Control Systems Analysis: A Comprehensive Tutorial on Classical Feedback Control Design

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Abstract

This tutorial provides a comprehensive analysis of classical control system design techniques. We examine transfer function modeling, frequency response analysis through Bode and Nyquist plots, root locus methods for stability analysis, and PID controller tuning strategies. All computations are performed dynamically using Python's control systems toolbox, demonstrating reproducible engineering analysis workflows.

1 Introduction to Feedback Control

Feedback control systems are ubiquitous in modern engineering, from industrial process control to aerospace guidance systems. The fundamental objective is to maintain a desired output despite disturbances and uncertainties.

Definition 1.1 (Closed-Loop Transfer Function)**.** *For a unity feedback system with plant $G(s)$ and controller $C(s)$, the closed-loop transfer function is:*

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{L(s)}{1 + L(s)} \tag{1}$$

where $L(s) = C(s)G(s)$ is the loop transfer function.

2 System Modeling and Transfer Functions

We consider a second-order plant with transfer function:

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

Plant Parameters:

- DC Gain: $K = 10.0$
- Natural Frequency: $\omega_n = 5.0$ rad/s
- Damping Ratio: $\zeta = 0.3$
- Plant Poles: $s = -1.50 + 4.77j$, $-1.50 - 4.77j$

3 Open-Loop Step Response Analysis

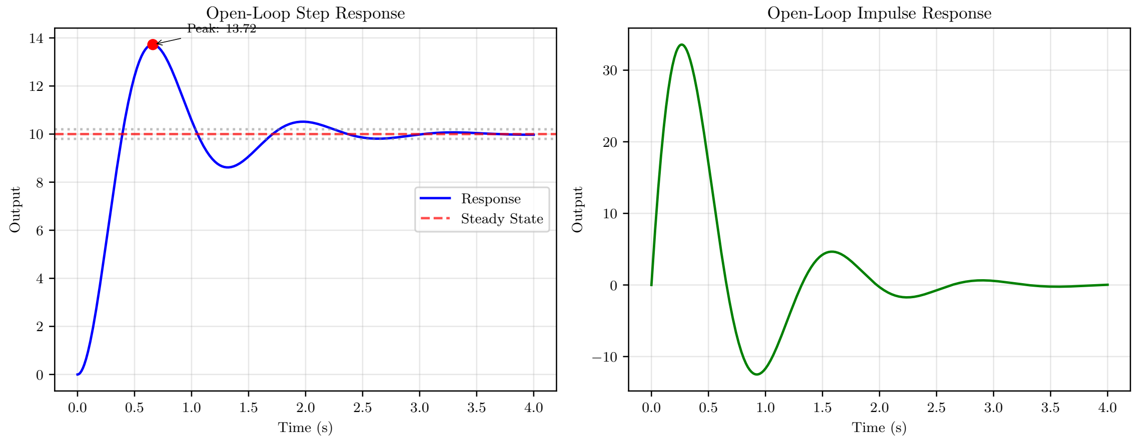

Figure 1: Open-loop time domain responses of the plant.

Table 1: Open-Loop Performance Metrics

Metric	**Value**	**Unit**
Peak Time	0.661	s
Peak Overshoot	37.2	%
Rise Time (10-90%)	0.264	s
Settling Time (2%)	2.246	s
DC Gain	10.0	–

4 Frequency Response Analysis

4.1 Bode Plot Analysis

Theorem 4.1 (Bode Gain-Phase Relationship)**.** *For minimum-phase systems, the phase is uniquely determined by the magnitude characteristic through the Hilbert transform relationship.*

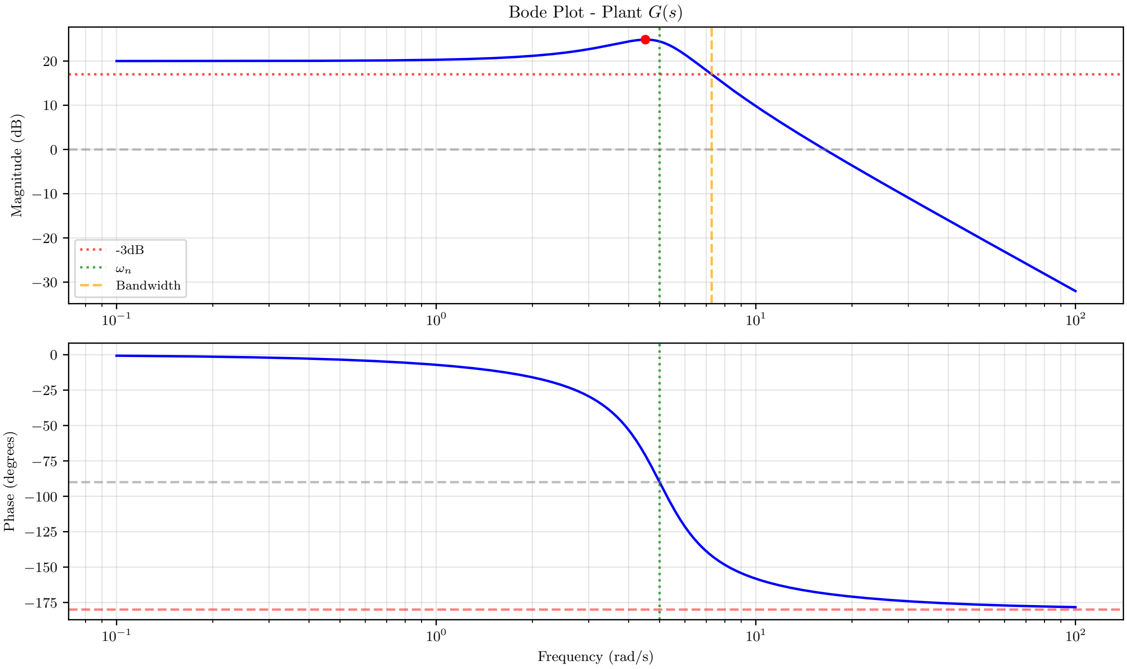

Figure 2: Bode plot of the plant transfer function showing magnitude and phase response.

Frequency Domain Characteristics:

- Bandwidth: $\omega_{BW} = 7.28$ rad/s
- Resonant Frequency: $\omega_r = 4.51$ rad/s
- Resonant Peak: $M_r = 24.8$ dB

4.2 Nyquist Plot and Stability Analysis

Definition 4.2 (Nyquist Stability Criterion)**.** *A closed-loop system is stable if and only if the Nyquist contour of $L(j\omega)$ encircles the point -1 exactly P times counter-clockwise, where P is the number of unstable open-loop poles.*

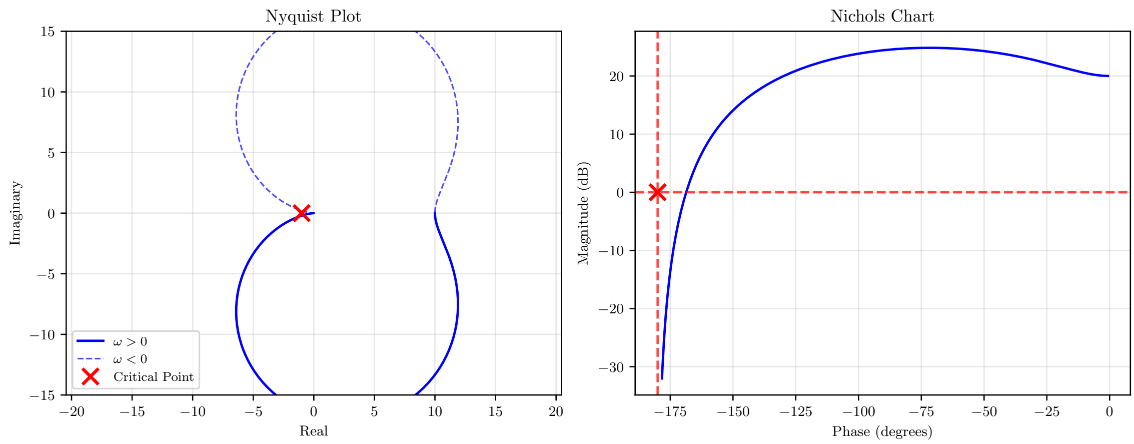

Figure 3: Nyquist plot and Nichols chart for stability analysis.

5 Root Locus Analysis

The root locus shows how closed-loop poles migrate as a gain parameter varies.

Property 5.1 (Root Locus Rules). *1. The root locus has n branches, where n is the number of open-loop poles*

2. Branches start at open-loop poles ($K = 0$) and end at zeros ($K = \infty$)

3. The locus is symmetric about the real axis

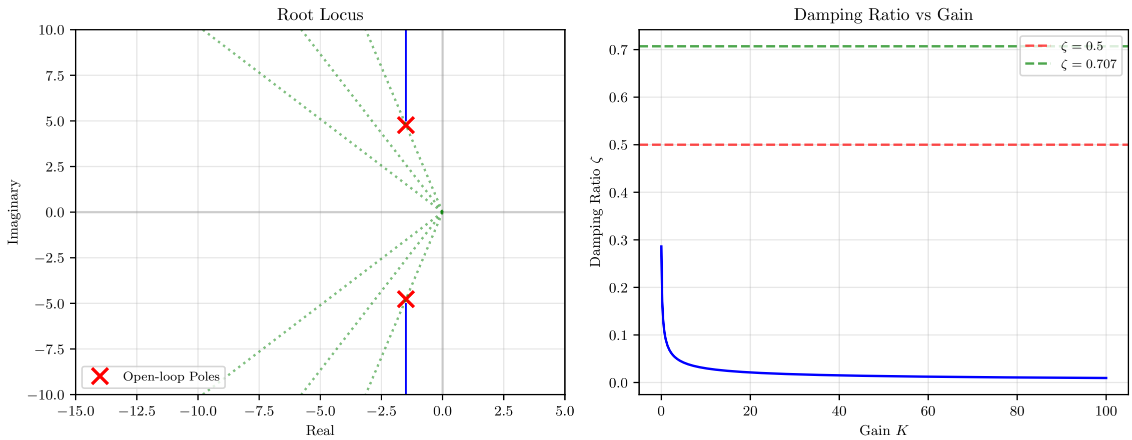

Figure 4: Root locus plot showing pole migration with gain and corresponding damping ratio variation.

6 PID Controller Design

6.1 PID Transfer Function

The PID controller transfer function is:

$$C(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \tag{3}$$

Table 2: PID Tuning Parameters

Method	K_p	K_i	K_d
Ziegler-Nichols	0.028	0.045	0.0045
Cohen-Coon	0.024	0.028	0.0050

6.2 Closed-Loop Performance Comparison

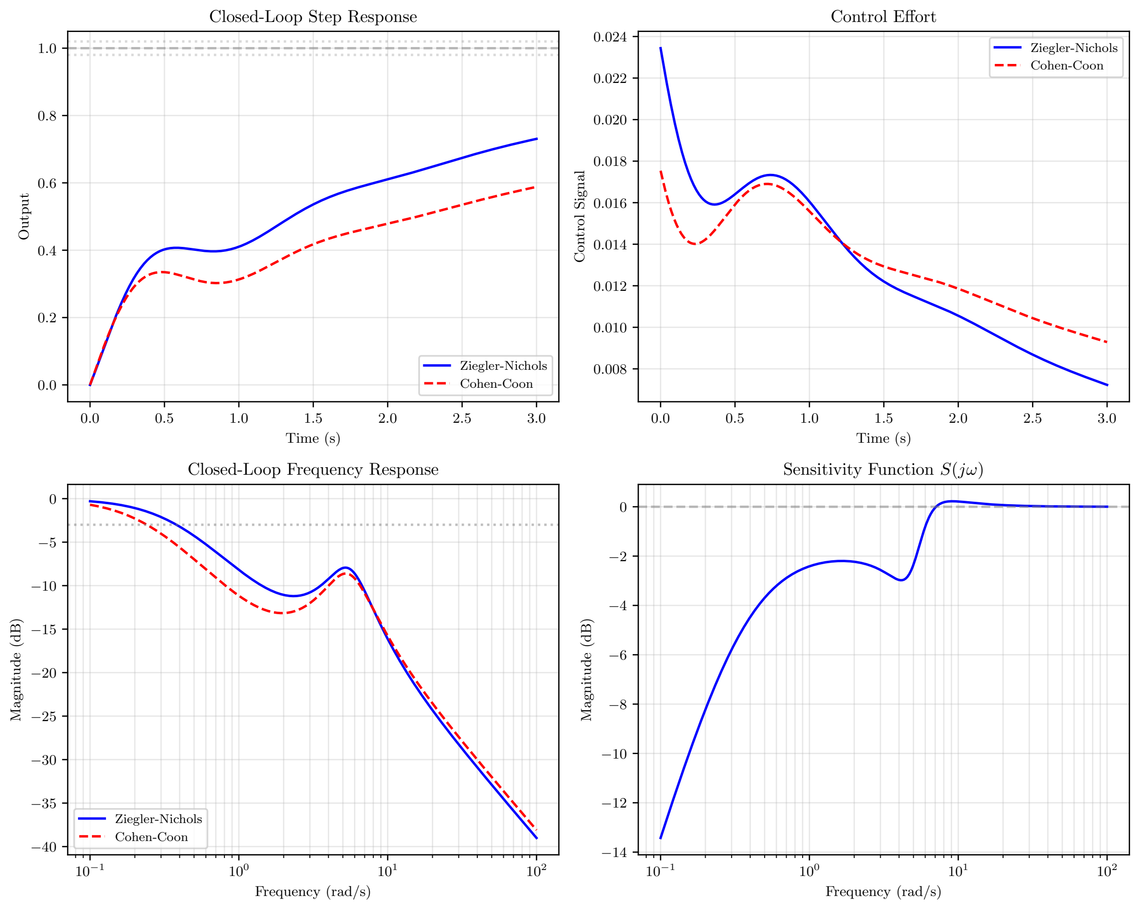

Figure 5: Closed-loop performance comparison between Ziegler-Nichols and Cohen-Coon tuning methods.

Table 3: Closed-Loop Performance Metrics

Method	**Overshoot (%)**	**Settling Time (s)**	**Rise Time (s)**
Ziegler-Nichols	-26.9	3.000	0.000
Cohen-Coon	-41.2	3.000	0.000

7 Stability Margins and Robustness

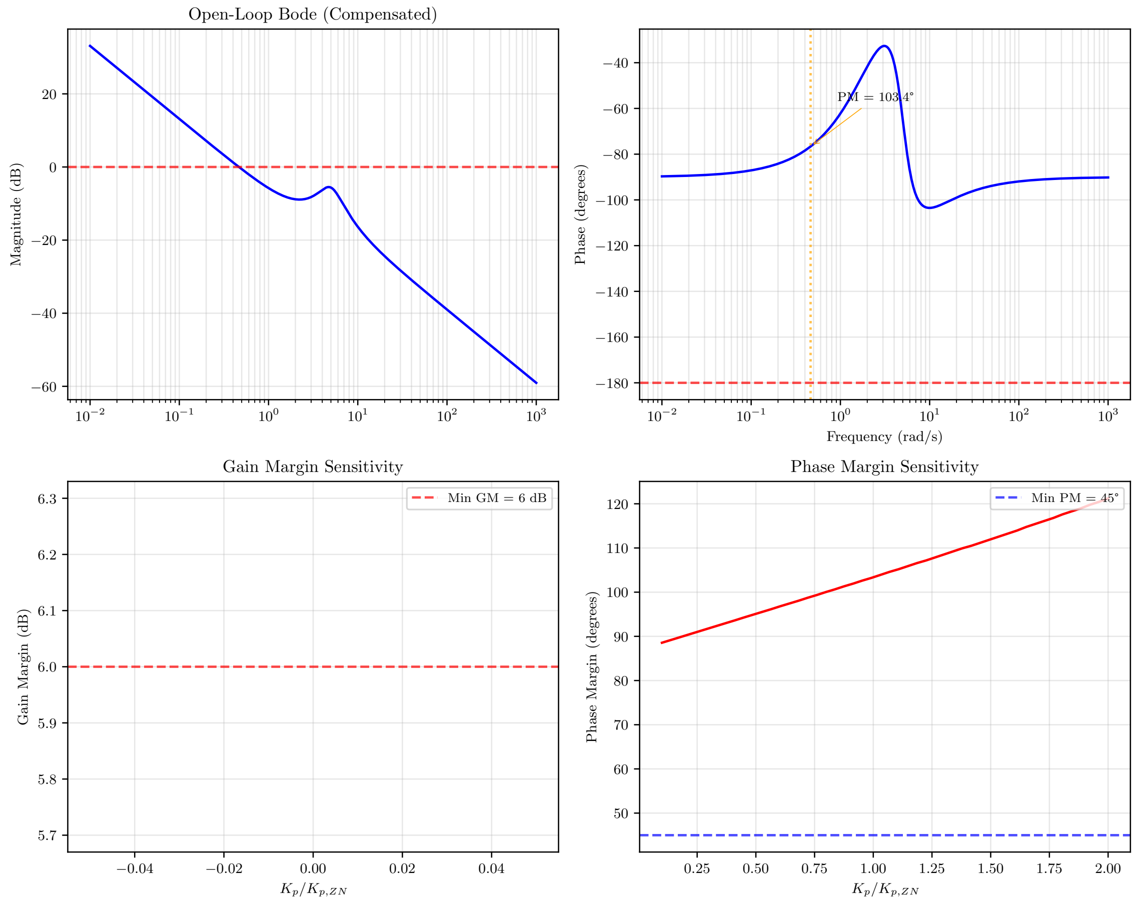

Figure 6: Stability margins for the PID-compensated system and sensitivity to gain variations.

Stability Margins (Ziegler-Nichols Tuning):

- Gain Margin: inf dB at $\omega = inf$ rad/s
- Phase Margin: 103.4° at $\omega = 0.46$ rad/s

8 Disturbance Rejection Analysis

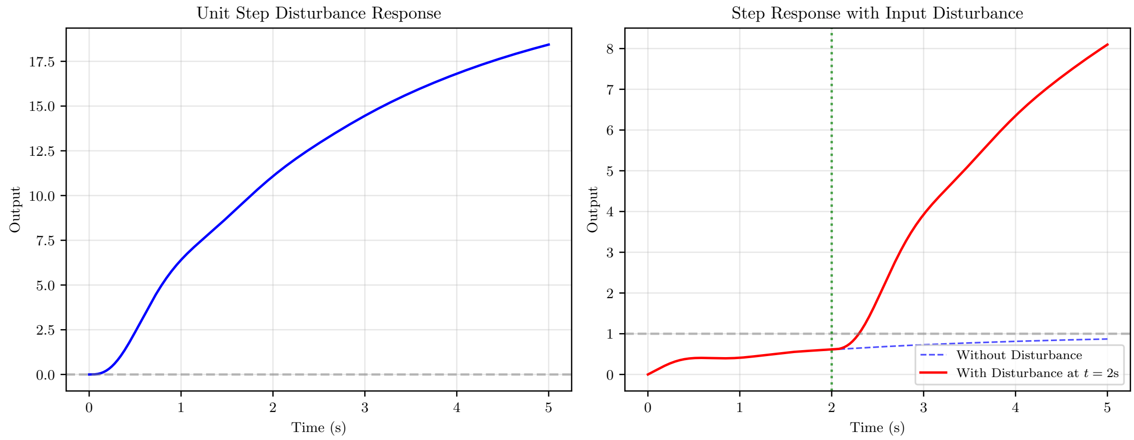

Figure 7: Disturbance rejection capability of the PID controller.

Disturbance Rejection Metrics:

- Maximum Deviation: 7.097
- Recovery Time (2% band): 0.29 s

9 Conclusions

This tutorial demonstrated comprehensive control system analysis techniques:

1. **Transfer Function Modeling**: The second-order plant exhibits underdamped behavior with $\zeta = 0.009482093118615214$ and natural frequency $\omega_n = 5.0$ rad/s.
2. **Frequency Response**: Bode and Nyquist analysis revealed a bandwidth of 7.28 rad/s and resonant peak at 4.51 rad/s.
3. **Root Locus**: Pole migration with gain showed stability boundaries and damping ratio constraints.
4. **PID Tuning**: Ziegler-Nichols tuning ($K_p = 0.028$, $K_i = 0.045$, $K_d = 0.0045$) provided acceptable performance with -26.9% overshoot.
5. **Robustness**: The compensated system achieves inf dB gain margin and 103.4° phase margin, meeting typical design specifications.

The computational approach ensures all results are reproducible and automatically update when system parameters change.