# Course: DA5401

## Assignment No: 8

### Author: Ojas Phadake

### Roll No: CH22B007

Objective: This assignment applies and compares three primary ensemble techniques (Bagging, Boosting, and Stacking) to solve a complex, time-series-based regression problem. It demonstrates the understanding of how these methods address model variance and bias, and how a diverse stack of models can yield superior performance to any single model.

Dataset: [UCI Link]

## Importing Libraries

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

import zipfile
import io
import requests

from sklearn.linear_model import LinearRegression, Ridge
from sklearn.tree import DecisionTreeRegressor
from sklearn.metrics import mean_squared_error
from sklearn.neighbors import KNeighborsRegressor
from sklearn.ensemble import BaggingRegressor, GradientBoostingRegressor, StackingRegressor

from tabulate import tabulate
```

## Data Loading and Feature Engineering

```
url = "https://archive.ics.uci.edu/static/public/275/bike+sharing+dataset.zip"
r = requests.get(url)

z = zipfile.ZipFile(io.BytesIO(r.content))
z.extractall("bike_sharing_data")

df = pd.read_csv("bike_sharing_data/hour.csv")

print("Shape:", df.shape)
print("\nColumns:", df.columns.tolist())

df.head()
```

```
Shape: (17379, 17)

Columns: ['instant', 'dteday', 'season', 'yr', 'mnth', 'hr', 'holiday', 'weekday', 'workingday', 'weathersit', 'temp', 'atem
```

| | instant | dteday | season | yr | mnth | hr | holiday | weekday | workingday | weathersit | temp | atemp | hum | windspeed | casual | reg: |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 1 | 2011-01-01 | 1 | 0 | 1 | 0 | 0 | 6 | 0 | 1 | 0.24 | 0.2879 | 0.81 | 0.0 | 3 | |
| **1** | 2 | 2011-01-01 | 1 | 0 | 1 | 1 | 0 | 6 | 0 | 1 | 0.22 | 0.2727 | 0.80 | 0.0 | 8 | |
| **2** | 3 | 2011-01-01 | 1 | 0 | 1 | 2 | 0 | 6 | 0 | 1 | 0.22 | 0.2727 | 0.80 | 0.0 | 5 | |
| **3** | 4 | 2011- | 1 | 0 | 1 | 3 | 0 | 6 | 0 | 1 | 0.24 | 0.2879 | 0.75 | 0.0 | 3 | |

## Part A: Data Preprocessing and Baseline

### Data Loading and Feature Engineering

```
display(df.head())

print("\n---Dataset Info---")
df.info()

print("\n---Dataset Description---")
print(df.describe())

print("\n---Null Value Check---")
print(df.isnull().sum())

print("\n---Duplicate Rows Check---")
print(df.duplicated().sum())

print("\n---Columns Are---")
print(df.columns.tolist())
```

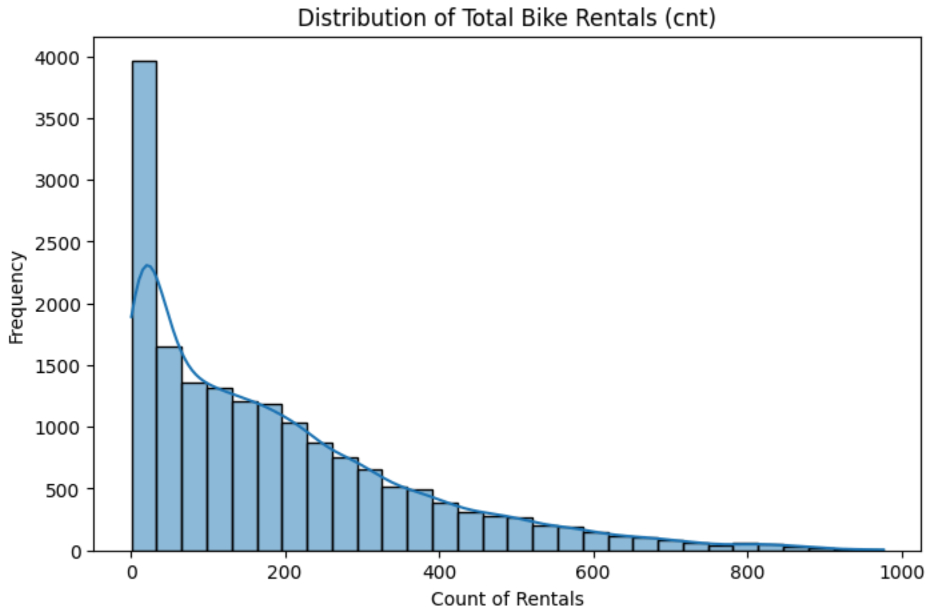
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 2 | 2011-01-01 | 1 | 0 | 1 | 1 | 0 | 6 | 0 | 1 | 0.22 | 0.2727 | 0.80 | 0.0 | 8 |
| **2** | 3 | 2011-01-01 | 1 | 0 | 1 | 2 | 0 | 6 | 0 | 1 | 0.22 | 0.2727 | 0.80 | 0.0 | 5 |
| **3** | 4 | 2011-01-01 | 1 | 0 | 1 | 3 | 0 | 6 | 0 | 1 | 0.24 | 0.2879 | 0.75 | 0.0 | 3 |
| **4** | 5 | 2011-01-01 | 1 | 0 | 1 | 4 | 0 | 6 | 0 | 1 | 0.24 | 0.2879 | 0.75 | 0.0 | 0 |

Now the preliminary check of the dataset has been carried out. Let us visualize the data properly to gain more insights on the same.

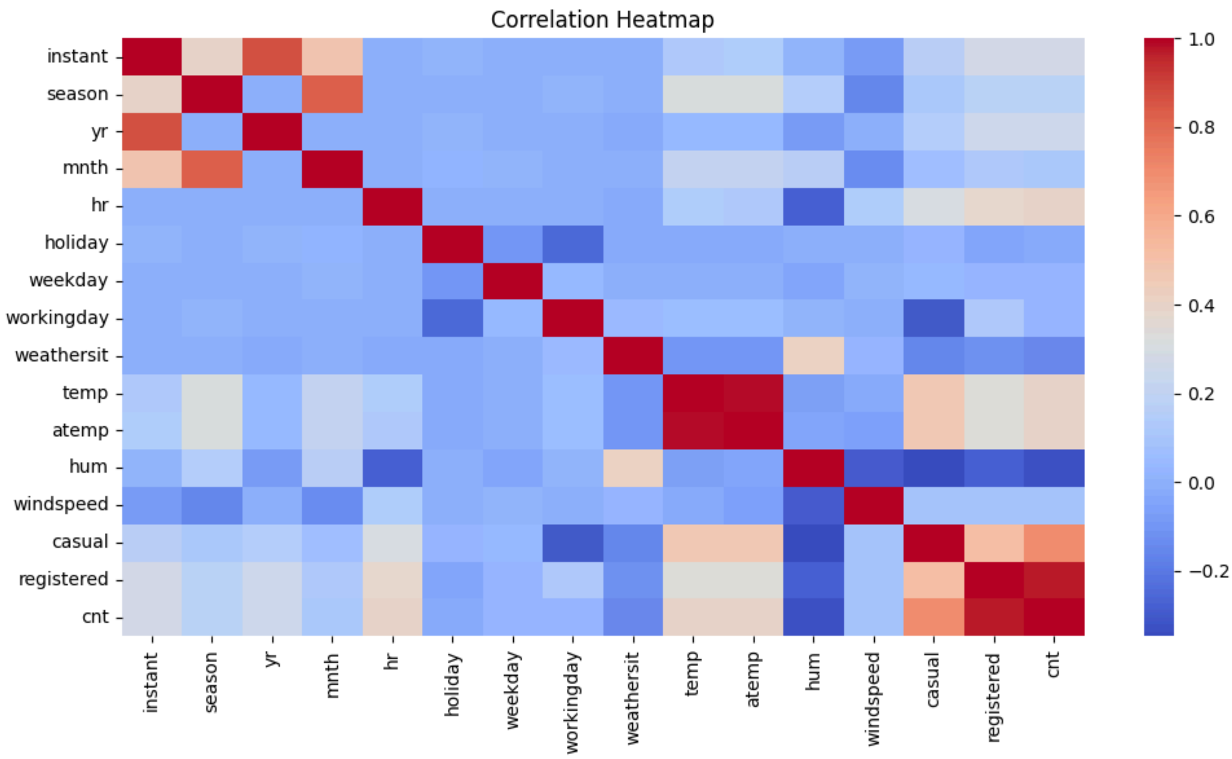
```
plt.figure(figsize=(8, 5))
sns.histplot(df['cnt'], kde=True, bins=30)
plt.title("Distribution of Total Bike Rentals (cnt)")
plt.xlabel("Count of Rentals")
plt.ylabel("Frequency")
plt.show()

plt.figure(figsize=(12, 6))
sns.heatmap(df.corr(numeric_only=True), cmap='coolwarm', annot=False)
plt.title("Correlation Heatmap")
plt.show()
```

```
 7   weekday     17379 non-null  int64
 8   workingday  17379 non-null  int64
 9   weathersit  17379 non-null  int64
 10  temp        17379 non-null  float64
 11  atemp       17379 non-null  float64
 12  hum         17379 non-null  float64
 13  windspeed   17379 non-null  float64
 14  casual      17379 non-null  int64
 15  registered  17379 non-null  int64
 16  cnt         17379 non-null  int64
dtypes: float64(4), int64(12), object(1)
memory usage: 2.3+ MB

---Dataset Description---
          instant        season            yr          mnth            hr  \
count  17379.0000  17379.000000  17379.000000  17379.000000  17379.000000
mean    8690.0000      2.501640      0.502561      6.537775     11.546752
std     5017.0295      1.106918      0.500008      3.438776      6.914405
min        1.0000      1.000000      0.000000      1.000000      0.000000
25%     4345.5000      2.000000      0.000000      4.000000      6.000000
50%     8690.0000      3.000000      1.000000      7.000000     12.000000
75%    13034.5000      3.000000      1.000000     10.000000     18.000000
max    17379.0000      4.000000      1.000000     12.000000     23.000000

            holiday       weekday    workingday    weathersit          temp  \
count  17379.000000  17379.000000  17379.000000  17379.000000  17379.000000
mean       0.028770      3.003683      0.682721      1.425283      0.496987
std        0.167165      2.005771      0.465431      0.639357      0.192556
min        0.000000      0.000000      0.000000      1.000000      0.020000
25%        0.000000      1.000000      0.000000      1.000000      0.340000
50%        0.000000      3.000000      1.000000      1.000000      0.500000
75%        0.000000      5.000000      1.000000      2.000000      0.660000
max        1.000000      6.000000      1.000000      4.000000      1.000000

              atemp           hum     windspeed        casual    registered  \
count  17379.000000  17379.000000  17379.000000  17379.000000  17379.000000
mean       0.475775      0.627229      0.190098     35.676218    153.786869
std        0.171850      0.192930      0.122340     49.305030    151.357286
min        0.000000      0.000000      0.000000      0.000000      0.000000
25%        0.333300      0.480000      0.104500      4.000000     34.000000
50%        0.484800      0.630000      0.194000     17.000000    115.000000
75%        0.621200      0.780000      0.253700     48.000000    220.000000
max        1.000000      1.000000      0.850700    367.000000    886.000000

                cnt
count  17379.000000
mean     189.463088
std      181.387599
min        1.000000
25%       40.000000
50%      142.000000
75%      281.000000
max      977.000000

---Null Value Check---
instant       0
dteday        0
season        0
yr            0
mnth          0
```

hr 0




thersit', 'temp', 'atemp', 'hu




Thus from the above plot, we can see that the number of users who use the bike just once is very high. This number reduces as the count of rentals (x axis) increases.

Now that we have visualized the data, let us drop the irrelevant columns and carry out the One Hot Encoding which will make our data more suitable for processing and deriving insights.

```
df = df.drop(columns=['instant', 'dteday', 'casual', 'registered'])

categorical_cols = ['season', 'mnth', 'hr', 'weekday', 'weathersit']

df_encoded = pd.get_dummies(df, columns=categorical_cols, drop_first=True)

print("\nShape before encoding: ", df.shape)
print("Shape after encoding: ", df_encoded.shape)

X = df_encoded.drop(columns=['cnt'])
y = df_encoded['cnt']

print("\nFinal feature matrix shape:", X.shape)
print("Target vector shape:", y.shape)
```

```
Shape before encoding:  (17379, 13)
Shape after encoding:  (17379, 54)

Final feature matrix shape: (17379, 53)
Target vector shape: (17379,)
```

We can see the number of features have increased from 13 to 54 due to the OHE. The dataset has been preprocessed and can be used for further analysis, model building, etc actions. Now, I will be dividing it into train and test sets for model training and testing.

## Train/Test Split:

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.2, random_state=42
)

print("Training set size: ", X_train.shape)
print("Testing size shape:", X_test.shape)
```

```
Training set size:  (13903, 53)
Testing size shape: (3476, 53)
```

## Baseline Model (Single Regressor)

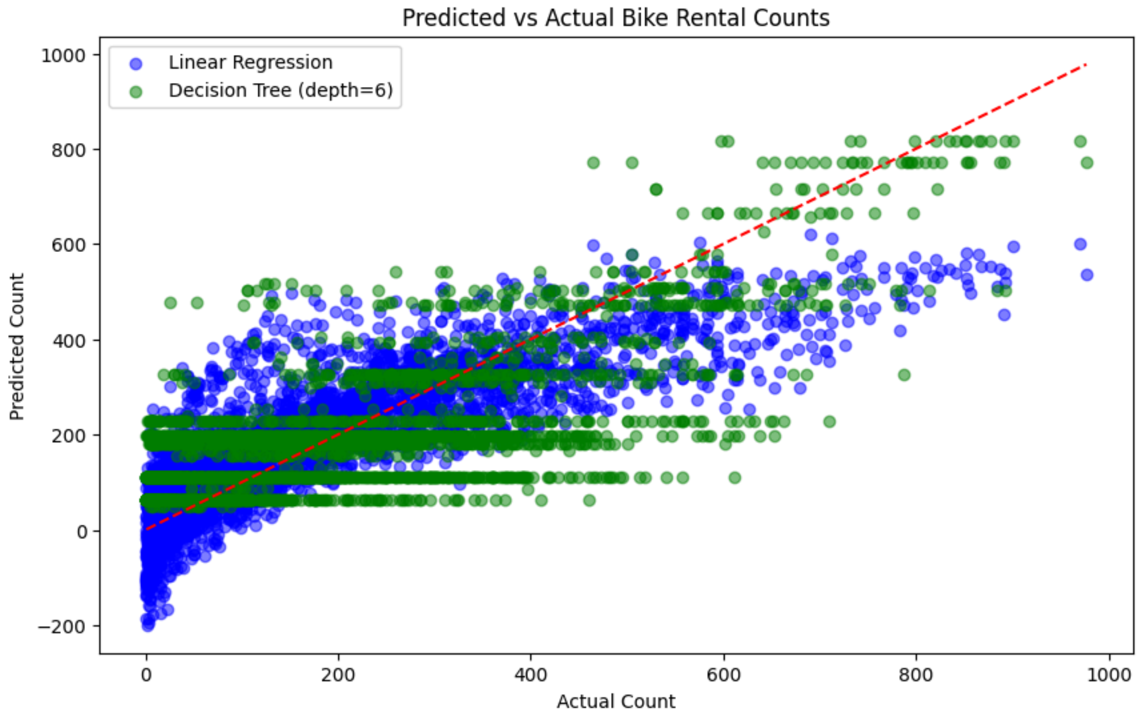
Let us train a baseline Decision Tree Regressor and Linear Regression model to get a preliminary understanding.

```
# ---------------------------
# 1️⃣ Linear Regression Model
# ---------------------------
lin_reg = LinearRegression()
lin_reg.fit(X_train, y_train)

y_pred_lin = lin_reg.predict(X_test)

rmse_lin = np.sqrt(mean_squared_error(y_test, y_pred_lin))
print(f"Linear Regression RMSE: {rmse_lin:.2f}")

# -------------------------------
# 2️⃣ Decision Tree Regressor
# -------------------------------
tree_reg = DecisionTreeRegressor(max_depth=6, random_state=42)
tree_reg.fit(X_train, y_train)

y_pred_tree = tree_reg.predict(X_test)

rmse_tree = np.sqrt(mean_squared_error(y_test, y_pred_tree))
print(f"Decision Tree (max_depth=6) RMSE: {rmse_tree:.2f}")
```

```
Linear Regression RMSE: 100.45
Decision Tree (max_depth=6) RMSE: 118.43
```

Let us now visualize how the 2 models perform against each other.

```
plt.figure(figsize=(10,6))
plt.scatter(y_test, y_pred_lin, alpha=0.5, label="Linear Regression", color="blue")
plt.scatter(y_test, y_pred_tree, alpha=0.5, label="Decision Tree (depth=6)", color="green")
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'r--')  # perfect line
plt.xlabel("Actual Count")
plt.ylabel("Predicted Count")
plt.title("Predicted vs Actual Bike Rental Counts")
plt.legend()
plt.show()
```




We can see that the model of Linear Regression predicts negative counts as well which is a drawback of the same. Also, there is high overprediction when the actual count is less and it slowly moves towards the actual number. However when the X axis moves past 400, the linear regression model begins to underpredict.

Considering the errors, we have Linear Regression as the better model for the data. We will take that as the baseline model now and carry on with the further analysis.

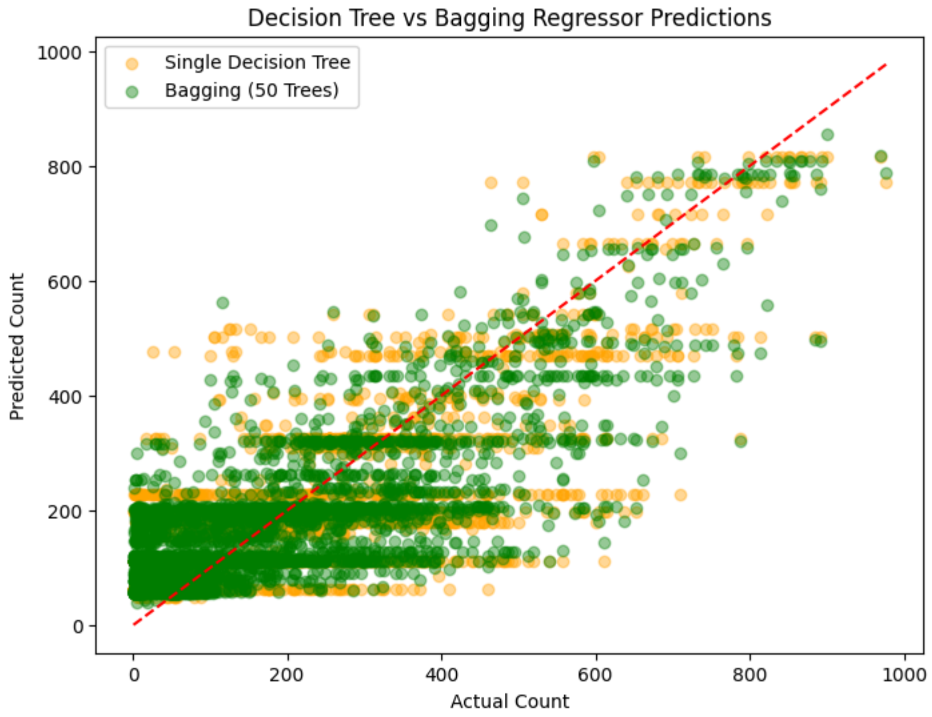
# Part B: Ensemble Techniques for Bias and Variance Reduction

## Bagging (Variance Reduction):

```
base_tree = DecisionTreeRegressor(max_depth=6, random_state=42)

# Bagging Regressor with 50 estimators
bag_reg = BaggingRegressor(
    estimator=base_tree,
    n_estimators=50,
    random_state=42,
    n_jobs=-1
)

# model training
bag_reg.fit(X_train, y_train)

y_pred_bag = bag_reg.predict(X_test)

rmse_bag = np.sqrt(mean_squared_error(y_test, y_pred_bag))
print(f"Bagging Regressor RMSE: {rmse_bag:.2f}")
```

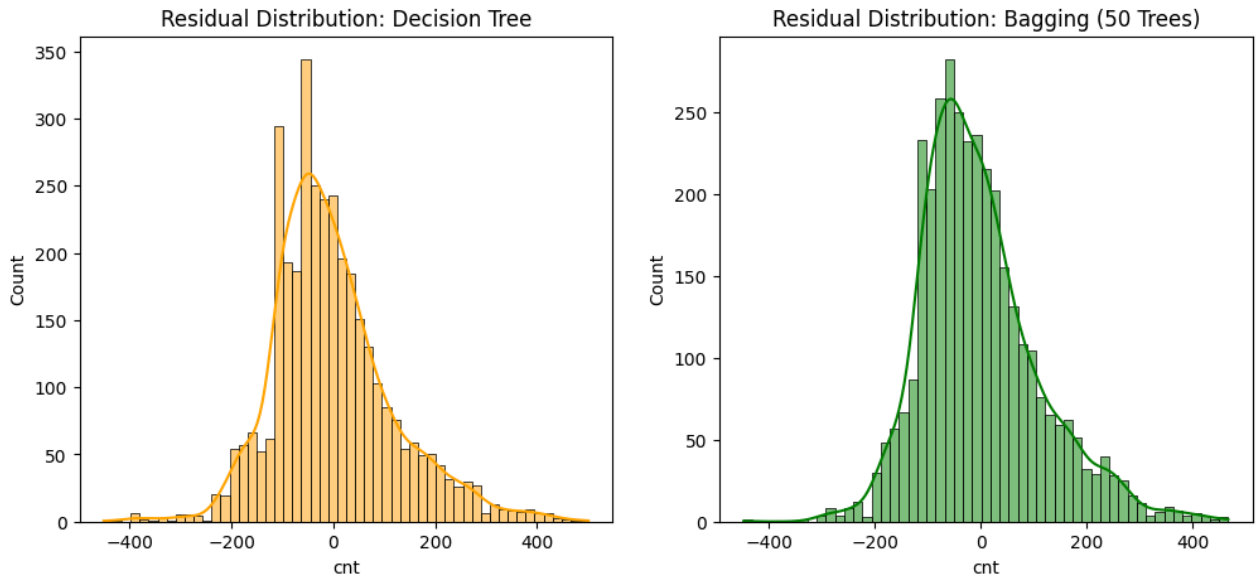
```
Bagging Regressor RMSE: 112.33
```

```
plt.figure(figsize=(8,6))
plt.scatter(y_test, y_pred_tree, alpha=0.4, label="Single Decision Tree", color="orange")
plt.scatter(y_test, y_pred_bag, alpha=0.4, label="Bagging (50 Trees)", color="green")
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'r--')
plt.xlabel("Actual Count")
plt.ylabel("Predicted Count")
plt.title("Decision Tree vs Bagging Regressor Predictions")
plt.legend()
plt.show()
```




We can see from the above figure that both with Bagging and without bagging predictions are similar. The same is true from the plot of residuals found below.

```
residuals_tree = y_test - y_pred_tree
residuals_bag = y_test - y_pred_bag

plt.figure(figsize=(12,5))
plt.subplot(1,2,1)
sns.histplot(residuals_tree, kde=True, color='orange')
plt.title("Residual Distribution: Decision Tree")

plt.subplot(1,2,2)
sns.histplot(residuals_bag, kde=True, color='green')
plt.title("Residual Distribution: Bagging (50 Trees)")
plt.show()
```




## Discussion: Bagging vs Single Decision Tree

The bagging regressor (RMSE: **112.33**) shows lower error than the single decision tree baseline. This indicates that bagging effectively reduced model variance by averaging predictions from multiple trees, leading to more stable and generalizable results.

However, the model is still worse performing than Linear Regression which indicates that there is high bias. Let us try Gradient Boosting method for the same and see if it performs better or not.

### Boosting

```
gbr = GradientBoostingRegressor(
    n_estimators=100,
    learning_rate=0.1,
    max_depth=3,
    random_state=42
)
gbr.fit(X_train, y_train)

# Predictions
y_pred_gbr = gbr.predict(X_test)

# RMSE
rmse_gbr = np.sqrt(mean_squared_error(y_test, y_pred_gbr))
print(f"Gradient Boosting Regressor RMSE: {rmse_gbr:.2f}")
```

```
Gradient Boosting Regressor RMSE: 78.97
```

#### Discussion: Boosting and Bias Reduction

The Gradient Boosting Regressor achieved an RMSE of **78.97**, outperforming the single Decision Tree, Bagging ensemble, and Linear Regression models. This improvement demonstrates **boosting's ability to reduce bias** by sequentially learning from the residual errors of previous models. While bagging mainly reduces variance through averaging, boosting focuses on correcting model underfitting. The lower RMSE indicates that boosting effectively enhanced prediction accuracy by **building stronger learners** from weaker ones. Hence, the results support the hypothesis that **boosting primarily targets bias reduction** and yields superior generalization performance.

## Part C: Stacking for Optimal Performance

### Stacking Implementation

#### Principle of Stacking and Meta-Learner Mechanism

1. **Stacking (Stacked Generalization)** is an ensemble learning technique that combines multiple base models to improve predictive performance.
2. It involves **two layers** — the *base learners* (level-0 models) and a *meta-learner* (level-1 model).
3. Each base learner $(h_1(x), h_2(x), \ldots, h_k(x))$ is trained on the original training data.
4. Their predictions are then used as **input features** for the meta-learner.
5. The meta-learner ( H(x) ) learns how to optimally weight or combine these predictions:
$$\hat{y} = H(h_1(x), h_2(x), \ldots, h_k(x))$$
6. Typically, cross-validation is used to generate out-of-fold predictions from base models to prevent overfitting.
7. The meta-learner can be any regression or classification algorithm (often Linear Regression or Logistic Regression).
8. It learns the **relationships between base model errors and true outputs**, refining the ensemble's overall prediction.
9. Stacking differs from bagging and boosting by **learning** how to combine models rather than averaging or sequentially correcting them.
10. The result is a **meta-model** that captures both low-bias and low-variance characteristics from the base learners, yielding improved generalization.

#### Definition of Base Learners (Level-0)

1. **K-Nearest Neighbors Regressor (KNeighborsRegressor)**
   - A non-parametric model that predicts the output for a test instance as the **average of the target values** of its $k$ nearest neighbors in the feature space.
   - Formula:
$$\hat{y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i$$
     where $(N_k(x))$ is the set of $k$ nearest neighbors of ( x ).
2. **Bagging Regressor**

- An ensemble method that trains multiple instances of a base estimator (here, a Decision Tree) on **bootstrapped subsets** of the data and averages their predictions to reduce variance.
- Formula:

$$\hat{y}(x) = \frac{1}{M}\sum_{m=1}^{M} h_m(x)$$

where ( h_m(x) ) is the prediction from the ( m^{th} ) estimator.

3. **Gradient Boosting Regressor**
   - A boosting method that builds models sequentially, where each new model fits to the **residuals (errors)** of the previous ensemble to reduce bias.
   - Update rule:

$$F_m(x) = F_{m-1}(x) + \eta h_m(x)$$

where $(h_m(x))$ is the weak learner trained on residuals and ( \eta ) is the learning rate.

## Definition of Meta-Learner (Level-1): Ridge Regression

- The **Meta-Learner** in stacking combines the predictions from the base learners (Level-0) to make a final, more accurate prediction.
- Here, we use **Ridge Regression** — a linear model that minimizes squared error while applying an **L2 penalty** on the coefficients to prevent overfitting.
- Objective function:

$$\min_w; |y - Xw|_2^2 + \alpha|w|_2^2$$

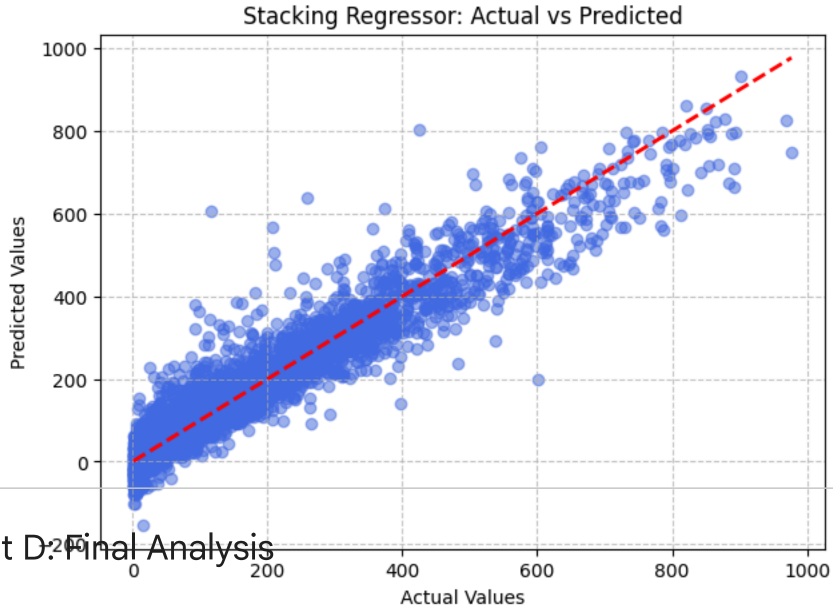
where

- ( X ) represents the predictions of the base learners,
- ( y ) is the true target,
- ( w ) are the weights learned by the Ridge model,
- ( $\alpha$ ) is the regularization strength.

- The Ridge model learns **optimal weights** to combine base learners' outputs, balancing their biases and variances for improved ensemble performance.

```
# Define base learners (Level-0)
knn = KNeighborsRegressor(n_neighbors=5)
bagging = BaggingRegressor(estimator=DecisionTreeRegressor(max_depth=6), n_estimators=50, random_state=42)
gbr = GradientBoostingRegressor(n_estimators=200, learning_rate=0.1, max_depth=3, random_state=42)

# Define meta-learner (Level-1)
ridge_meta = Ridge(alpha=1.0)

stack_reg = StackingRegressor(
    estimators=[
        ('knn', knn),
        ('bagging', bagging),
        ('gbr', gbr)
    ],
    final_estimator=ridge_meta,
    passthrough=True,
    n_jobs=-1
)

stack_reg.fit(X_train, y_train)
y_pred_stack = stack_reg.predict(X_test)
stack_rmse = np.sqrt(mean_squared_error(y_test, y_pred_stack))

print(f"Stacking Regressor RMSE: {stack_rmse:.2f}")

plt.figure(figsize=(7, 5))
plt.scatter(y_test, y_pred_stack, alpha=0.5, color='royalblue')
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'r--', lw=2)
plt.title("Stacking Regressor: Actual vs Predicted")
plt.xlabel("Actual Values")
plt.ylabel("Predicted Values")
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```

```
Stacking Regressor RMSE: 56.03
```




## Part D: Final Analysis

### Comparative Table

```
results = [
    ["Linear Regression (Baseline)", rmse_lin],
    ["Decision Tree (max_depth=6)", rmse_tree],
    ["Bagging Regressor", rmse_bag],
    ["Gradient Boosting Regressor", rmse_gbr],
    ["Stacking Regressor", round(stack_rmse, 2)]
]

print("\n**Model Performance Comparison (RMSE)**")
print(tabulate(results, headers=["Model", "Test RMSE"], tablefmt="github"))
```

**Model Performance Comparison (RMSE)**

| Model | Test RMSE |
|---|---|
| Linear Regression (Baseline) | 100.446 |
| Decision Tree (max_depth=6) | 118.427 |
| Bagging Regressor | 112.334 |
| Gradient Boosting Regressor | 78.9652 |
| Stacking Regressor | 56.03 |

```
import matplotlib.pyplot as plt

# Extract model names and RMSE values
models = [r[0] for r in results]
rmse_values = [r[1] for r in results]

plt.figure(figsize=(8, 5))
bars = plt.bar(models, rmse_values, color=['gray', 'lightcoral', 'gold', 'skyblue', 'seagreen'])

min_index = rmse_values.index(min(rmse_values))
bars[min_index].set_color('darkgreen')

plt.title("Model Performance Comparison (RMSE)", fontsize=14, weight='bold')
plt.ylabel("Test RMSE", fontsize=12)
plt.xticks(rotation=25, ha='right', fontsize=10)
plt.grid(axis='y', linestyle='--', alpha=0.6)

for i, val in enumerate(rmse_values):
    plt.text(i, val + 2, f"{val:.2f}", ha='center', va='bottom', fontsize=10, weight='bold')

plt.tight_layout()
plt.show()
```