above equation we get the following value of poles in open left half complex plane:

$$\begin{aligned}
 p1 &= -0.1161 - 1.0305i \\
 p2 &= -0.3425 - 0.9788i \\
 p3 &= -0.5517 - 0.8781i \\
 p4 &= -0.7333 - 0.7333i \\
 p5 &= -0.8781 - 0.5517i \\
 p6 &= -0.9788 - 0.3425i \\
 p7 &= -1.0305 - 0.1161i \\
 p8 &= -1.0305 + 0.1161i \\
 p9 &= -0.9788 + 0.3425i \\
 p10 &= -0.8781 + 0.5517i \\
 p11 &= -0.7333 + 0.7333i \\
 p12 &= -0.5517 + 0.8781i \\
 p13 &= -0.3425 + 0.9788i \\
 p14 &= -0.1161 + 1.0305i
 \end{aligned}$$

The plot of the poles of the magnitude response of the Analog Lowpass filter plotted in python is as follows:-



Poles of Magnitude response

The Transfer Function of the Analog Lowpass Filter:

$$H_{analog,LPF}(s_L) =$$

$$\frac{\Omega_c^N}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)(s_L - p_7)(s_L - p_8)(s_L - p_9)(s_L - p_{10})(s_L - p_{11})(s_L - p_{12})(s_L - p_{13})(s_L - p_{14})}$$

2.6 Analog Bandstop Transfer Function

Now we need to transform the Analog Lowpass filter back to Analog Bandstop filter using the same transformation we used earlier.

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Thus

$$s_L = \frac{0.525s}{0.6234 + s^2}$$

Substituting this value of s_L into the above Analog Lowpass Filter Transfer Function to get the Analog Bandstop Filter Transfer function i.e $H_{analog,BSF}(s)$. Numerator and Denominator coefficients are as follows:

2.7 Peer Review

I have reviewed the assignment of Kushal Gajbe, Roll No. 210070048. His phase response is too smooth to be that of real life filter. Also it does not match (atleast the nature of his phase response) of the assignment which was given as reference. Magnitude response is fine. There seems to be calculation error in order and passband/stopband edges.

Powers of s in Denominator	Coefficients		Powers of s in Numerator	Coefficients
s^{28}	1		s^{28}	1
s^{27}	4.5216		s^{26}	5.44
s^{26}	15.6636		s^{24}	13.7438
s^{25}	38.1214		s^{22}	21.3649
s^{24}	78.2437		s^{20}	22.8332
s^{23}	133.00100		s^{18}	17.7472
s^{22}	198.57304		s^{16}	10.34
s^{21}	258.8133		s^{14}	4.5949
s^{20}	302.5689		s^{12}	1.5625
s^{19}	316.1823		s^{10}	0.404
s^{18}	299.8353		s^8	0.07866
s^{17}	257.2226		s^6	0.01111657
s^{16}	201.52		s^4	0.00108005
s^{15}	143.6732		s^2	6.45e-05
s^{14}	93.8095		s^0	1.79e-06
s^{13}	55.8353			
s^{12}	30.4367			
s^{11}	15.09			
s^{10}	6.8393			
s^9	2.8028			
s^8	1.0423			
s^7	0.3465			
s^6	0.1033			
s^5	0.0268			
s^4	0.00614			
s^3	0.001164			
s^2	0.0001859			
s^1	2.08e-05			
s^0	1.79e-05			

Powers of s in Denominator	Coefficients	Powers of s in Numerator	Coefficients
z^{-28}	1	z^{-28}	1
z^{-27}	9.57	z^{-27}	12.32
z^{-26}	50.54	z^{-26}	84.5647
z^{-25}	187.8297	z^{-25}	408.8093
z^{-24}	544.4017	z^{-24}	1539.5987
z^{-23}	1297.2673	z^{-23}	4755.4093
z^{-22}	2625.6999	z^{-22}	12439.4012
z^{-21}	4611.614	z^{-21}	28134.1051
z^{-20}	7134.3600	z^{-20}	55891.89107
z^{-19}	9826.3641	z^{-19}	98477.3500
z^{-18}	12143.8409	z^{-18}	155140.5032
z^{-17}	13542.3158	z^{-17}	219772.7785
z^{-16}	13681.73244	z^{-16}	28109.5449
z^{-15}	12555.8230	z^{-15}	325473.9455
z^{-14}	10482.1612	z^{-14}	281090.54498
z^{-13}	7964.1211	z^{-13}	219772.7785
z^{-12}	5503.5802	z^{-12}	155140.5032
z^{-11}	3453.45056	z^{-11}	98477.35009
z^{-10}	1962.1101	z^{-10}	155140.5032
z^{-9}	1004.2234	z^{-9}	219772.778
z^{-8}	461.69327	z^{-8}	281090.54498
z^{-7}	188.60517	z^{-7}	325473.9455
z^{-6}	67.7939	z^{-6}	12439.40124
z^{-5}	21.1230	z^{-5}	4755.4093
z^{-4}	5.58500	z^{-4}	1539.5987
z^{-3}	1.12327	z^{-3}	408.8093
z^{-2}	0.20558	z^{-2}	84.5647
z^{-1}	0.0245	z^{-1}	12.32868
z^0	000163	z^0	1.0000887