Optics: Gaussian Beam Propagation

Computational Science Templates

November 24, 2025

1 Introduction

Gaussian beams are fundamental to laser optics and optical communication. They represent the lowest-order transverse electromagnetic mode (TEM_{00}) of optical resonators. This analysis computes the propagation characteristics of a Gaussian beam, including beam waist, divergence, Rayleigh range, and focuses on practical applications like focusing and optical system design using ABCD matrix methods.

2 Mathematical Framework

2.1 Electric Field and Intensity

The Gaussian beam electric field amplitude:

$$E(r,z) = E_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w(z)^2}\right) \exp\left(i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right) \tag{1}$$

The intensity profile is:

$$I(r,z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w(z)^2}\right) \tag{2}$$

2.2 Key Parameters

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2} \quad \text{(beam width)} \tag{3}$$

$$z_R = \frac{\pi w_0^2}{\lambda} \quad \text{(Rayleigh range)} \tag{4}$$

$$R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right] \quad \text{(wavefront curvature)} \tag{5}$$

$$\psi(z) = \arctan\left(\frac{z}{z_R}\right) \quad \text{(Gouy phase)} \tag{6}$$

2.3 Beam Quality Factor

The M^2 parameter characterizes beam quality:

$$M^2 = \frac{\pi w_0 \theta}{\lambda} \tag{7}$$

where θ is the far-field divergence half-angle. For an ideal Gaussian beam, $M^2 = 1$.

3 Environment Setup

4 Basic Beam Propagation

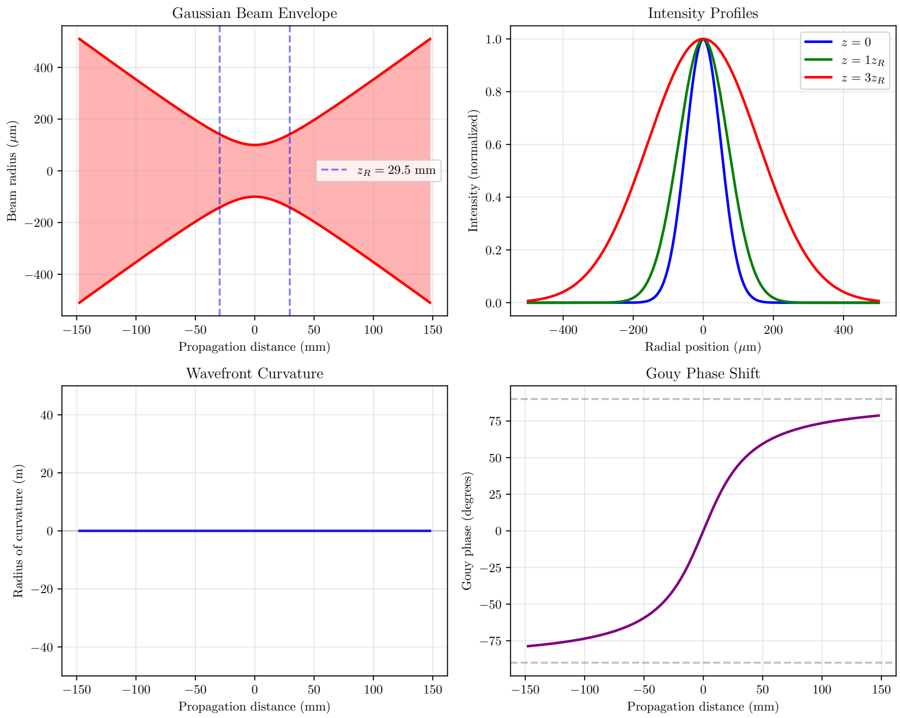

Figure 1: Basic Gaussian beam propagation: envelope, intensity profiles, wavefront curvature, and Gouy phase.

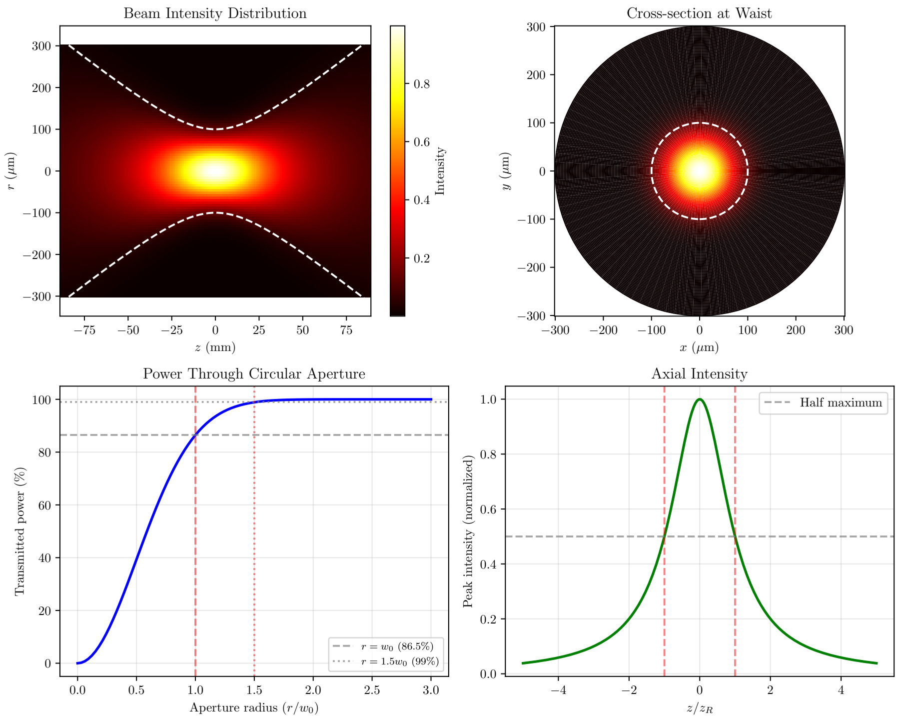

Figure 2: 2D beam visualization: intensity distribution, cross-section, power transmission, and axial intensity.

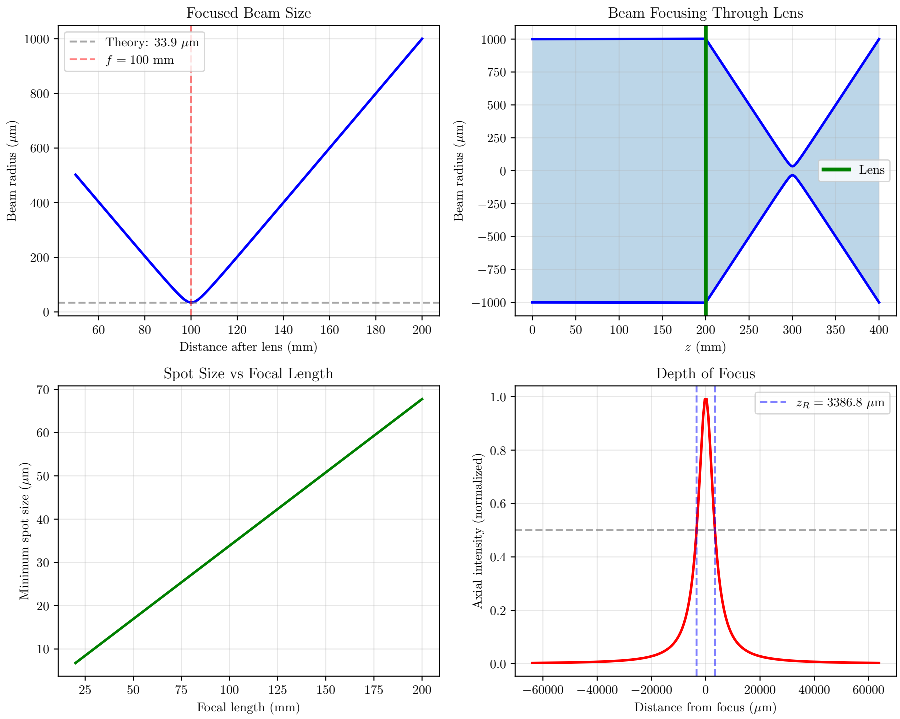

Figure 3: Gaussian beam focusing: spot size, beam propagation through lens, and depth of focus.

5 2D Beam Propagation Visualization

6 Focusing with a Lens

7 Higher-Order Hermite-Gaussian Modes

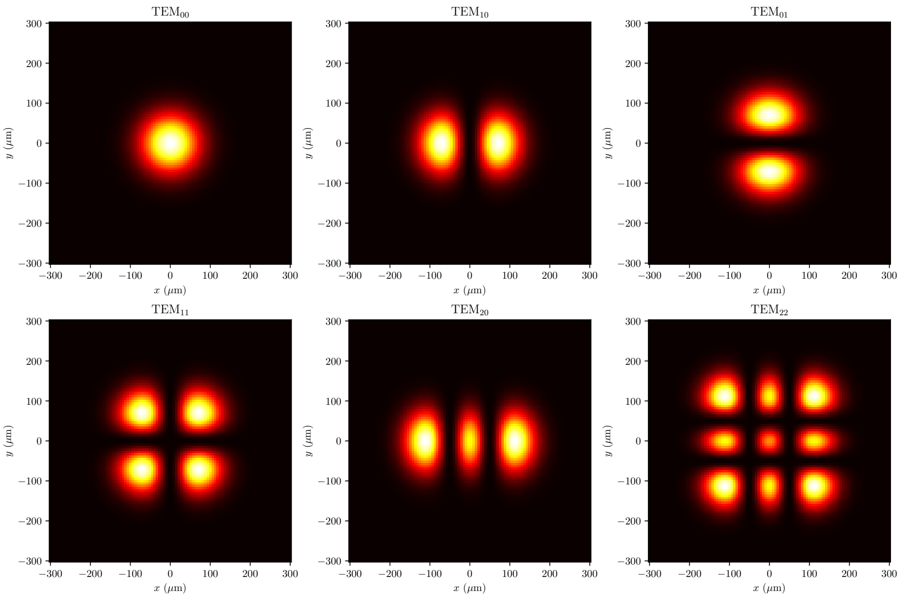

Figure 4: Hermite-Gaussian transverse modes TEM_{mn} intensity patterns.

8 Beam Quality Analysis

9 Optical Resonator Stability

10 Results Summary

11 Statistical Summary

- **Wavelength**: $\lambda = 1064$ nm
- **Beam waist**: $w_0 = 100\ \mu$m
- **Rayleigh range**: $z_R = 29.53$ mm
- **Divergence half-angle**: $\theta = 194.05$ mrad
- **Depth of focus**: $2z_R = 59.05$ mm

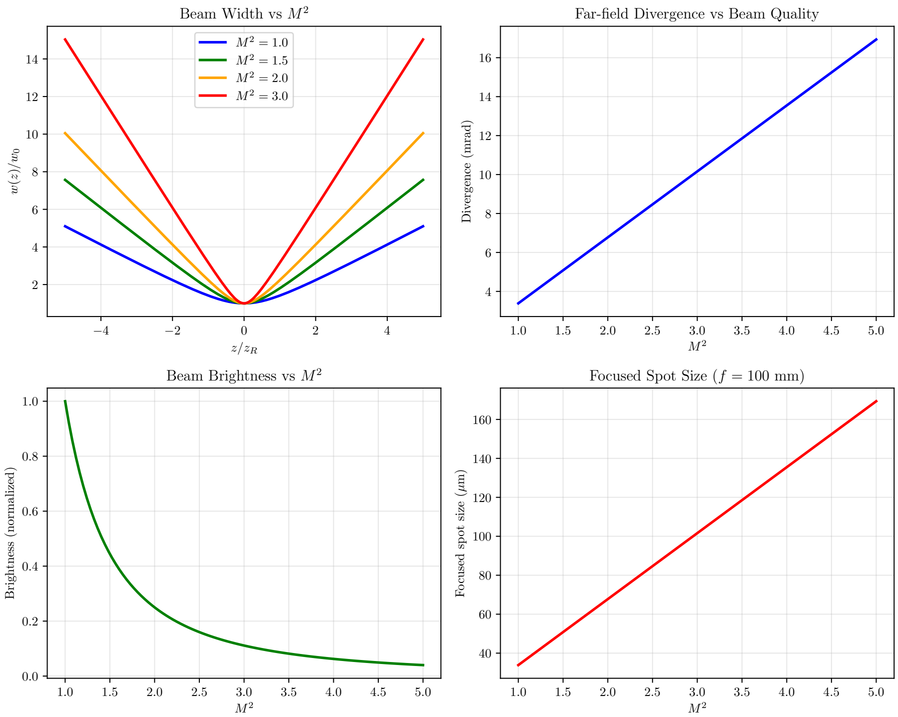

Figure 5: Beam quality analysis: effect of M^2 on propagation, divergence, brightness, and focusing.

Table 1: Summary of Gaussian Beam Parameters

Parameter	Symbol	Value
Wavelength	λ	1064 nm
Beam waist	w_0	100 μm
Rayleigh range	z_R	29.53 mm
Divergence half-angle	θ	0.1941°
Depth of focus	$2z_R$	59.05 mm
Beam quality factor	M^2	3.0
Input waist (focusing)	$w_{0,in}$	1 mm
Focal length	f	100 mm
Focused spot size	$w_{0,focus}$	33.9 μm
Focused Rayleigh range	$z_{R,focus}$	3386.8 μm

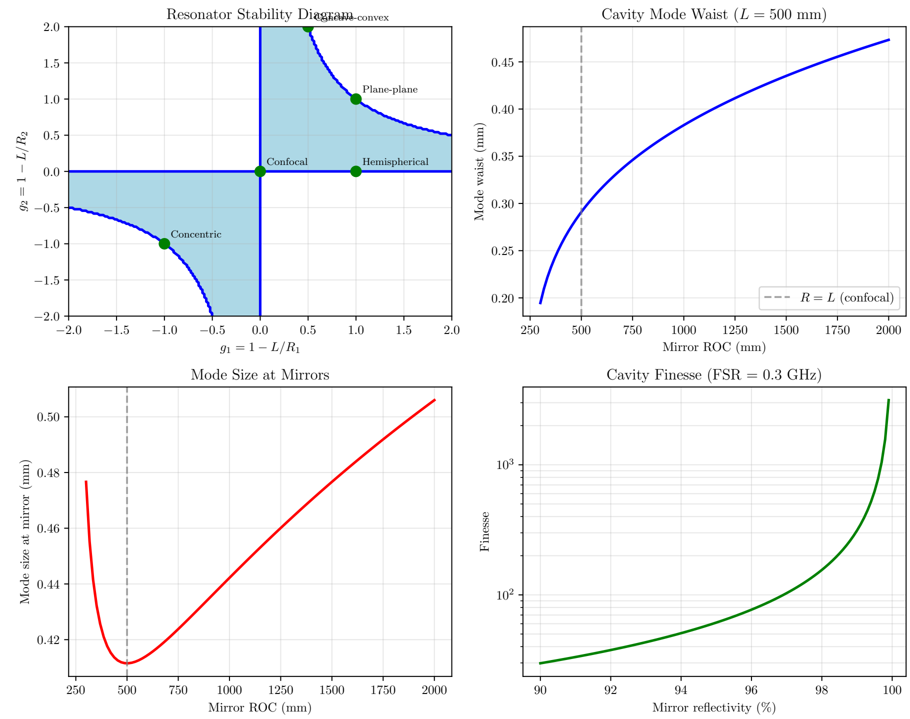

Figure 6: Optical resonator analysis: stability diagram, mode sizes, and cavity finesse.

- **Power in $1/e^2$ radius**: 86.5%
- **Focused spot (diffraction limit)**: $w_0' = 33.9\ \mu$m
- **Confocal cavity FSR**: 0.3 GHz

12 Conclusion

Gaussian beam propagation is characterized by the Rayleigh range z_R, where the beam width increases by $\sqrt{2}$. The product $w_0 \cdot \theta = \lambda/\pi$ is minimum for an ideal Gaussian beam, representing the diffraction limit. Understanding beam propagation is essential for optical system design, including laser focusing, fiber coupling, and resonator mode analysis. The ABCD matrix formalism provides a powerful tool for analyzing complex optical systems. Higher-order Hermite-Gaussian modes exhibit characteristic multi-lobed intensity patterns and are important in resonator analysis and mode-selective applications.