Stellar Evolution: From Main Sequence to Stellar Remnants

A Comprehensive Analysis of the HR Diagram and Nuclear Burning Stages

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Abstract

This comprehensive analysis explores stellar structure and evolution through the Hertzsprung-Russell diagram. We derive the fundamental stellar relations including the mass-luminosity relation, main sequence lifetime, and Stefan-Boltzmann law. The analysis covers all major evolutionary phases from pre-main sequence contraction through hydrogen and helium burning to white dwarf, neutron star, and black hole endpoints. We generate synthetic stellar populations to visualize the main sequence, red giant branch, horizontal branch, and white dwarf cooling sequence, and explore the physics governing each evolutionary stage.

1 Introduction

The Hertzsprung-Russell (HR) diagram, independently developed by Ejnar Hertzsprung and Henry Norris Russell in the early 1900s, provides a powerful tool for understanding stellar populations and evolution. By plotting stellar luminosity against effective temperature (or equivalently, color or spectral type), the HR diagram reveals the fundamental relationships governing stellar structure.

Definition 1 (Hertzsprung-Russell Diagram) *A scatter plot of stellar luminosity L versus effective temperature T_{eff} (with temperature decreasing to the right), showing distinct regions occupied by different stellar types and evolutionary stages.*

2 Theoretical Framework

2.1 Stellar Luminosity

The luminosity of a star is determined by its radius and effective temperature:

Theorem 1 (Stefan-Boltzmann Law)

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}$ $W\ m^{-2}\ K^{-4}$ *is the Stefan-Boltzmann constant.*

2.2 Mass-Luminosity Relation

For main sequence stars, luminosity scales strongly with mass:

Theorem 2 (Mass-Luminosity Relation)

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha \tag{2}$$

where $\alpha \approx 3.5$ *for stars with* $0.5 < M/M_\odot < 20$, *with variations at the extremes.*

Remark 1 (Mass-Luminosity Variations)

- *Low mass* ($M < 0.5M_\odot$)*:* $\alpha \approx 2.3$
- *Intermediate mass:* $\alpha \approx 4$
- *High mass* ($M > 20M_\odot$)*:* $\alpha \approx 1$ *(Eddington limit)*

2.3 Main Sequence Lifetime

Nuclear burning timescale depends on fuel supply and consumption rate:

$$\tau_{\text{MS}} \approx \frac{M}{L} \propto M^{1-\alpha} \approx \frac{10^{10}}{(M/M_\odot)^{2.5}} \text{ years} \tag{3}$$

2.4 Stellar Spectral Classification

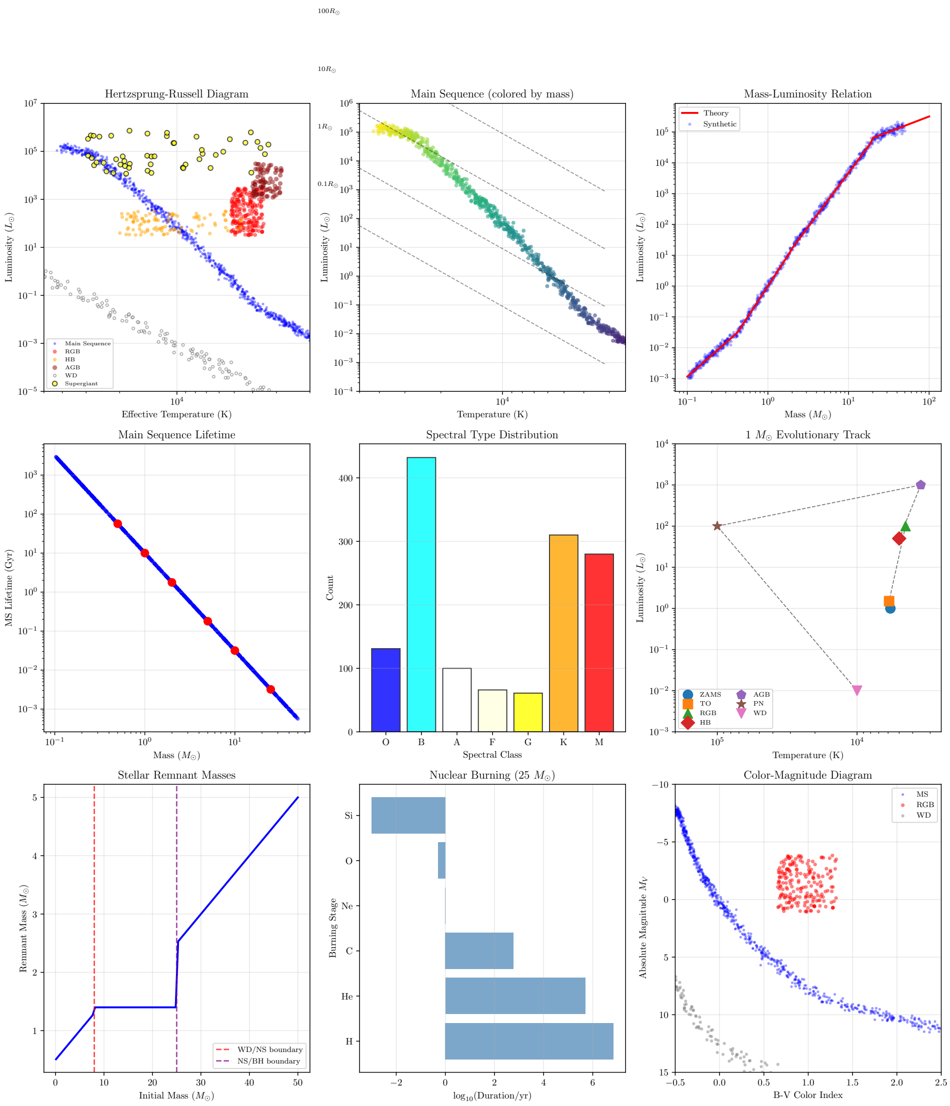
Definition 2 (Spectral Types) *The Harvard classification scheme orders stars by temperature:*

O - B - A - F - G - K - M

("Oh Be A Fine Girl/Guy Kiss Me")

Temperature ranges: O (>30,000 K), B (10,000-30,000 K), A (7,500-10,000 K), F (6,000-7,500 K), G (5,200-6,000 K), K (3,700-5,200 K), M (<3,700 K)

3 Computational Analysis



4 Results and Analysis

4.1 Stellar Population Statistics

4.2 Main Sequence Properties

Example 1 (Solar Evolution) *The Sun is a G2V main sequence star with the following properties:*

- *Mass:* $M = 1M_\odot$
- *Luminosity:* $L = 1L_\odot =$ *3.828e+26 W*

Table 1: Synthetic Stellar Population Statistics

Evolutionary Stage	Count	Fraction (
Main Sequence	800	58.0
Red Giant Branch	200	14.5
Horizontal Branch	100	7.2
AGB	80	5.8
White Dwarf	150	10.9
Supergiant	50	3.6
Total	1380	100

Table 2: Main Sequence Lifetimes

Mass (M odot) odot)	Luminosity (L Temperature (K)	Lifetime (Gyr)	
0.5	0.1	4086	56.57
1.0	1.0	5778	10.00
2.0	16.0	8171	1.77
5.0	419.0	12920	0.18
10.0	4743.0	18272	0.03
25.0	80000.0	28890	0.00

- *Effective temperature: $T_{eff} = 5778$ K*
- *Main sequence lifetime: $\tau_{MS} \approx 10$ Gyr*
- *Current age: 4.6 Gyr (middle of MS phase)*

5 Evolutionary Phases

5.1 Pre-Main Sequence

Stars form from collapsing molecular clouds and contract along Hayashi tracks (nearly vertical in HR diagram) before reaching the main sequence.

5.2 Main Sequence

Core hydrogen burning via pp-chain (low mass) or CNO cycle (high mass). Stars spend ~90% of their lives on the main sequence.

5.3 Red Giant Branch

After core hydrogen exhaustion:

1. Hydrogen shell burning begins
2. Core contracts, envelope expands
3. Surface cools to ∼4000 K
4. Luminosity increases to ∼100-1000 $L_\odot$

5.4 Helium Burning

Remark 2 (Helium Flash) *In low-mass stars ($M < 2M_\odot$), helium ignition is degenerate and occurs explosively (helium flash), though the energy is absorbed internally.*

Stars on the Horizontal Branch burn helium in the core and hydrogen in a shell.

5.5 Asymptotic Giant Branch

Double-shell burning (H and He) produces thermal pulses and significant mass loss.

6 Stellar Endpoints

6.1 White Dwarfs

Final state for $M < 8M_\odot$:

- Supported by electron degeneracy pressure
- Mass limit: Chandrasekhar mass $M_{Ch} = 1.4M_\odot$
- Cooling timescale: ∼10 Gyr

6.2 Neutron Stars

Core-collapse remnants for $8 < M/M_\odot < 25$:

- Supported by neutron degeneracy pressure
- Typical mass: $1.4M_\odot$
- Radius: ∼10 km

6.3 Black Holes

For $M > 25M_\odot$, no known physics can halt collapse.

7 Nuclear Burning Stages

Table 3: Nuclear Burning in Massive Stars (25 M
odot)

Stage	Temperature (K)	Duration	Products
H rightarrowHe times 10^7	1.5 7 Myr	He	
He rightarrowC, O times 10^8	1 500 kyr	C, O	
C rightarrowNe, Mg times 10^8	5 600 yr	Ne, Mg	
Ne rightarrowO, Mg times 10^9	1.2 1 yr	O, Mg	
O rightarrowSi, S times 10^9	1.5 6 months	Si, S	
Si rightarrowFe times 10^9	2.7 1 day	Fe	

8 Limitations and Extensions

8.1 Model Limitations

1. **Single stars**: Binaries not included
2. **Solar metallicity**: Population II not modeled
3. **No rotation**: Rotating stars evolve differently
4. **Simplified tracks**: No detailed stellar models

8.2 Possible Extensions

- Binary evolution and mass transfer
- Metallicity effects on evolution
- Stellar rotation and magnetic fields
- Detailed nucleosynthesis yields

9 Conclusion

This analysis demonstrates the power of the HR diagram for stellar astrophysics:

- Main sequence stars follow the mass-luminosity relation
- MS lifetime: $\tau \propto M^{-2.5}$
- Red giants: cool but luminous ($L \sim 100L_{\odot}$, $T \sim 4000$ K)
- White dwarfs: hot but dim ($L \sim 0.01L_{\odot}$)
- Stellar endpoints depend on initial mass

Further Reading

- Kippenhahn, R., Weigert, A., & Weiss, A. (2012). *Stellar Structure and Evolution*. Springer.
- Prialnik, D. (2009). *An Introduction to the Theory of Stellar Structure and Evolution*. Cambridge University Press.
- Salaris, M. & Cassisi, S. (2005). *Evolution of Stars and Stellar Populations*. Wiley.