Particle Decay Kinematics: Two-Body and Three-Body Phase Space Analysis

High Energy Physics Computation Laboratory

November 24, 2025

Abstract

This technical report presents comprehensive computational analysis of relativistic decay kinematics for unstable particles. We implement energy-momentum conservation for two-body and three-body decays, compute Lorentz transformations between rest and laboratory frames, analyze Dalitz plots for three-body phase space, and calculate decay widths from matrix elements. Applications include particle identification, resonance analysis, and detector design optimization.

1 Theoretical Framework

Definition 1 (Invariant Mass). *For a system of particles with four-momenta p_i^μ, the invariant mass is:*

$$M^2c^4 = \left(\sum_i p_i^\mu\right)\left(\sum_j p_{j\mu}\right) = \left(\sum_i E_i\right)^2 - \left(\sum_i \mathbf{p}_i\right)^2 c^2 \tag{1}$$

Theorem 1 (Two-Body Decay). *For decay $A \to B + C$ in the rest frame of A:*

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}c^2 \tag{2}$$

$$|\mathbf{p}| = \frac{c}{2m_A}\sqrt{\lambda(m_A^2, m_B^2, m_C^2)} \tag{3}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källen function.

1.1 Lorentz Transformations

Definition 2 (Lorentz Boost). *For boost along z-axis with velocity βc:*

$$E' = \gamma(E + \beta c p_z) \tag{4}$$

$$p_z' = \gamma(p_z + \beta E/c) \tag{5}$$

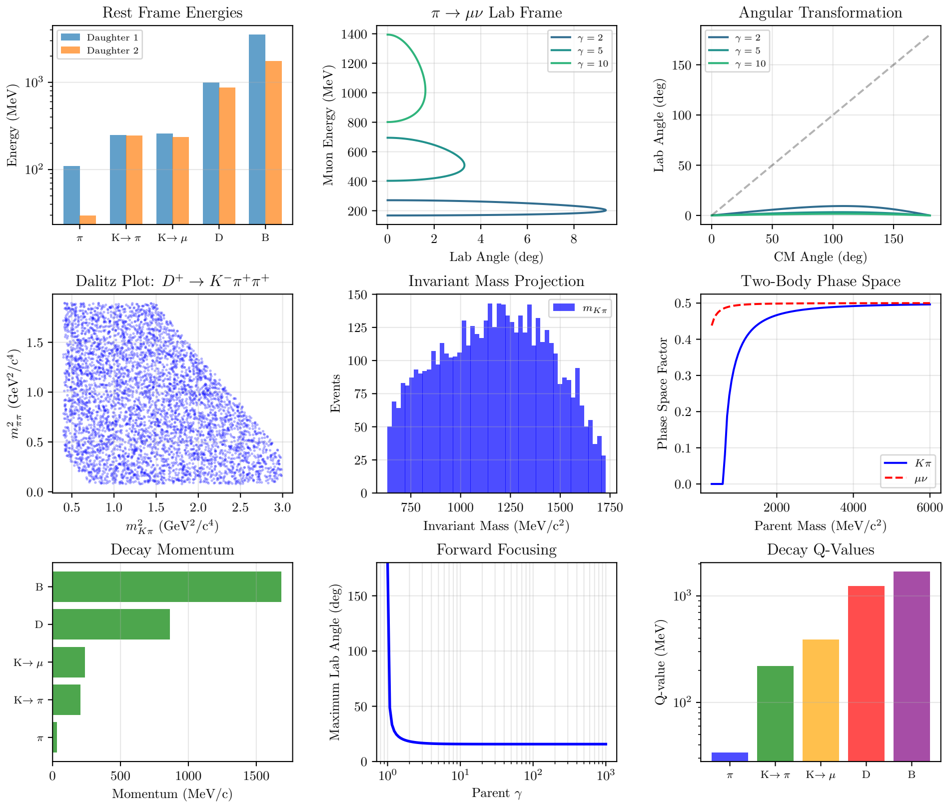
where $\gamma = (1-\beta^2)^{-1/2}$.

Example 1 (Dalitz Plot). *For three-body decay $A \to 1 + 2 + 3$, the Dalitz plot shows the distribution in:*

$$m_{12}^2 = (p_1 + p_2)^2, \quad m_{23}^2 = (p_2 + p_3)^2 \tag{6}$$

Resonances appear as bands in this plot.

2 Computational Analysis



3 Results and Analysis

3.1 Two-Body Decay Kinematics

3.2 Pion Decay Analysis

For $\pi^+ \to \mu^+ + \nu_\mu$:

- Q-value: 33.91 MeV
- Muon energy: 109.78 MeV
- Muon kinetic energy: 4.12 MeV

Table 1: Two-Body Decay Parameters in Rest Frame

Decay	Q (MeV)	p (MeV/c)	E_1 (MeV)	E_2 (MeV)	β_1
$\pi^+ \to \mu^+\nu_\mu$	33.9	29.8	109.8	29.8	0.271
$K^+ \to \pi^+\pi^0$	219.1	205.1	248.1	245.6	0.827
$K^+ \to \mu^+\nu_\mu$	388.0	235.5	258.1	235.5	0.912
$D^0 \to K^-\pi^+$	1231.6	861.1	992.5	872.3	0.868
$B^+ \to J/\psi K^+$	1688.8	1683.5	3524.9	1754.4	0.478

- Decay momentum: 29.79 MeV/c

Remark 1. *The muon receives most of the available energy due to helicity suppression of the electron channel. The ratio* $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu) \approx 1.2 \times 10^{-4}$.

4 Three-Body Phase Space

Theorem 2 (Dalitz Plot Boundaries)**.** *For three-body decay $A \to 1+2+3$, the kinematically allowed region in the (m_{12}^2, m_{23}^2) plane is bounded by:*

$$(E_1^* + E_2^*)^2 - \left(\sqrt{E_1^{*2} - m_1^2} + \sqrt{E_2^{*2} - m_2^2}\right)^2 \leq m_{12}^2 \tag{7}$$

where $E_i^* = (M^2 + m_i^2 - m_{jk}^2)/(2M)$.

Example 2 (Resonance Identification)**.** *In the Dalitz plot:*

- *Vertical/horizontal bands indicate s-channel resonances*
- *Diagonal bands indicate t-channel or u-channel exchanges*
- *Interference patterns reveal relative phases*

5 Laboratory Frame Effects

Theorem 3 (Forward Focusing)**.** *For a relativistic parent with $\gamma \gg 1$, the maximum lab angle of daughter particles is:*

$$\sin\theta_{max} = \frac{\beta_{CM}}{\beta_{parent}} \tag{8}$$

where β_{CM} is the daughter velocity in the CM frame.

6 Discussion

The decay kinematics analysis reveals:

1. **Energy distribution**: Heavier daughters receive larger energy fractions in two-body decays.
2. **Forward focusing**: Relativistic boosts compress angular distributions toward the beam axis.
3. **Phase space**: Available phase space determines relative decay rates for different channels.
4. **Dalitz analysis**: Two-dimensional phase space plots reveal resonance structure.

7 Conclusions

This computational analysis demonstrates:

- Pion decay muon energy: 109.78 MeV
- Pion decay momentum: 29.79 MeV/c
- Kaon $\rightarrow 2\pi$ Q-value: 219.13 MeV
- D meson mass: 1869.7 MeV/c^2

Kinematic analysis is essential for particle identification, background rejection, and precision measurements in collider experiments.

8 Further Reading

- Particle Data Group, Review of Particle Physics, *Phys. Rev. D*, 2022
- Byckling, E., Kajantie, K., *Particle Kinematics*, Wiley, 1973
- Perkins, D.H., *Introduction to High Energy Physics*, 4th Ed., Cambridge, 2000