Ocean Wave Dynamics: Dispersion, Spectra, and Coastal Processes

Physical Oceanography Templates

November 24, 2025

1 Introduction

Ocean surface waves transport energy across vast distances and undergo transformations as they approach coastlines. This template covers wave dispersion relations, spectral representations, shoaling, and refraction processes.

2 Mathematical Framework

2.1 Linear Wave Theory

The surface elevation for a monochromatic wave:

$$\eta(x,t) = a\cos(kx - \omega t) \tag{1}$$

2.2 Dispersion Relation

Deep water waves satisfy:

$$\omega^2 = gk\tanh(kh) \tag{2}$$

In deep water ($kh \gg 1$): $\omega^2 = gk$, giving phase speed $c = \sqrt{g/k}$.

In shallow water ($kh \ll 1$): $\omega^2 = gk^2h$, giving $c = \sqrt{gh}$.

2.3 Group Velocity

Energy propagates at the group velocity:

$$c_g = \frac{\partial\omega}{\partial k} = \frac{c}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right) \tag{3}$$

2.4 Pierson-Moskowitz Spectrum

Fully developed sea spectrum:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\beta \left(\frac{\omega_0}{\omega}\right)^4\right] \tag{4}$$

where $\alpha = 8.1 \times 10^{-3}$, $\beta = 0.74$, and $\omega_0 = g/U_{19.5}$.

2.5 JONSWAP Spectrum

Fetch-limited spectrum with peak enhancement:

$$S_J(\omega) = S_{PM}(\omega) \cdot \gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]} \tag{5}$$

2.6 Shoaling and Refraction

Wave height transformation during shoaling:

$$\frac{H}{H_0} = \sqrt{\frac{c_{g0}}{c_g}} = K_s \tag{6}$$

Snell's law for wave refraction:

$$\frac{\sin\theta}{c} = \frac{\sin\theta_0}{c_0} \tag{7}$$

3 Environment Setup

4 Dispersion Relation Analysis

5 Wave Spectra

6 Random Sea Surface Simulation

7 Wave Shoaling and Refraction

8 Breaking Wave Criterion

9 Results Summary

9.1 Wave Properties

9.2 Spectral Parameters

9.3 Coastal Transformation

9.4 Physical Summary

- Deep water wavelength (T=10s): 156.1 m

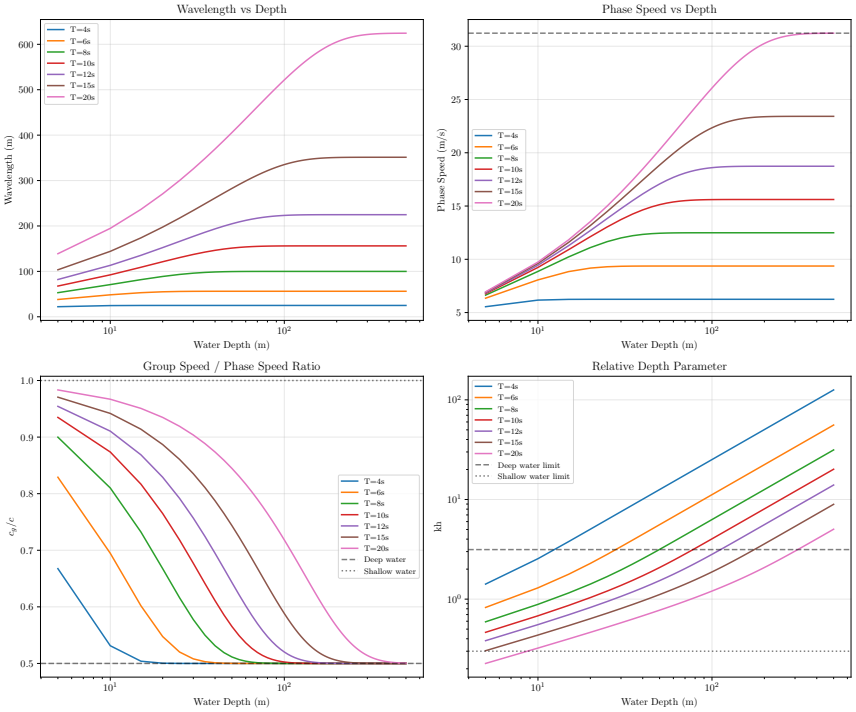

Figure 1: Wave dispersion characteristics

Table 1: Wave characteristics for T=10s

Property	Deep Water	Shallow (10m)
Wavelength (m)	156.1	92.4
Phase speed (m/s)	15.6	9.2
Group speed (m/s)	7.8	8.1

Table 2: Wave spectrum characteristics

Parameter	Value
Wind speed	15 m/s
Significant wave height	10.95 m
Peak period	15.3 s
Maximum fetch	1000 km

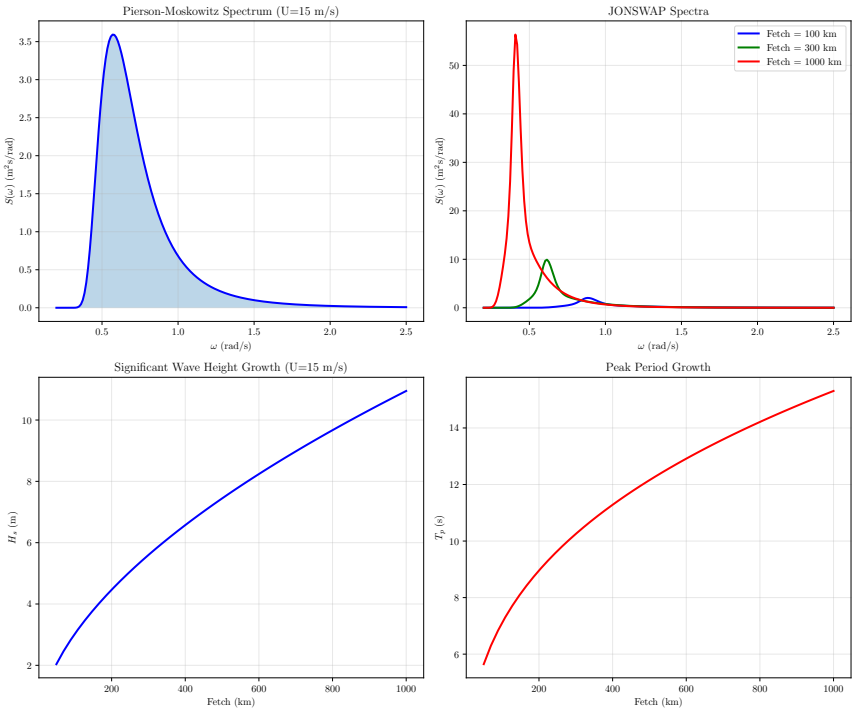

Figure 2: Ocean wave spectra and wave growth

Table 3: Wave transformation during shoaling

Parameter	Value
Deep water height	2.0 m
Maximum shoaled height	2.72 m
Maximum shoaling coefficient	1.36
Refraction angle (deep)	30°

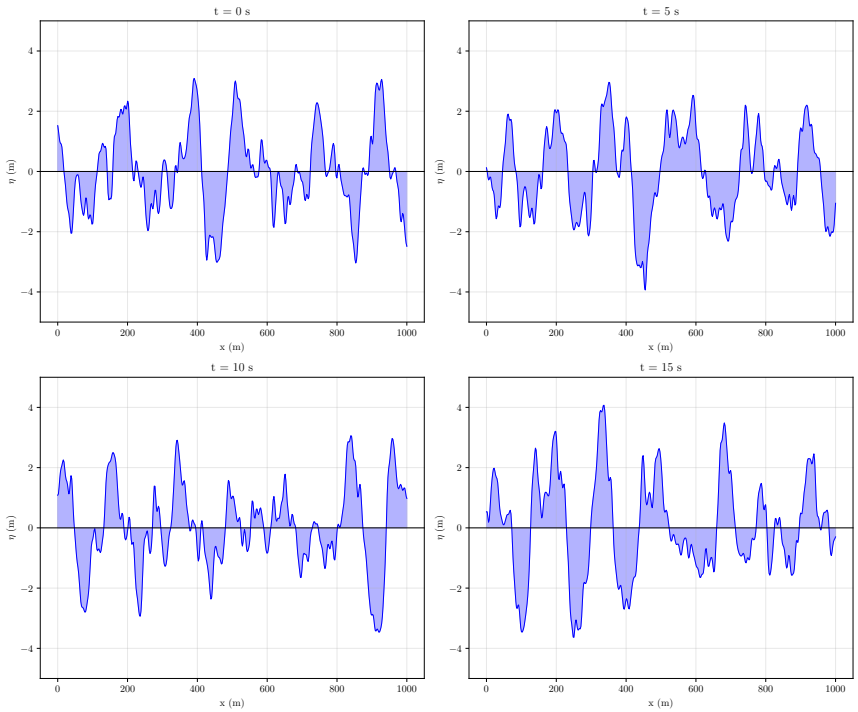

Figure 3: Random sea surface time evolution

- JONSWAP H_s: 10.95 m
- Maximum shoaling coefficient: 1.36
- Approach angle: 30 degrees

10 Conclusion

This template demonstrates fundamental ocean wave dynamics. The dispersion relation governs how wave properties vary with depth, transitioning from deep to shallow water behavior. Spectral representations (Pierson-Moskowitz, JONSWAP) describe the energy distribution in random seas and wave growth with fetch. Shoaling causes wave height to increase as waves approach shore, while refraction bends wave crests to align with depth contours. These processes are essential for coastal engineering and understanding surf zone dynamics.

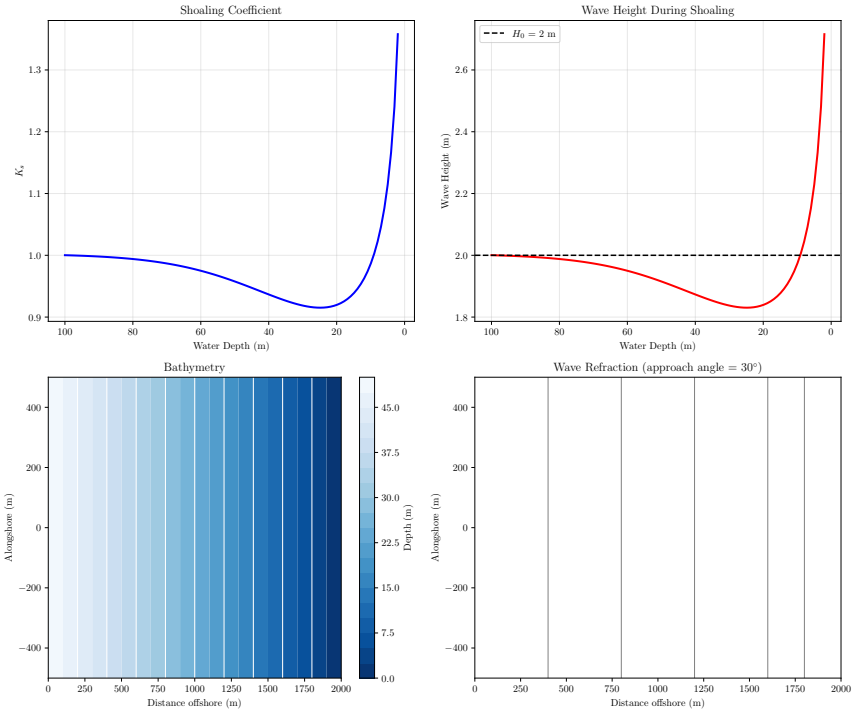

Figure 4: Wave shoaling and refraction

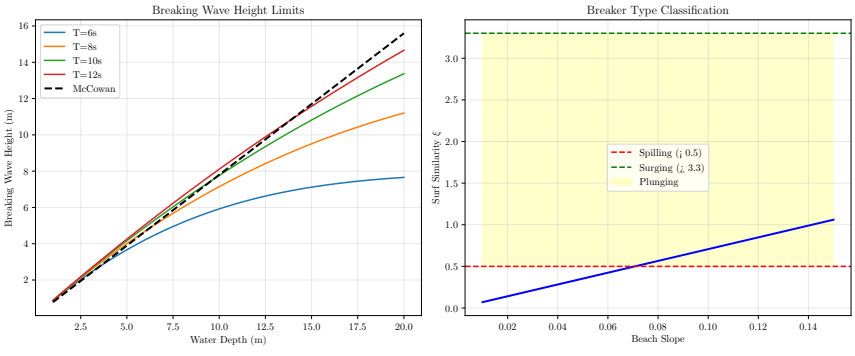

Figure 5: Wave breaking criteria and breaker types