Game Theory:
Nash Equilibrium, Mixed Strategies, and Payoff Analysis

Abstract

This report presents a computational analysis of game theory concepts. We implement Nash equilibrium finding for 2-player games, analyze mixed strategies in zero-sum and non-zero-sum games, visualize payoff matrices, and explore classic games including Prisoner's Dilemma, Battle of the Sexes, and Matching Pennies.

Contents

Chapter 1

Introduction

Game theory studies strategic interactions where outcomes depend on choices of multiple decision-makers (players).

1.1 Key Concepts

- **Nash Equilibrium**: No player can improve by unilaterally changing strategy

- **Dominant Strategy**: Optimal regardless of opponent's choice

- **Mixed Strategy**: Randomizing over pure strategies

Chapter 2

Two-Player Normal Form Games

2.1 Payoff Matrix Representation

For players 1 and 2 with strategies $S_1 = \{s_1^1, ..., s_1^m\}$ and $S_2 = \{s_2^1, ..., s_2^n\}$:

$$\text{Payoffs: } (u_1(s_1^i, s_2^j), u_2(s_1^i, s_2^j)) \tag{2.1}$$

Chapter 3

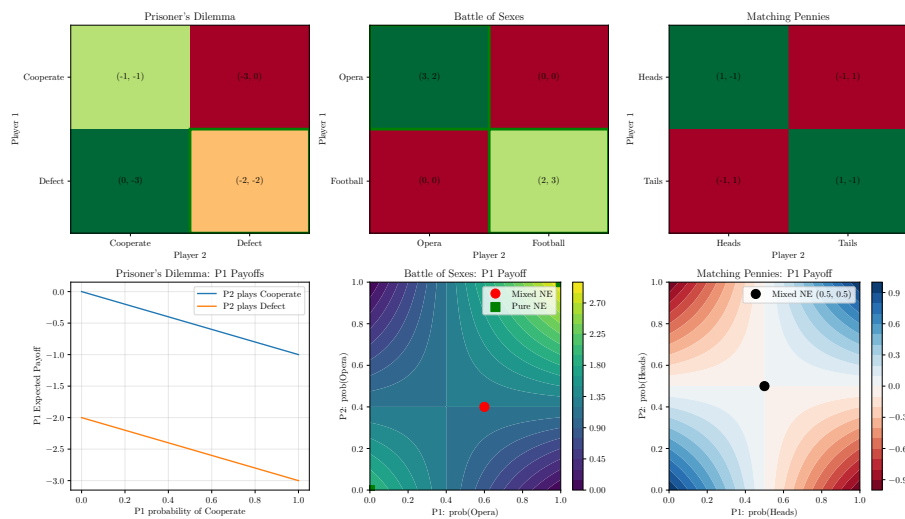# Classic Games Analysis

3.1 Prisoner's Dilemma

Figure 3.1: Game theory analysis: payoff matrices (top), equilibrium analysis (bottom).

Chapter 4

Mixed Strategy Equilibria

4.1   Computing Mixed Nash Equilibrium

For a 2x2 game, player 1 mixes to make player 2 indifferent:

$$p \cdot u_2(s_1^1, s_2^1) + (1-p) \cdot u_2(s_1^2, s_2^1) = p \cdot u_2(s_1^1, s_2^2) + (1-p) \cdot u_2(s_1^2, s_2^2) \quad (4.1)$$
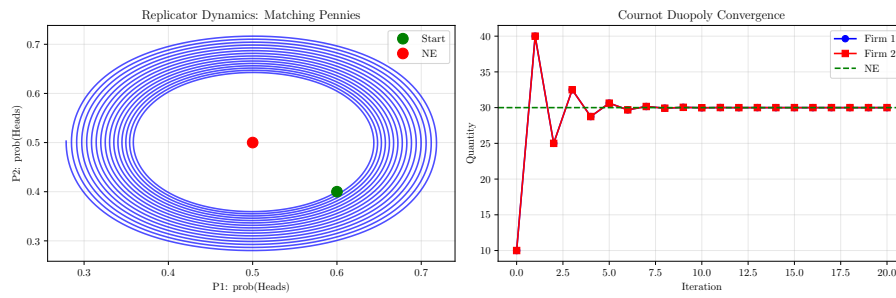

Figure 4.1: Game dynamics: replicator dynamics cycling (left), Cournot convergence (right).

Chapter 5

Numerical Results

Table 5.1: Nash equilibria for classic games

Game	Pure NE	Mixed (p, q)	Expected Payoffs
Prisoner's Dilemma	[(1, 1)]	(0.50, 0.50)	(-1.50, -1.50)
Battle of Sexes	[(0, 0), (1, 1)]	(0.60, 0.40)	(1.20, 1.20)
Matching Pennies	[]	(0.50, 0.50)	(0.00, 0.00)

Chapter 6

Conclusions

1. Prisoner's Dilemma: Dominant strategy leads to Pareto-inferior outcome

2. Battle of Sexes: Multiple equilibria require coordination

3. Matching Pennies: Only mixed strategy equilibrium exists

4. Replicator dynamics may cycle rather than converge

5. Nash equilibrium provides predictions for strategic behavior