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SML - Assignment 1

A1) Let's compute $\frac{P(x|w_1)}{P(x|w_2)}$ first

$$\frac{P(x|w_1)}{P(x|w_2)} = \frac{\frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left\{\frac{-(x-\mu_1)^2}{2\sigma_1^2}\right\}}{\frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left\{\frac{-(x-\mu_2)^2}{2\sigma_2^2}\right\}}$$

$$= \exp\left\{\frac{(x-\mu_2)^2-(x-\mu_1)^2}{2}\right\}$$

$$= \exp\left\{\frac{2x(\mu_1-\mu_2)+\mu_2^2-\mu_1^2}{2}\right\}$$

Now, $\frac{P(x|w_1)}{P(x|w_2)} > \frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}} \cdot \frac{P(w_2)}{P(w_1)}$

for D.B.

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i) For 0-1 loss,

$$\frac{P(x|w_1)}{P(x|w_2)} > 3$$

$$\Rightarrow \frac{21-6x}{2} > \ln(3)$$

$$\Rightarrow x < \frac{21-2\ln(3)}{6}$$

$$\Rightarrow \boxed{x < 3.134}$$ D.B.

→ chose w_1
else w_2

ii) $$\frac{P(x|w_1)}{P(x|w_2)} > 2$$

$$\Rightarrow \frac{21-6x}{2} > \ln(2)$$

D.B.

$$\Rightarrow \boxed{x < 3.269}$$ → chose w_1
else w_2

iii) For cancer predicted I'll probably not chose the 0-1 loss since false negatives can be more harmful than false positives.

A2) We have

$$Y = A^T X + B$$

$$= \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + 5$$

$$Y = 2X_1 - X_2 + 2X_3 + 5$$

Now $\mu_Y = E[Y] = 2\mu_{X_1} - \mu_{X_2} + 2\mu_{X_3} + 5$

$$\boxed{\mu_Y = 32}$$

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A3) Assuming $P(w_1) = P(w_2) = 1/2$, we have the following condition to decide w_1:

$$P(x|w_1) > P(x|w_2)$$

$$\Rightarrow \frac{1}{\pi b} \cdot \frac{1}{1 + \left(\frac{x-a_1}{b}\right)^2} > \frac{1}{\pi b} \cdot \frac{1}{1 + \left(\frac{x-a_2}{b}\right)}$$

$$\Rightarrow 1 + \left(\frac{x-a_2}{b}\right)^2 > 1 + \left(\frac{x-a_1}{b}\right)^2$$

$$\Rightarrow (x-a_2)^2 > (x-a_1)^2$$

$$\Rightarrow x^2 - 2a_2x + a_2^2 > x^2 - 2a_1x + a_1^2$$

$$\Rightarrow 2x(a_1-a_2) > (a_1-a_2)(a_1+a_2)$$

D.B:

Case 1 : $a_1 > a_2$

$$x > \frac{a_1+a_2}{2} \Rightarrow \text{Chose } w_1$$

Case 2 : $a_1 < a_2$

$$x < \frac{a_1+a_2}{2} \Rightarrow \text{Chose } w_1$$

iii) P(error)

$$= \int \min\{ P(w_1|x), P(w_2|x)\} P(x) dx$$

$$= \int \min\{ P(x/w_1) P(w_1), P(x/w_2) P(w_2)\}$$

$$= \int \min\{ P(x/w_1), P(x/w_2)\} \cdot \frac{1}{2}$$

From part (i) we know that

$P(x/w_1) > P(x|w_2)$ when

$$x < \frac{a_1 + a_2}{2} = 4$$

Hence

$$P(\text{error}) = \int_{-\infty}^{4} \frac{P(x|w_2)}{2} dx + \int_{4}^{\infty} \frac{P(x|w_1)}{2} dx$$

$$= \frac{1}{2\pi} \int_{-\infty}^{4} \frac{dx}{1+(x-5)^2} + \frac{1}{2\pi} \int_{4}^{\infty} \frac{dx}{1+(x-3)^2}$$

$$= \frac{1}{2\pi} \left[\tan^{-1}(x-5) \Big|_{-\infty}^{4} + \tan^{-1}(x-3) \Big|_{4}^{\infty} \right]$$

$$= \boxed{\frac{1}{4}}$$ overall error rate

A4) a) We have,

$$P(a) = \theta^{a}(1-\theta)^{1-a}$$

$$P(b) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} = N($$

Given $X = \begin{bmatrix} a \\ b \end{bmatrix}$,

$$\text{Cov}(X) = \begin{bmatrix} \theta(1-\theta) & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

This means a and b are independent, so PDF of X is given by

$$P(X) = P(a) \cdot P(b)$$

$$= \frac{\theta^{a}(1-\theta)^{1-a}}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

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b) For N iid samples,

$$Q(x) = P(x_1, x_2, \dots, x_n)$$

$$= \prod_{i=1}^{n} P(x_i)$$

$$Q(x) = \theta^{\sum_{i=1}^{n} a_i} (1-\theta)^{\sum_{i=1}^{n} 1-a_i} \prod_{i=1}^{n} N(\mu_i, \sigma_i^2)$$

$$\ln(Q(x)) = \sum_{i=1}^{n} a_i \ln(\theta)$$

$$+ n - \sum_{i=1}^{n} a_i \ln(1-\theta)$$

$$+ \theta$$

$$\frac{d \ln(Q(x))}{d\theta} = \frac{\sum_{i=1}^{n} a_i}{\theta} + n - \sum_{i=1}^{n} a_i \left(\frac{-1}{1-\theta}\right)$$

$$= \sum_{i=1}^{n} a_i - n\theta = 0$$

$$\Rightarrow \boxed{\theta = \frac{\sum_{i=1}^{n} a_i}{n}}$$