Stress Analysis: Mohr's Circle and Failure Theories

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Abstract

This report presents computational analysis of stress states in solid mechanics. We examine stress transformation using Mohr's circle, principal stresses, von Mises equivalent stress, and common failure theories. Python-based computations provide quantitative analysis with dynamic visualization.

Contents

1 Introduction to Stress Analysis

Stress analysis is fundamental to mechanical design and failure prediction. This analysis covers:

- Plane stress transformation
- Mohr's circle construction
- Principal stress calculation
- von Mises yield criterion
- Failure theories (Tresca, Rankine, Mohr-Coulomb)

2 Plane Stress State

For a 2D stress state, the stress tensor is:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix} \tag{1}$$

2.1 Stress Transformation

Stresses on a rotated plane (angle θ from x-axis):

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta) \tag{2}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta) \tag{3}$$

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Figure 1: Mohr's circle construction and stress transformation with angle.

Table 1: Principal Stress Results

Parameter	Value	Units
σ_1 (max principal)	87.1	MPa
σ_2 (min principal)	-47.1	MPa
τ_{max}	67.1	MPa
Principal angle θ_p	13.3	degrees

3 3D Stress State and von Mises Stress

For a general 3D stress state, the von Mises equivalent stress is:

$$\sigma_{VM} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \tag{4}$$

For plane stress ($\sigma_3 = 0$):

$$\sigma_{VM} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \tag{5}$$

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Figure 2: Yield surfaces and von Mises equivalent stress analysis.

von Mises stress: $\sigma_{VM} = 117.9$ MPa, Safety factor = 2.12

4 Failure Theories

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Figure 3: Comparison of failure theories: von Mises, Tresca, and Rankine.

5 Stress Concentration

Stress concentration factors modify nominal stress:

$$\sigma_{max} = K_t \cdot \sigma_{nom} \tag{6}$$

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Figure 4: Stress concentration factors for common geometries.

6 Beam Bending Stress

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Figure 5: Cantilever beam analysis: moment, shear, and stress distributions.

Maximum bending stress at root: $\sigma_{max} = 60.0$ MPa

7 Conclusions

This analysis demonstrates key aspects of stress analysis:

1. Mohr's circle provides graphical stress transformation
2. Principal stresses define maximum and minimum normal stresses
3. von Mises criterion is appropriate for ductile materials
4. Tresca is more conservative than von Mises by up to 15%
5. Stress concentration must be considered at geometric discontinuities
6. Beam bending produces linear stress distribution through depth