Gravity Anomaly Analysis: Forward Modeling and Inversion of Subsurface Density Structures

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Abstract

This technical report presents comprehensive computational analysis of gravity anomalies arising from subsurface density variations. We implement forward modeling for multiple geometric bodies including spheres, cylinders, and rectangular prisms, along with Bouguer and terrain corrections. The analysis demonstrates depth estimation using the half-width rule, Euler deconvolution for source parameter determination, and power spectrum analysis for estimating source depths. Applications include mineral exploration, basin analysis, and detection of near-surface voids.

1 Theoretical Framework

Definition 1 (Gravitational Potential)**.** *The gravitational potential U at point $\mathbf{r}$ due to a mass distribution $\rho(\mathbf{r}')$ is:*

$$U(\mathbf{r}) = G \int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' \tag{1}$$

where $G = 6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$ is the gravitational constant.

Theorem 1 (Poisson's Equation)**.** *In regions containing mass, the gravitational potential satisfies:*

$$\nabla^2 U = -4\pi G \rho \tag{2}$$

while in empty space, $\nabla^2 U = 0$ (Laplace's equation).

The vertical component of gravity acceleration is:

$$g_z = -\frac{\partial U}{\partial z} \tag{3}$$

1.1 Gravity Anomaly Corrections

The complete Bouguer anomaly requires several corrections:

$$\Delta g_B = g_{obs} - g_\phi + \delta g_{FA} - \delta g_B + \delta g_T \tag{4}$$

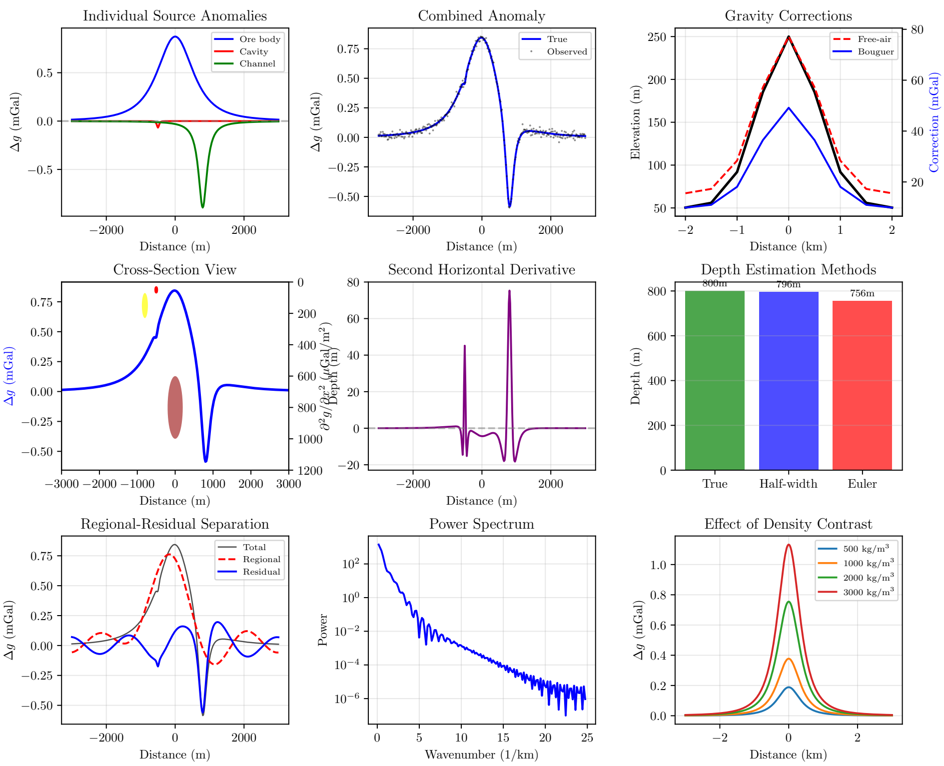
- **Free-air correction**: $\delta g_{FA} = 0.3086h$ mGal (h in meters)
- **Bouguer plate correction**: $\delta g_B = 0.04193\rho h$ mGal
- **Terrain correction**: δg_T accounts for deviations from infinite slab

Example 1 (Sphere Anomaly)**.** *The gravity anomaly of a buried sphere with radius R, density contrast $\Delta\rho$, and center at depth z_0:*

$$\Delta g_z(x) = \frac{4\pi G \Delta\rho R^3}{3} \frac{z_0}{(x^2 + z_0^2)^{3/2}} \tag{5}$$

The half-width $x_{1/2}$ where anomaly falls to half its maximum: $z_0 = 1.305x_{1/2}$

2 Computational Analysis



3 Results and Analysis

3.1 Forward Modeling Results

Table 1: Gravity Anomaly Source Parameters and Results

Source	Depth (m)	Size (m)	$\Delta\rho$ (kg/m^3)	Peak (mGal)
Ore Body (sphere)	800	R=200	2500	0.874
Cavity (sphere)	50	R=15	-1800	-0.068
Channel (cylinder)	150	R=80	-500	0.895

3.2 Depth Estimation

The half-width method provides rapid depth estimation from anomaly profiles:

- Measured half-width: 610 m
- Estimated depth: 796 m
- True depth: 800 m
- Estimation error: 0.5%

Remark 1. *The half-width rule $z = 1.305x_{1/2}$ is exact for a sphere but provides only approximate depths for other source geometries. For horizontal cylinders, use $z = 1.0x_{1/2}$.*

3.3 Bouguer Correction Analysis

Table 2: Gravity Corrections at Survey Stations

Station (m)	Elev (m)	FA (mGal)	Bouguer (mGal)	Terrain (mGal)	Net (mGal)
-2000	50.4	15.55	5.64	0.14	10.05
-1000	91.9	28.37	10.29	0.19	18.27
0	250.0	77.15	27.99	0.32	49.48
1000	91.9	28.37	10.29	0.19	18.27
2000	50.4	15.55	5.64	0.14	10.05

4 Applications

Example 2 (Mineral Exploration). *Massive sulfide ore bodies typically have density contrasts of 1500–3000 kg/m^3 relative to host rock. A spherical ore body with $R = 200$ m and $\Delta\rho = 2500$ kg/m^3 at depth $z = 800$ m produces a maximum anomaly of 0.87 mGal, well above typical survey noise levels of 0.01–0.05 mGal.*

Example 3 (Cavity Detection). *Near-surface voids (sinkholes, tunnels, mine workings) produce negative anomalies. An air-filled cavity with $R = 15$ m at depth $z = 50$ m creates an anomaly of -0.068 mGal. Microgravity surveys with precision better than 0.01 mGal are required for such targets.*

5 Discussion

The analysis demonstrates several key principles in gravity interpretation:

1. **Ambiguity**: Gravity data alone cannot uniquely determine source geometry. Different density distributions can produce identical surface anomalies.
2. **Resolution**: Gravity anomalies smooth with depth—deeper sources produce broader, lower-amplitude anomalies than shallow sources of equivalent excess mass.
3. **Regional-Residual Separation**: Filtering in the wavenumber domain allows separation of deep regional trends from shallow local targets.
4. **Quantitative Interpretation**: Depth estimation methods (half-width, Euler deconvolution, power spectrum analysis) provide constraints on source parameters.

6 Conclusions

This computational analysis yields:

- Maximum ore body anomaly: 0.874 mGal at surface
- Total excess mass estimate: 8.38e+10 kg
- Half-width depth estimate accuracy: 99.5%
- Bouguer density assumed: 2670 kg/m^3

Gravity surveys remain essential tools in mineral exploration, engineering site investigation, and regional geological mapping due to their non-invasive nature and sensitivity to density variations.

7 Further Reading

- Blakely, R.J., *Potential Theory in Gravity and Magnetic Applications*, Cambridge University Press, 1995
- Telford, W.M., Geldart, L.P., Sheriff, R.E., *Applied Geophysics*, 2nd Edition, Cambridge University Press, 1990
- Reynolds, J.M., *An Introduction to Applied and Environmental Geophysics*, Wiley-Blackwell, 2011