2017

quark

SHIFTING PARADIGMS

qSAT

(Quark Scholastic Aptitude Test)

(For Level 2: Students of Class 11th and 12th)

Time: 2 Hours **Maximum Marks = 200**

Read all the Instructions carefully.

Instructions:

A. General

- This booklet is your question paper and it has **12 pages**
- A blank page is provided at the end for your rough work (don't use any additional sheet for rough work).
- Use of blank papers, slide rules, calculators or any electronic gadget in any form is not allowed.
- **The answer sheet is separately provided**. Make sure you fill in your details on the answer sheet.

B. Question Paper Format

- The Question paper contains **25 questions divided into 10 sections**.
- All section contains different number of questions.

C. Marking Scheme

- The marking scheme of each section is different.
- Adjacent to the section name the marking scheme for that section is mentioned. **(Format: No. of Questions x marks per Question = Total marks of that section, negative marks for each question)**

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SECTION 1: WARM UP (5x 5 =20; -3 for incorrect)

Q.1. There is a cube of side 1cm length. What are the maximum number of faces of the cube you can see directly? (no mirror, only you and the cube)

Q.2. Given the following series-
1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 2, 1, 1, 2, 1, v, w, x, y, z …
Find the value of 2vw + xy - z + xz?

Q.3. Given the following series-
4, 6, 8, 9, 10, 12, x, y, z.
Find the value of 2y - x – z?

Q.4. You have 100 kgs tomatoes, which are 99 percent water by weight. You let them dehydrate until they are 98 percent water. The weight of the remaining tomatoes is x, then find the sum of prime factors of x.

Q.5. If we try to find out numerals in place of the following letters to make the following true, then we get two unique solution sets. The sum of values of all the letters in both the solutions is S. Then find the value of [S/10] where [x] is the greatest integer less than or equal to x.

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   TWO
+ THREE
 SEVEN
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 TWELVE
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SECTION 2: SHERLOCK IS BACK (1 x 6 = 6; - 3 for incorrect)

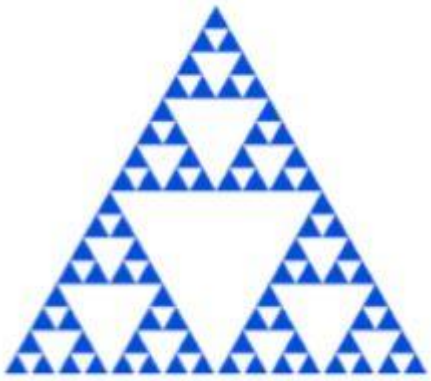
Once again, after so many years of wait, when everyone had thought Sherlock had a great Reichenbach Fall, he is back now but is trapped in a maze. This time, Moriarty has been quite careful in the designing of the maze. Moriarty knows that Sherlock is a grandmaster at chess, so instead he modifies the game of chess and makes it 3 dimensional. Now suppose the king is allowed to move to a square which has one side adjacent with its current position. It can also go inside the volume of the cube. If each face of the cube has 8 faces, in how many ways the king can go from one corner of the cube to the diagonally opposite end? If the answer of this question is x, then find the number of prime factors in the sum of digits of x.

SECTION 3: FINANCIAL CRISIS (1 x 7 =7; -4 for incorrect)

Its New Year time and Sherlock is still solving crimes but this is a pretty easy one. Moriarty is out of town for vacations so his men are not doing a good job and work is becoming too easy for Sherlock. So, Sherlock has also decided to go for a vacation and left his assistant Watson to solve the problems. Moriarty's men have sent him 12 coins and a weighing scale. One of the coins is not of the same weight as all the others. Your job is to find out the minimum number of weighing required to find out the coin which has the different weight.

SECTION 4: STILL TRAPPED (1 x 7 = 7, -4 for incorrect)

Though the mastermind Moriarty had modified the game of chess, he was still in doubt whether or not he will be successful or not because after all, Sherlock was a grandmaster at chess. So Moriarty decided to design another puzzle. In order to solve a mystery crime, Sherlock worked out that Moriarty was fond of counting figures. Hence help Sherlock count the number of triangles in the following figure. If the number of triangles are x, then find the sum of the digits of x.



SECTION 5: CIRCULAR CIRCUMFERENCE (1 x 10 = 10; -6 for incorrect)

Sherlock, Watson and his team are sitting on a round table and discussing the possible strategies to eradicate Moriarty forever. But Moriarty as we all know, doesn't keep quiet for long. He has trapped Sherlock again and asked him to solve the following puzzle. Moriarty's men have given each of Sherlock's team members a switch and a light is placed on the top of everyone. Each switch if pressed once, will toggle the light of three persons, the one who pressed it and the two adjacent ones. Initially all lights are switched off. Help Sherlock solve the following-

Q.1) If there are 1000 members in Sherlock's team, then find the minimum number of times Sherlock's team has to press the switch to turn on all the lights. If the answer is x, then find the sum of digits of the number of factors of x (including x).

Q.2) If there are 3000 members in Sherlock's team, then find the minimum number of times Sherlock's team has to press the switch to turn on all the lights. If the answer is x, then find the sum of digits in the sum of all factors of x (including x).

Q.3) Now define a function f such that f(x) = Minimum number of presses required to turn on all the lights if there are x people in the team. Now if

$$\sum(i) = x \quad (\text{i=5 to i=20})$$

Then find the value of [x/20] where [x] denotes the greatest integer smaller than or equal to x.

SECTION 6: LET THE MATHNESS BEGIN (5 x 8 = 40; -4 for incorrect)

Sherlock has been very easily solving all the questions uptil now, (yeah, that's why you're here, pun intended). Moriarty now wants Sherlock to solve some easy math problems to continue his further investigation. He has kept 4 hostages and each question will free each one of them. Help Sherlock save lives of all the hostages

.

Q.1) Define a function f such that f(x) = x! which means f(x) = product of all natural numbers between 1 and x inclusive. Define another function

$$C(n,r) = \frac{f(n)}{f(r)f(n-r)}$$

Now suppose $C(17,4)+C(17,3)+C(18,3)+C(19,3) = C(n,r)$
Find the minimum value of n-r.

Q.2) Moriarty likes decimal expansion of fractions. So, help him find the 49th digit after the decimal point in the decimal expansion of 1/9801.

Q.3) You have 7 points strictly inside a hexagon of side length 6 namely P0, P1, P2…. P6. Define a function D(x,y) = Distance between x and y. If the value of the following is x,

$$\max(\min\left(D(P_i,P_j)\right) \text{ for all } i \text{ not equal to } j \text{ and } 0 \le i,j \le 6$$

Find the value of 3x-10.

Q.4) Given the equation:

$$(1+\frac{1}{x})^{x+1} = (1+\frac{1}{1999})^{1999}$$

If the sum of all real x that satisfy the above equation is S, then find the number of trailing zeroes in the value of [S/5] where [x] is the greatest integer less than or equal to x.

Q.5) Given the equation

$$(x+y)(y+z)(z+x) = (2017)\text{^}n$$

If x,y and z are positive integers then find the number of possible values of n.

SECTION 7: SUMMER IN ENGLAND (2 x 8 =16; -4 for incorrect)

Sherlock was free for some time in July so he started watching television. He was impressed by the class of Sachin Tendulkar and by the elegance of Roger Federer. Therefore, he started watching the ICC Champions Trophy and Wimbledon to cheer the genius. However, Moriarty is a smartass and traps Sherlock in his on strengths. Help Sherlock solve the following puzzles
There is no controversy, ambiguity etc… (And all other stuff that you generally claim when you don't understand the question ☺)

Q.1) A cricket match can obviously not be played without a ball. Suppose the number of ways in which we can use 18 identical balls in 15 different cricket matches is x, then find the number of prime factors of x.

Q.2) There are 18 identical tennis balls to be used in the Wimbledon final starring Roger Federer and Rafa Nadal. There are 15 identical boxes. Suppose the number of ways in which we can use these 18 identical balls in 15 empty boxes is x, then find the number of trailing zeroes in [x/100] where [x] is the greatest integer less than or equal to x.

SECTION 8: SHERLOCK MEETS ADLER (3 x 10 = 30; -5 for incorrect)

One of the brilliant cases that Sherlock encountered is that of Irene Adler. She is a beautiful lady who has won the field' medal in Mathematics but has a criminal record and has crafted a deadly murder case. In order to get past Adler, she puzzles Sherlock in polynomials and asks her to solve the following to give him the next hint. Help Sherlock solve the following problems.

Consider the following cubic polynomial:

$$x^3 - 9x^2 + 24x + c$$

Q.1) Find the number of integral values of c such that the following cubic polynomial has exactly 3 real zeroes.
Suppose c is chosen in a way that the given polynomial has 3 distinct real roots. If x,y,z are the distinct real roots of the given polynomial

Q.2) Then find the number of possible values of [x]+[y]+[z]?

Q.3) The sum of all possible values of [x]+[y]+[z] is S. Then find the value of [S/2] where [x] is the smallest integer less than or equal to x.

SECTION 9: SHERLOCK AND MYCROFT (3 x 10 = 30; -5 for incorrect)

Sherlock Holmes has the following conversation with his brother Mycroft and both of them bombard each other with a set of problems. Help Sherlock win the verbal attacks and solve the following problems.

Sherlock: He?

Mycroft: Obviously?

Sherlock: Why? Size of the hat?

Mycroft: Don't be silly. Some women have large heads too

(Sherlock flinches slightly, possibly at Mycroft's insult to his intelligence.)

Mycroft: No he's recently had his hair cut. You can see the little hairs adhering to the perspiration stains on the inside.

(Sherlock looks down at the hat, pouting slightly)

Sherlock: Some women have short hair too.

Mycroft: Balance of probability Sherlock!

A fair die is rolled m+n times where m, n are natural numbers. In the questions that follow, [x] represents the greatest integer less than or equal to x. Answer the following-

Q.1) If m=10, n=17 then the probability to get 10 consecutive 3's can be expressed as p/q where p and q are relatively co-prime. Then value of [p/10] is

Q.2) If m = 17, n = 10, then the probability to get 17 consecutive 4's can be expressed as p/q where p and q are relatively co-prime. Then value of p is

Q.3) If m = 30, n = 19, then the probability to get 30 consecutive 5's can be expressed as 101/q where q>101. Let the maximum power of 6 in the denominator q be S, then find the sum of digits of S.

SECTION 10: THE FINAL PROBLEM! (1 x 15 = 15; -8 for incorrect)

To everyone's surprise, Jim Moriarty, the mastermind who trapped Sherlock in New York is back. He exclaims, "Miss me? Did you miss me?" Sherlock is stunned and cannot believe what is happening around. However, Moriarty has come up with a case which adds to the surprise element of Sherlock as he is in a state of shock. Help Sherlock solve the final problem.

Q. Define a function f such that

$$f(a,b) = a^3 + b^3 + 3(a^2+b^2) - 700(a+b)^2 \text{ and given that } a \geq b$$

Suppose a,b are rational numbers such that a+b is a positive integer. Then for all solutions (a,b) for the equation

$$\boldsymbol{f(a,b) = 0}$$

If the value of $\sum \boldsymbol{a+b}$ can be expressed as a four digit number PQRS where P denotes the thousandth place and S denotes the ones place. Find the value of P-Q+R-S.

--The End---

Rough Work