Explorations in Partition Theory and Topology

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November 22, 2025

1 Integer Partitions

The partition function $p(n)$ represents the number of distinct ways of representing n as a sum of natural numbers. For large n, this computation is non-trivial.

Using SageMath, we calculate the number of partitions for $n = 100$:

```
n = 100
partitions = number_of_partitions(n)
factors = factor(partitions)
```

The number of partitions of 100 is:

$$p(100) = 190569292$$

This number can be factored into primes as:

$$2^2 \cdot 43 \cdot 59 \cdot 89 \cdot 211$$

This demonstrates Sage's ability to handle arbitrary-precision integers, which would overflow standard types in C++ or basic Python.

2 Topological Surfaces

We visualize the behavior of the function $f(x, y) = \sin(x^2 + y^2)$ near the origin.

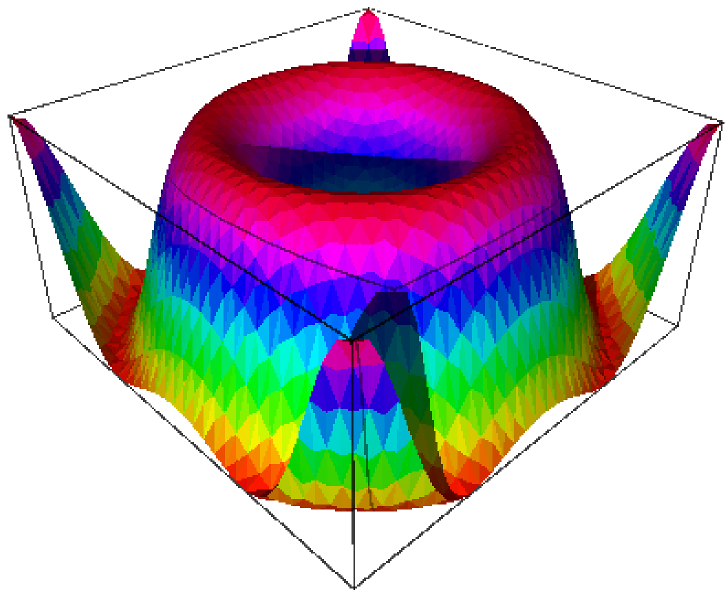

Figure 1: 3D Plot of $f(x, y)$ computed by SageMath.

3 Additional Symbolic Computation

SageMath excels at symbolic manipulation. Here we demonstrate polynomial factorization and calculus:

```
# Polynomial factorization
poly = x^4 - 1
factored = factor(poly)

# Symbolic integration
integral_result = integrate(sin(x)^2, x)
```

The polynomial $x^4 - 1$ factors as:

$$\left(x^2 + 1\right)(x + 1)(x - 1)$$

The indefinite integral of $\sin^2(x)$ is:

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right) + C$$