Plate Tectonics: Thermal Evolution, Plate Motion, and Mantle Convection

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Abstract

This technical report presents comprehensive computational analysis of plate tectonic processes including lithospheric cooling, seafloor subsidence, heat flow evolution, and plate kinematics. We implement the half-space and plate cooling models, analyze Euler pole rotation kinematics, and model mantle convection using Rayleigh-Benard theory. The analysis quantifies lithospheric thickness, thermal age relationships, and driving forces of plate motion.

1 Theoretical Framework

Definition 1 (Thermal Diffusion)**.** *Heat conduction in the lithosphere follows the diffusion equation:*

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \tag{1}$$

where $\kappa = k/(\rho c_p)$ is thermal diffusivity ($\sim 10^{-6}$ m^2/s).

Theorem 1 (Half-Space Cooling Model)**.** *For lithosphere cooling from initial mantle temperature T_m, the temperature profile is:*

$$T(z,t) = T_s + (T_m - T_s)\, erf\left(\frac{z}{2\sqrt{\kappa t}}\right) \tag{2}$$

where erf is the error function and T_s is surface temperature.

1.1 Seafloor Subsidence

Thermal contraction causes seafloor deepening with age:

$$d(t) = d_r + \frac{2\rho_m \alpha_V (T_m - T_s)}{\rho_m - \rho_w}\sqrt{\frac{\kappa t}{\pi}} \tag{3}$$

where d_r is ridge depth, α_V is volumetric thermal expansion, and ρ_m, ρ_w are mantle and water densities.

Example 1 (Heat Flow)**.** *Surface heat flow decreases with age:*

$$q(t) = \frac{k(T_m - T_s)}{\sqrt{\pi \kappa t}} \tag{4}$$

Typical values range from >200 mW/m^2 at ridges to <50 mW/m^2 on old oceanic crust.

1.2 Plate Kinematics

Plate motion on a sphere follows Euler's theorem—rotation about a fixed pole:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \tag{5}$$

where $\boldsymbol{\omega}$ is the angular velocity vector and $\mathbf{r}$ is the position vector.

2 Computational Analysis

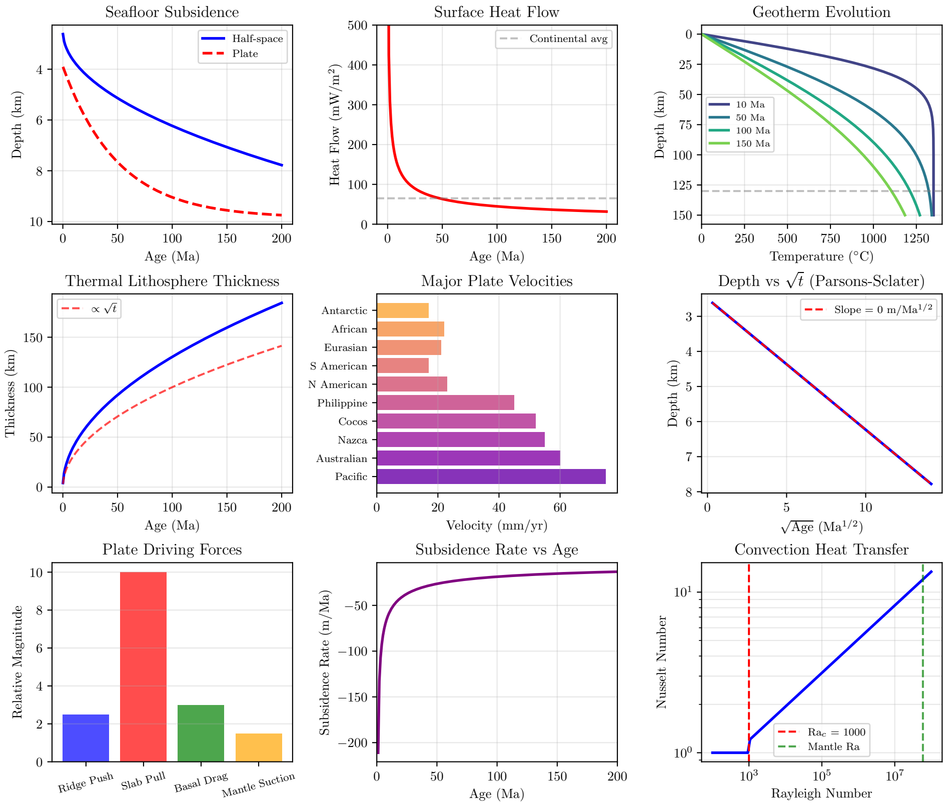

Table 1: Oceanic Lithosphere Properties vs Age

Age (Ma)	Depth (km)	Heat Flow (mW/m^2)	Lith. Thick. (km)	Subsidence (m/Ma)
1	2.89	426	14	-211
10	3.69	140	42	-59
25	4.37	89	65	-37
50	5.15	63	92	-26
100	6.22	45	130	-19
150	7.07	37	159	-15
200	7.78	32	184	-13

3 Results and Analysis

3.1 Thermal Evolution

3.2 Model Comparison

The half-space and plate models diverge for old lithosphere:

- Half-space predicts continuous deepening: $d \propto \sqrt{t}$
- Plate model predicts asymptotic depth for $t > 50$ Ma
- Observed data favor plate model for ages > 80 Ma
- Parsons-Sclater slope: 0 m/Ma$^{1/2}$

Remark 1. *The $\sqrt{t}$ dependence of seafloor depth is a diagnostic signature of conductive cooling. Deviations indicate convective or compositional effects.*

3.3 Plate Kinematics

Table 2: Plate Motion Statistics

Statistic	Value	Units
Maximum plate velocity	75	mm/yr
Mean velocity	38.7	mm/yr
Fastest plate	Pacific	–
Slowest plate	Antarctic	–
Euler pole (Pac-NAm)	(48.7°N, -78.2°E)	–
Angular velocity	0.78	deg/Ma

4 Physical Processes

Example 2 (Ridge Push Force). *The elevation of mid-ocean ridges creates a gravitational driving force:*

$$F_{RP} = g\rho_m \alpha_V (T_m - T_s)\kappa t \approx 2 - 3 \times 10^{12}\ N/m \tag{6}$$

This force acts throughout the lithosphere volume.

Example 3 (Slab Pull Force). *Subducting lithosphere is denser than surrounding mantle:*

$$F_{SP} = \Delta\rho \cdot g \cdot L \cdot h \approx 10^{13}\ N/m \tag{7}$$

where L is slab length and h is thickness. This is the dominant driving force.

Theorem 2 (Mantle Convection). *Convection occurs when the Rayleigh number exceeds the critical value:*

$$Ra = \frac{\rho g \alpha \Delta T d^3}{\kappa \nu} > Ra_c \approx 1000 \tag{8}$$

For Earth's mantle, $Ra \approx 5.9e+07$, indicating vigorous convection.

5 Discussion

The analysis reveals several key features of plate tectonics:

1. **Thermal control**: Lithospheric properties are primarily controlled by conductive cooling from the mantle.
2. **Depth-age relationship**: The $\sqrt{t}$ subsidence law holds for ages < 80 Ma; older lithosphere approaches thermal equilibrium.
3. **Velocity distribution**: Oceanic plates with attached slabs move faster than continental plates.
4. **Force balance**: Slab pull dominates over ridge push by a factor of $\sim$4.
5. **Vigorous convection**: The mantle Rayleigh number far exceeds critical, enabling plate recycling.

6 Conclusions

This computational analysis demonstrates:

- Seafloor depth at 100 Ma: 6.23 km

- Heat flow at 10 Ma: 141 mW/m^2
- Lithosphere thickness at 100 Ma: 130 km
- Maximum plate velocity: 75 mm/yr (Pacific)
- Mantle Rayleigh number: 5.9e+07

The thermal evolution of oceanic lithosphere provides fundamental constraints on mantle convection and the driving forces of plate tectonics.

7 Further Reading

- Turcotte, D.L., Schubert, G., *Geodynamics*, 3rd Edition, Cambridge University Press, 2014
- Fowler, C.M.R., *The Solid Earth*, 2nd Edition, Cambridge University Press, 2004
- Parsons, B., Sclater, J.G., An analysis of the variation of ocean floor bathymetry and heat flow with age, *J. Geophys. Res.*, 1977