Pulsar Timing: From Spin-down to Gravitational Wave Detection

A Comprehensive Analysis of Pulsar Astrophysics

High-Energy Astrophysics Division
Computational Science Templates

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Abstract

This comprehensive analysis explores pulsar timing theory and applications, from basic spin-down physics to gravitational wave detection via pulsar timing arrays. We derive the fundamental pulsar equations including period derivatives, characteristic ages, and magnetic field strengths. The analysis covers the P-$\dot{P}$ diagram for pulsar classification, timing residuals analysis, binary pulsar systems, and the search for nanohertz gravitational waves. We examine synthetic pulsar populations representing normal pulsars, millisecond pulsars, and magnetars, and explore their evolutionary relationships.

1 Introduction

Pulsars are rapidly rotating, highly magnetized neutron stars that emit beams of electromagnetic radiation. Discovered in 1967 by Jocelyn Bell Burnell and Antony Hewish, pulsars have become invaluable tools for precision astrophysics, from testing general relativity to detecting gravitational waves.

Definition 1 (Pulsar) *A pulsar is a rotating neutron star with a dipolar magnetic field misaligned with its rotation axis. The lighthouse effect produces periodic pulses as the beam sweeps past Earth with each rotation.*

2 Theoretical Framework

2.1 Spin-down Physics

Pulsars lose rotational energy through magnetic dipole radiation and particle winds:

Theorem 1 (Spin-down Luminosity) *The rate of rotational energy loss is:*

$$\dot{E} = -\frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = I\Omega\dot{\Omega} = \frac{4\pi^2 I\dot{P}}{P^3} \tag{1}$$

where $I \approx 10^{45}$ g cm^2 is the neutron star moment of inertia, P is the period, and $\dot{P}$ is the period derivative.

2.2 Characteristic Age

The characteristic age assumes constant braking index $n = 3$ (pure magnetic dipole):

$$\tau_c = \frac{P}{(n-1)\dot{P}} = \frac{P}{2\dot{P}} \tag{2}$$

Remark 1 (Age Limitations) *The characteristic age equals the true age only if:*

- *Birth period $P_0 \ll P$ (current period)*
- *Braking index $n = 3$ (magnetic dipole)*
- *Magnetic field is constant*

For young pulsars, τ_c often overestimates the true age.

2.3 Magnetic Field Strength

Equating spin-down luminosity to magnetic dipole radiation:

Theorem 2 (Surface Magnetic Field) *The equatorial surface field strength is:*

$$B = \sqrt{\frac{3c^3 I P\dot{P}}{8\pi^2 R^6 \sin^2\alpha}} \approx 3.2 \times 10^{19}\sqrt{P\dot{P}}\ G \tag{3}$$

where $R \approx 10$ km is the neutron star radius and α is the inclination angle.

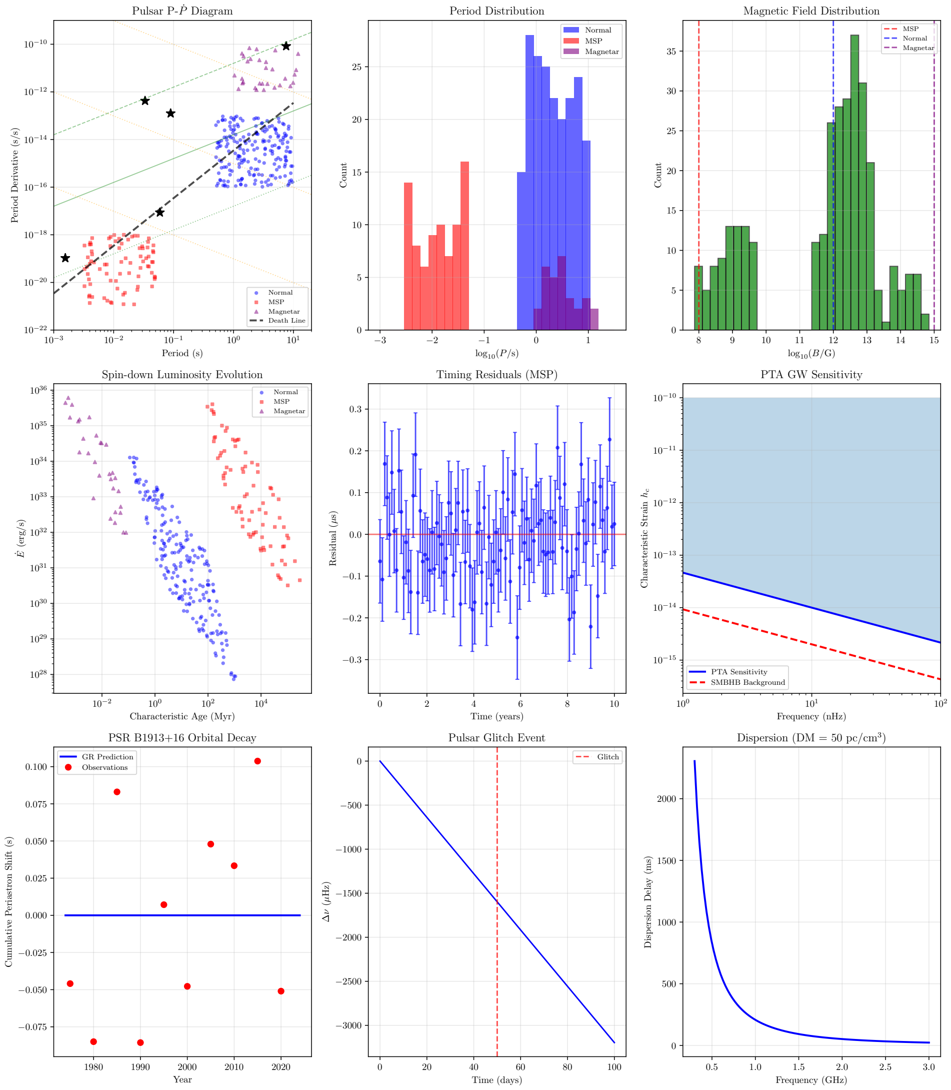
2.4 Braking Index

The braking index describes the spin-down mechanism:

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2} \tag{4}$$

- $n = 3$: Pure magnetic dipole radiation
- $n = 1$: Particle wind dominated
- $n = 5$: Gravitational wave emission

3 Computational Analysis



4 Results and Analysis

4.1 Pulsar Population Statistics

4.2 Famous Pulsars

Example 1 (The Crab Pulsar) *The Crab pulsar (PSR B0531+21) is a young pulsar in the Crab Nebula supernova remnant:*

- *Period:* $P = 33.4$ *ms*
- *Period derivative:* $\dot{P} =$ *4.21e-13 s/s*

Table 1: Pulsar Population Statistics

Population	Count	$\langle P \rangle$ (s)	$\langle B \rangle$ (G)	$\langle \tau_c \rangle$ (Myr)
Normal	200	2.202	2.99e+12	9.6
MSP	80	0.013	9.72e+08	3526.5
Magnetar	30	2.879	1.60e+14	0.0

Table 2: Properties of Notable Pulsars

Pulsar	P (ms)	$\dot{P}$ (s/s)	B (G)	τ_c (yr)
Crab (B0531+21)	33.40	4.21e-13	3.79e+12	1258
Vela (B0833-45)	89.30	1.25e-13	3.38e+12	11325
PSR B1937+21	1.56	1.05e-19	4.10e+08	235528580
PSR B1913+16	59.00	8.63e-18	2.28e+10	108380096
SGR 1806-20	7550.00	8.30e-11	8.01e+14	1442

- *Characteristic age: τ_c = 1258 years*
- *True age (SN 1054): 970 years*
- *Surface magnetic field: B = 3.79e+12 G*
- *Spin-down luminosity: $\dot{E}$ = 4.46e+38 erg/s*

5 Pulsar Evolution

5.1 Evolutionary Tracks

Pulsars evolve through the P-$\dot{P}$ diagram:

1. **Birth**: Upper left, short period, high $\dot{P}$
2. **Spin-down**: Move right as period increases
3. **Death line**: Stop emitting when B/P^2 drops below threshold
4. **Recycling**: Binary accretion spins up to MSP phase

5.2 Millisecond Pulsar Formation

Remark 2 (Recycling Scenario) *MSPs are "recycled" pulsars:*

- *Originally normal pulsars in binary systems*
- *Accretion from companion spins them up*

- *Magnetic field buried by accreted material*
- *Final state:* $P \sim 1 - 10$ *ms,* $B \sim 10^8 - 10^9$ *G*

6 Pulsar Timing Arrays

6.1 Gravitational Wave Detection

Pulsar timing arrays use the correlated timing residuals of many MSPs to detect low-frequency gravitational waves:

Definition 2 (Hellings-Downs Curve) *The angular correlation of timing residuals between pulsar pairs due to a stochastic GW background:*

$$\Gamma(\theta) = \frac{3}{2}x \ln x - \frac{x}{4} + \frac{1}{2} + \frac{1}{2}\delta_{12} \tag{5}$$

where $x = (1 - \cos\theta)/2$ *and* θ *is the angular separation.*

6.2 Sources in PTA Band

- Supermassive black hole binaries (SMBHB)
- Cosmic strings
- Primordial gravitational waves
- Individual continuous sources

7 Binary Pulsars

7.1 Tests of General Relativity

Binary pulsars provide precision tests of GR through post-Keplerian parameters:

- Periastron advance: $\dot{\omega}$
- Gravitational redshift: γ
- Orbital decay: $\dot{P}_b$
- Shapiro delay: r, s

Remark 3 (Hulse-Taylor Pulsar) *PSR B1913+16 provided the first indirect evidence for gravitational waves through its orbital decay, matching GR predictions to 0.3%.*

8 Timing Phenomena

8.1 Glitches

Sudden spin-up events caused by transfer of angular momentum from the superfluid interior:

$$\frac{\Delta\nu}{\nu} \sim 10^{-9} - 10^{-6} \tag{6}$$

8.2 Dispersion

Interstellar plasma delays lower frequencies:

$$\Delta t = 4.149 \times 10^3 \cdot \mathrm{DM} \cdot \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \text{ s} \tag{7}$$

where DM is the dispersion measure in pc cm^{-3} and ν is in MHz.

9 Limitations and Extensions

9.1 Model Limitations

1. **Vacuum dipole**: Real magnetospheres are plasma-filled
2. **Constant B**: Field may decay over time
3. **Point dipole**: Higher multipoles exist
4. **Rigid rotation**: Differential rotation possible

9.2 Possible Extensions

- Full magnetosphere models (force-free, MHD)
- Timing noise characterization
- Pulsar wind nebula evolution
- Equation of state constraints from masses

10 Conclusion

This analysis demonstrates the rich physics of pulsar timing:

- P-$\dot{P}$ diagram reveals distinct populations and evolution
- Characteristic ages provide evolutionary timescales
- MSPs are precision clocks with $\sigma_{\mathrm{rms}} \lesssim 100$ ns

- PTAs are sensitive to nanohertz gravitational waves
- Binary pulsars test GR with unprecedented precision

Further Reading

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- Hobbs, G. & Dai, S. (2017). A review of pulsar timing array gravitational wave research. *National Science Review*, 4, 707.