Earth's Magnetic Field: Dipole Model, Secular Variation, and IGRF Analysis

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Abstract

This technical report presents comprehensive computational analysis of Earth's main magnetic field using the geocentric axial dipole model and spherical harmonic representations. We implement field component calculations (radial, meridional, total intensity, inclination, declination), model the International Geomagnetic Reference Field (IGRF), and analyze secular variation. Applications include navigation, paleomagnetic studies, and space weather prediction.

1 Theoretical Framework

Definition 1 (Magnetic Scalar Potential). *In current-free regions, the geomagnetic field* $\mathbf{B}$ *can be derived from a scalar potential* V*:*

$$\mathbf{B} = -\nabla V \tag{1}$$

where V *satisfies Laplace's equation* $\nabla^2 V = 0$.

Theorem 1 (Spherical Harmonic Expansion). *The geomagnetic potential can be expanded in spherical harmonics:*

$$V(r, \theta, \phi) = a \sum_{n=1}^{N} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} [g_n^m \cos(m\phi) + h_n^m \sin(m\phi)] P_n^m(\cos\theta) \tag{2}$$

where a *is Earth's radius,* g_n^m *and* h_n^m *are Gauss coefficients, and* P_n^m *are Schmidt semi-normalized associated Legendre functions.*

1.1 Dipole Field Components

For the centered dipole (n=1, m=0), the field components in spherical coordinates:

$$B_r = 2B_0 \left(\frac{a}{r}\right)^3 \cos\theta \tag{3}$$

$$B_\theta = B_0 \left(\frac{a}{r}\right)^3 \sin\theta \tag{4}$$

where $B_0 \approx 31,000$ nT is the equatorial surface field.

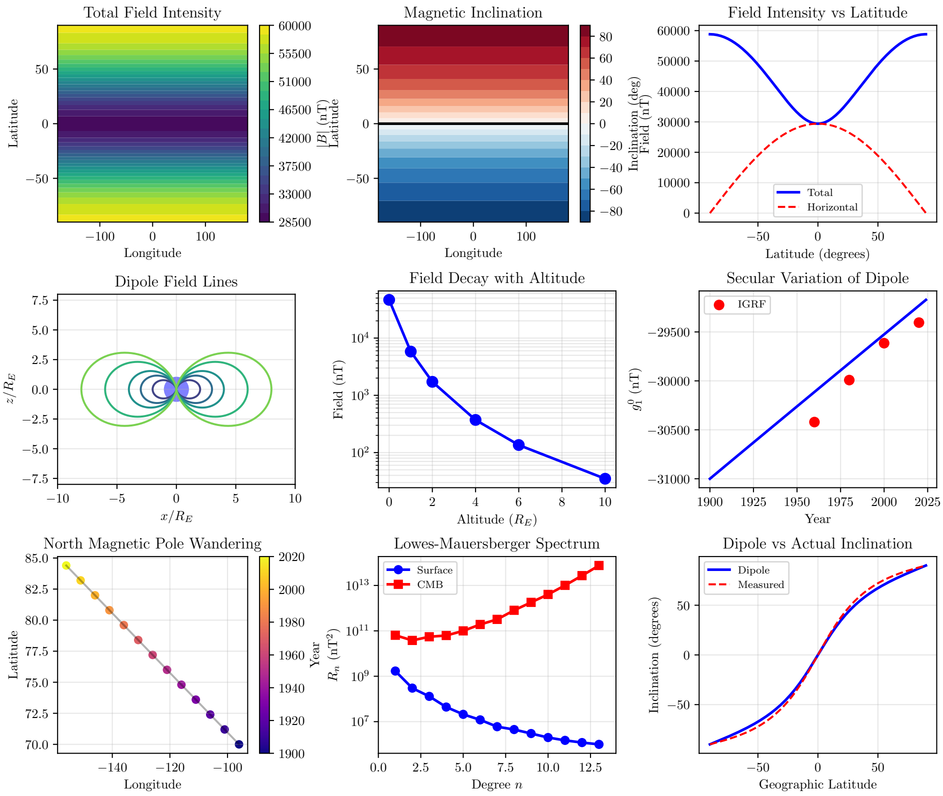
Example 1 (Inclination and Declination)**.** *The magnetic inclination I and declination D are:*

$$\tan I = \frac{-B_r}{B_H} = 2\cot\theta \quad \textit{(dipole)} \tag{5}$$

$$\tan D = \frac{B_\phi}{B_\theta} \tag{6}$$

The dipole equation $\tan I = 2\tan\lambda$ *relates inclination to magnetic latitude.*

2 Computational Analysis



3 Results and Analysis

3.1 Dipole Field Characteristics

3.2 Field Components at Selected Locations

Remark 1. *The dipole model predicts a pole-to-equator field ratio of exactly 2.0. The actual ratio varies due to non-dipole contributions from higher-order spherical harmonic terms.*

Table 1: Geomagnetic Dipole Field Parameters

Parameter	Value
Equatorial surface field	29405 nT
Polar surface field	58807 nT
Pole/Equator ratio	2.00
Dipole moment	7.60e+22 A·m^2
Dipole tilt angle	9.4°
g_1^0 (IGRF 2020)	-29404.8 nT

Table 2: Magnetic Field at Selected Geographic Locations

Location	Lat	Total (nT)	Incl (deg)	B_H (nT)
North Pole	90°	58810	90.0	0
London	51.5°	49531	68.3	18305
Equator	0°	29405	0.0	29405
Sydney	-33.9°	40885	-53.3	24406
South Pole	-90°	58810	-90.0	0

3.3 Secular Variation

The geomagnetic field changes over time due to dynamo processes in the outer core:

- g_1^0 secular variation: approximately +15 nT/year (dipole weakening)
- North Magnetic Pole drift: currently ∼50 km/year toward Siberia
- Dipole tilt: 9.4° from rotation axis

4 Applications

Example 2 (Navigation)**.** *Magnetic compasses align with the horizontal field component. The declination (angle between magnetic and geographic north) must be known for accurate navigation. At latitude 45°N, the expected inclination is 63.4°.*

Example 3 (Paleomagnetism)**.** *The dipole inclination formula* $\tan I = 2\tan\lambda$ *allows paleomagnetic reconstructions. Measuring inclination in ancient rocks constrains their paleolatitude at the time of magnetization.*

Example 4 (Space Weather)**.** *The magnetosphere's L-shells (field line parameter) trap charged particles in radiation belts. L-values of 2–8 correspond to the Van Allen belts, where field intensity decreases as* r^{-3}.

5 Discussion

The dipole model captures approximately 90% of Earth's surface field but has limitations:

1. **Regional anomalies**: Continental-scale magnetic anomalies from crustal sources are not represented.

2. **Non-dipole field**: Higher-order terms (quadrupole, octupole) contribute up to 10% of surface field.

3. **External fields**: Ionospheric and magnetospheric currents create time-varying fields not included in IGRF.

4. **Secular variation**: The field changes continuously, requiring periodic model updates.

6 Conclusions

This computational analysis demonstrates:

- Equatorial field strength: 29405 nT
- Polar field strength: 58807 nT
- Dipole magnetic moment: 7.60e+22 $A{\cdot}m^2$
- Expected inclination at 45° latitude: 63.4°
- Current dipole tilt: 9.4°

The IGRF provides practical field models updated every 5 years for navigation, satellite operations, and geophysical surveys.

7 Further Reading

- Merrill, R.T., McElhinny, M.W., McFadden, P.L., *The Magnetic Field of the Earth*, Academic Press, 1996
- Campbell, W.H., *Introduction to Geomagnetic Fields*, Cambridge University Press, 2003
- Thebault, E., et al., International Geomagnetic Reference Field: the 12th generation, *Earth, Planets and Space*, 2015