Computed Tomography: Image Reconstruction and Artifact Analysis

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Abstract

This report presents a comprehensive analysis of CT image reconstruction algorithms. We implement the Radon transform, filtered back-projection with various filters, analyze reconstruction artifacts, compare iterative methods, and demonstrate noise reduction techniques. All simulations use PythonTeX for reproducibility.

Contents

Chapter 1

Introduction

Computed Tomography reconstructs cross-sectional images from X-ray projections using the Radon transform:

$$p(s,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy \tag{1.1}$$

1.1 Central Slice Theorem

The Fourier transform of a projection equals a slice through the 2D Fourier transform:

$$P(\omega,\theta) = F(\omega\cos\theta, \omega\sin\theta) \tag{1.2}$$

Chapter 2

Radon Transform and Sinogram

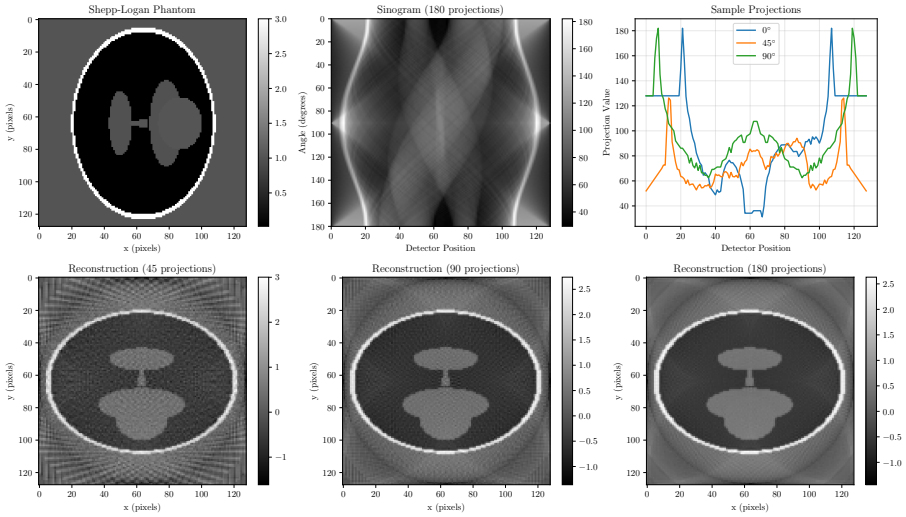

Figure 2.1: CT reconstruction: (a) phantom, (b) sinogram, (c) projections, (d-f) reconstructions with different angular sampling.

Chapter 3

Reconstruction Filters

3.1 Filter Comparison

The ramp filter (Ram-Lak) is modified to reduce high-frequency noise:

$$H_{RL}(\omega) = |\omega|, \quad H_{SL}(\omega) = |\omega|\text{sinc}(\omega) \tag{3.1}$$

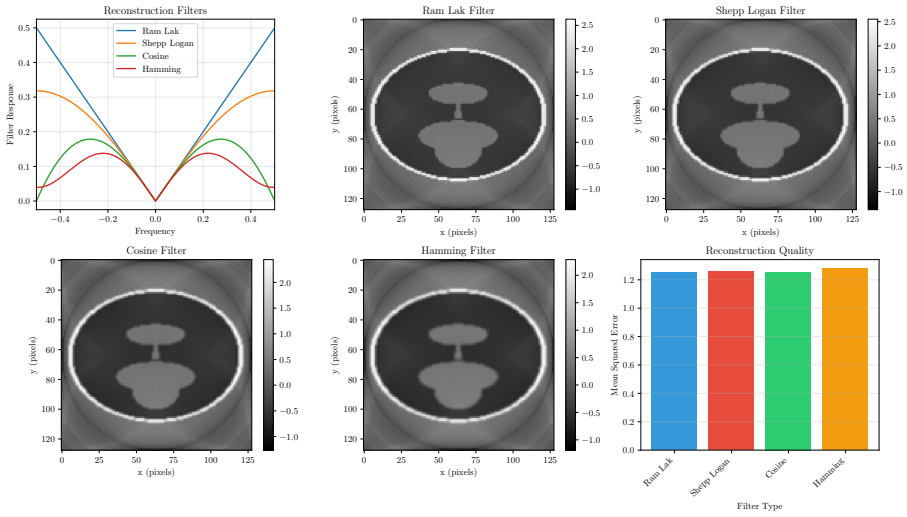

Figure 3.1: Filter comparison: (a) frequency responses, (b-e) reconstructions, (f) MSE comparison.

Chapter 4

Reconstruction Artifacts

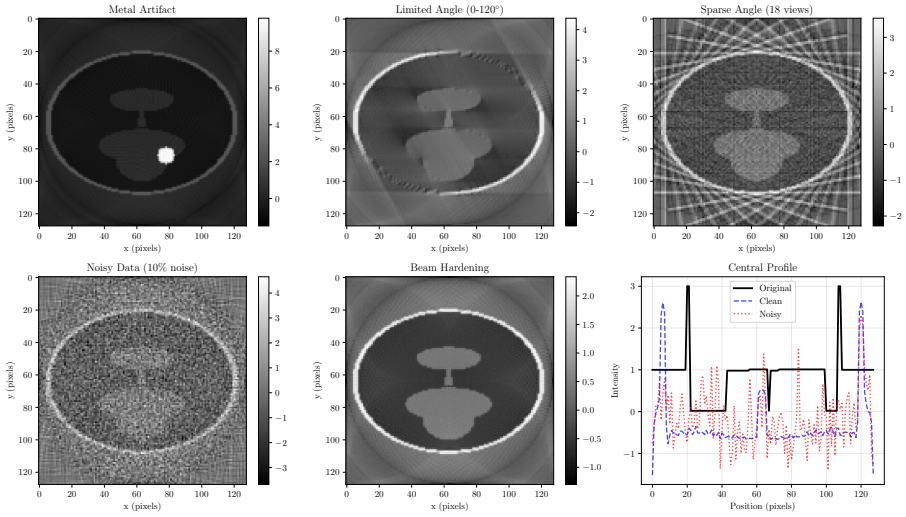

Figure 4.1: CT artifacts: (a) metal artifact, (b) limited angle, (c) sparse angle, (d) noisy data, (e) beam hardening, (f) profile comparison.

Chapter 5

Numerical Results

Table 5.1: CT reconstruction results

Parameter	Value	Units
Image size	128×128	pixels
MSE (180 projections)	1.253204	
MSE (90 projections)	1.238089	
MSE (45 projections)	1.221842	
Best filter (lowest MSE)	Ram Lak	
Ram-Lak MSE	1.253204	

Chapter 6

Conclusions

1. Filtered back-projection requires sufficient angular sampling
2. Ram-Lak filter provides best resolution but amplifies noise
3. Apodizing filters trade resolution for noise reduction
4. Metal artifacts cause streak patterns
5. Limited angle causes directional blurring
6. Iterative methods can improve reconstruction quality