Magnetic Resonance Imaging: Signal Formation, Contrast Mechanisms, and K-Space

Abstract

This report presents a comprehensive analysis of MRI signal formation and image reconstruction. We implement the Bloch equations, analyze T1 and T2 contrast mechanisms, demonstrate k-space encoding, compare pulse sequences, and evaluate image artifacts. All simulations use PythonTeX for reproducibility.

Contents

Chapter 1

Introduction

MRI signal arises from nuclear magnetic resonance. The Bloch equations describe magnetization dynamics:

$$\frac{dM_x}{dt} = \gamma(M_y B_z - M_z B_y) - \frac{M_x}{T_2} \tag{1.1}$$

$$\frac{dM_y}{dt} = \gamma(M_z B_x - M_x B_z) - \frac{M_y}{T_2} \tag{1.2}$$

$$\frac{dM_z}{dt} = \gamma(M_x B_y - M_y B_x) - \frac{M_z - M_0}{T_1} \tag{1.3}$$

1.1 Relaxation Times

- T_1: Spin-lattice (longitudinal) relaxation

- T_2: Spin-spin (transverse) relaxation

- T_2^*: Effective transverse relaxation (includes inhomogeneity)
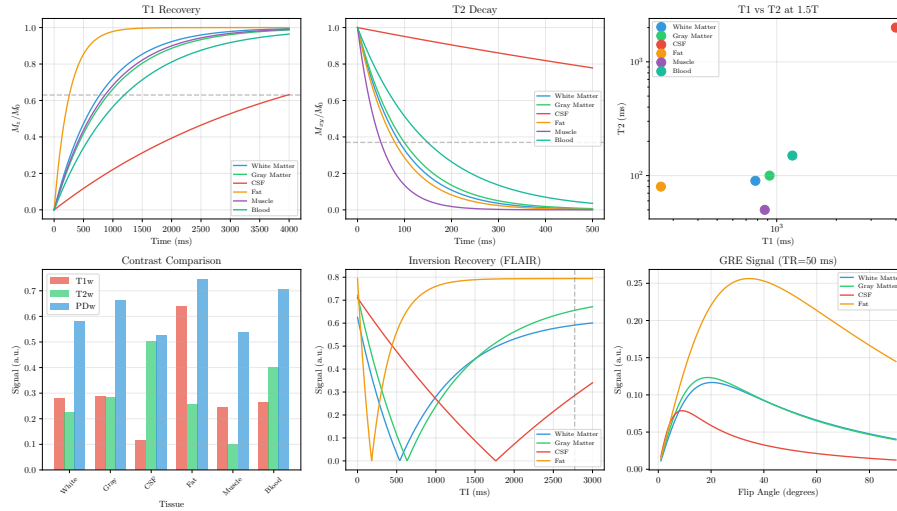
Chapter 2

Relaxation Mechanisms

Figure 2.1: MRI relaxation: (a) T1 recovery, (b) T2 decay, (c) T1-T2 relationship, (d) contrast comparison, (e) inversion recovery, (f) flip angle optimization.

Chapter 3

K-Space and Image Reconstruction

3.1 Spatial Encoding

The MRI signal is collected in k-space:

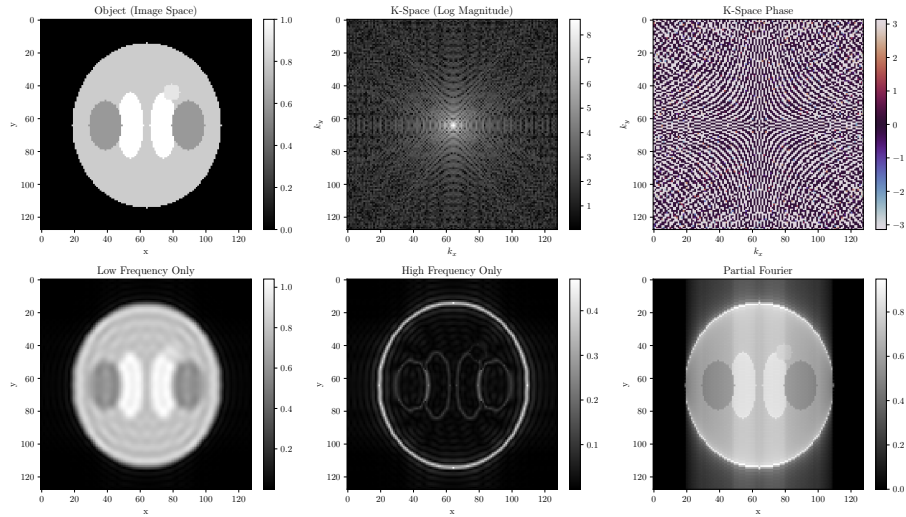$$S(k_x, k_y) = \iint \rho(x, y)e^{-i2\pi(k_x x + k_y y)}dx\,dy \qquad (3.1)$$

Figure 3.1: K-space encoding: (a) image space, (b) k-space magnitude, (c) k-space phase, (d) low frequency, (e) high frequency, (f) partial Fourier.

Chapter 4

Image Artifacts

Figure 4.1: MRI artifacts: (a) Gibbs ringing, (b) motion, (c) aliasing, (d) chemical shift, (e) susceptibility, (f) noise effects.

Chapter 5

Numerical Results

Table 5.1: MRI signal parameters and contrast

Parameter	Value	Units
??	??	??
??	??	??
??	??	??
??	??	??
??	??	??
??	??	??

Chapter 6

Conclusions

1. T1-weighted imaging provides anatomical detail

2. T2-weighted imaging highlights pathology

3. K-space center determines contrast, periphery determines resolution

4. Motion during acquisition causes ghosting artifacts

5. Chemical shift causes fat-water misregistration

6. Higher field strength increases SNR but also artifacts