Vibration Analysis: SDOF Systems and Modal Analysis

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November 24, 2025

Abstract

This report presents computational analysis of mechanical vibrations including free and forced response of single degree of freedom (SDOF) systems, damping effects, frequency response, and modal analysis. Python-based computations provide quantitative analysis with dynamic visualization.

Contents

1 Introduction to Mechanical Vibrations

Vibration analysis is essential for:

- Machine design and reliability
- Structural dynamics and earthquake engineering
- Noise and vibration control
- Condition monitoring and diagnostics

2 SDOF System Fundamentals

2.1 Equation of Motion

For a mass-spring-damper system:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{1}$$

Natural frequency and damping ratio:

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_n} \tag{2}$$

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Figure 1: Free vibration: time response with envelope decay and phase portrait.

Table 1: SDOF System Parameters

Parameter	Value	Units
Natural frequency ω_n	10.00	rad/s
Natural frequency f_n	1.59	Hz
Damping ratio ζ	0.200	–
Damped frequency ω_d	9.80	rad/s

3 Effect of Damping

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Figure 2: Effect of damping ratio on free vibration and logarithmic decrement measurement.

4 Forced Vibration Response

4.1 Harmonic Excitation

For $F(t) = F_0 \sin(\omega t)$, the steady-state response is:

$$X = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}, \quad r = \frac{\omega}{\omega_n} \tag{3}$$

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Figure 3: Frequency response function showing magnitude and phase for various damping ratios.

4.2 Resonance Analysis

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Figure 4: Resonance characteristics: peak location, quality factor, bandwidth, and time response.

5 Transmissibility

Force transmitted to foundation:

$$TR = \frac{F_T}{F_0} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}} \tag{4}$$

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Figure 5: Force transmissibility and vibration isolation efficiency.

6 Two-DOF System (Modal Analysis)

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Figure 6: Modal analysis of 2-DOF system: mode shapes, time response, and FFT.

Natural frequencies: $\omega_1 = 10.00$ rad/s, $\omega_2 = 17.32$ rad/s

7 Conclusions

This analysis demonstrates key aspects of vibration analysis:

1. SDOF systems are characterized by natural frequency and damping ratio
2. Logarithmic decrement provides experimental damping measurement
3. Resonance occurs near natural frequency with phase shift of 90 degrees
4. Vibration isolation requires $r > \sqrt{2}$
5. MDOF systems have multiple natural frequencies and mode shapes
6. FFT analysis identifies frequency content from time domain data