Efficient Scheduling for Operating Rooms in a Hospital

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Motivation

- Health care is a basic need for all humans and hence hospitals need to allocate their resources and time efficiently.
- According to an NHS watchdog report, nearly 300,000 more operations could be carried out through better management of surgical lists. [1]
- Moreover, due to the pandemic there is a serious backlog, furthering the cause for efficient scheduling.

Problem Statement

- The objective of this project is to schedule operations as efficiently as possible given data about the upcoming surgeries and availability of operating rooms.
- The scheduling happens in two phases:
    - Operation tasks are collected and availability of operating rooms are determined.
    - A scheduling problem is solved to form the sequence of operations in the most efficient manner.

Previous Work

Key-points of “Scheduling operating theatres: Mixed integer programming vs. constraint programming” [2]:

- The daily scheduling of an operating theatre is a highly constrained problem. These constraints concern the priority of operations, the affinities between surgical team members, renewable and non-renewable resources, the surgical team’s preferences/availability, etc. [2]

Previous Work

- Each resource limitation included in an OR scheduling model leads to an increase in its complexity, in terms of the number of variables and/or number of constraints and the computational time required to solve the optimization problem. [2]
- The major difficulty encountered in solving a combinatorial optimization problem is the **explosion in the number of combinations** as the number of variables increase. [2]

Previous Work

- The authors use two models to solve this highly constrained problem:
  - Constraint programming
  - Mixed integer programming

- **Constraint programming (CP)** is a paradigm for solving combinatorial problems that draws on a wide range of techniques from artificial intelligence, computer science, and operations research.

Previous Work

- For the mixed integer problem, the authors produced about 27 constraints, and the objective function is to minimize the makespan of the operating theatre.

Data

- Surgery Waiting List:
  - Case ID
  - Consultant ID
  - Procedure
  - Speciality
  - Expected Duration
  - Target Deadline

CaseID	ConsultantID	Procedure	Speciality	Expected Duration	TargetDeadline
1	C011	Cataract Surgery	Ophthalmology	45	07/07/2020
2	C011	Vitrectomy	Ophthalmology	70	17/07/2020
3	C011	Cataract Surgery	Ophthalmology	45	05/06/2020
4	C011	Cataract Surgery	Ophthalmology	45	28/06/2020

Data

- OR Space Availability:
  - Session ID
  - Date
  - Start time
  - End time
  - Duration
  - Speciality

SessionID	Date	Start	End	Duration	ConsultantID	Specialty
1001	03/06/2020	08:30:00	18:00:00	570	C011	Ophthalmology
1002	10/06/2020	08:30:00	18:00:00	570	C011	Ophthalmology
1003	17/06/2020	08:30:00	18:00:00	570	C011	Ophthalmology
1004	25/06/2020	08:30:00	13:00:00	270	C011	Ophthalmology

Approach - Introduction (Mixed Integer LP)

- The objective is to maximize the summation of utilization of every operating theatre given upcoming surgeries.

$$\text{maximize} \sum_{n=1}^{m} utilization_{ORn}$$

$$\text{utilization}_{ORn} = \frac{1}{OR_{duration}} \sum a_{surgery,OR} OR_{duration}$$

Approach

- Important considerations:
  - The surgery’s start time must be after the start time of the OR.
  - The surgery must end before the OR does.
  - A surgery can be assigned to one case at max.
  - Either the surgery gets assigned or not (i.e. 1 or 0)
  - Surgeries in the same OR cannot overlap.
  - The surgery must be completed before the deadline.

Approach - Introduction

- The objective is to maximize the utility of every OR, which is the amount of time the operating room is being utilized for. For example, let’s consider OR1 is open for 120 minutes, the goal is to use OR1 for 120 minutes to have maximum utilization.
- We assume that the utilization must be between 0% and 75% because we want to allot 25% of the OR time for sanitization of surgical equipment.

Approach - Model Variables

Model variables:

$surgery_n$: Number of upcoming surgeries

$surgery_{duration}$: Duration of the surgery

OR_n : Number of available operating rooms allocated to a surgeon

$OR_{duration}$: Amount of time the OR is available for

OR_{start} : Start time of the operating theatre

OR_{end} : End time of the operating theatre

$surgery_{deadline}$: Deadline of the surgery

$utilization_{max}$: Maximum operating theatre utilization

OR_{date} : Date on which the operating theatre is available

Approach - Decision Variables

Decision variables:

$utilization_{OR_n}$: Utilization of the operating theatre

$$a_{surgery,OR} = \begin{cases} 1, & \text{if surgery is assigned to a OR} \\ 0, & \text{otherwise} \end{cases}$$

$surgery_{start}$: Start time of the surgery

Approach - Decision Variables

$$a_{surgery,OR} \in \{0,1\}$$

$$0 \leq utilization_{ORn} \leq utilization_{max}$$

$$0 \leq surgery_{start} \leq 1440$$

Approach - Constraints

Constraints:

1. If a surgery is allocated to a operating room, then the start time of the surgery must be later than the start time of the operating room availability

$$\text{if } a_{surgery,OR} = 1 \text{ then } surgery_{start} \geq OR_{start}$$

2. The end time of the surgery must be before the end time of the operating room

$$\text{if } a_{surgery,OR} = 1 \text{ then } surgery_{start} + surgery_{duration} \leq OR_{start} + OR_{duration}$$

Approach - Constraints

3. Surgeries can be assigned to at most one operating session

$$\sum a_{surgery,OR} \leq 1$$

4. Surgery must be completed before the deadline

$$\text{if } a_{surgery,OR} = 1 \text{ then } OR_{date} \leq surgery_{deadline}$$

5. Surgeries can't overlap

$$a_{surgery1,\, time1} + a_{surgery2,\, time2} = 2, \text{ then}$$

$$[surgery1_{start} + surgery1_{duration} \leq surgery2_{start}] \vee$$

$$[surgery2_{start} + surgery2_{duration} \leq surgery1_{start}$$

Big M

Larger than any reasonable value that a continuous variable may take

Let **x** be a continuous variable and **y** a binary variable,

And we want to model “If y is chosen, then x<=5”

Then consider the following constraint:

$$x \leq 5 + M(1 - y)$$

Approach - Code

Our code can be found on this link:

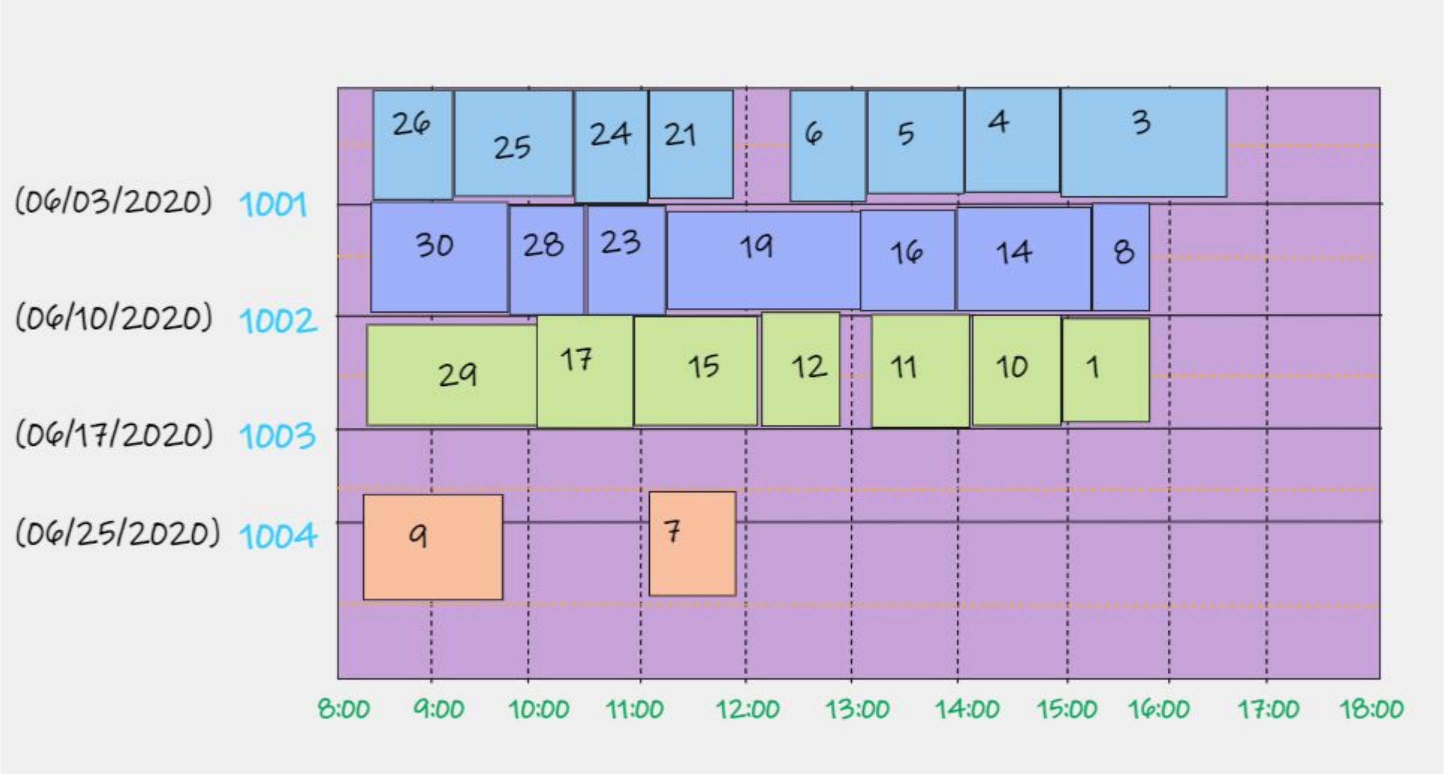
https://github.com/Ojasvi-97/Scheduling_LP_Project.git

Results - Schedule

When utilization is set to 0.75 for each operating room.

	OR	Surgery ID	OR_date	Surgery Start Time	Surgery End Time
0	1003	1	06/17/2020	14:52:30	15:37:30
1	1001	3	06/03/2020	14:52:30	15:37:30
2	1001	4	06/03/2020	14:07:30	14:52:30
3	1001	5	06/03/2020	13:22:30	14:07:30
4	1001	6	06/03/2020	12:37:30	13:22:30
5	1004	7	06/25/2020	11:07:30	11:52:30
6	1002	8	06/10/2020	15:07:30	15:37:30
7	1004	9	06/25/2020	8:30:00	9:40:00
8	1003	10	06/17/2020	14:07:30	14:52:30
9	1003	11	06/17/2020	13:22:30	14:07:30
10	1003	12	06/17/2020	12:05:00	12:50:00
11	1002	14	06/10/2020	13:57:30	15:07:30
12	1003	15	06/17/2020	10:55:00	12:05:00
13	1002	16	06/10/2020	13:12:30	13:57:30
14	1003	17	06/17/2020	10:10:00	10:55:00
15	1002	19	06/10/2020	11:10:00	13:10:00
16	1001	21	06/03/2020	11:10:00	11:55:00
17	1002	23	06/10/2020	10:25:00	11:10:00
18	1001	24	06/03/2020	10:25:00	11:10:00
19	1001	25	06/03/2020	9:15:00	10:25:00
20	1001	26	06/03/2020	8:30:00	9:15:00
21	1002	28	06/10/2020	9:40:00	10:25:00
22	1003	29	06/17/2020	8:30:00	10:10:00
23	1002	30	06/10/2020	8:30:00	9:40:00

Results - Gantt Chart

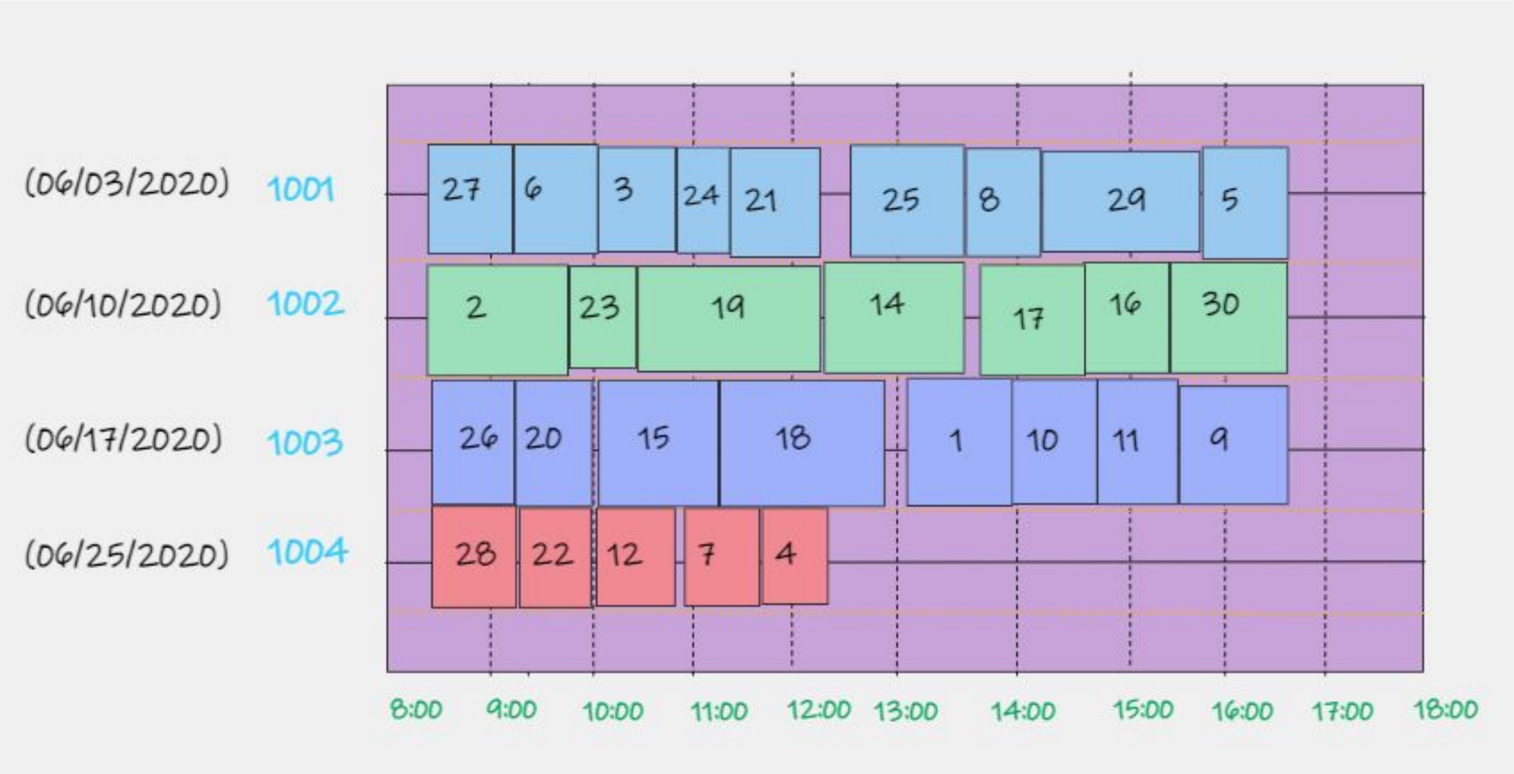


Results - Schedule

When utilization is set to 0.85 for each operating room.

	OR	Surgery ID	OR_date	Surgery Start Time	Surgery End Time
0	1003	1	06/17/2020	13:09:30	13:54:30
1	1002	2	06/10/2020	8:30:00	9:40:00
2	1001	3	06/03/2020	10:00:00	10:45:00
3	1004	4	06/25/2020	11:34:30	12:19:30
4	1001	5	06/03/2020	15:49:30	16:34:30
5	1001	6	06/03/2020	9:15:00	10:00:00
6	1004	7	06/25/2020	10:49:30	11:34:30
7	1001	8	06/03/2020	13:39:30	14:09:30
8	1003	9	06/17/2020	15:24:30	16:34:30
9	1003	10	06/17/2020	13:54:30	14:39:30
10	1003	11	06/17/2020	14:39:30	15:24:30
11	1004	12	06/25/2020	10:00:00	10:45:00
12	1002	14	06/10/2020	12:25:00	13:35:00
13	1003	15	06/17/2020	10:00:00	11:10:00
14	1002	16	06/10/2020	14:39:30	15:24:30
15	1002	17	06/10/2020	13:54:30	14:39:30
16	1003	18	06/17/2020	11:10:00	12:50:00
17	1002	19	06/10/2020	10:25:00	12:25:00
18	1003	20	06/17/2020	9:15:00	10:00:00
19	1001	21	06/03/2020	11:30:00	12:15:00
20	1004	22	06/25/2020	9:15:00	10:00:00
21	1002	23	06/10/2020	9:40:00	10:25:00
22	1001	24	06/03/2020	10:45:00	11:30:00
23	1001	25	06/03/2020	12:29:30	13:39:30
24	1003	26	06/17/2020	8:30:00	9:15:00
25	1001	27	06/03/2020	8:30:00	9:15:00
26	1004	28	06/25/2020	8:30:00	9:15:00
27	1001	29	06/03/2020	14:09:30	15:49:30
28	1002	30	06/10/2020	15:24:30	16:34:30

Results - Gantt Chart



Future Work

- To incorporate real-world factors like:
  - Patients preferences
  - Elective vs. non-elective surgeries
  - Affinity between surgeons
  - Surgeons workload, and many more...

- To solve this problem using constraint programming

Conclusion

- MIP is an incredibly powerful technique for scheduling
- The increasing # of constraints slows down the optimizer
- MIP formulation can be thought of as a method of **reduction**

References

1. nationalhealthexecutive.com/News/operating-room-inefficiencies-costing-the-nhs-300000-operations-a-year
2. Tao Wang, Nadine Meskens, David Duvivier (2015), Scheduling operating theatres: Mixed integer programming vs. constraint programming, European Journal of Operational Research.