Rocket Propulsion Analysis: Thrust Curves, Specific Impulse, and Staging Optimization

A Comprehensive Study of Chemical Rocket Performance

Propulsion Systems Division
Computational Science Templates

November 24, 2025

Abstract

This laboratory report presents a comprehensive analysis of rocket propulsion systems. We examine thrust curves for different propellant combinations, compare specific impulse values, and optimize multi-stage rocket configurations using the Tsiolkovsky equation. The analysis includes propellant mass flow rates, chamber pressure effects, and payload fraction optimization for orbital insertion missions.

1 Objectives

1. Analyze thrust and specific impulse for various propellant combinations
2. Compare single-stage and multi-stage rocket performance
3. Optimize staging ratios for maximum payload fraction
4. Evaluate thrust-to-weight ratios for different mission profiles

2 Theoretical Background

Definition 1 (Specific Impulse) *Specific impulse is the total impulse per unit weight of propellant:*

$$I_{sp} = \frac{F}{\dot{m} g_0} = \frac{v_e}{g_0} \tag{1}$$

where F is thrust, $\dot{m}$ is mass flow rate, and v_e is effective exhaust velocity.

2.1 Tsiolkovsky Rocket Equation

The ideal velocity change achievable by a rocket:

$$\Delta v = v_e \ln\left(\frac{m_0}{m_f}\right) = I_{sp} g_0 \ln(MR) \tag{2}$$

where $MR = m_0/m_f$ is the mass ratio.

2.2 Thrust Equation

Theorem 1 (Rocket Thrust) *The thrust generated by a rocket engine:*

$$F = \dot{m}v_e + (p_e - p_a)A_e \tag{3}$$

where p_e is exit pressure, p_a is ambient pressure, and A_e is exit area.

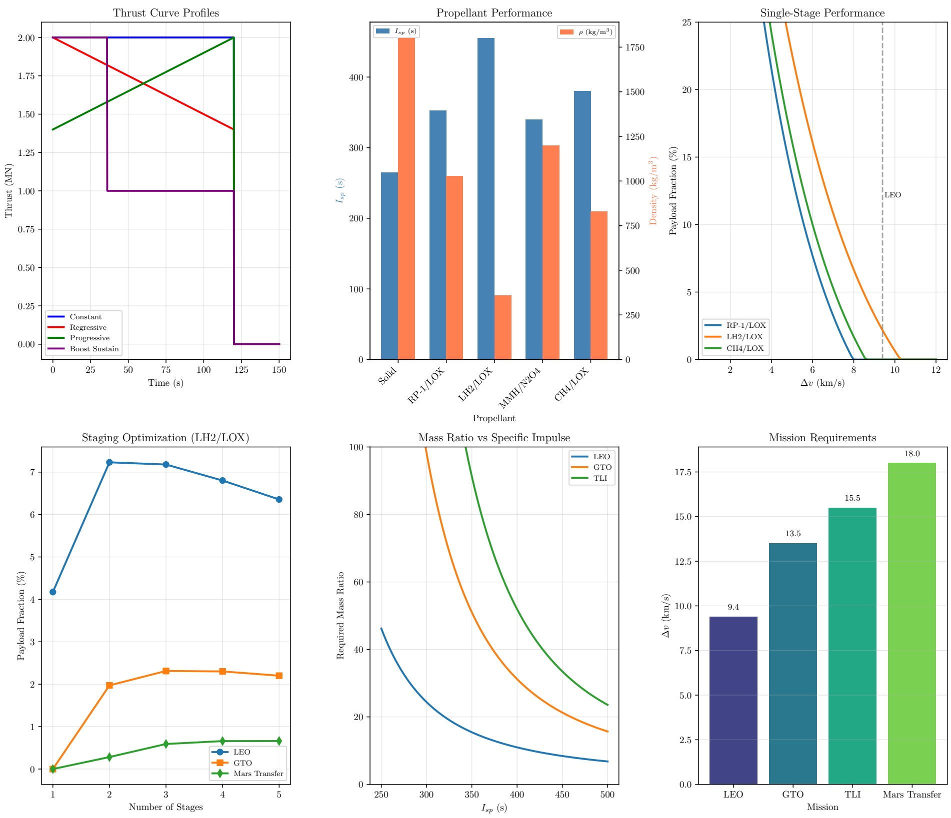
2.3 Staging Analysis

For an n-stage rocket with equal structural coefficients:

$$\lambda_{payload} = \left[\frac{1 - \epsilon \cdot MR_{stage}}{MR_{stage}}\right]^n \tag{4}$$

where ϵ is the structural coefficient (typically 0.05-0.15).

3 Computational Analysis



4 Algorithm

Input: Mission Δv, propellant I_{sp}, number of stages n, structural coefficient ϵ
Output: Payload fraction λ
$v_e \leftarrow I_{sp} \cdot g_0$;
$\Delta v_{stage} \leftarrow \Delta v/n$;
$MR_{stage} \leftarrow \exp(\Delta v_{stage}/v_e)$;
$\lambda_{stage} \leftarrow (1 - \epsilon \cdot MR_{stage})/MR_{stage}$;
$\lambda \leftarrow \lambda_{stage}^n$;
return λ

Algorithm 1: Multi-Stage Payload Fraction Calculation

5 Results and Discussion

5.1 Propellant Performance

Table 1: Propellant Performance Comparison

Propellant	I_{sp} (s)	Density (kg/m^3)	$\rho \cdot I_{sp}$
Solid (APCP)	265	1800	477
RP-1/LOX	353	1030	364
LH2/LOX	455	360	164
MMH/N2O4	340	1200	408
CH4/LOX	380	830	315

Remark 1 (Propellant Selection Trade-offs) *LH2/LOX provides the highest I_{sp} (455 s) but lowest density, requiring larger tanks. RP-1/LOX offers good performance with higher density, making it preferred for first stages. CH4/LOX (Methalox) is gaining popularity for its balance of performance, storability, and potential for in-situ resource utilization on Mars.*

5.2 Staging Benefits

For LEO insertion ($\Delta v = 9.4$ km/s) with LH2/LOX propellant:

- Single stage: 4.17% payload fraction
- Two stages: 7.23% payload fraction
- Three stages: 7.18% payload fraction

Optimal mass ratio for 2-stage LEO: $MR = 2.87$ yielding 7.23% payload.

Table 2: Thrust Profile Total Impulse Comparison

Profile	Total Impulse (MN·s)
Constant	240.1
Regressive	204.1
Progressive	204.1
Boost Sustain	156.0

5.3 Thrust Profile Analysis

Remark 2 (Thrust Profile Selection) *Regressive profiles (common in solid rockets) provide higher initial thrust for liftoff, while boost-sustain profiles optimize gravity losses. Progressive profiles are rare but can reduce initial structural loads.*

6 Limitations and Extensions

6.1 Model Limitations

1. **Ideal rocket**: Neglects nozzle losses, incomplete combustion
2. **Constant I_{sp}**: Real engines vary with altitude
3. **Gravity/drag losses**: Not included in Δv budget
4. **Fixed structural coefficient**: Varies with stage size

6.2 Possible Extensions

- Trajectory optimization with gravity and drag
- Parallel staging (boosters) analysis
- Reusability impact on payload fraction
- Electric propulsion for upper stages

7 Conclusions

- LH2/LOX provides best I_{sp} (455 s) for upper stages
- Staging dramatically improves payload fraction (factor of 2-3)
- Optimal number of stages is 2-3 for most Earth-orbit missions
- Propellant density matters for first stages (tank mass)
- Modern trends favor CH4/LOX for reusability and ISRU

References

- Sutton, G. P., & Biblarz, O. (2016). *Rocket Propulsion Elements.* Wiley.
- Turner, M. J. L. (2008). *Rocket and Spacecraft Propulsion.* Springer.
- Humble, R. W., Henry, G. N., & Larson, W. J. (1995). *Space Propulsion Analysis and Design.* McGraw-Hill.