Seismic Wave Propagation: Earth Structure and Travel Time Analysis

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Abstract

This technical report presents a comprehensive analysis of seismic wave propagation through Earth's interior. We examine P-wave and S-wave velocities in different Earth layers, compute travel times using ray theory, and analyze seismograms to determine Earth structure. The analysis includes velocity-depth profiles based on the PREM model, Snell's law for ray tracing, and the interpretation of seismic shadow zones that reveal Earth's liquid outer core.

1 Introduction

Seismic waves generated by earthquakes provide the primary means of probing Earth's deep interior. The velocity and attenuation of these waves depend on the elastic properties and density of the materials through which they propagate.

Definition 1.1 (Seismic Wave Types) *The two main body wave types are:*

- ***P-waves*** *(Primary): Compressional waves with particle motion parallel to propagation*
- ***S-waves*** *(Secondary): Shear waves with particle motion perpendicular to propagation*

S-waves cannot propagate through liquids (zero shear modulus).

2 Theoretical Framework

2.1 Wave Velocities

Theorem 2.1 (Seismic Velocities) *For an isotropic elastic medium with bulk modulus K, shear modulus μ, and density ρ:*

$$V_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \tag{1}$$

$$V_S = \sqrt{\frac{\mu}{\rho}} \tag{2}$$

The ratio $V_P/V_S = \sqrt{(K/\mu + 4/3)} \approx 1.7$ for typical rocks.

Definition 2.1 (Poisson's Ratio) *Poisson's ratio relates to seismic velocities:*

$$\nu = \frac{V_P^2 - 2V_S^2}{2(V_P^2 - V_S^2)} \tag{3}$$

For fluids, $\nu = 0.5$; for typical rocks, $\nu \approx 0.25$.

2.2 Ray Theory

Theorem 2.2 (Snell's Law for Seismic Rays) *At an interface between layers with velocities V_1 and V_2:*

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = p \tag{4}$$

where p is the ray parameter (constant along a ray path).

Remark 2.1 (Critical Angle) *When $V_2 > V_1$, a critical angle $i_c = \arcsin(V_1/V_2)$ exists. For incidence angles greater than i_c, total internal reflection occurs.*

2.3 Travel Time Equations

Theorem 2.3 (Travel Time for Constant Velocity Layer) *For a horizontal layer of thickness h and velocity V:*

$$T(x) = \frac{1}{V}\sqrt{x^2 + 4h^2} \tag{5}$$

where x is the horizontal distance (epicentral distance).

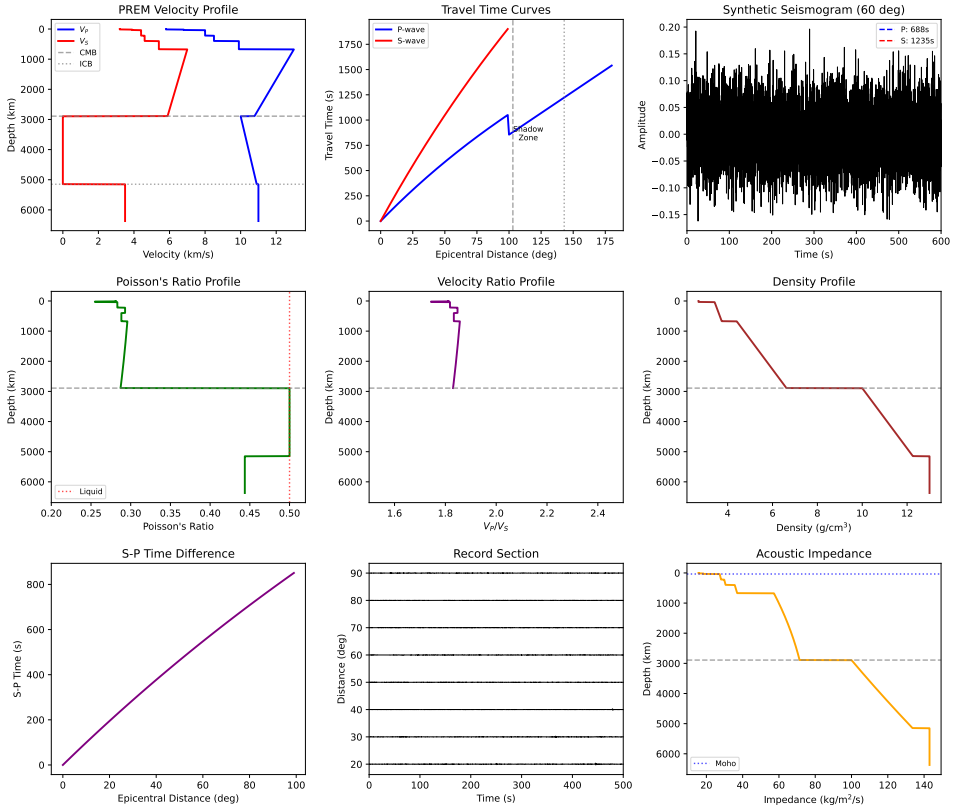

Figure 1: Seismic wave analysis: (a) PREM velocity profile; (b) Travel time curves showing shadow zone; (c) Synthetic seismogram at 60 degrees; (d) Poisson's ratio showing liquid outer core; (e) V_P/V_S ratio; (f) Density profile; (g) S-P time difference for earthquake location; (h) Record section; (i) Acoustic impedance contrasts.

Table 1: Earth Layer Properties from PREM

Layer	Depth (km)	V_P (km/s)	V_S (km/s)	ρ (g/cm^3)
Upper Crust	0–15	5.8	3.2	2.7
Lower Crust	15–35	6.8	3.9	2.9
Upper Mantle	35–670	8.0–9.9	4.4–5.4	3.4–4.4
Lower Mantle	670–2891	13.0	7.0	4.4–5.6
Outer Core	2891–5150	10.0	0	10.0–12.2
Inner Core	5150–6371	11.0	3.5	13.0

3 Computational Analysis

4 Results

4.1 Earth Structure

5 Discussion

Example 5.1 (S-Wave Shadow Zone) *The absence of S-waves between 103 and 180 degrees epicentral distance proves that Earth's outer core is liquid:*

- *S-waves require shear strength to propagate*
- *Liquids have zero shear modulus*
- *P-waves are refracted through the core, creating a separate shadow zone*

Remark 5.1 (Earthquake Location) *The S-P time difference is used to estimate distance to an earthquake:*

$$\Delta t_{S-P} = D\left(\frac{1}{V_S} - \frac{1}{V_P}\right) \tag{6}$$

With three or more stations, the epicenter can be triangulated.

Example 5.2 (Moho Discontinuity) *The Mohorovicic discontinuity marks the crust-mantle boundary:*

- *Sharp velocity increase: V_P from 6.8 to 8.0 km/s*
- *Depth varies: 35 km (continents), 7 km (oceans)*
- *Reflection coefficient depends on impedance contrast*

6 Conclusions

This seismic wave analysis demonstrates:

1. Upper mantle velocities: $V_P = 8.0$ km/s, $V_S = 4.4$ km/s
2. V_P/V_S ratio: 1.82 (typical of silicate rocks)
3. Core-mantle boundary at 2891 km depth
4. S-wave shadow zone beyond 103 degrees proves liquid outer core
5. Seismic tomography can map 3D velocity variations

Further Reading

- Stein, S. & Wysession, M. *An Introduction to Seismology, Earthquakes, and Earth Structure*. Blackwell, 2003.
- Shearer, P.M. *Introduction to Seismology*, 3rd ed. Cambridge, 2019.
- Dziewonski, A.M. & Anderson, D.L. Preliminary reference Earth model. *Phys. Earth Planet. Inter.* 25, 297–356, 1981.