# Qubit Operations and Quantum Gates: Pauli Gates, Superposition, Entanglement, and Bell States

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**Abstract**

This report explores fundamental qubit operations and quantum gates. We implement Pauli gates (X, Y, Z), demonstrate superposition using Hadamard gates, create entangled Bell states, and visualize quantum states on the Bloch sphere. Matrix representations and state evolution are simulated using PythonTeX.

# Contents

# Chapter 1

# Introduction

A qubit is the fundamental unit of quantum information:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle , \quad |\alpha|^2 + |\beta|^2 = 1 \tag{1.1}$$

## 1.1 Computational Basis

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.2}$$

# Chapter 2

# Pauli Gates

## 2.1 Definitions

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{(NOT gate)} \tag{2.1}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{2.2}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{(Phase flip)} \tag{2.3}$$

Properties: $X^2 = Y^2 = Z^2 = I$

## 2.2 Gate Operations

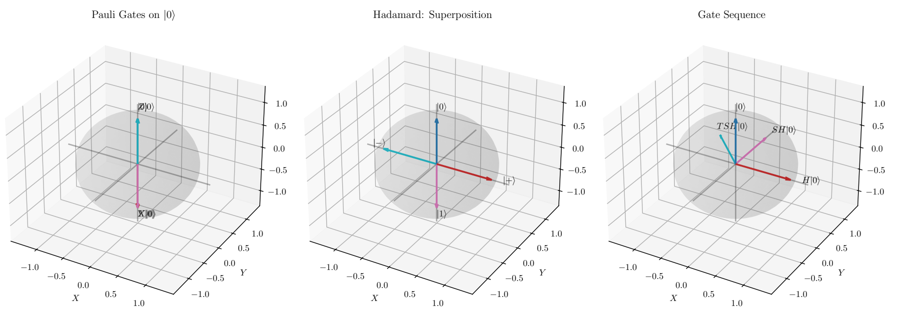


Figure 2.1: Bloch sphere visualization: (a) Pauli gates, (b) Hadamard creating superposition, (c) gate sequence.

# Chapter 3

# Superposition

The Hadamard gate creates superposition:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle \tag{3.1}$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle \tag{3.2}$$

# Chapter 4

# Entanglement and Bell States

## 4.1 Bell States

The four maximally entangled Bell states:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{4.1}$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \tag{4.2}$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{4.3}$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{4.4}$$

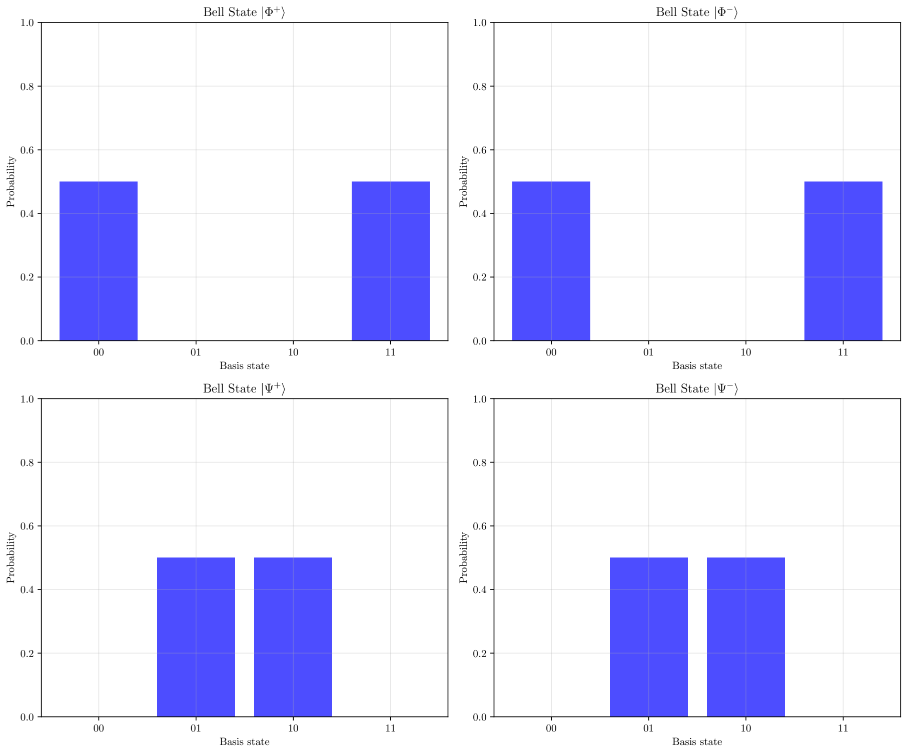


Figure 4.1: The four Bell states showing equal superposition of correlated basis states.

# Chapter 5

# Quantum Circuit Analysis

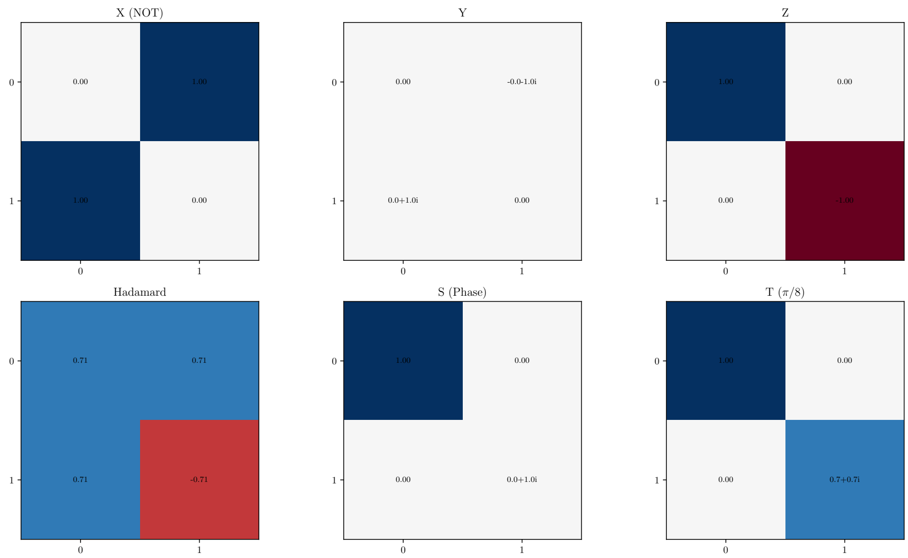


Figure 5.1: Single-qubit gate matrices (real part shown, complex values annotated).

# Chapter 6

# Measurement and Probabilities

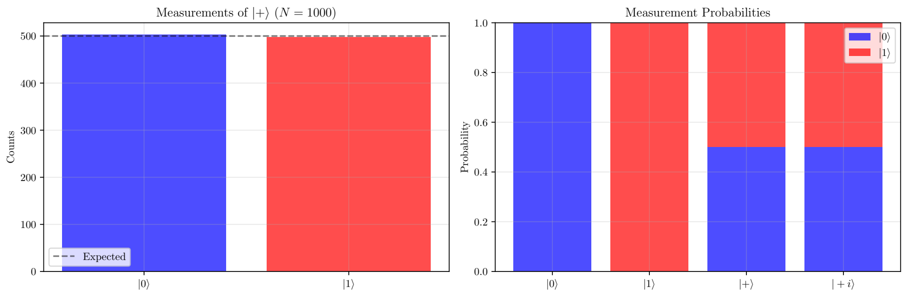


Figure 6.1: Quantum measurements: simulated counts (left), theoretical probabilities (right).

# Chapter 7

# Summary

Table 7.1: Summary of quantum gates

| Gate | Function | Example |
|---|---|---|
| $X$ | Bit flip | $X\|0\rangle = \|1\rangle$ |
| $Y$ | Bit+phase flip | $Y\|0\rangle = i\|1\rangle$ |
| $Z$ | Phase flip | $Z\|1\rangle = -\|1\rangle$ |
| $H$ | Superposition | $H\|0\rangle = \|+\rangle$ |
| CNOT | Entanglement | Control-target flip |

Purity of reduced density matrix for $|\Phi^+\rangle$: 0.500 (maximum entanglement = 0.5)

## Chapter 8

# Conclusions

1. Pauli gates rotate states on Bloch sphere axes
2. Hadamard creates equal superposition
3. Bell states are maximally entangled two-qubit states
4. CNOT gate enables entanglement between qubits
5. Measurement collapses superposition to basis states