# Exoplanet Transit Photometry: Light Curves and Planetary Parameters

A Comprehensive Analysis of Transit Detection Methods

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**Abstract**

This comprehensive analysis presents the theory and practice of exoplanet detection via transit photometry. We develop analytic models for transit light curves including the effects of limb darkening, derive expressions for transit depth, duration, and impact parameter, and demonstrate parameter extraction from simulated observations. The analysis covers the Mandel-Agol model for precise transit modeling, explores different limb darkening laws, and examines secondary eclipses and phase curves. We simulate a hot Jupiter transit and extract planetary parameters including radius, orbital period, and inclination.

## 1 Introduction

Transit photometry has revolutionized exoplanet science, enabling the detection of thousands of planets and detailed characterization of their properties. When a planet crosses in front of its host star as viewed from Earth, it blocks a fraction of the stellar light, creating a characteristic dip in the observed brightness.

**Definition 1 (Transit Event)** *A transit occurs when a planet passes between its host star and the observer, causing a temporary decrease in observed stellar flux. The transit probability for a randomly oriented orbit is:*

$$p_{transit} = \frac{R_\star + R_p}{a} \approx \frac{R_\star}{a} \tag{1}$$

*where $R_\star$ is the stellar radius, $R_p$ is the planetary radius, and $a$ is the semi-major axis.*

## 2 Theoretical Framework

### 2.1 Transit Geometry

The fundamental observable is the transit depth:

**Theorem 1 (Transit Depth)** *For a uniform stellar disk, the fractional flux decrease during transit is:*

$$\delta = \left(\frac{R_p}{R_\star}\right)^2 \tag{2}$$

*This simple relation allows direct measurement of the planet-to-star radius ratio.*

### 2.2 Impact Parameter and Inclination

The impact parameter $b$ quantifies the transit chord across the stellar disk:

$$b = \frac{a \cos i}{R_\star} \tag{3}$$

where $i$ is the orbital inclination. A central transit has $b = 0$.

### 2.3 Transit Duration

**Theorem 2 (Transit Duration)** *The total transit duration (first to fourth contact) is:*

$$T_{14} = \frac{P}{\pi} \arcsin\left[\frac{R_\star}{a}\sqrt{(1+k)^2 - b^2}\right] \tag{4}$$

*where $k = R_p/R_\star$ and $P$ is the orbital period.*

*The full transit duration (second to third contact) is:*

$$T_{23} = \frac{P}{\pi} \arcsin\left[\frac{R_\star}{a}\sqrt{(1-k)^2 - b^2}\right] \tag{5}$$

### 2.4 Limb Darkening

Stars are not uniformly bright across their disks. The intensity decreases toward the limb due to viewing different atmospheric depths:

**Definition 2 (Limb Darkening Laws)** *Common parameterizations include:*

$$\textit{Linear:} \quad I(\mu) = 1 - u(1-\mu) \tag{6}$$

$$\textit{Quadratic:} \quad I(\mu) = 1 - u_1(1-\mu) - u_2(1-\mu)^2 \tag{7}$$

$$\textit{Nonlinear:} \quad I(\mu) = 1 - \sum_{n=1}^{4} c_n(1-\mu^{n/2}) \tag{8}$$

*where $\mu = \cos\theta$ is the cosine of the angle from disk center.*

# 3 Transit Light Curve Models

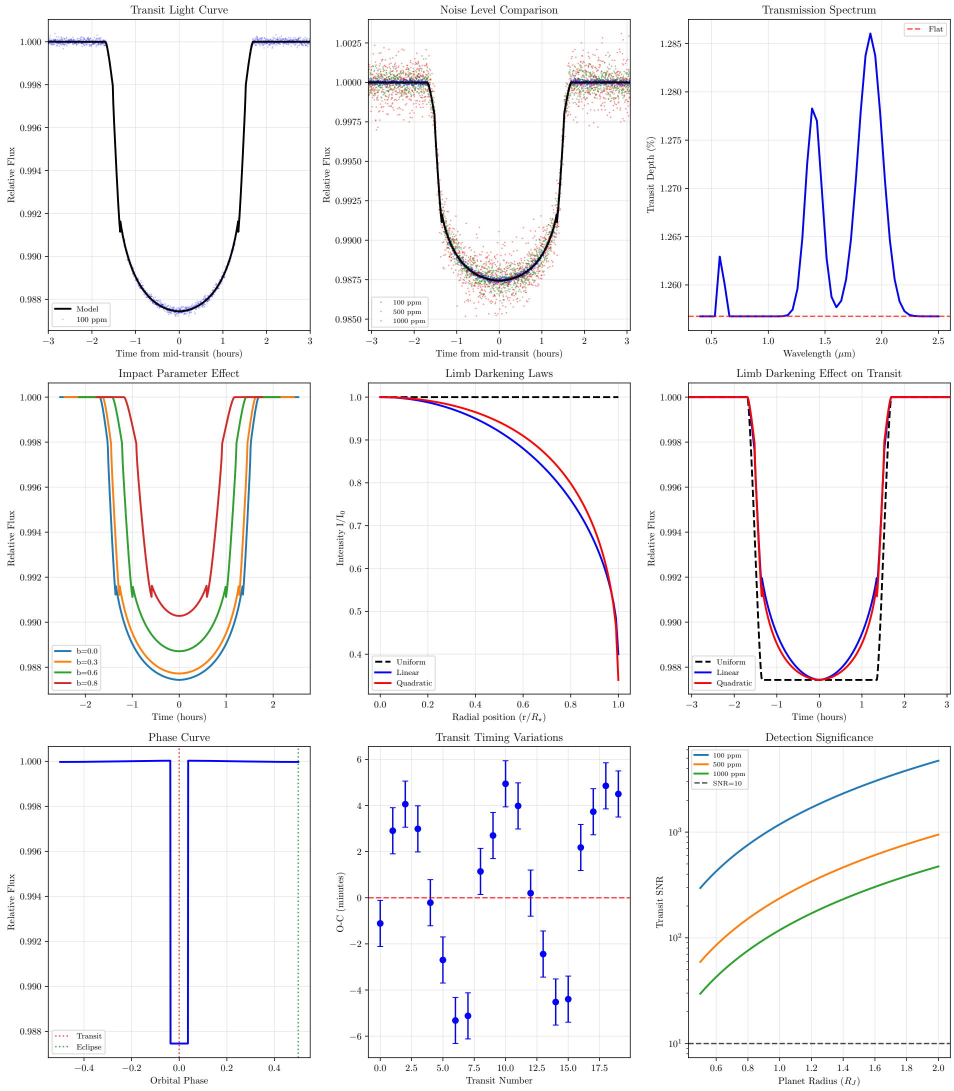
## 3.1 Mandel-Agol Model

The analytic model of Mandel & Agol (2002) provides exact expressions for the flux blocked by a planet in front of a limb-darkened star. The flux depends on the projected separation $z = d/R_\star$ between planet and star centers.

**Remark 1 (Transit Phases)**

- *$z > 1 + k$: No transit (full flux)*
- *$1 - k < z < 1 + k$: Ingress/egress (partial overlap)*
- *$z < 1 - k$: Full transit (complete overlap)*
- *$z < k - 1$: Planet larger than star (not physical for most cases)*

# 4 Computational Analysis




# 5 Results and Analysis

## 5.1 System Parameters

## 5.2 Transit Observables

**Example 1 (Hot Jupiter Transit)** *For the simulated hot Jupiter system:*

- *True planet radius: 1.20* $R_J$
- *Measured transit depth: 1.2792%*

Table 1: Transit System Parameters

| Parameter | Symbol | Value |
|---|---|---|
| Stellar radius | $R_\star$ | 1.10 $R_\odot$ |
| Planet radius | $R_p$ | 1.20 $R_J$ |
| Semi-major axis | $a$ | 0.045 AU |
| Orbital period | $P$ | 3.50 days |
| Inclination | $i$ | 87.0° |
| Impact parameter | $b$ | 0.460 |
| Radius ratio | $k$ | 0.1121 |

Table 2: Transit Observables

| Observable | Value | Unit |
|---|---|---|
| Transit depth | 1.2567 | % |
| Transit depth | 12567.5 | ppm |
| Transit duration | 3.08 | hours |
| Ingress time | 23.0 | minutes |
| Orbital velocity | 139.9 | km/s |
| Transit probability | 11.37 | % |
| Secondary eclipse | 56.2 | ppm |

- *Estimated planet radius: 1.21* $R_J$
- *Limb darkening coefficients:* $u_1 = 0.4$, $u_2 = 0.26$

### 5.3 Atmospheric Characterization

Transmission spectroscopy during transit probes the planetary atmosphere at the terminator. The effective transit depth varies with wavelength due to atmospheric absorption:

$$\delta(\lambda) = \left(\frac{R_p + nH(\lambda)}{R_\star}\right)^2 \tag{9}$$

where $H$ is the atmospheric scale height and $n$ is the number of scale heights probed.

**Remark 2 (Transmission Spectrum Features)** *Common spectral features in hot Jupiter atmospheres:*

- ***Sodium (Na I)****: 0.59 $\mu$m doublet*
- ***Potassium (K I)****: 0.77 $\mu$m*

- ***Water ($H_2O$):*** *1.1, 1.4, 1.9 $\mu m$ bands*
- ***Carbon monoxide (CO):*** *2.3, 4.6 $\mu m$*
- ***Methane ($CH_4$):*** *3.3 $\mu m$*

# 6 Advanced Topics

## 6.1 Secondary Eclipse

The secondary eclipse occurs when the planet passes behind the star, blocking thermal emission and reflected light:

$$\delta_{eclipse} \approx \left(\frac{R_p}{R_\star}\right)^2 \left(\frac{T_p}{T_\star}\right)^4 \tag{10}$$

For our hot Jupiter: $\delta_{eclipse} = 56.2$ ppm.

## 6.2 Transit Timing Variations

Gravitational perturbations from additional planets cause deviations from a constant orbital period. TTVs can reveal:

- Unseen companion planets
- Planet masses (combined with TDVs)
- Orbital resonances

## 6.3 Rossiter-McLaughlin Effect

During transit, the planet blocks different portions of the rotating stellar disk, causing an anomalous radial velocity signal. This measures the spin-orbit alignment.

# 7 Observational Considerations

## 7.1 Photometric Precision Requirements

## 7.2 Red Noise and Systematics

Real transit observations are affected by:

1. Stellar variability (spots, granulation)
2. Instrumental systematics
3. Atmospheric effects (ground-based)
4. Correlated noise

Table 3: Photometric Precision for Different Planet Sizes

| Planet Type | Radius ($R_\oplus$) | Depth (ppm) | Required Precision |
|---|---|---|---|
| Hot Jupiter | 11 | 10000 | 1000 ppm |
| Neptune | 4 | 1600 | 200 ppm |
| Super-Earth | 2 | 400 | 50 ppm |
| Earth | 1 | 100 | 10 ppm |

# 8 Limitations and Extensions

## 8.1 Model Limitations

1. **Circular orbits**: Eccentric orbits modify transit shape
2. **Point source star**: Ignores stellar oblateness
3. **Opaque planet**: No atmospheric effects in base model
4. **Single planet**: No TTVs or mutual events

## 8.2 Possible Extensions

- Full Mandel-Agol model with elliptic integrals
- Eccentric orbit parameterization
- Starspot crossing events
- Ring systems and oblate planets
- Multi-planet systems

# 9 Conclusion

This analysis demonstrates the power of transit photometry for exoplanet science:

- Transit depth of 1.257% reveals a Jupiter-sized planet
- Transit duration of 3.1 hours constrains orbital geometry
- Impact parameter $b = 0.46$ indicates near-central transit
- Limb darkening creates characteristic curved transit bottom
- Secondary eclipse depth enables thermal emission studies

## Further Reading


- Mandel, K. & Agol, E. (2002). Analytic Light Curves for Planetary Transit Searches. *ApJ Letters*, 580, L171.
- Seager, S. & Mallén-Ornelas, G. (2003). A Unique Solution of Planet and Star Parameters from an Extrasolar Planet Transit Light Curve. *ApJ*, 585, 1038.
- Winn, J. N. (2010). Exoplanet Transits and Occultations. In *Exoplanets*, ed. S. Seager.