Number system: (5M)

Prime no greater than 3 can be represented as 6n-1 and 6n+1.

* Prime numbers: 2, 3, 5, 7, 11, 13 ....
But not all the nos which are in the form of 6n+1 and 6n-1 are prime nos. Every prime no must be in form of 6n+1 and 6n-1.
Ex: 25 can also be represented as 6n+1. but not prime number

* Sum of the first natural numbers = $\frac{n(n+1)}{2}$

$1+2+3+4+5+6+\ldots+n = \frac{n(n+1)}{2}$

* Sum of first even natural numbers =

$= 2+4+6+8+\ldots\ldots 2n$

$= 2(1+2+3+4+5\ldots n)$

$= \frac{2(n(n+1))}{2} = n(n+1)$

* Average of first 'n' Natural even numbers: $A_{N_E} = \frac{n(n+1)}{n} = (n+1)$

* Sum of first 'n' odd natural numbers:

$= 1+3+5+7+9\ldots\ldots(2n-1)$

$= 2-1+4-1+6-1+8-1\ldots\ldots (2n-1+1)$

$= (2+4+6+8+\ldots\ldots) - (n)$

$= n(n+1) - n$

$= n(n+1-1)$

$= n^2$

avg of first n nat. odd no.

* $A_{N_O} = \frac{n^2}{n} = n.$

* Arithmetic progression:

$a, a+d, a+2d, a+3d \ldots\ldots a+(n-1)d$

a = 1st term

d = common difference

$a_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[a + a + (n-1)d]$

$= \frac{n}{2}[\text{1st term} + \text{last term}]$

Average : $\frac{\frac{n}{2}[2a+(n-1)d]}{n}$

$= \frac{2a+(n-1)d}{2}$

or, $\frac{(\text{1st} + \text{last term})}{2}$

* Geometric progression:

$a, ar, ar^2, ar^3 \ldots\ldots ar^{n-1}$

a = first term

r = common ratio $= \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

$a_n = ar^{n-1}$

$S_n = \frac{a(r^n-1)}{r-1}$ or $(r<1)$ $S_n = \frac{a(1-r^n)}{(1-r)}$

$S_\infty = 0<r<1$, $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots$

$= \frac{1\left(1-\left(\frac{1}{3}\right)^4\right)}{1-\frac{1}{3}}$ ↑ if power increases the value become lower

$= \left(\frac{a}{1-r}\right)$

Q. What is the sum of max sum of given series?

$= 44 + 42 + 40 + 38 + \dots + 4 + 2$

$= \overset{1}{2} + \overset{2}{4} + \overset{3}{6} + \dots + \overset{22}{44}$ $\quad [n(n+1)]$

$= 22(23) = 506.$

Q. What is the avg. of all the multiples, between 2 and 198

$= 10 + 201 \dots + 190 / 19$

$= \frac{19}{2}\left[\frac{10+190}{2}\right] = 95 = 100 \quad \left[\because \frac{1st + last}{2}\right]$

Q. Find out the sum of these series?

$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \infty$

$\frac{1}{1-\frac{1}{4}} = \frac{4}{3}$ Ans. $\quad \left[\frac{a}{1-\frac{1}{r}} \quad \frac{a}{1-r}\right]$

Q. Sum of given series $4 + 44 + 444 + \dots$

$= 4(1 + 11 + 111 + 1111 + \dots)$

$= 4\left(\frac{9}{9} + \frac{99}{9} + \frac{999}{9} + \dots\right)$

$= \frac{4}{9}\left[10 - 1 + 10^2 - 1 + 10^3 - 1 + 10^4 - 1 \dots\right)$

$= \frac{4}{9}\left[(10 + 10^2 + 10^3 \dots 10^n) - n\right)\Big]$

$= \frac{4}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$

$= \frac{4}{9}\left[\frac{10(10^n - 1) - 9n}{9}\right]$

$= \frac{4}{81}\left[10^{n+1} - 10 - 9n\right]$

most important steps in these kind of Questions.

432145067

Check by options, assuming the $n=1$. and sum = 4

Go by options

Q. $10 + 84 + 734 + \ldots\ldots n$

Options:

a) $\frac{9(9^n+1)}{10} + 1$ for $n=2$ $\frac{9(81+1)}{10} + 1$

$= \frac{9(82)}{10} + 1 = \frac{9(82)}{10} + 10 = \frac{9(82)}{10} + \frac{10}{10}$

b) $\frac{9[9^n-1]}{8} + 1$

$= \frac{9[9^2-1]}{8} + 1 = \frac{9[80]}{8} + 1$

c) $\frac{9[9^n-1]}{8} + n$

$= \frac{9[9^2-1]}{8} + 2 = 91$

d) $\frac{9[9^n-1]}{8} + n^2$

$= \frac{9[9^2-1]}{8} + n^2 = 90 + 2^2 = 90 + 4$

$= 94$

$= 9 + 1 + 9^2 + 3 + 9^3 + 5 + \ldots\ldots$

$= [9 + 9^2 + 9^3 + \ldots 9^n] + [1 + 3 + 5 + 7 + \ldots]$

$= \frac{9(9^n - 1)}{8} + (n^2)$

$= \frac{9^{n+1} - 9 + 8n^2}{8}$

Q. $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$

$= \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \frac{1}{5\times6} + \frac{1}{6\times7} + \frac{1}{8\times7} + \frac{1}{8\times9} + \frac{1}{9\times10}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} - \ldots - \frac{1}{9} + \frac{1}{9} - \frac{1}{10}$

$= 1 - \frac{1}{10} = \frac{9}{10}$

Q. $\left\{\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}\cdots\frac{1}{98\times99}\right\}$

$= \left\{1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}-\cdots\frac{1}{98}-\frac{1}{99}\right\}$

Write no by parts. $= 1-\frac{1}{99} = \frac{98}{99}$ Ans.

Q. $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}\cdots\frac{1}{103\times107}$

$= \frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}\right]$

$\frac{1}{3}-\frac{1}{7} = \frac{7-3}{3\times7} = \frac{4}{3\times7}$

$\frac{1}{7}-\frac{1}{11} = \frac{11-7}{7\times11} = \frac{4}{7\times11}$

$= \frac{1}{4}\left[\frac{4}{3\times7}+\frac{4}{7\times11}+\frac{4}{11\times15}\cdots\right]$

$= \frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-\frac{1}{15}\cdots\frac{1}{103}-\frac{1}{107}\right]$

First step in Gate examination. $= \frac{1}{4}\left[\frac{1}{3}-\frac{1}{107}\right] = \frac{1}{4}\left[\frac{107-3}{3\times107}\right] = \frac{1}{4}\left[\frac{104}{3\times107}\right]$ (104 = 26)

$= \frac{26}{321}$ Ans.

Q. $\frac{1}{2\times7}+\frac{1}{7\times12}+\cdots\cdots+\frac{1}{1002\times1007}$

$= \frac{1}{5}\left[\frac{1}{2}-\frac{1}{1007}\right]$ = Ans.

Q. $a_n = \frac{1}{n} - \frac{1}{n+2}$. $S_{50} = a_1 + a_2 + a_3 + a_4 + a_5 \cdots a_{50}$

$\frac{1}{1} - \frac{1}{3}$, $\frac{1}{2} - \frac{1}{4}$, 1

$= \frac{n+2-n}{n(n+2)}$

$= \frac{2}{n(n+2)} = 2\left[\frac{1}{n(n+2)}\right]$

$= 2\left[\frac{1}{1\times3} + \frac{1}{2\times4} + \frac{1}{3\times5} + \frac{1}{4\times6} + \cdots\right]$

$= 2\left[\frac{1}{1\times3} + \frac{1}{3\times5} \cdots + \frac{1}{47\times49} + \frac{1}{2\times4} \cdots + \frac{1}{48\times50}\right]$

$= \frac{2}{2}\left[1 - \frac{1}{49} + \frac{1}{2} - \frac{1}{50}\right]$

$1 - \frac{1}{3} = \frac{2}{3}$

$\frac{1}{2} - \frac{1}{4} = \frac{4-2}{2\times4}$

$\left[\frac{3}{2} - \frac{1}{49} - \frac{1}{50}\right] = \frac{3}{2} - \left[\frac{50+49}{50\times49}\right]$

$= \frac{3}{2} - \left[\frac{99}{50\times49}\right]$

$= \frac{3\times50\times49 - 99}{2\times50\times49}$.

$= 2[$ $=$

front expansions $a_{48} = \frac{1}{48} - \frac{1}{50}$

open the bracket $= 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} \cdots\cdots \frac{1}{50} - \frac{1}{52}$ $a_{49} = \frac{1}{49} - \frac{1}{51}$.

$= \left(1 + \frac{1}{2}\right) - \left(\frac{1}{51} + \frac{1}{52}\right)$

Q. $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \cdots \frac{1}{\sqrt{80}+\sqrt{81}}$

$= \frac{1-\sqrt{2}}{1-2\,|} + \frac{\sqrt{2}-\sqrt{3}}{4-3\,|} \cdots \frac{\sqrt{80}-\sqrt{81}}{6400-(81)^2}$

$= \frac{\sqrt{2}-1}{1} + \frac{\sqrt{3}-\sqrt{2}}{5\,1} \cdots \frac{\sqrt{81}-\sqrt{80}}{1}$ ↳ for $\sqrt{80}-\sqrt{79}$

$= -1+\sqrt{81} = -1+9 = 8$

A.) 7 B.) 8 C.) 9 d.) 10

Q. concept

Concept of unit digit :- (2M) always.

Take unit digits of the numbers.

Q. $2358 \times 539 \times 276 \times 8389 = \ldots 8$

$7 \times 2 = 14$, 12, 18

Q. $593 * 476 + 853 \ldots . 1$

18, $8+5=13$, 11

Note: (x^p)'s unit digit = (x^R)'s unit digit where 'R' is the remainder when we divide the power (p) by 4.

$2^{46} \Rightarrow \ldots 2^2 \Rightarrow \ldots 4$

$\frac{46}{4} = R = 2$

Q $2^{1397867} = 2^3 = \ldots 8$

$1397867/4 = R = 3.$

2^{1383}

or

$1383 = \underline{1300} + 83.$

↳ already divisible by 4

Just divide $83/4 = R = 3. = \ldots 8$

Q $3^{147} = 47/4 = 3. = \ldots 7.$

Q. $(7\text{/}12)^{854} = 2^{854} = R = 2^2 = \ldots 4$

For unit digit the other two doesn't matter

Q. $(3\text{/}17)^{338} = 7^2 = 49 = \ldots 9.$

Note: If power is multiple of 4 then the unit digit of that expansion will be as same as power 4 of that no.

* Note: If the unit digit of any no is 0, 1, 5, 6, then whatever will be the power the unit digit is always 0, 1, 5, 6 respectively.

Ex: $5^{623} = \ldots 5$

$7^{859} = 7^3 = 49 \times 7 = 63 = 3 = \ldots 3.$

$8^{996} = 8^4 = \ldots 6$

* $2^{16} = 2^0 = 1$

Not possible

$\therefore 2^{16} = 2^4 = 6$

or $\ldots 6.$

$2^{8564} = 2^4$

$= \ldots 6.$

GATE QUESTION:

Q. $211^{870} + 146^{127} \times 3^{424}$

$= 1 + 6 \times 3^{4}$.

$= 1 + 6 \times 1$.

$= 7$.

or.

$\ldots 1 + \ldots 6 * \ldots 1$.

$= \ldots 7$. Ans.

$4^1 = 4 \quad 4^3 = 64$

$4^2 = 16 \quad 4^4 = 256$

$4^{odd} = \ldots\ldots 4$.

$4^{even} = \ldots\ldots 6$

Q. $(2171)^{7} + (2172)^{9} + (2176)^{11} + (2174)^{13}$.

$= 1 + 2 + 3^{3} + 4$.

$= 1 + 2 + 7 + 4$

$= 4$. Ans.

1119.

$9^{odd} = \ldots\ldots 9$

$9^{even} = \ldots\ldots 1$

$even^{odd/even} = even$.

$Odd^{odd/even} = odd$

$(444)^{869} = \ldots\ldots 6$. (376, 7777)

$(9999)^{8888} = \ldots\ldots 1$.

Q $(2)^{42^{43}} = 2^4 = 6$. ∴ $2^{42 \times 42 \times 42 \times 42 \ldots} = 2^4 = \ldots 6$.

→ 21 × 2 × 2 × 21 → to make it even. → to make it nine

Q $3^{36^{89}} = 3^{36 \times 36 \times 36 \times 36 \ldots} = 3^4$

Q $3^{85^{79}}$

$1! = 1$ Note: if $n \geq 4$ $n!$ will always be multiple of 4

$2! = 2$

$3! = 6$

$4! = 24$.

$= 3^{79!} = 3^{4\times(\,)} = 3^{2\times 2\times\ldots} = 9^{2\times\ldots} = (9)^{\text{even}} = \ldots 1.$

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

$5! = 120$

Note: if $n \geq 5$ then the unit digit of $n!$ is always 0.

Ex. $(79!)^{76} = \emptyset \ldots 0$

Ex. $(3896831!)^{3896391} = 0$

*. Highest no which can divide a no. - HCF AND LCM:

- When we find the ratio of 2 numbers the common no will get cancelled and that common no is the HCF of given numbers.

- When we multiply the ratios to the HCF we will get numbers

5. $(15:20)$ $(3:4)$

$5\times3 \quad 5\times4 \quad 5\times7$

$15, 20, 35.$

$= 5\times7\times4\times3 = 420$

$16, 16, 24, 30, 32.$

already there $= 8\times4\times5\times3$

$= 480$

Q. $28, 42, 64, 16.$ (7×4, 4×6, 2×3, 2×8, 2×2)

$= 8\times8\times7\times3.$

$= 64\times21.$

$= 1344.$

Q 24, 30, 43, 50

= $43 \times 10 \times 5 \times 3 \times 4 = 25800$

Q L

Note: $LCM = HCF \times a \times b$

GATE Q The ratio of two nos. is 3:4 and there HCF is 5 find the LCM = 60

Q

Note: $I^{st}\ NO \times II^{nd}\ NO = HCF \times LCM$

$= HCF \times HCF \times a \times b$

second no.

First no

Q Two nos are in ratio 3:4 if there LCM is 240 the smaller no is what: 60

Q The HCF of two nos is 5 and their sum is 75 how many pairs of nos are possible for this

$a(75-a) = 5(a)$

$= 5a + 5b = 75$.

$= a + b = 15$

Ans: 4.

1 14 4 11 7 8

2 13

5 10

5 12

6 9

15 60

Q. Where a no is divided by 15, 20, 35 each time a remainder is 8. That smallest no is what,

$7 \times 5 \times 4 \times 3 = 420$ [Smallest no divided by 15, 20, 35)]

$= 428$ Ans

Q. What is the smallest natural no which when divided by 20, 42, and 76 leaves a remainder 7.

2 | 76
2 | 38
19

$= 19 \times 4 \times 3 \times 7 \times 5$

$= 7980$

$= 7980 + 7 = 7987.$

Q Find that greatest no of 4 digit such that when we divide that no, by 5, 6, 8, 10, 12, 24 it will always gives a remainder 2.

$6 \times 4 \times 5 = 120$

$120 \overline{)9999} \; 83$

960

399

360

39

$9999 - 39 = 9960 + 2$

$= 9962.$ Ans.

Q Find the least no which when divide by 24, 32, 36 and gives the remainder ~~19~~ 19, 27, 31.

$LCM = 6 \times 6 \times 2 \times 2^2 = 288$

24, 32, 36

- 19, 27, 31

5, 5, 5

$\therefore 288 - 5 = 283$ Ans.

**** diviaing the no gives remainder 0. to make the remainder 19, 27, 31 the 288 should be reduced by 5

* No of trailing zeroes in the end :-

~~How many~~ max power of 2 divides 101

101

$\frac{10}{2} = 5$

$\frac{5}{2} = 2$

$\frac{2}{2} = 1$

$5 + 2 + 1 = 8$

15!

$\frac{15}{3} = 5$

$\frac{5}{3} = 1$

$5 + 1 = 6$

$\frac{15}{2} = 7$

3 → 11

1

45 | 45!

$\frac{45}{5} = 15$ $\quad = 21 > 3^{21}$

4 4

1

Q. 45! for 6^x

→ See the smallest value of power out of the both and find the max power.

$\frac{45}{2} = 22$ $\quad \frac{45}{3}$

11 $\quad$ 15

5 $\quad$ 5

2 $\quad$ 1

1 $\quad$ 21

for 2's 41.

$\frac{45!}{3^{21} \times 2^{41}} = \frac{45!}{(2\times3)^{21} \times 2^{20}}$

or just find the power of 3.

$= 6^{(21)}$ → max power which divide 45!

Q.

Note: To find out the no of trailing zeroes we have to find out no of 2's and no of 5's ultimately we have to find out the no of pairs of 2 and 5 in that product.

1. 100!

→ Find the no of 5's only.

no of 5's.

$\frac{100}{5} = 20$

4

24 zeroes.

no of 2's.

50

25

12

6

3

1

97.

2. 165!

52

6

51

39

Q. 1000!

200	500
40	250
8	125
248	62
1	31
249	15
	7
	3
	1

Q. 491 × 531!

141	29	153	30
	5		6
	1		1
	35	+	37

Ans. (72) zeroes.

Q. Find out the trailing zeroes in 316, 317, 318 ... 1200

$$\frac{1200!}{316 \cdot 315!} = 1200$$

1200 → 600, 300, 150, 30, 15, 7, 1 → 1103

315 → 63, 12, 2 → 77

1103 − 87 = 1016

* 1200 = 240, 48, 9, 1

298 − 77 = 221

$\frac{1200!}{315!}$ ← no. of zeroes will get subtracted

* 315 = 63, 12, 2

Q. $10^{13} + 10^{29} = 10^{13}(1 + 10^{16})$

$= 1\_\_\_.10^{13}$

∴ 13 zeroes in the end.

Q. $10^{613} + 10^{1293}$ = 613 trailing zeroes.

Q. $9 + 9 * 213 + 8 \times 2$.

$9 + 18 + 3 + 16$

$21 + 9 + 16$.

$25 + 21 = 46$.

Open the eqn in factorial form.

i) $10^{280}(280!)$

$= 10^{280}$ [$280 = 56$ (11) (2)] $- 69$.

$= 280 + 69 = 349$ Ans

Question: $10 \times 20 \times 30 \times 40 \times 50 \times \ldots\ldots \times 2800$

$10^{280}(1 \times 2 \times 3 \times 4 \times 5 \times 6 \ldots 280)$

Q. i) $1^2 \times 2^3 \times 3^4 \times 4^5 \times 5^6 \ldots\ldots \times 28^{29}$.

Finding no of 5 only →

$28! \times 28! \; 5^6 \times 5^{11} \times 5^{16} \times 5^{21} \times (5^2)^{26}$.

$= 6 + 11 + 16 + 21 + 52$

$= 106$

Q. $3 \quad 3 \times 6^{121} \cdot 3^2 \; f$

$(3^{123} - 3^{122} - 3^{121}) \times (2^{121} - 2^{120} - 2^{119})$

$= 3^{121}(3^2 - 3 - 1) \times 2^{119}(2^2 - 2 - 1)$

$= 3^{121} \times 5 \times 2^{119} \times 1$

↓

one zero (trailing)

Q $1125 \times 56 \times 25 \quad 125 \times 24 \times 5$.

↓ prime factorise

5×225

$5^3 \times 5^2 \times 5$

$\Rightarrow 5^3 \times 3 \times 2^2 \times 3^2 \times 5^3 \times 2^3 \times 3 \times 5$

= no of trailing zeroes = 5 as no

$n(2) < n(5)$.

consider no of 2's also.

FACTORS

24 = 1, 2, 5, 4, 6, 8, 12, 24.

→ If a natural no can be written in

$N = p^x \times q^y \times r^z$ where p, q, r : Prime nos.

No of factors of 'N' = $(x+1)(y+1)(z+1)\ldots\ldots$

→ $36 = 6 \times 6 = 2^2 \times 3^2 = (3)(3) = 9.$

$360 = 10 \times 36 = 10 \times 9 \times 4 = 5 \times 2^3 \times 3^2.$

$360 = (1+1)(4)(3) = 24.$

Open in terms of the prime factors or squares. →

$1440 = 10 \times 12^2 = 2 \times 5 \times (4 \times 3)^2 = 2 \times 5 \times (2^2 \times 3)$

$= 2^5 \times 3^2 \times 5$

$= 36.$

→ $10,500 = 21 \times 5 \times 100$

$= 7 \times 3 \times 5 \times 10^2.$

$= 7 \times 3 \times 5 \times (2 \times 5)^2$

$= 2^2 \times 5^3 \times 3 \times 7$

$= 48.$

Keep on trying prime nos. →

2014

$= 2 \times 1007.$

$= 2 \times 19 \times 3$

$= 8.$

~~keep on~~ → 2020

2×1010

~~2×101~~ 202×10

$= (2 \times 5) \times 202$

$= (2^2 \times 5) \times (101)$

$= 12.$

Product of factors:

$24 = 1 \times 2 \times 3 \times 4 \times 6 \times 8 \times 12 \times 24$

$= 24^{4}$

$= (24)^{8/2}$

$P_n = (N)^{n/2}$.

n: no of factors of 'N'

(*) Q. $360 = (6)^2 \times (2 \times 5)$

$= 2^3 \times 3^2 \times 5^1$.

$= 4 \times 4 \times 3 \times 4 \times 3 \times 2$

$= 12\ 24$

$(360)^{12}$: Product of factors of 360.

Perfect sqr have odd ~~multiples~~ factors.

Q. As Note: As we know the no of factors of any perfect square no is odd, so we take the sqr. root of the perfect sqr to eliminate 1/2 from the power

$(36)^{9/2} = (6^2)^{9/2} = 6^9$.

Q. Find out the product of all the factors of ~~7056~~ 7056.

$7056 = 8 \times 882$

$= 8 \times 2 \times 441$

$= 2^4 \times 9 \times 49$

$= 2^4 \times 3^2 \times 7^2$.

$= 45$.

$(84)^{45}$.

$(7056)^{45/2} = \left[(86)^2\right]^{45/2}$.

Ø.

Q. $a + a^2b^3$ is odd 2M, 2018.

• check by options.

✗ i) a and b both are odd

✗ ii) a and b both are even

✗ iii) a is even and b is odd.

→ iv) a is odd and b is even.

Q. $a-b$ is even which of the following will always be even 1M 2017.

✗ A) ab.

✗ b) a^2+b^2+1.

✗ c) (a^2+b+1)

✓ d) $ab-b$.

Ø. DIVISIBILITY RULES :-

2 → unit digit even.

3 → sum of all digits must be divisible by 3

4 → last 2 digits must be divisible by 4.

5 → (0, 5) as unit digit

6 → should be divisible by 2, 3

8 → last 3 digits must be divisible by 8

9 → sum of all digits must be divisible by 9.

11 → The diff b/w the sum of the digits of odd places and the sum of digits of even places is equal to either 0 or multiple of 11.

Q If 715 9*425 is divisible by 3 then the smallest whole no in place of * is what.

= 1 · 2 Gate IM

Q How many values can x take if 78765x4 is divisible by 12. [ans = 2] (8,2)

78

78765[]4 ⇒ 37

2 ⇒ 39 ←

5 ⇒ 42

8 ⇒ 45 ←

Q 12×2 = 24. 78765[24]]160

12×7 = 84. 78765[84]]160

Q Find the value of x-y if 7448x24y is divisible by 72 (or divisible by 8×9) x>y.

8

8 ⇒ 45

×1 240+y / 8 7448[]24[] ⇒ 29

7 0 ⇒ 36

Q 4266[][] how many combinations of x and y are possible if the given no is divisible by 9. 45. (5 and 9)

y = 0, 5.

x = 0, 0)

1) (0,0)

145. (9,0) (4,5)]145

4266[][] ⇒ 18

27.

Q (543845)/11.

8

5 4 3 4 = 8-8 = 0

8

Q 97506/11 = 9 7 3 0 6

18-7 = 11/11 = 0.

2M

7 : Subtract the twice of unit digit from other numbers

ex: 1484

148 − 2×4 = 140/7

884

88 − 8 = 80/7 ×

13 : Add the twice (four times) of unit digit in other digits.

Ex: 1352

135 + 2×4 = 143

14 + 12 = 26

17 : Subtract the five times of unit digit in from other digits

ex: 323

32 − 15 = 17

1785

178 − 25 = 153

15 − 3×5 = 0

19 : Add the three (two) times of unit digit

21793

2179 + 6 = 2188 2185

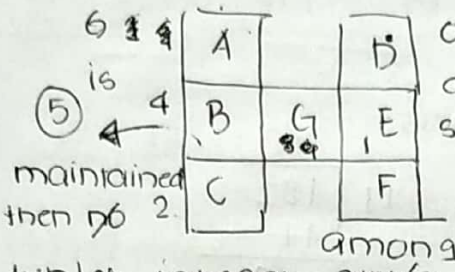
218 + 10 = 228

2+6 = 30

22+16 = 38

2M 2018.

If all the alphabets given in the Boxes are representing upto integer a unique integer from (1 to 9) and in such a way ABC = BGE = DEF



only the ABC and BGE will be divisible but can't say with DEF. similarly on (A) Posn only ABC is divisible by but BGE and DEF will need 5.

4/18
12
2×6.
3×4.
3×

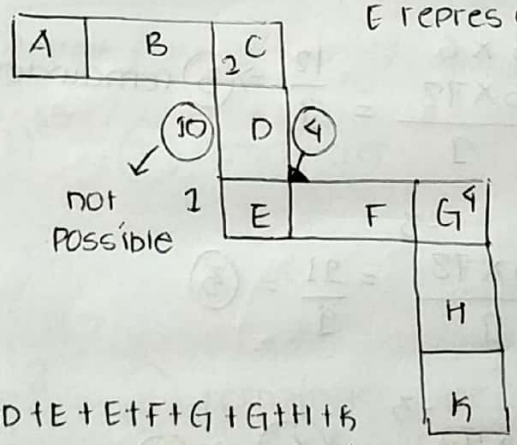
Which integer among the following option can't be represented by any letter.

i) 4 ii) 5 iii) 6 iv) 9.

↳ as can be disintegrated in 2×2 — But this needs a 5. — 2×3 others do will compensate. — 3×3.

Q. Each of a letter represent a unique integer from 1 to 9. and the letters are positioned in the figure such that a+b+c = c+d+e = e+f+g = g+h+k = 13. Which integer does E represent.

4



x i) 1
ii) 4
x iii) 6
x iv) 7

= A+B+C+C+D+E+E+F+G+G+H+K
= 13×4

If I put 1
E+G = 6
↓
2, 4
4 2
3 3 repeating.
1 5
5, 1

= A+B+2C+D+2E+F+2G+H+K = 52.

= A+B+C ... +K = $\frac{9(10)}{2}$ = 45.

= C+E+G = 7.

= 7 could not, 6 could not, 1

REMAINDER THEOREM:

(-2) 13 (+3) → +ve remainder

negative remainder ← 13/5

Separate remainders:

$= \frac{13 \times 14}{11} = \frac{182}{11}$ (2) (5)

$11 \overline{)182}$ (16, 11, 72, 66, 6)

Why?

$\frac{(11+2)(11+3)}{11} = \frac{(11\times3)+(11\times11)+(11\times2)+(2\times3)}{11}$

$= 6$

Negative remainders:

$\frac{13 \times 14}{11} = \frac{72}{11} = 6$ (-9, -8)

Q. $\frac{65 \times 78}{9} = \frac{12}{9} \Rightarrow 3$ remainder (2, 6)

$\frac{65 \times 78}{9} = \frac{21}{9} = 3$ (-7, -3)

$\frac{65 \times 78}{9} = -6 \Rightarrow 3$ (2, -3, +)

Q. $\frac{2 \times (-1) \times (-4) \times (-1) \times 192 \times 57 \times 1953 \times 1956}{19} = -4 \Rightarrow 15 - 8 \Rightarrow +11$ R.

Q. $\frac{2 \quad 15 \quad -2 \quad -3 \quad +1 \quad +2}{70 \times 100 \times 65 \times 1735 \times 87} = +24 \Rightarrow 7$

17

Q. $\frac{1! + 2! + 3! + 4! + 5! + 6! \ldots\ldots + 78!}{10}$

$= \frac{① + ② + ⑥ + ④}{10}$

$= \frac{12}{10} \quad \frac{13}{10} = ③$

Q. $\frac{① + ② + ⑥}{12.}$ → divisible by 0

$= \frac{9}{12} = ⑨$ remainder.

Q. $\frac{2^{65}}{3} = \frac{\ldots 8}{3} = ②$

or

$\frac{2^{-1} \times 2^{-1} \times 2^{-1} \times 2^{-1} \times 2^{-1} \ldots 2^{-1}}{3.} = (-1)^{65} = -1 = ②$ remainder

Q. $\frac{(4)^{65}}{3} = (1)^{65} = ①$ remainder.

Q. $\frac{2^{63}}{9} = \frac{\ldots 8}{9} = ⑧$ remainder. or $\left(\frac{2^3}{9}\right)^{21} = \left(\frac{8}{9}\right)^{21} = (-1)^{21}$

⑧ ④

$= ⑧$ remainder.

Q. $\frac{2^{63} \times 2^2}{9} = \frac{32}{9} = ⑤$

Q. What will be the remainder $\frac{3^{68}}{82}$

$= \frac{(3^4)^{15} \cdot 3^8}{82} = \frac{(81)^{15} \cdot 3^8}{82} = \frac{(81)^{17}}{82} = (-1)^{17} = -1$

$\frac{(81)^{15} \cdot (3^4)^2}{82}$ $= 81$

Q. $\frac{2^{63}}{} \overset{45}{\Rightarrow} \frac{(63)^{45}}{4} = (-1)^{45} = -1 = 3$ remainder

$2^3 = 8$ ---- 8

Q. $3^{81}{}^{54} \Rightarrow \frac{(81)^{54}}{4} \Rightarrow (1)^{54} = 1 = 1$

$3^{81} =$ ---- 43.

Note: Last 2 digits of any integer will be equal to the the remainder when we divide that no by 100.

Q. $\frac{48}{15} = R \Rightarrow 3$

$\downarrow \times 3$

$\frac{16}{5} \Rightarrow R \Rightarrow 1.$

Q. $\frac{2^{76}}{96} = \frac{2^{76}}{32 \times 3} = \frac{2^{76}}{2^5 \times 3} = \frac{2^{71}}{3} = \frac{(-1)^{71}}{3} = -1$

$= 2$

Q. Find out the last two digit $11279 \times 31239 \times 36 \times 65 \times 84 \times 96$.

$$\frac{11279 \times 31239 \times 36 \times 65 \times 84 \times 96}{100 \quad 50 \quad 4}$$

$$25$$

$2 \times 2 \times 5 = 20$.

$= -1 \times -1 \times -1 \times -2 \times -1 \times +1$

$= -2 + 5 = (3) \times 20 = 60$ Ans.

Q. $(x^a) \times (x^b) = x^{a+b}$.

Q. $q^{-p} = \frac{1}{r}$, $r^{-b} = \frac{1}{s}$ $s^{-c} = \frac{1}{q}$ find out the product of a.b.c 2018 2M.

$= q^a = r$

$= q^a r^b = s$

$= s^c = q$

$= s^{ac} = r$

$= r^{abc} = r$

$\boxed{abc = 1}$

Q. $p^{-x} = \frac{1}{q}$, $q^{-y} = \frac{1}{r}$, $r^{-z} = \frac{1}{p}$

$p^x = q$, $q^y = r$, $r^z = p$.

$r^{zx} = q$

$q^{xyz} = q \quad (1 = xyz)$.

TIME & WORK

* Think in the ways of eating chapatis.

Q. A = 10 => one day = 1/10

B = 15 => B " = 1/15

$A+B = \frac{1}{10}+\frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$] one day work of A and B

A = 10 L.C.M (T.W) = 30

B = 15

$\begin{array}{r} 3 \\ +\ 2 \\ \hline 5 \end{array}$

NO of Days = $\frac{30}{5} = 6$.

NO. of Days = $\frac{\text{T.W}}{\text{O.D.W}}$

Q. A = 6 T.W = 12 chapatis O.D.W

B = 4 Total days = 2 days 2

C = 12 3

1

6 chapatis

Q.

Note: To find out the no of days of any person we must know the one day work. of

Q. A+B = 12 days B = 1/30 T.W = 60 [C LCM (12,30)]

B = 30 days units

A = ? A = 1/x

O.D.W

A+B = 5 chapatis

B = 2 chapatis

A = 3 chapatis

NO. Of. days = $\frac{60}{3} = 20$ days

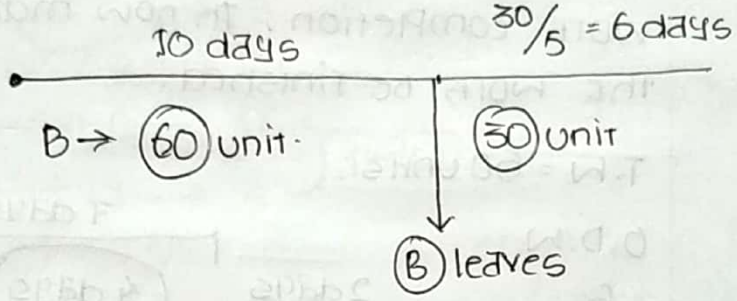
Q. A = 18 days
B = 15 days

B works alone for the first 10 days and then he leaves the work, in how many days remaining work would be done by A.

A = 12 × 6 = 90 | 0.D.W = 5
B = 15 × 5 = 90 | 6

(T.W) = 90

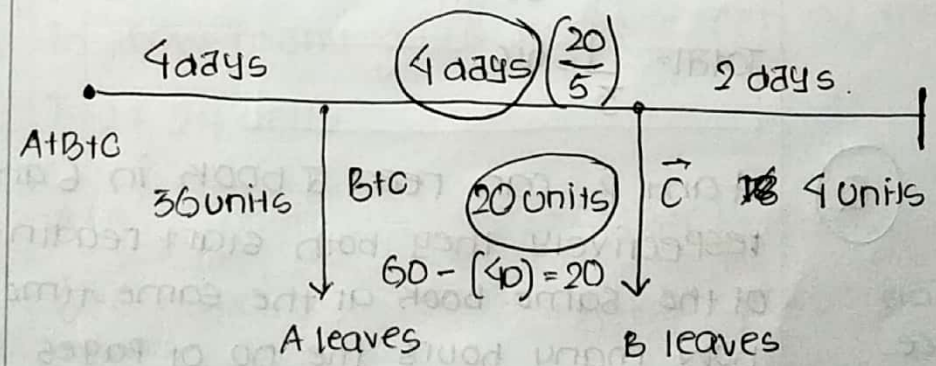
* Line concept is important



Q. All start the work together but after 4 days A leaves the work, B also leaves the work after some days and remaining work is done by C in 2 days.
How many days are required to finish the work?

A = 15 days 0.D.W 4
B = 20 days 3
C = 30 days. 2
A+B+C = 9 days unit.

T.W = 60 unit.

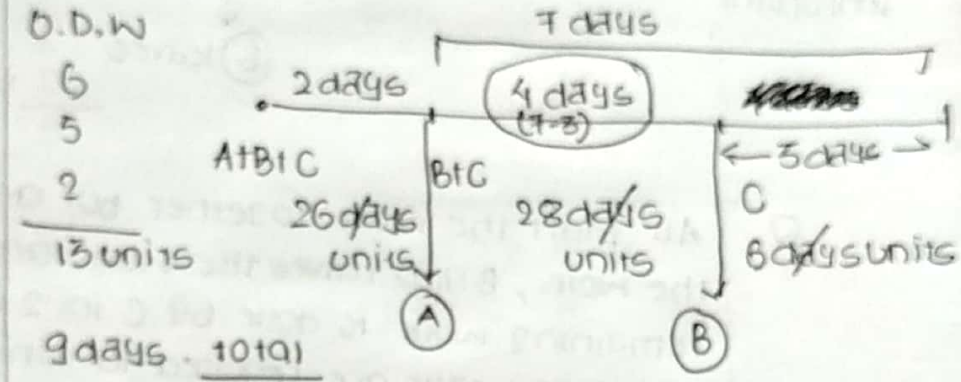


Total = 10 days.

Q. A = 10 days
B = 15 days
C = 30 days
all start the work together but A leaves the work B4 7days of work completion B also leaves the work before 3 days of work completion. In how many days will the work be finished

T.W = 60 units.

O.D.W
6
5
2
13 units



= 9 days. total

Q. GATE IM

	O.H.W	T.W = 4hr units
X = 2 hr	2 units	
Y = 4 hr	1 units	
	3 units	

Total = $\frac{4}{3}$ = hrs

Q. A and B can read a book in 6 and 4 hours respectively they both start reading copies of the same book at the same time after how many hours the no of pages is to be read by A will be twice of the no of pages is to be read by B.

Check Question ones once.

A=6 2 Pages T.W = 12 Pages in book.

B=4 3 Pages

5 Pages

1 — (A) 2 Pages read → 10 pages rem

(B) 3 Pages read → 9 pages rem

2 → 4 → 8

→ 6 → 6

3 → 6 → 6

→ 9 → 3 } twice

4

1 — (A)

2

3

4

Soln. A = 6. 2 Ø.4.

B = 4 3.

After 't' hrs

remaining pages of A after t time

$= (12-2t) = 2(12-5t)$] remaining pages of B after t time

$= 12-2t = 24-6t$

$= 4t = 24-12$

$t = 12/3 = 3.$

Q. If A+B = 12 days, B+C = 8 days, C+A = 6 days in how many days B alone can do that work.

T.W = 24 units

A+B = 2 units

B+C = 3 units

C+A = 4 units

9 units

∴ 2(A+B+C) = 9 units

A+B+C = 4.5 units.

B = 0.5 units.

$B = \frac{24 \times 10}{5} 2 = 48$ days.

$A = \frac{12}{24 \times 10} 2$

$= 24$ days

$C = \frac{24}{2.5} = \frac{48}{5} = 9\frac{3}{5}$

$A+B+C = \frac{45}{8} \times \frac{16}{24} \times \frac{2}{3} = \frac{16}{3} = 5\frac{1}{3}$

Q. If A+B can do a work in 10 days,

B+C = 15 days T.W = 60 days

C+A = 20 days

C = ?

O.D.W
6
4
3
13 units

A+B+C = $\frac{13}{2}$ = 6.5 units.

B+C = 6.5 units

C = 0.5 units.

$\frac{60}{0.5}$ = days = 120 days

Q.

#

$\times\frac{1}{2}$ → 5M = 200 days → $\times 2$.

$\times 2$ → 10M = 100 days → $\times\frac{1}{2}$.

20M = 50 days

$M_1 D_1 = M_2 D_2$.

$M_1 D_1 H_1 = M_2 D_2 H_2$

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

Q. M_1 = 12 men M_2 = 48 men

D_1 = 8 days D_2 = ?

$12 \times 8 = 48 \times x$

$\frac{12 \times 8}{2} = 48 \times x$

2 = x days.

Q. 20 Men can do a work in 9 days by working daily 12 hours. How many men will be required to compete the work in 24 days by working daily 6 hours.

$$\frac{20 \times 9 \times 12}{24 \times 6} = M_1$$

$$\frac{15}{2}$$

$$\frac{20 \times 9 \times 12}{24 \times 6} = M_1$$

$$= 15 \text{ days} = M_1$$

Q. 32 Men can make 400 tiles in 25 days by working daily 6 hours then how many tiles would be made by 30 men in 24 days by working daily 5 hours.

$$\frac{32 \times 400 \times 25 \times 6}{400 \times 24 \times 5 \times 30} = \frac{1}{W_2}$$

$$300 \text{ tiles} = W_2.$$

Q. 7 Machine takes 7 min to make 7 toys and the same rate how many would it take for 100 M to make 100 toys.

$$\frac{7 \times 7 \times 7}{7 \times} = \frac{100M \times h}{100}$$

7 mins

Q. 4M 6 Days.
3W 16 days.

if a team has 1M and 2W how long will it take to do the work?

$$= 4M \times 6 = 3W \times 16$$

$$= 24M = 48W$$

$$= \boxed{M = 2W}$$

$$= (1M + 2W) \times D_5 = 4M \times 6.$$

$$= 2M \times D_5 = 4M \times 6$$

$$= D_5 = 12.$$

A

Q.

$$= 1M \times \frac{1}{24} + 2W \times \frac{1}{48} = \frac{1}{12}$$ Ans (12 days).

$\Rightarrow$ A = 6 days

$\Rightarrow$ A's $\frac{1}{6}$ days unit

$A = (a_1, a_2, a_3, a_4)$

$\Rightarrow a_1 = \frac{1}{6 \times 4}$

$\Rightarrow a_1 = \frac{1}{24}$

$B = (b_1, b_2, b_3)$

$\Rightarrow$ B = 16 days

$\Rightarrow b_1 = \frac{1}{16 \times 3} = \frac{1}{48}$

$\Rightarrow 1 \times \frac{1}{24} + \frac{1}{48} \times 2 = 1$ day work.

$= \frac{2}{24} = \frac{1}{12}$ = one day work.

Q. 3M = 16 days
6W = 16 days
12M and 8W together

$$= 12 \times \frac{1}{48} + 8 \times \frac{1}{96} \Rightarrow \frac{1}{4} + \frac{1}{12} \Rightarrow \frac{3+1}{12} \Rightarrow \frac{1}{3}.$$

$\Rightarrow$ 3 days.

$= 3$ days.

Q. 20 skilled workers can build a wall in 5 days
20 25 semi skilled workers can build a same wall in 8 days.
30 unskilled workers = 10 days
(25 + 6 skilled + 5 Unskilled) workers.

$$= \frac{1}{50} + \frac{3}{6} \times \frac{25}{25 \times 4} + \frac{8}{8} \times \frac{1}{600}$$

$$= \frac{1}{50} + \frac{3}{100} + \frac{1}{600}$$

$$= \frac{12 + 18 + 1}{600} = \frac{31}{600}$$

$$= 2 \times \frac{1}{100} + 6 \times \frac{1}{200} + 5 \times \frac{1}{300}$$

$$= \frac{12 + 18 + 10}{600}$$

$$= \frac{1}{15} \frac{40}{600} = 15 \text{ days}.$$

combined categories Question :- 12 men and 18 women can ⇒ 10 days
(9 men and 18 women can ⇒ 12 days
2 men and 3) ll together can do that work = ?

$$= (12M + 18W) \times 10 = (9M + 18W) \times 12 \text{ days}$$

$$= 120M + 180W = 108M + 216W$$

$$= 12M = 36W$$

$$M = 3W$$

$= (2M+3W) \times D_3 = (12M+18W) \times 10$

$= (6+3) \times D_3 = (36+18) \times 10$

$= D_3 = (2+1) \times 10$

$= D_3 = 30$ days 60 days.

checking ratios

$= (2M+3M) \times D_3$

$\times 6 \quad \times 6 \quad \times 6$

$= (12M+18W) \times 10$

Q. $(6M+8W)$ can do a work in 10 days

$(26M+48W) \times 2$

$(7M+5W)$

$= (6M+8W) \times 10 = (26M+48W) \times 2.$

$= 60M+80W = 52M+96W$

$= 8M = 16W$

$= M = 2W$

$(7M + 3W) \times D = (6M + 8W) \times 10$

$17 \times D = 20 \times 10$

$D = 40$ days. $\frac{200}{17}$ days.

$= 12\frac{2}{17}$.

Q.

Q. $(x$ bullocks $+ y$ tractors$) \times 8$ days.

$\left(\frac{x}{2} + 2y\right) \times 5$ days

$= 8x + 8y = \frac{5}{2}x + 10y$.

$= \frac{11}{2}x = 2y$

$= 11x = 4y$. Or $\frac{x}{y} = \frac{4}{11}$ [put the values directly]

$= (x + 0) \times D = 8x + 8y$. only bullocks

$= \frac{4y}{11} \times D = \frac{8 \times 4y}{11} + 8y$.

$= \frac{44y \times D}{44} = \frac{32y + 88y}{4}$

$D = 19$ days.

$\therefore \quad xD = (x+4)8$

$\Rightarrow \quad 4D = (15)8$

$\Rightarrow \quad D = 30 \text{ days.}$

Q. $\left(1200^{M} + 500^{W}\right) \times 12.$

$\left(900^{M} + 250^{W}\right) \times 13$

How many men would be needed to build the bridge in one week.

$\Rightarrow \quad (1200M + 500W) \times 2 = (900^{M} \times 3 + 250W \times 3)$

$\Rightarrow \quad 2400M + 1000W = 2700M + 750W$

$\Rightarrow \quad 250W = 300M$

$\Rightarrow \quad 5W = 6M$

$\Rightarrow \quad \frac{M}{W} = \frac{5}{6}.$

$MD = (900M + 250W) \times 5$.

$M = (900 \times 5 + 250 \times 6) \times 3$.

$= 4500 + 30$

~~$xM \times 1 = (900M +$~~

$= xM \times 1 = (1200M + 500W) \times 2$.

$= xM \times 1 = 3600M + 1000W$.

$=$ 3600M. Ans.

Q.

Q. 50 15M in 40 days but after every 10 days All start the work together but after every 10 days 5 men will leave the work in How many days will the work be finished?

Ans. $50 \times 40 = 2000$U

M	50	45	40	35	30
D	10	10	10	10	10

$500 + 450 + 400 + 350 + 300$

= 50 days. it would be completed.

Q. A contractor has a contract to develope a road 120 days He put 100 men on work but after 45 days He finds only $\frac{1}{4}$th work has been completed then how many more man are required to finish the work on.

T.W = ~~100 x 120 = 12000 units~~.

M	100	
D	45	75
	3000	9000
	4500	1500 + = 10500

but done 3000

$\frac{10500}{75}$

$$75 \overline{)10500} \quad 140$$

75

300

300

00

Q.

Ans: i) $\frac{100 \times 45}{1/4} = \frac{M_2 \times 75}{3/4}$.

$M_2 = 180$.

ii) $M \times 75 = 13500$

$45 \times 100 = 4500$.

$\frac{1}{4} = 4500$

$1W = 4500 \times 4$.

$W = 18000$

$R.W = \frac{13500}{75} = 180$

$180 - 100 = 80$ Men.

Q.

Q. A contract has to be completed ~~calculated~~ in 52 days and 125 robots were employed. But after 39 days only $5/7^{th}$ work is completed. Each robot

operational for 7 hours a day. Then how many more robots are to be completed work.

$$= \frac{125 \times 39 \times 7}{5 \times 7} = \frac{R_2 \times 13 \times 7}{2 \times 7}$$

$$= \frac{25 \times 39 \times 2}{13} = R_2$$

$$= 150 = R_2.$$

$$= 150 - 100 = 50 \text{ Robots.}$$

Q. P, Q, R, S are working on a project and Q can finish the task in 25 days, working alone for 12 hours a day. R → 50 days, working alone for 12 hours. Q → 12hr/day → ~~six~~ but took sick leave in beginning for 2 days. R worked 18 hours a day on all the days. What is the ratio of work done by Q and R after 7 days from the start.

Ans. : Q = 300 hrs

= T.W = 600

: R = 600 hrs

$= 4 \times 5 \times 2 : 3 \times 7 \times 1$

$= \boxed{20 : 21}$

Q. II: $\frac{1 \times 12 \times 5}{25 \times 12} : \frac{1 \times 18 \times 7}{50 \times 12}$

Q R

20 : 21.

Q. S, M, E, F are working in shift in a team to finish a project M works with twice the eff of others but for half days as many as E worked. S and M has 6 hours ~~swift~~ shift in a day whereas E and F has 12 hours shift. what is the ratio of contribution of M to contribution of E in the project.

S	M	E	F.
x	2x6 :	12x2	x

1 : 2.

ALTERNATE WORK

Q.

A = 10 days

B = 15 days

Then in how many days will the work be finished if they work alternat-e?

T.W = 30

A : 10 3

B : 15 2.

3 2 3 2 3 2 3 2 3 2

3 2 → 30

NOTE: multiply the one day work of both all to a number such that either we get the total work or just less than the total work. Then multiply that number with no of alternate days.

= A : 10 3

B : 15 2

5 × (6) = 30

↳ 2 = 12 days

Q.

Q.

Q.

Q.

Q. A = 10 3
B : 15 2
C : 30 1
6 × 5
ToW = 30. ↓ ×3
15 days.
alternate work?

Q. A : 15 days 4
B : 20 days 3
7 → 8
↓ ×2
16 days. > 56 unit.
ToW = 60.
A start the work.
then again A comes : 4 + 56 = 60 unit.
16 + 1 = 17 days.

Q. If B start the work
= 56 + 3 + 1/4 A.
= 59 work + 1/4 th day.
= 17 days + 1/4 th day.
= 17 1/4 days.

Q. A : 10 from start to end 3
B : 15 ; alternate } with him 2
C : 30 ; alternate. 1

A+B : 5

A+C : 4

9 x 3 = 27

↓ 2 days

6 days = 27

rem works = 3.

time taken = $\frac{3}{5}$

∴ Total time taken = $6\frac{3}{5}$

5 work (A+B) = 1

1 w → $\frac{1}{5}$

3 → $\frac{1}{5} \times 3$

$\frac{3}{5}$

Q. What is the Question.

A : 3

A+B : 5

A+B+C : 6

14 x 2 = 28

rem = 2. ×3 → 6 days

Time taken = 2/3 = ~~6~~

Total = 6 + 2/3 = 6 2/3.

CAPACITY: RELATED QUESTIONS

$C \propto \frac{1}{D}$

C A : B

2 : 3

D 3 : 2.

Q. A's capacity is 40% higher than B's capacity if B can do this work in 14 days, then in how many days can A do this work?

	A		B
C:	140		100
	7	:	5
D ⇒	5x	:	7x
	10	:	14

Q. A can do 50% more work as B can do in same time, B alone can do a piece of work in 20 days then in how many ~~hours~~ days can A and

B finish the work?

	A		B
	150		100
C:	3	:	2.
D:	2	:	3x
	⇓		⇓
	40/3 days		20 days

$x = \frac{20}{3}$.

T. W 40 units.

A = 3

B = $\frac{2}{5}$

$\Rightarrow \frac{40}{85}$ = 8 days.

hours 8 × 24 = 192

B :	A+B.
2 :	5
5x :	2x
↓	↓
20	8

PIPE AND SISTER CISTERN

Q. A tap can fill the tank in 4 hours and other tap can empty the tank in 5 hours in how many hours will the tank be full if they both open together if initially they were empty?

T.W = 20

A : 4 5

B : 5 -4

1 unit

To fill 20 unit = $\frac{20}{1}$ = 20 hrs.

Q. (Question verification.)

X + Y : 5 and 4 hours.

T.W 20 hrs

X : 5

Y : 4

Each pipe X & Z can empty the fully filled tank in 20 hours. If All the pipes are opened together how many ~~tish~~ times the tank would be full?

Ans

X : 5 4

Y : 4 5

Z : 20 1/8

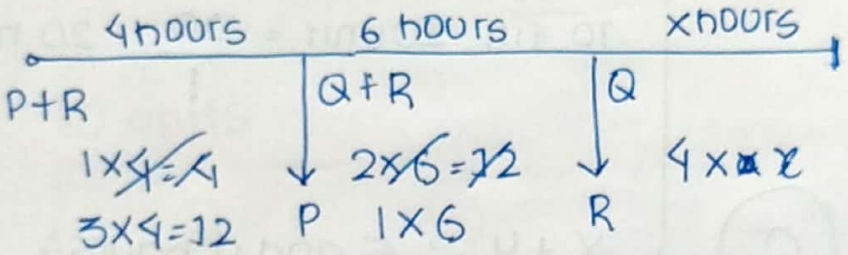
$\frac{20}{8}$ = 2.5 hrs.

Q. P : 6 ✗ 6 ToW = ~~12~~36.

Q : ~~8~~9 ~~3~~ 4

R : 12 +~~1~~ 3

Initially P and R are opened for 4 hours then P is closed and Q is opened., after 6 more hours R is closed. Total time taken to fill the tank?



$12 + 6 + 4x = 36$

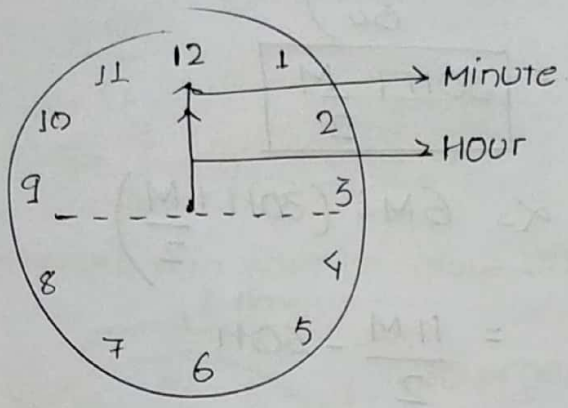
$18 + 4x = 36$

$x = \frac{36 - 18}{4}$

$x = \frac{18}{2} \frac{9}{4} = 4.5$ hours.

14.5 hours total.

CLOCK



Minute

$60M = 360°$

$1M = 6°$

HOUR

$12 hrs \rightarrow 360°$

$1 hr \rightarrow 30°$

$60 min \rightarrow 30°$

$1 min \rightarrow \frac{30}{60} = \frac{1}{2}°$

Q1. Ex Between 4:20 — Angle

$\alpha = \Phi_2 - \Phi_1$

$\Phi_1 = 20 \times 6 = 120°$

$\Phi_2 = 4 hr\ 20 min = 4 + \frac{20}{60} = \frac{13}{3} hr$

$= \frac{13}{3} \times 30 = 130°$

$\alpha = 10°$

If we talk about HH:MM

Q2. $\alpha = \phi_2 - \phi_1$.

4:40

$\phi_1 = 4 + \frac{40}{60} = \frac{14}{3}$

$= \frac{14}{3} \times 30 = 140°$

$\phi_2 = 40 \times 6 = 240°$

$\alpha = 240 - 140 = 100°$

Ex: 5:36

$\alpha = \left|30\times5 - 11\left(\frac{36}{2}\right)\right|$

$= |150 - 11\times18|$

$\Rightarrow |150 - 198|$

$\Rightarrow 48°$

(1) Angle made by Minute hand

$= \boxed{6M°}$

(2) $H = \left(H + \frac{M}{60}\right)$ hr.

$H° = \left(H + \frac{M}{60}\right)\times30$

$= \boxed{30H + \frac{M}{2}}$

(3) $\alpha = 6M - \left(30H + \frac{M}{2}\right)$

$= \frac{11M}{2} - 30H$

$\alpha = 30H + \frac{M}{2} - 6M$

$= \left[30H - \frac{11M}{2}\right]$

$\Rightarrow \boxed{\alpha = \left|30H - \frac{11M}{2}\right|}$

$30H - \frac{11}{2}M = \phi$

$\Rightarrow \frac{11M}{2} = 30H - \theta$.

$\Rightarrow M = \frac{2}{11}(30H - \theta)$

$\Rightarrow \theta = \frac{11}{2}M - 30H$

$\Rightarrow M = \frac{2}{11}(30H + \theta)$

$\Rightarrow M = \frac{2}{11}(30H \pm \theta)$

Q What time between 4-5 the minute hand and hour hand will coincide

$$M = \frac{2}{11}(30 \times 4 \pm 0)$$

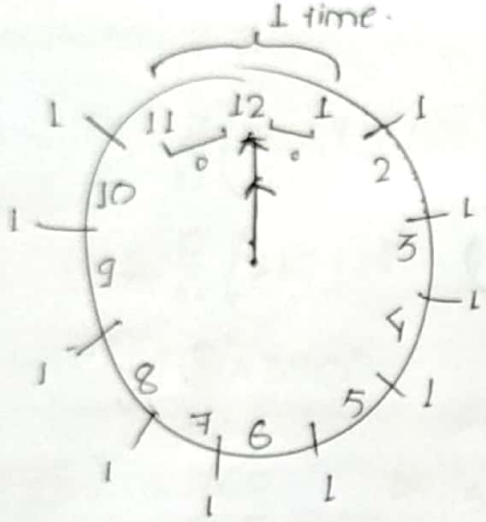
$$= \frac{2}{11}(120)$$

$$= \frac{240}{11}$$

$$M = 21\frac{9}{11} \text{ min.}$$

We can say $4 : 21\frac{9}{11}$ the clock will coincide

NOTE: Overlap will happen only once in the clock between 1 hour.



10 times + Once during 11 and 12.

1 hour = 1 time

12 hour = 11 times

24 hour = 22 times

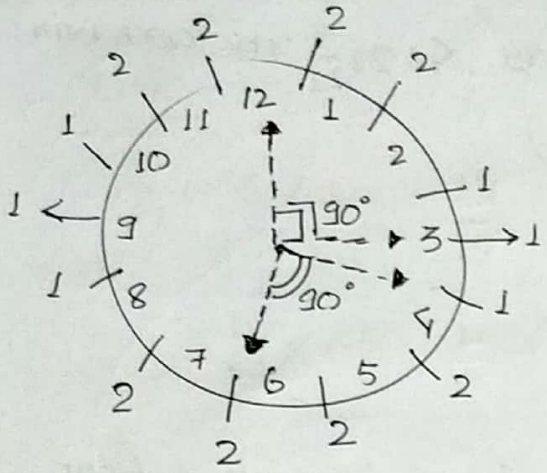
Ex At what time between 7-8 o clock the min hand and hour hand will coincide.

$$M = \frac{2}{11}(30 \times 7 \pm 0)$$

Time : $7 : 38\frac{2}{11}$ min

$$M = \frac{2}{11}(210) = \frac{420}{11} = 38\frac{2}{11}.$$

90° degrees.



1 hour = 2 times
12 hour = 22 times
24 hour = 44 times

between 10 - 11.

$$M = \frac{2}{11}(10 \times 30 \pm 90)$$

$$= \frac{2}{11}(300 + 90) \text{ or } \frac{2}{11}(300 - 90)$$

$$= \frac{780}{11} \text{ or } \frac{420}{11}.$$

Q. between 5 and 6 how many times clock hands will be at 90°

$$M = \frac{2}{11}(150 \pm 90)$$

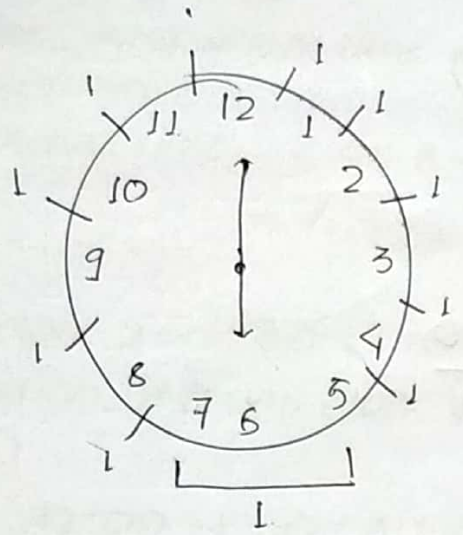
$$= \frac{2}{11}(240) \text{ or } \frac{2}{11}(60)$$

$$= \frac{480}{11} \text{ or } \frac{120}{11}$$

$$= 43\frac{1}{11} \text{ or } 10\frac{10}{11}.$$

$$= 5:43\frac{7}{11} \text{ or } 5:10\frac{10}{11}.$$

180° degree



1 hour = 1 time.

12 hour = 11 times

24 hour = 22 times

between 7-8

$$M = \frac{2}{11}(30 \times 7 \pm 180^\circ)$$

$$= \frac{2}{11}(210 + 180) \text{ or } \frac{2}{11}(30)$$

$$= \frac{2}{11} \times 390^\circ \text{ or } \frac{60}{11}$$

$$= \frac{780}{11} \text{ or } 5\frac{5}{11}$$

$$= 70\frac{10}{11} \text{ or } 5\frac{5}{11}.$$

$$\downarrow$$

$$8:10\frac{10}{11} \text{ or } 7:05\frac{5}{11}$$

between 3 and 4.

$$M = \frac{2}{11}(210 \pm$$

$$= \frac{2}{11}(90 \pm 180)$$

$$= \frac{2}{11}(-90) \text{ or } \left(\frac{2}{11} \times 270\right)$$

$$= \frac{2}{11} \cdot 180 \text{ or } \frac{540}{11}$$

$$= -16\frac{4}{11} \text{ or } 49\frac{1}{11}.$$

$$= 2 \downarrow \qquad 3:49\frac{1}{11}.$$

couldn't be taken

$2:43\frac{37}{11}$ and $3:49\frac{1}{11}$

NOTE: $H < 6 \quad \theta \rightarrow +180$.

$H < 6 \quad \theta \rightarrow -180$

At what time between 6 and 7 the angle of clock would be closest to 60°.

2M GATE:- 14.

i) 6:45
ii) 6:22
iii) 6:36
iv) 6:15

check the difference

$$M: \frac{2}{11}(6\times30 + 60^\circ)$$

$$= \frac{2}{11}(180 \pm 60)$$

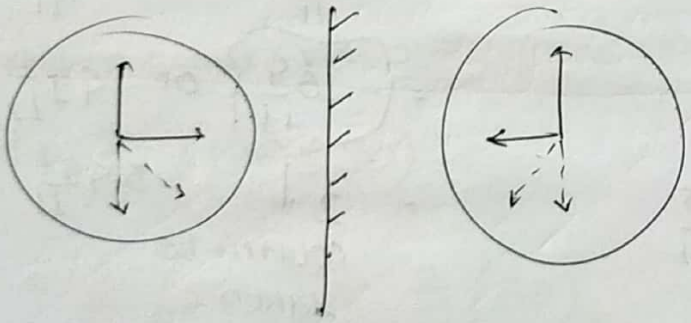
$$= \frac{2}{11}(120) \text{ or } \frac{480}{11}$$

$$= \frac{240}{11} \text{ or } \frac{480}{11}$$

$$= 21\frac{9}{11} \text{ or } 43\frac{7}{11}.$$

MIRROR CLOCK

→ Actual Time + Mirror Time = 12 hours.



Mirror

A = 2:00
M = 10:00

A = 2:15
M = 9:45
→ 12 hrs.

$$\begin{array}{r} 12:00 \\ -\ 2:15 \\ \hline 9:45 \end{array}$$

Q Before 2 hour the ~~mirror~~ current time is 3:15 then what is actual mirror

→ First find actual time as you are doing operations a/q to the current time

→ Actual time : 8:45 P → −2hour

⇒ 6:45

GATE - 16 2M

Q. Before 2 and Quarter hour back when seen in the mirror the reflection of a wall clock w/o no. markings seem to show 1:30

12:00 − 1:30 = Actual time = 10:30.

10:30 + 2:15 = 12:45

Q. GATE - 18 A clock loses 16 min in 24 hours and the clock was synchronized to the correct time at 5:00 AM on 5th December. What will be the true time when the clock indicates 10:PM on 8 December of the same year.

20hrs 24 hrs = 60 min

$1 - \frac{60}{24} \times 20$

$= 6\frac{12}{24}$ hrs

$= 50$ min.

3 hours 50 min.

1:50 AM

17 $\frac{60}{24} \times 17$

$\frac{12 \times 50}{24}$ 50 $\frac{40}{3}$

$16 \times 3 + \frac{40}{3} = \frac{144+40}{3} = \frac{184}{3}$

$= 61\frac{1}{3}$

$11.\frac{1}{3}$.

23:44 hrs of defective watch ⇒ 24 hrs of correct clock

$23\frac{11}{15}$ ——— ⇒ 24 hrs

1 ——— ⇒ $24/23\frac{11}{15}$

$89hr = \frac{24}{356} \times 15 \times 89$

$= 90 hrs$

89 hrs of DW = 90 hrs of CC.

11 PM of C.C.

* solving

$\Rightarrow$ 24 hrs = 16 min.

$\Rightarrow$ 1 hrs = $\frac{16}{24}$ min

$\Rightarrow$ 89 hrs = $\frac{16}{24} \times 89$ min

$= 59\frac{1}{3}$ min $\longrightarrow$ $\frac{22}{7} = 3.141.5$

GATE 2018:

A faulty wall clock ~~[illegible]~~ is now to gain $\frac{15}{50}$ min is every 24 hrs. It is synchronized to the correct time at 9 AM on 11 July what will be the correct time to the nearest minute. when the clock shows 2PM on 15 July.

~~24×4 + 5 = 96 + 15 = 101 hrs.~~

~~24 → 50 min 15 min~~

$1 \rightarrow \frac{15}{24}$

$101 \rightarrow \frac{15}{24} \times 101 = \frac{1515}{24} \frac{505}{6}$

$= 63\frac{1}{8}$

$3 + 15\frac{1}{8}$

OR

24 : 15 min of f.c = 24.

$\frac{97}{1584} - 24$

$1 = \frac{24 \times 15}{97}, \frac{96}{97}$

$101 = \frac{24 \times 15}{97} \times 101.$

$101 \text{ hrs} = \frac{96 \times 101}{97}$

$= 99.958... hrs.$

12:45 PM.
12:58 PM closest one
1:00 PM
× 2:00 PM

TIME AND DISTANCE

$D = S \cdot T$ $\quad S = \frac{D}{T}$ and $T = \frac{D}{S}$

$\Rightarrow x$ km/hr $= x \times \frac{5}{18}$ m/sec

$\Rightarrow x$ m/sec $= x \times \frac{18}{5}$ km/hr

Q If boy goes from home to school with a speed of 8km/hr and returns back with speed of 12 km/h if he takes total 10 hrs. find out the distance btw home and school?

i) ~~$8x = 12y$~~ ~~$12x = 24$~~

ii) Time taken: $\frac{x}{8} + \frac{x}{12} = 10$

$= \frac{3x + 2x}{24} = 10$

$= 5x = 240$

$= x = 48$ km

$D = \frac{S_1 \times S_2}{S_1 + S_2} \times T.T$

Q. If a boy goes with the speed of 30km/hr then he will reach the target at by 10 min late. If he goes 40 km/hr then he reaches 20 min earlier. Distance?

~~$30\left(\frac{x}{30} + \frac{1}{6} \times \frac{60}{1}\right)$~~

$= \frac{x}{30} - \frac{x}{40} = \frac{30}{60}$ hrs

Time difference

= 30 min.

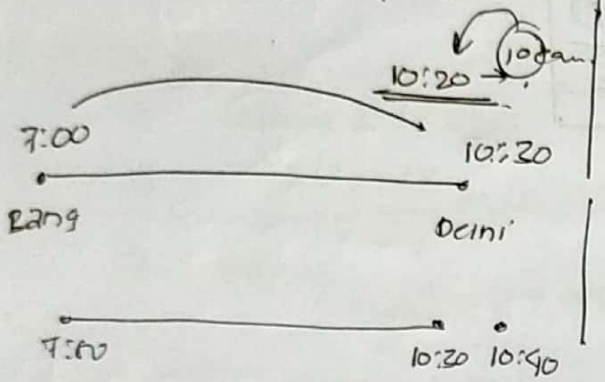
case1 case2

$\Rightarrow \frac{x}{30} - \frac{x}{40} = \frac{1}{2}$

$\Rightarrow \frac{4x - 3x}{120} = \frac{1}{2}$

$\Rightarrow \frac{x}{120} = \frac{1}{2}$.

$\Rightarrow x = 60$ kms.



$T_1 = \frac{60 \text{ kms}}{30} = 2$ hrs.

became 10 min late.

$\therefore T_E = 1:50$ min you should reach at in.

T case 2.

$= \frac{60}{40} = 1\frac{1}{2} = 1$ 30min

became 20min earlier.

T_E 1:50min. $\therefore$

Q. If a boys goes to school speed of 20km/hr then he will get late by 10min if he goes with the speed of 30 km/hr then he reaches 10 min earlier. What should be his exact speed to reach on time?

$\Rightarrow \frac{x}{20} - \frac{x}{30} = \frac{20}{60}$

$\Rightarrow \frac{3x - 2x}{60} = \frac{20}{60}$

$\Rightarrow x = 20$ kms.

he need to reach at.

$\Rightarrow \frac{20 \text{ kms}}{20} \rightarrow 1$ hour $\Rightarrow$ 60min.

$\therefore \frac{20 \times 60}{50/60} = \frac{20 \times 66}{55} = 24$ km/hr.

$\Rightarrow \frac{x}{S_1} - \frac{cx}{S_2} = T.D \Rightarrow x\left(\frac{S_2 - S_1}{S_1 \times S_2}\right) = T.D$

$x = \frac{T.D(S_1 \times S_2)}{(S_2 - S_1)}$

Q. If a man goes at the speed of 40 km/hr then he will get late by 30 min but if he goes with a speed of 60 km/hr then he will get late only by 10 min. speed?

$$\Rightarrow \frac{x}{40} - \frac{x}{60} = \frac{20}{60}$$

$$\Rightarrow \frac{3x - 2x}{120} = \frac{20}{60}$$

$$= \frac{x}{120} = \frac{20}{60}$$

$$\Rightarrow x = 40 \text{ km}$$

$\Rightarrow S$

TE → 30 min he should

$$T_1 = \frac{40}{40} = 1 \text{ hr} \quad (60-30)$$

$$T_2 = \frac{40}{60} = \frac{2}{3} \times 60 = 240 \text{ min}$$

$$TE = \frac{1}{2} \text{ hr} = (40 - 10) = 30 \text{ mins.}$$

$$\text{Speed} = \frac{40 \times 2}{1} = 80 \text{ km/hr.}$$

Q. A man covers certain distance by scooter if he increases his speed by 3 km/hr then he will take 40 min less than usual time but if he reduces the speed 2 km/hr 40 min more than usual time. Distance.

$D = y$

$S = x$ km/hr

$T = \frac{y}{x}$

$\Rightarrow$ (1) $S_1 = x + 3$

$\Rightarrow T_1 = \frac{y}{x+3}$

$$\Rightarrow \frac{y}{x} - \frac{y}{x+3} = \frac{40}{60} = \frac{2}{3} \text{ hr.}$$

(2) $S_2 = x - 2$

$T_2 = \frac{y}{x-2}$

$$\frac{y}{x-2} - \frac{y}{x} = \frac{40}{60} = \frac{2}{3} \text{ hr.}$$

$$\Rightarrow \frac{y}{x} - \frac{y}{x+3} = \frac{y}{x-2} - \frac{y}{x}$$

$$\Rightarrow \frac{x+3-x}{x(x+3)} = \frac{x-x+2}{(x-2)x}$$

$$\Rightarrow 3(x-2) = 2(x+3)$$

$$\Rightarrow 3x - 6 = 2x + 6.$$

$$\Rightarrow x = 12.$$

$\Rightarrow \frac{x(x+3)}{3} \cdot \frac{2}{3} = \frac{x(x-2)}{2} \cdot \frac{2}{3} = 0$

$\Rightarrow 2x+6 = 3x-6.$

$\Rightarrow \boxed{12 = x}$

$\Rightarrow \frac{y}{12} - \frac{y}{15} = \frac{2}{3}$

$\Rightarrow \frac{5y - 4y}{60} = \frac{2}{3}$

$\Rightarrow y = 40 \text{ km}.$

Q. A man covers a certain distance by car if he increase the speed by 6km/hr then he will take 4hr less than usual time if he ~~increase~~ decrease the speed by 6 km/hr, then he will take 6 hours more to complete the journey find out the distance.

$D = y$

$S = x$

$T = \frac{y}{x}$

$\Rightarrow \frac{x(x+6)}{6} \times 4 = \frac{x(x-6)}{6} \times 6$

$\Rightarrow 4x(x+6) = 6x(x-6)$

$\Rightarrow 4x^2 + 4x + 24 = 6x - 36$

$\Rightarrow 60 = 2x$

$\Rightarrow x = 30 \text{ km/hr}.$

$\Rightarrow \frac{y}{x} - \frac{y}{x+6} = 4$

$\Rightarrow y\left(\frac{x+6-x}{x(x+6)}\right) = 4.$

$\Rightarrow y\left(\frac{6}{30 \times 36}\right) = 4$

$y = 120 \times 6$

$= 720 \text{ km}.$

Q. Excluding stoppage the speed of a bus is 45 km/hr and including stoppage the speed of a bus is 36 km/hr for how many minutes does the bus stop per hour?

S. $T_E = \frac{x}{45}$ 100 80. $S.T = \frac{x}{36} - \frac{x}{45}$

$T_I = \frac{x}{36}$. $= \frac{x}{180}$ hrs.

In $\frac{x}{36}$ hrs the bus stops for $= \frac{x}{180}$ hr.

1 hrs $= \frac{x}{\frac{180}{5} \times \frac{x}{36}} = \frac{1}{5}$ hrs. $= \frac{1}{5} \times 60 = 12$ min.

$T_E = \frac{x}{S_E}$

$T_I = \frac{x}{S_I}$

$S.T = \frac{x}{S_I} - \frac{x}{S_E}$

$= \frac{x(S_E - S_I)}{S_I S_E}$

proof.

In $\frac{x}{S_I}$ hr $\longrightarrow \frac{x(S_E - S_I)}{S_I S_E}$

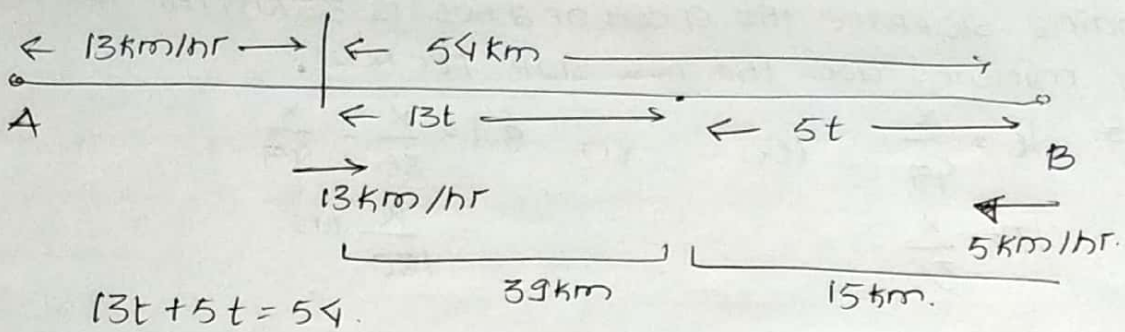
1 hr $\longrightarrow \frac{x(S_E - S_I)}{S_I S_E \cdot \frac{x}{S_I}} = \frac{S_E - S_I}{S_E}$

One hour stopping Time = $\boxed{\frac{S_E - S_I}{S_E}}$

Q. The distance between two cities city A and city B is 67 km at 7AM a boy starts at city A towards city B with the speed of 13 km/hr after 1 hour a girl starts from city B towards city A with a speed of 5 km/hr.

→ At what time they will meet?

→ At what distance from city A will they meet.



$13t + 5t = 54$.

$t = (3)$

D = 54
S = 18
T = 3.

$\Rightarrow \frac{54 - x}{13} = \frac{x}{5}$

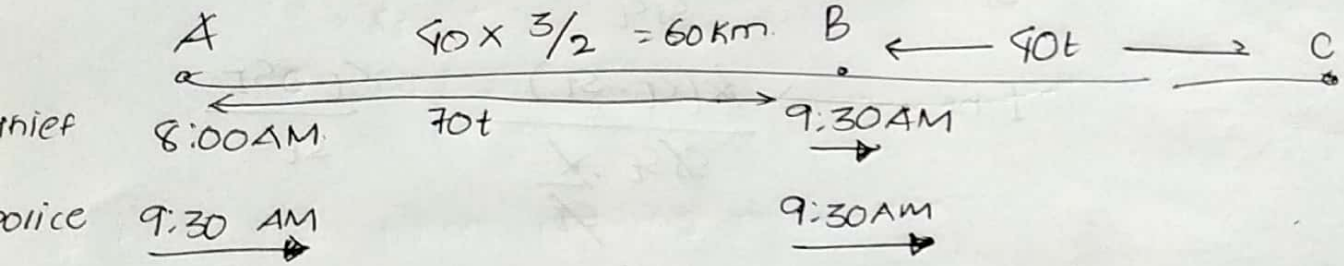
$\Rightarrow 270 - 5x = 13x$

$\Rightarrow 18x = 270$

$\Rightarrow x = \frac{270}{18} = 15$

Q.

A thief steals a car at 8 AM and he drove the car at 40 km per hour the theft was found at 9:30 AM and police starts chasing him with the speed of 70 km/hour, At what time will the thelf be caught?



$70t - 40t = 60$

$t = 2$ hrs.

R.S = 30

D = 60

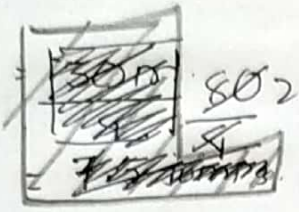
t = 2 hrs.

Q. Distance between a dog and a rabbit is 30M when rabbit takes 5 jumps dog takes 4 jumps in the same direction. Rabbit covers 2 m per jump dog covers 4m per jumps how many jumps does the dog require to catch the rabbit?

Rabbit = 10m/sec.
Dog = 16m/sec.

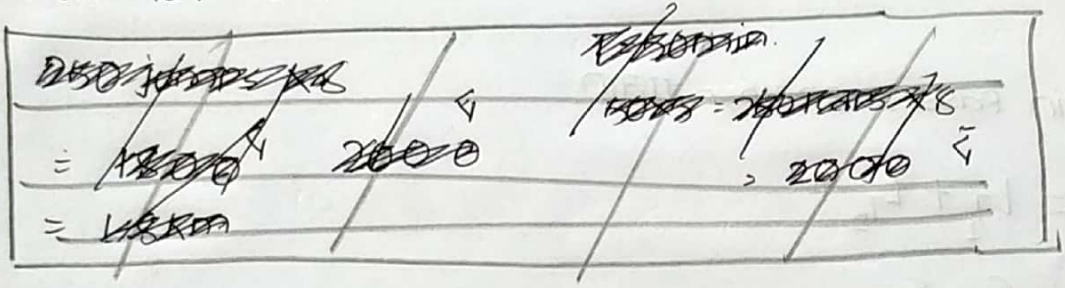
5sec.

Dog = 16×5 = 80

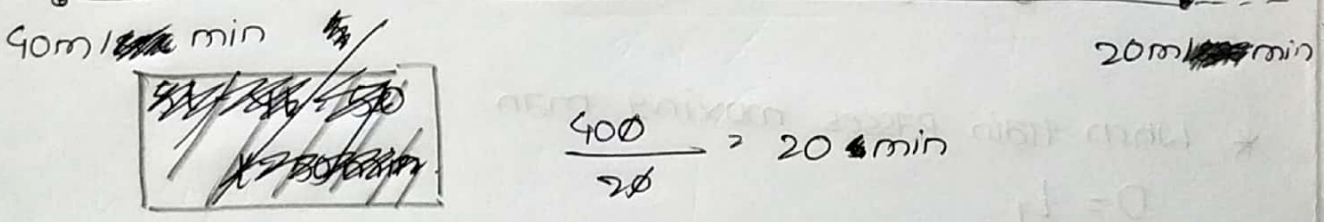


20 jumps.

Q. A tiger is 50 leaps of its own behind a deer the tiger takes 5 jumps/min deer takes 4 jumps/min. If the tiger, cover 8m and 3m per leap respectively, what distance and deer the tiger have to ran to catch the deer.



50 leap ×8 = 400m.



40m/min

20m/min

$\frac{400}{20}$ = 20 min

⇒ 5 40×20 = 800m.

Q. Two trains started at 7AM from the same point the first train travelled North at speed of 80 km/hr and the second train travelled south at speed of 100 km/hr the time at which they were 540 km apart.

3hrs. 18 540/180 = 3hr.

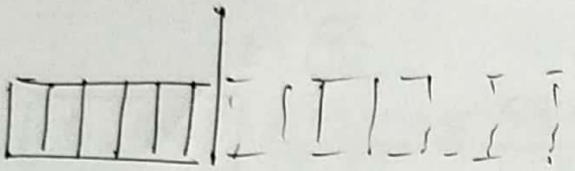
or at 10:00 AM.

TRAIN RELATED QUESTIONS

* When train passes a stationary object or person

$D = L_T$ (length of train)

$S = S_T$ (speed of train)



* When train passes a tunnel or platform / tunnel / Bridge

$D = L_T + L_B$.

$S = S_T$.

* When train passes another train.

$D = L_{T_1} + L_{T_2}$

$S_S = S_{T_1} - S_2$.

$S_O = S_1 + S_2$

* When train passes moving man.

$D = L_T$

$S_S = S_1 - S_2$

$S_O = S_1 + S_2$.