Digital Filter Design: FIR and IIR Filters

Computational Signal Processing

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1 Introduction

Digital filters are fundamental components in signal processing systems. This document explores the design and analysis of both Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters. We implement various design methods including windowing techniques, frequency sampling, Parks-McClellan algorithm for FIR filters, and Butterworth, Chebyshev, and elliptic designs for IIR filters. The analysis includes frequency response characteristics, phase response, group delay, and filter stability assessment.

2 Mathematical Framework

2.1 FIR Filter Design

An FIR filter of order N has transfer function:

$$H(z) = \sum_{k=0}^{N} b_k z^{-k} \tag{1}$$

The ideal lowpass filter impulse response is:

$$h_d[n] = \frac{\sin(\omega_c n)}{\pi n} \tag{2}$$

Window-based design multiplies this by a window function $w[n]$:

$$h[n] = h_d[n] \cdot w[n] \tag{3}$$

2.2 IIR Filter Design

An IIR filter transfer function:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{4}$$

Butterworth magnitude response:

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}} \tag{5}$$

Chebyshev Type I response with ripple ϵ:

$$|H(j\omega)|^2 = \frac{1}{1 + \epsilon^2 T_N^2\left(\frac{\omega}{\omega_c}\right)} \tag{6}$$

where T_N is the Chebyshev polynomial of order N.

3 Environment Setup

4 FIR Filter Design: Window Method

5 FIR Frequency Response Analysis

6 Parks-McClellan Optimal FIR Design

7 IIR Filter Design: Butterworth

8 IIR Filter Comparison: Butterworth, Chebyshev, Elliptic

9 Bandpass and Bandstop Filter Design

10 Filter Stability Analysis

11 Real-Time Filtering Application

12 Results Summary

12.1 FIR Window Method Performance

12.2 IIR Filter Comparison

12.3 Butterworth Order Analysis

13 Statistical Summary

Key filter design metrics:

- FIR filter length: **??** taps

- Parks-McClellan passband ripple: **??** dB
- Parks-McClellan stopband attenuation: **??** dB
- RMS error (lfilter): **??**
- RMS error (filtfilt): **??**
- SNR improvement: **??** dB

14 Conclusion

This computational analysis demonstrates comprehensive digital filter design techniques for both FIR and IIR filters. FIR filters designed with window methods provide linear phase response but require higher orders for sharp transitions. Parks-McClellan optimal design achieves equiripple behavior with minimal order. IIR filters (Butterworth, Chebyshev, Elliptic) offer sharper transitions with lower orders but introduce phase distortion. The choice of filter type depends on the specific application requirements for phase linearity, transition sharpness, and computational efficiency.