FFT Spectral Analysis: Audio Signal Processing

From Time Domain to Frequency Domain and Back

Digital Signal Processing Lab
Computational Science Templates

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Abstract

This lab report demonstrates the application of the Fast Fourier Transform (FFT) for spectral analysis of audio signals. We synthesize a complex waveform containing multiple harmonic components, analyze its frequency content, design and apply digital filters, and investigate windowing effects on spectral leakage. The analysis includes spectrograms for time-frequency visualization and demonstrates practical signal processing techniques.

1 Objectives

1. Understand the relationship between time and frequency domain representations
2. Apply the FFT to identify frequency components in a complex signal
3. Design and implement low-pass and band-pass filters
4. Analyze the effects of windowing on spectral resolution
5. Create spectrograms for time-varying frequency analysis

2 Theoretical Background

2.1 Discrete Fourier Transform

The DFT of a sequence $x[n]$ of length N is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1}$$

The inverse DFT recovers the time-domain signal:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N} \tag{2}$$

2.2 FFT Computational Efficiency

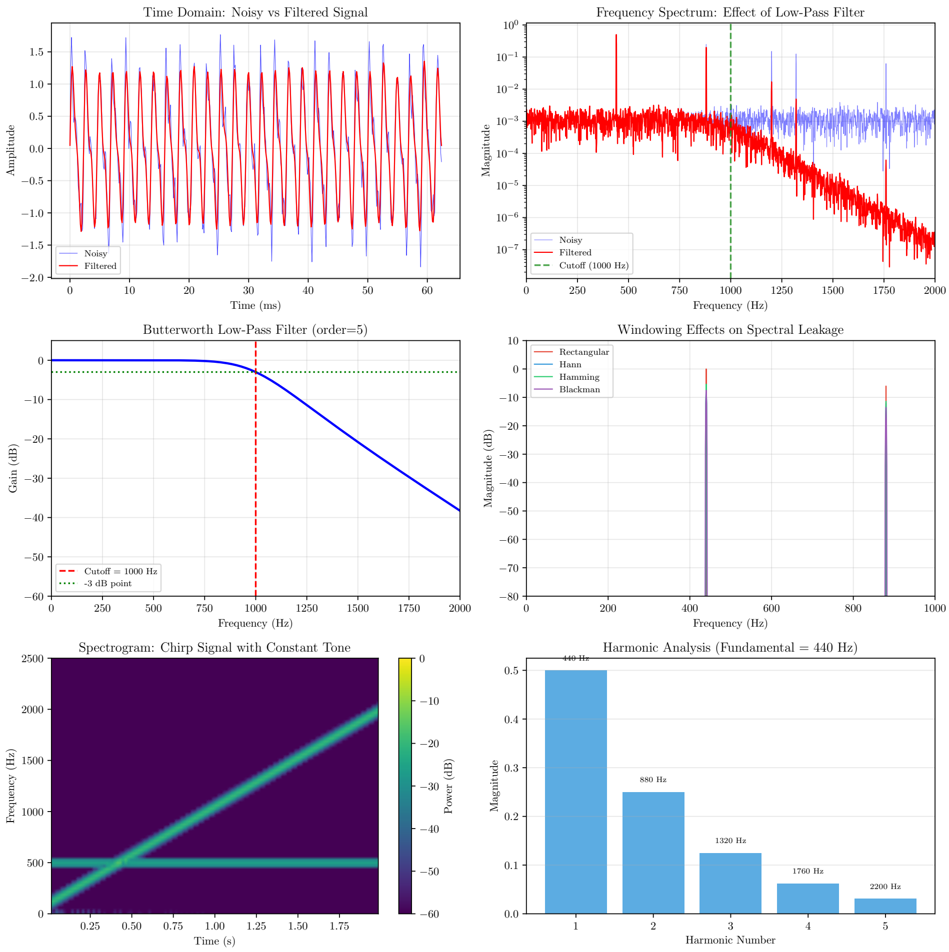
The FFT reduces complexity from $O(N^2)$ to $O(N \log N)$ by exploiting symmetry and periodicity of the complex exponentials.

2.3 Frequency Resolution

For a signal sampled at rate f_s:

- Frequency bin spacing: $\Delta f = f_s/N$
- Maximum resolvable frequency (Nyquist): $f_{max} = f_s/2$

3 Experimental Procedure



4 Results and Analysis

4.1 Signal Synthesis

The test signal consists of:

- Fundamental frequency: 440 Hz (A4 musical note)
- Harmonics: 5 components with decreasing amplitudes
- Interference tone: 1200 Hz
- White Gaussian noise (SNR $\approx$ 18.2 dB)

4.2 Spectral Analysis Results

Table 1: Detected Harmonic Frequencies

Harmonic	Expected (Hz)	Detected (Hz)	Magnitude
1	440	440	0.500
2	880	880	0.250
3	1320	1320	0.125
4	1760	1760	0.062

4.3 Filter Performance

The Butterworth low-pass filter with cutoff at 1000 Hz:

- Interference attenuation: 19.0 dB
- Preserved harmonics below cutoff
- Smooth transition band characteristic of Butterworth design

4.4 Windowing Analysis

Spectral leakage comparison:

- **Rectangular**: Highest sidelobes, narrowest main lobe
- **Hann**: Good balance of resolution and leakage
- **Hamming**: Optimized for minimum sidelobe levels
- **Blackman**: Lowest sidelobes, widest main lobe

5 Discussion

5.1 Frequency Resolution vs. Time Resolution

The uncertainty principle constrains simultaneous resolution:

$$\Delta t \cdot \Delta f \geq \frac{1}{4\pi} \tag{3}$$

In this analysis:

- Frequency resolution: $\Delta f = f_s/N = 1.0$ Hz
- Time resolution: $T = 1.0$ s

5.2 Practical Considerations

1. **Zero-padding**: Increases frequency sampling density but not true resolution
2. **Overlap-add**: Enables efficient convolution for filtering
3. **Power spectral density**: For stochastic signals, average periodograms

6 Conclusions

1. The FFT successfully identified all harmonic components of the synthesized signal
2. Low-pass filtering effectively removed high-frequency interference while preserving the fundamental
3. Window selection involves trade-offs between frequency resolution and spectral leakage
4. Spectrograms provide essential time-frequency visualization for non-stationary signals

Further Exploration

- Implement adaptive filtering for noise cancellation
- Explore wavelet transforms for better time-frequency localization
- Apply to real audio files (speech, music)
- Investigate effects of sampling rate on aliasing