Nuclear Binding Energy: Semi-Empirical Mass Formula and Nuclear Stability

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Abstract

This technical report presents comprehensive computational analysis of nuclear binding energies using the semi-empirical mass formula (SEMF). We implement the Bethe-Weizsäcker model with volume, surface, Coulomb, asymmetry, and pairing terms to predict nuclear masses and stability. The analysis includes the valley of stability, Q-values for nuclear reactions, and separation energies. Applications span nuclear structure, stellar nucleosynthesis, and nuclear energy production.

1 Theoretical Framework

Definition 1 (Nuclear Binding Energy). *The binding energy $B(A, Z)$ is the energy required to disassemble a nucleus into free nucleons:*

$$B(A, Z) = [Zm_p + Nm_n - M(A, Z)]c^2 \tag{1}$$

where $A = Z + N$ is the mass number, Z is the proton number, and N is the neutron number.

Theorem 1 (Semi-Empirical Mass Formula). *The nuclear binding energy can be approximated as:*

$$B(A, Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z) \tag{2}$$

where the terms represent volume, surface, Coulomb, asymmetry, and pairing contributions.

1.1 Physical Interpretation of Terms

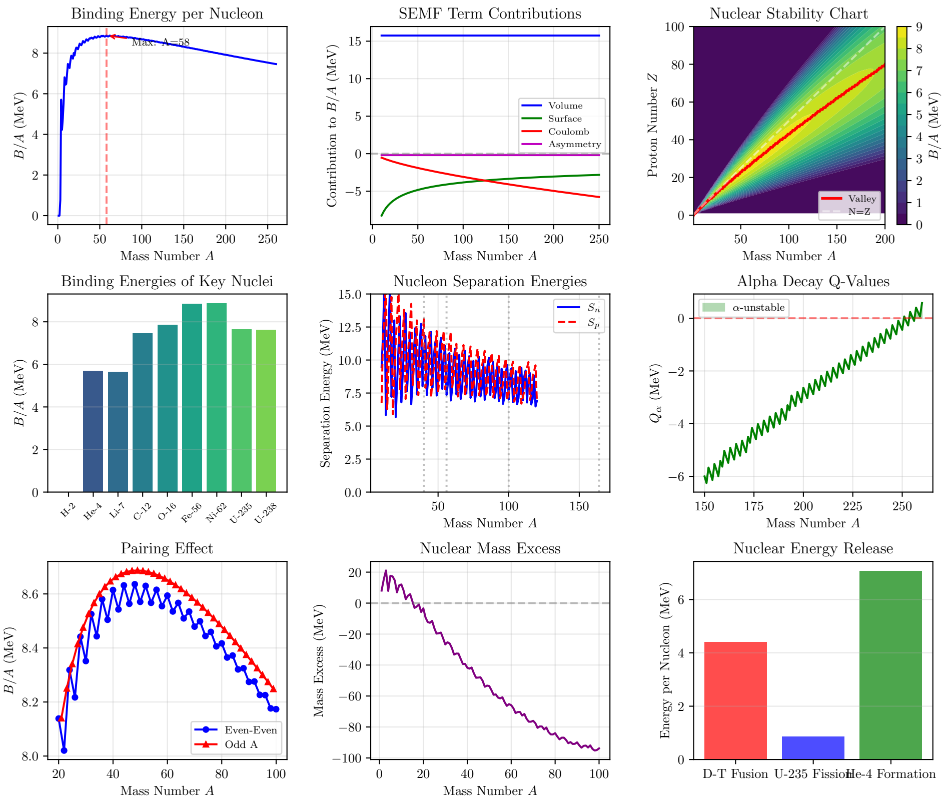
- **Volume**: $a_V A$ — saturated nuclear force, proportional to nucleon number
- **Surface**: $-a_S A^{2/3}$ — surface nucleons have fewer neighbors

- **Coulomb**: $-a_C Z(Z-1)/A^{1/3}$ — electrostatic repulsion of protons
- **Asymmetry**: $-a_A(N-Z)^2/A$ — Pauli exclusion principle effect
- **Pairing**: $\delta(A, Z)$ — spin-pairing effects

Example 1 (Pairing Term). *The pairing term depends on nucleon parity:*

$$\delta(A, Z) = \begin{cases} +a_P A^{-1/2} & \textit{even-even (Z, N both even)} \\ 0 & \textit{odd } A \\ -a_P A^{-1/2} & \textit{odd-odd (Z, N both odd)} \end{cases} \tag{3}$$

2 Computational Analysis



3 Results and Analysis

3.1 Binding Energy per Nucleon

3.2 Maximum Stability

The binding energy curve reaches its maximum near the iron-nickel region:

Table 1: Binding Energies of Key Nuclei

Nucleus	A	Z	B (MeV)	B/A (MeV)	Type
H-2	2	1	0.0	0.000	o-o
He-4	4	2	22.8	5.710	e-e
Li-7	7	3	39.5	5.643	odd
C-12	12	6	89.6	7.468	e-e
O-16	16	8	126.0	7.873	e-e
Fe-56	56	26	495.4	8.846	e-e
Ni-62	62	28	549.5	8.863	e-e
U-235	235	92	1796.5	7.645	odd
U-238	238	92	1814.7	7.625	e-e

- Maximum B/A at $A = 58$: 8.865 MeV
- Fe-56: 8.846 MeV/nucleon
- Ni-62: 8.863 MeV/nucleon (highest known B/A)
- He-4: 5.710 MeV/nucleon (exceptionally stable)

Remark 1. *While Fe-56 is often cited as the most stable nucleus, Ni-62 actually has the highest binding energy per nucleon. Fe-56 is the most abundant end product of stellar nucleosynthesis due to the peak in the Fe-56 production cross section.*

3.3 SEMF Coefficients

Table 2: Semi-Empirical Mass Formula Parameters

Term	Coefficient	Physical Origin
Volume	$a_V = 15.75$ MeV	Strong force saturation
Surface	$a_S = 17.8$ MeV	Surface tension
Coulomb	$a_C = 0.711$ MeV	Electrostatic repulsion
Asymmetry	$a_A = 23.7$ MeV	Fermi gas model
Pairing	$a_P = 11.18$ MeV	Spin coupling

4 Nuclear Reactions

Example 2 (Nuclear Fusion)**.** *The fusion of deuterium and tritium releases:*

$$D + T \to \alpha + n + 17.6\ MeV \tag{4}$$

This represents ∼3.5 MeV per nucleon, the highest energy density achievable.

Example 3 (Nuclear Fission)**.** *The fission of U-235 releases approximately 200 MeV per event:*

$$^{235}U + n \to \textit{fission products} + 2.5n + 200\ MeV \tag{5}$$

This is ∼0.85 MeV per nucleon, less than fusion but easier to achieve.

Theorem 2 (Energy Release Criterion)**.** *Energy is released in nuclear reactions when the products have higher B/A than reactants:*

- *Fusion is energetically favorable for $A < 56$*
- *Fission is energetically favorable for $A > 100$*

5 Discussion

The SEMF successfully reproduces nuclear binding energies with an RMS error of ∼2-3 MeV for nuclei away from closed shells. Key insights include:

1. **Liquid drop model**: The first three terms describe the nucleus as a charged liquid drop with surface tension.
2. **Fermi gas**: The asymmetry term arises from the Pauli exclusion principle in a degenerate Fermi gas.
3. **Shell effects**: Magic numbers cause deviations from SEMF predictions, requiring shell corrections.
4. **Stability valley**: The competition between Coulomb and asymmetry terms determines the valley of stability.

6 Conclusions

This computational analysis demonstrates:

- Maximum binding energy per nucleon: 8.865 MeV at $A = 58$
- Volume term dominates: $a_V = 15.75$ MeV
- Alpha decay Q-value for U-235: -0.58 MeV
- Energy release in fusion exceeds fission by factor of ∼4 per nucleon

The SEMF provides a quantitative foundation for understanding nuclear stability, decay modes, and energy production in nuclear reactions.

7 Further Reading

- Krane, K.S., *Introductory Nuclear Physics*, Wiley, 1987
- Heyde, K., *Basic Ideas and Concepts in Nuclear Physics*, 3rd Edition, IOP Publishing, 2004
- Wong, S.S.M., *Introductory Nuclear Physics*, 2nd Edition, Wiley-VCH, 2004