Robotics: PID Motor Control and Tuning

Computational Robotics Templates

November 24, 2025

Abstract

This document presents a comprehensive analysis of Proportional-Integral-Derivative (PID) control for robotic motor systems. We implement continuous and discrete PID controllers, analyze the effects of each gain component on system response, explore automatic tuning methods including Ziegler-Nichols and Cohen-Coon, and demonstrate applications to DC motor position and velocity control. The analysis includes stability margins, frequency response, and performance metrics for practical robotics applications.

1 Introduction

PID control is the most widely used feedback control strategy in industrial automation and robotics. Despite its simplicity, properly tuned PID controllers can achieve excellent performance for a wide range of systems. Understanding the role of each component and systematic tuning methods is essential for robotics engineers.

2 Mathematical Framework

2.1 PID Control Law

The continuous-time PID controller output is:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \tag{1}$$

where $e(t) = r(t) - y(t)$ is the error between reference $r(t)$ and output $y(t)$.

2.2 Transfer Function Form

In the Laplace domain:

$$C(s) = K_p + \frac{K_i}{s} + K_d s = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2}$$

where $T_i = K_p/K_i$ is the integral time and $T_d = K_d/K_p$ is the derivative time.

2.3 Discrete PID Controller

For digital implementation with sampling period T_s:

$$u[k] = K_p e[k] + K_i T_s \sum_{j=0}^{k} e[j] + K_d \frac{e[k] - e[k-1]}{T_s} \tag{3}$$

2.4 DC Motor Model

A DC motor with armature dynamics:

$$G(s) = \frac{\Theta(s)}{V(s)} = \frac{K_m}{s(Js + b)(Ls + R) + K_m K_b} \tag{4}$$

Simplified (neglecting inductance):

$$G(s) = \frac{K}{s(\tau s + 1)} \tag{5}$$

3 Computational Analysis

3.1 Effect of PID Gains

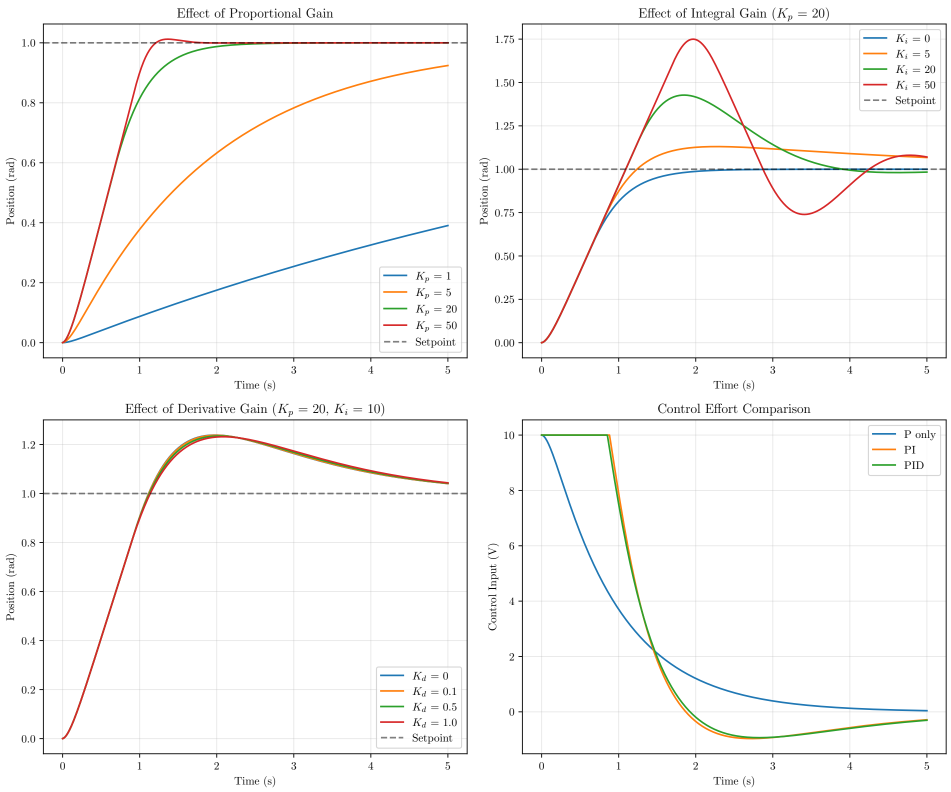

Figure 1: Effects of PID gains on system response: (a) proportional gain, (b) integral gain, (c) derivative gain, (d) control effort.

3.2 Performance Metrics Calculation

3.3 Ziegler-Nichols Tuning Method

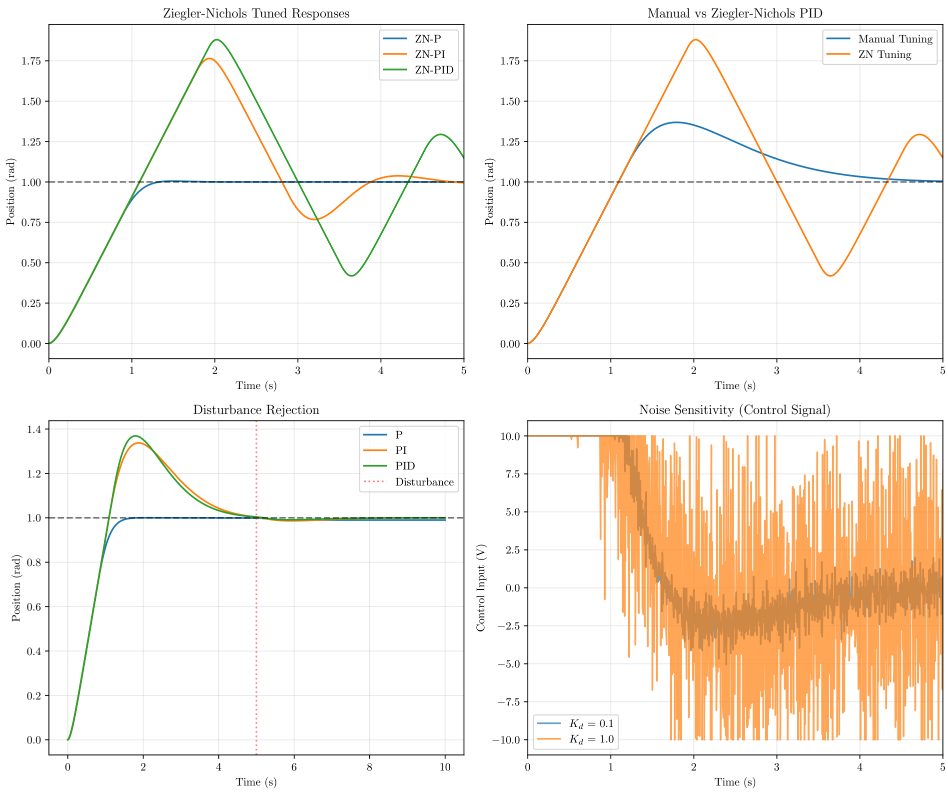

Figure 2: PID tuning analysis: (a) Ziegler-Nichols responses, (b) manual vs ZN tuning, (c) disturbance rejection, (d) noise sensitivity.

3.4 Discrete PID Implementation

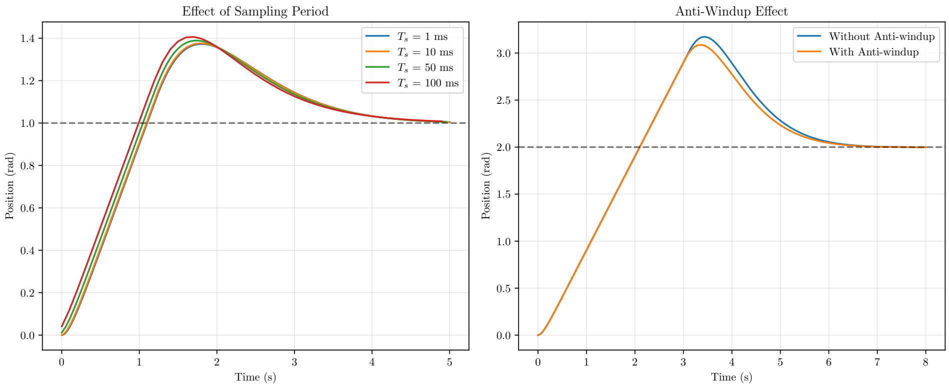

Figure 3: Discrete PID implementation: (a) effect of sampling period, (b) anti-windup mechanism.

3.5 Velocity Control Mode

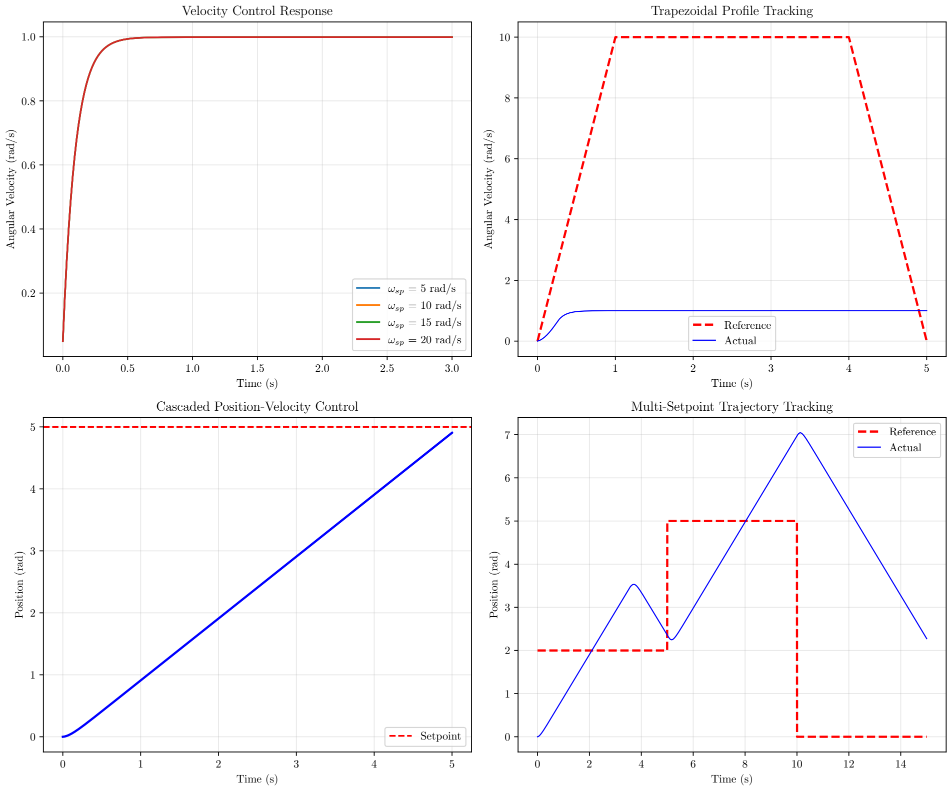

Figure 4: Velocity control modes: (a) step response, (b) profile tracking, (c) cascaded control, (d) multi-setpoint trajectory.

3.6 Frequency Response Analysis

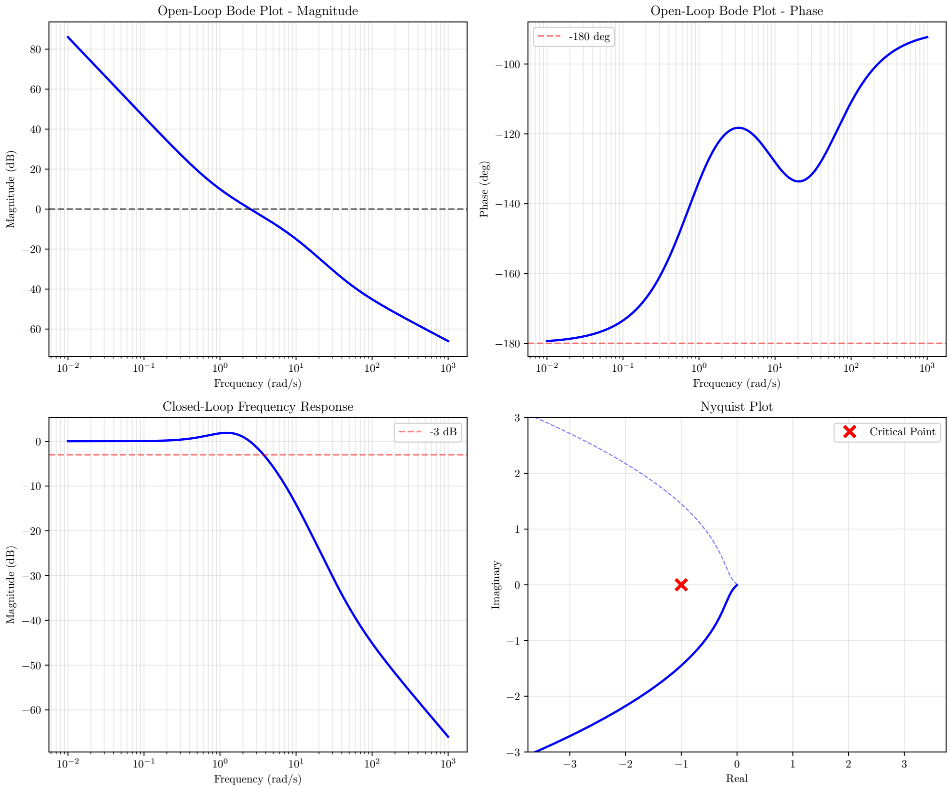

Figure 5: Frequency response analysis: (a) open-loop magnitude, (b) open-loop phase, (c) closed-loop response, (d) Nyquist plot.

4 Results and Discussion

4.1 Performance Metrics Comparison

Table 1: Controller Performance Comparison

Controller	Settling Time (s)	Overshoot (%)	SS Error	Gains
P	1.356	0.0	0.0000	$K_p = 30$
PI	4.412	33.8	0.000010	$K_p = 20$, $K_i = 15$
PID	4.277	36.9	0.000004	$K_p = 25$, $K_i = 20$, $K_d = 0.5$

4.2 Ziegler-Nichols Tuning Results

Using the ultimate gain method with $K_u = 80.0$ and $T_u = 0.50$ s:

Table 2: Ziegler-Nichols PID Parameters

Parameter	**Formula**	**Value**
K_p	$0.6K_u$	48.00
K_i	$1.2K_u/T_u$	192.00
K_d	$0.075K_uT_u$	3.0000

4.3 Stability Margins

The frequency response analysis reveals:

- Gain margin: -86.01 dB at 0.01 rad/s
- Phase margin: 60.86 degrees at 2.50 rad/s
- Closed-loop bandwidth: 3.78 rad/s

4.4 Motor Parameters

The simplified DC motor model parameters:

- Motor gain constant: $K = 0.0999$
- Time constant: $\tau = 0.0999$ s

5 Conclusion

This analysis demonstrated comprehensive PID controller design and tuning for robotic motor control. Key findings include:

1. Proportional control provides fast response but steady-state error
2. Integral action eliminates steady-state error but increases overshoot
3. Derivative action reduces overshoot and improves stability but amplifies noise
4. Ziegler-Nichols provides a good starting point but may require further tuning
5. Cascaded control architecture improves both position and velocity performance
6. Adequate stability margins (GM $>$ 6 dB, PM $>$ 45 deg) ensure robust operation

The methods presented form the foundation for practical PID implementation in robotic systems, with considerations for discrete implementation, anti-windup, and noise mitigation.