Epidemiological Modeling: SIR Dynamics
Parameter Sensitivity, Interventions, and Real-World Context

Computational Epidemiology Group
Computational Science Templates

November 24, 2025

Abstract

This report presents a comprehensive analysis of the SIR (Susceptible-Infected-Recovered) compartmental model for infectious disease dynamics. We examine the mathematical foundations, perform parameter sensitivity analysis, evaluate intervention strategies including vaccination and social distancing, and compare model predictions with historical outbreak data. The analysis demonstrates how simple mathematical models can inform public health policy.

1 Introduction

Compartmental models partition populations into discrete states based on disease status. The SIR model, introduced by Kermack and McKendrick (1927), remains a foundational tool in epidemiology.

Definition 1 (SIR Compartments)

- ***S*** *(Susceptible): Individuals who can contract the disease*
- ***I*** *(Infected): Individuals who have the disease and can transmit it*
- ***R*** *(Recovered): Individuals who have recovered and are immune*

2 Mathematical Framework

2.1 Model Equations

The SIR model is governed by three coupled ODEs:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

where β is the transmission rate and γ is the recovery rate.

2.2 Basic Reproduction Number

The basic reproduction number R_0 determines outbreak potential:

$$R_0 = \frac{\beta}{\gamma} \tag{4}$$

Remark 1 (Epidemic Threshold) *An epidemic occurs when $R_0 > 1$. The herd immunity threshold is $1 - 1/R_0$.*

2.3 Final Size Relation

The final epidemic size R_∞ satisfies:

$$R_\infty = 1 - S_0 \exp(-R_0 R_\infty) \tag{5}$$

3 Computational Analysis

4 Results

4.1 Baseline Epidemic Characteristics

4.2 Intervention Effectiveness

4.3 Key Findings

1. **Peak Reduction**:
    - Social distancing (50% reduction in β): **??**% peak reduction
    - Vaccination (60% coverage): **??**% peak reduction
    - Combined strategy: **??**% peak reduction
2. **Timing**: Higher R_0 leads to faster epidemics. Time to peak decreases from over 100 days for $R_0 = 1.5$ to under 30 days for $R_0 = 5$.
3. **Herd Immunity**: With $R_0 =$ **??**, **??**% immunity is needed to prevent outbreaks.

5 Model Limitations

- **Homogeneous mixing**: Real populations have structured contacts
- **Constant parameters**: β and γ may vary with behavior, seasons, mutations
- **No vital dynamics**: Does not include births, deaths, or waning immunity
- **Deterministic**: Does not capture stochastic extinction for small populations

6 Conclusion

The SIR model provides fundamental insights into epidemic dynamics:

1. R_0 determines outbreak severity and intervention needs
2. Combined interventions (vaccination + behavioral) are most effective
3. Timing of interventions critically affects outcomes
4. Models inform policy but require validation against data

Further Reading

- Keeling, M. J., & Rohani, P. (2008). *Modeling Infectious Diseases.* Princeton.
- Anderson, R. M., & May, R. M. (1991). *Infectious Diseases of Humans.* Oxford.