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1. The Three Species Problem

1.1. Introduction

We analyze the Lotka-Volterra Equations for three species X, Y and Z and observe their behavior, i.e., change in populations over time.
We use the forward Euler method with initial populations of X, Y and Z given as 2, 2.49 and 1.5 respectively. We use a timestep size of dt = 0.005s and calculate the populations up to T = 12s, outputting the values at every t = 0.5s (or 100 iterations).

1.2. Model and Theory

We use the following discretized governing equations to analyze the system –

$$x_{k+1} = x_k + dt*(a*x_k*(1 - x_k/20) - b*x_k*y_k - c*x_k*z_k)$$

$$y_{k+1} = y_k + dt*(y_k*(1 - y_k/25) - a*x_k*y_k - d*y_k*z_k)$$

$$z_{k+1} = z_k + dt*(b*z_k*(1 - z_k/30) - x_k*z_k - y_k*z_k)$$

where
a = 0.75; b = 1.5; c = 0.5; d = 1.25

1.3. Methods and Pseudocode

Pseudocode:
Define the coefficients of the Lotka-Volterra Equations
Set the time-stepping parameters
Calculate the integer number of steps
Set and display the initial conditions for the populations
Loop for each time step until the final time
    Set the discretized governing equations using the forward Euler method
    Update the populations of each species
    Display the values for every 100^{th} iteration
End loop

1.4. Calculations and Results

For 1(c) we have –

Set 1: (x_0, y_0, z_0) = (2.4, 3.2, 1.9)

The first 3 outputs are –

Time	X	Y	Z
0.0	2.40	3.20	1.90
0.5	0.53	1.67	0.78
1.0	0.20	1.53	0.62

And the final output is –

Time	X	Y	Z
12.0	0.00	24.97	0.00

Set 2: $(x_0, y_0, z_0) = (5.4, 2.1, 2.2)$

The first 3 outputs are –

Time	X	Y	Z
0.0	5.40	2.10	2.20
0.5	2.81	0.49	0.48
1.0	2.77	0.24	0.22

And the final output is –

Time	X	Y	Z
12.0	19.96	0.00	0.00

Set 3: $(x_0, y_0, z_0) = (3.5, 4.6, 5.8)$

The first 3 outputs are –

Time	X	Y	Z
0.0	3.50	4.60	5.80
0.5	0.66	0.60	2.60
1.0	0.36	0.13	3.40

And the final output is –

Time	X	Y	Z
12.0	0.00	0.00	30.00

Clearly, from each of the three sets above we see that only species of X, Y and Z continues to survive after a certain amount of time, i.e., one will always end up crowding out the other two.

For 1(d) we have -
When dt = 0.005, we have total time elapsed as 0.023s
When dt = 0.001, we have total time elapsed as 0.027s
When dt = 0.01, we have total time elapsed as 0.021s

Clearly, for values where $dt < 0.005$ the time taken is more and for values where $dt > 0.005$ the time taken is less.

For 1(e) we have –

When the initial conditions are the default values $(x_0, y_0, z_0) = (2, 2.49, 1.5)$ species Z survives. When the time stop (dt) is increased to $dt = 0.05$ we observe that species Y survives. The time step of 0.005 reflects a more accurate interpretation that when it is increased to 0.05 as the change in population is calculated more accurately due to a lower degree of error when using the forward Euler method.

1.5. Conclusion

We see that in this modelling three species X, Y and Z using the following Lotka-Volterra Equations predicts that one species always ends up crowding out the other two. The smaller our time step, the more accurate our final solution is.

2. Pocket Change Problem

2.1 Introduction

We find the average number of coins you can expect to receive in change, assuming the standard denominations for Quarters, Dimes, Nickels and Pennies as 25, 10, 5 and 1 respectively.

2.2 Model and Theory

We use a greedy approach to optimize the number of coins we need for a particular amount of change and compute the average for all possible values of change [0,99].

2.3 Methods and Pseudocode

<u>Pseudocode:</u>

```
Set the total count of coins
Loop for each possible value of change
    Set the count for each denomination
    Set the remaining amount of change
    While there is still change remaining
        Find the highest denomination that can be used to fill the change
            Update the remaining value and count for the denomination
            accordingly
    Increment the count of coins accordingly
Calculate the average
```

2.4 Calculations and Results

For 2(a) we observe that the average number of coins is **4.70.**

For 2(b) we can infer that the average number of coins decreases as it is **2.70.** This is because for any value that is not divisible by 5 in the range [0, 99] you would need a certain number of pennies to add to the coins you have already used to match that exact value. Since this varies between 1 and 4, we can infer that the average will be lower if we only choose values that are divisible by 5.

For 2(c) we observe that when the value of a quarter is 27 and a dime is 11, the average number of coins is **4.44** which is lower than the standard denominations. A disadvantage with such systems might be the fact that the assumption that all possible values of change are equally likely is probably erroneous, and on factoring that in the average value for such systems would actually decrease.

2.5 Conclusion

We observe that the average number of coins a person requires for all possible values of change, assuming each is equally likely is **4.70**.