Simulation and Modelling
Assignment E2

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1 Assignment E2

1.1 Algorithm to Generate Normal Deviates from Acceptance-Rejection Method

Let F be the distribution with probability density function f(x), from which we want to generate samples (in this case, F would be Normal Distribution). Let G be an alternate probability distribution, with density function g(x), from which we already have an efficient algorithm for generating, but also such that the function g(x) is near to f(x). And, let c be the bounding constant required to scale g(x) such that it can cover f(x) entirely. Now,

1. Generate a random variate X_i from G.
2. Generate $U_i \sim$ Uniform[0,1]
3. If
$$U_i \leq \frac{F(X_i)}{cg(X_i)}$$
then "accept" X_i; otherwise go back to Step 1 ("reject").

1.2 Generating 1000 Normal Deviates

I have attached a Python3 code file named ' rejection_method.py', that will generate 1000 Normal Random Deviates using the Acceptance-Rejection Method. The code has been written on the Algorithm mentioned in the above section. In my case, I have used Uniform Distribution as the alternate probability distribution (represented as G above).

As part of Exploratory Data Analysis, I have computed the mean, variance and standard deviation of the generated sample and have also plotted the generated sample against theoretical Normal Distribution.

1.3 Kolmogorov-Smirnov Goodness of Fit Test

I have attached a Python3 code file named 'ks-test.py', that will run the Kolmogorov-Smirnov Goodness of Fit test on the Generated Data. The program will compare the generated sample with Normal Distribution, and output whether the distribution of the generated sample is same as the Normal Distribution. Below id the algorithm on which the code was written.

H_0 - No significant difference between the sample distribution and the theoretical distribution.
H_1 - Significant difference exists between the sample distribution and the theoretical distribution.

Algorithm:
Let $Y_1, Y_2, Y_3 ... Y_N$ be N generated ordered data points and Let F be the theoretical cumulative distribution of the distribution (in this case, Normal Distribution) on which the Goodness of Fit is being conducted.

1. Compute
$$D^+ = \max_{1 \leq i \leq N} (\frac{i}{N} - F(Y_i))$$

2. Compute
$$D^- = \max_{1 \leq i \leq N} (F(Y_i) - \frac{i-1}{N})$$

3. Compute
$$D = \max\left(D^+, D^-\right)$$

4. Now compare D with the critical value, D_α for the specified significance level $\alpha = 0.05$ and the given sample size N. The critical value can be computed from $D_\alpha = \frac{1.36}{\sqrt{N}}$

5. If D $\leq D_\alpha$, conclude that no difference has been detected between the true distribution of the generated sample and the theoretical distribution (in this case, Normal Distribution).

1.4 Outputs

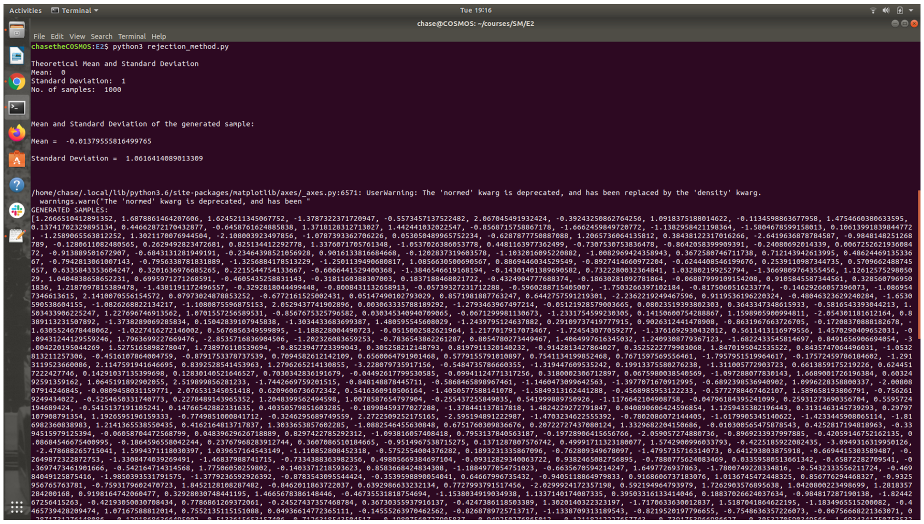

Figure 1: Output from generating the required 1000 normal random deviates

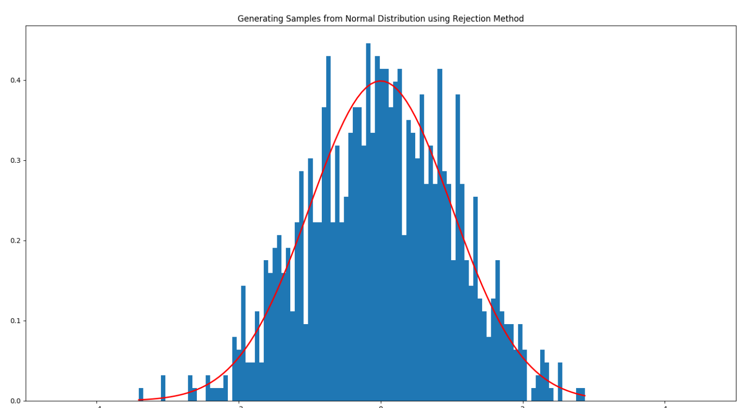

Figure 2: Plotting the generated 1000 normal random deviates against theoretical Normal Distribution

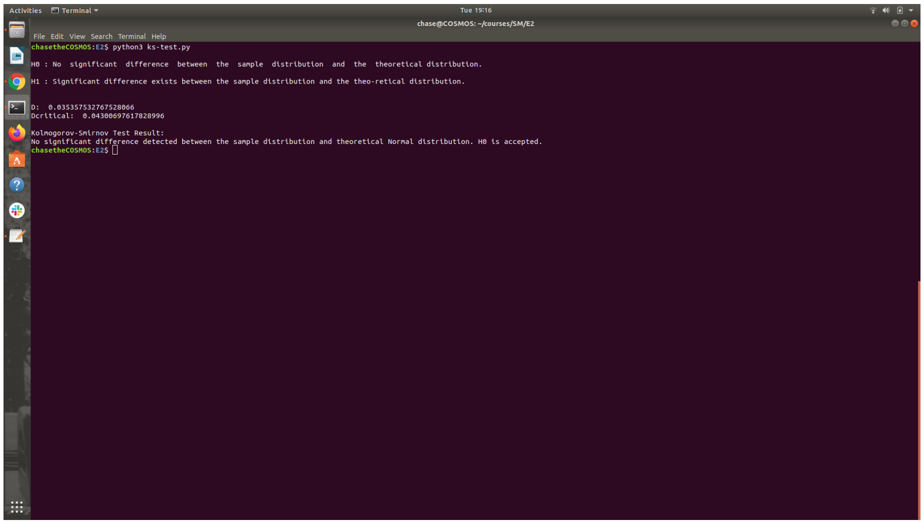

Figure 3: Output from running the Kolmogorov-Smirnov Test on the generated sample

Table A.8 Kolmogorov–Smirnov Critical Values

Degrees of Freedom (N)	$D_{0.10}$	$D_{0.05}$	$D_{0.01}$
1	0.950	0.975	0.995
2	0.776	0.842	0.929
3	0.642	0.708	0.828
4	0.564	0.624	0.733
5	0.510	0.565	0.669
6	0.470	0.521	0.618
7	0.438	0.486	0.577
8	0.411	0.457	0.543
9	0.388	0.432	0.514
10	0.368	0.410	0.490
11	0.352	0.391	0.468
12	0.338	0.375	0.450
13	0.325	0.361	0.433
14	0.314	0.349	0.418
15	0.304	0.338	0.404
16	0.295	0.328	0.392
17	0.286	0.318	0.381
18	0.278	0.309	0.371
19	0.272	0.301	0.363
20	0.264	0.294	0.356
25	0.24	0.27	0.32
30	0.22	0.24	0.29
35	0.21	0.23	0.27
Over 35	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

Source: F. J. Massey, "The Kolmogorov–Smirnov Test for Goodness of Fit," *The Journal of the American Statistical Association*, Vol. 46. © 1951, p. 70. Adapted with permission of the American Statistical Association.

Figure 4: Kolmogorov-Smirnov Critical Values. Reference: Discrete Event System Simulation - Jerry Banks