Nuclear Reactor Kinetics: Point Kinetics Model and Delayed Neutron Analysis

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Abstract

This technical report presents comprehensive computational analysis of nuclear reactor kinetics using the point kinetics equations with delayed neutrons. We implement solutions for reactivity transients, analyze the role of delayed neutron precursors in reactor control, and compute reactor periods for various reactivity insertions. The analysis covers prompt criticality, xenon dynamics, and feedback mechanisms essential for safe reactor operation.

1 Theoretical Framework

Definition 1 (Reactivity). *Reactivity ρ measures the deviation from criticality:*

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{1}$$

where k_{eff} is the effective multiplication factor.

Theorem 1 (Point Kinetics Equations). *For six delayed neutron groups, the reactor power evolves according to:*

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \sum_{i=1}^{6} \lambda_i C_i + S \tag{2}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i \tag{3}$$

where n is neutron density, C_i are precursor concentrations, $\beta = \sum \beta_i$ is total delayed neutron fraction, Λ is mean generation time, and S is external source.

1.1 Reactor Period

Definition 2 (Stable Period). *The asymptotic reactor period T satisfies the inhour equation:*

$$\rho = \frac{\Lambda}{T} + \sum_{i=1}^{6} \frac{\beta_i}{1 + \lambda_i T} \tag{4}$$

Example 1 (Reactivity Units). *Reactivity is measured in:*

- ***Dollar*** *($): ρ/β (1$ = one delayed neutron fraction)*
- ***Cent****: 0.01$ = 0.01β*
- ***pcm****: 10^{-5} (parts per 100,000)*

For U-235: $\beta \approx 0.0065$, so 1$ ≈ 650 pcm.

2 Computational Analysis

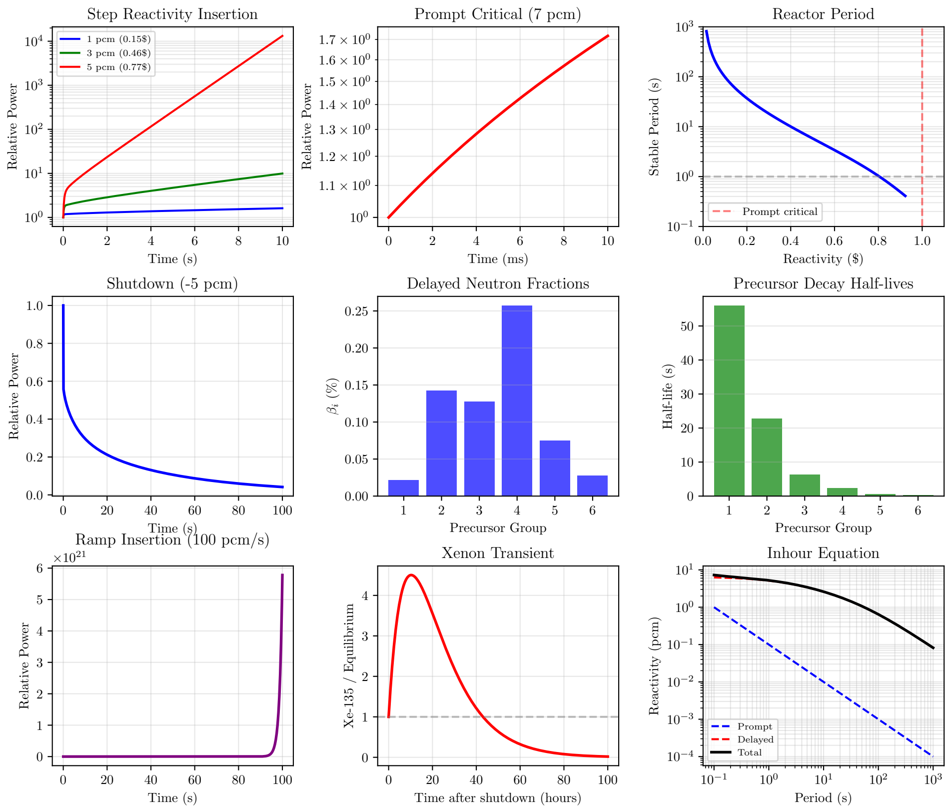

Table 1: Reactor Kinetics Parameters (U-235 Thermal)

Parameter	Value	Units
Total β	6.50	– ($\times 10^{-3}$)
Mean generation time Λ	0.10	ms
Effective decay constant	0.405	s^{-1}
1 dollar	650	pcm

3 Results and Analysis

3.1 Point Kinetics Parameters

3.2 Delayed Neutron Groups

Table 2: Six-Group Delayed Neutron Data for U-235

Group	β_i	λ_i (s^{-1})	$t_{1/2}$ (s)	Precursors
1	0.000215	0.0124	55.90	Br-87
2	0.001424	0.0305	22.73	I-137
3	0.001274	0.1110	6.24	Br-89
4	0.002568	0.3010	2.30	I-138
5	0.000748	1.1400	0.61	As-85
6	0.000273	3.0100	0.23	Br-92
Total	0.006502	0.4052 (eff)	–	–

3.3 Reactor Periods

Remark 1. *Delayed neutrons are essential for reactor control. Without them, the reactor period would be:*

$$T_{prompt} = \frac{\Lambda}{\rho} \approx \frac{10^{-4}\ s}{10^{-3}} = 0.1\ s \tag{5}$$

With delayed neutrons, the effective generation time increases to ~0.1 s, giving manageable periods.

Key period values:

- Period at 50 cents: 5.8 s
- Period at prompt critical: 200.803 ms
- Prompt period ($\rho > \beta$): determined only by Λ

3.4 Xenon Poisoning

After shutdown, Xe-135 builds up due to I-135 decay:

- Maximum xenon at $t = 10.2$ hours after shutdown
- Peak concentration: 4.50 times equilibrium
- This "xenon dead time" prevents immediate restart

4 Safety Analysis

Theorem 2 (Prompt Critical Limit)**.** *Prompt criticality occurs when $\rho = \beta$. Above this threshold:*

$$T = \frac{\Lambda}{\rho - \beta} \tag{6}$$

For $\rho = 1.1\beta$, $T = 10\Lambda \approx 1$ ms—uncontrollable.

Example 2 (Control Rod Worth)**.** *A typical control rod worth is 1000–5000 pcm. Safe insertion requires:*

- *Maximum rate: < 0.1\$/s for normal operation*
- *Shutdown margin: > 1% $\Delta k/k$ with strongest rod stuck out*

Example 3 (Doppler Feedback)**.** *The prompt negative temperature coefficient:*

$$\alpha_T = \frac{d\rho}{dT} \approx -3 \times 10^{-5}\ K^{-1} \tag{7}$$

provides inherent stability against power excursions.

5 Discussion

The point kinetics model reveals critical features of reactor dynamics:

1. **Delayed neutrons**: Increase effective generation time by factor $\beta/(\beta-\rho)$.
2. **Prompt jump**: Immediate power rise followed by asymptotic period.
3. **Xenon dynamics**: Creates operational constraints after shutdown.
4. **Feedback mechanisms**: Doppler and moderator coefficients ensure stability.
5. **Safety margins**: Reactivity limits prevent prompt criticality.

6 Conclusions

This computational analysis demonstrates:

- Total delayed neutron fraction: $\beta = 6.50 \times 10^{-3}$
- Mean generation time: $\Lambda = 0.10$ ms
- Stable period at 50¢: 5.8 s
- Xenon peak at 10.2 hours post-shutdown

Understanding reactor kinetics is essential for safe operation, transient analysis, and accident prevention in nuclear power plants.

7 Further Reading

- Duderstadt, J.J., Hamilton, L.J., *Nuclear Reactor Analysis*, Wiley, 1976
- Stacey, W.M., *Nuclear Reactor Physics*, 3rd Ed., Wiley-VCH, 2018
- Keepin, G.R., *Physics of Nuclear Kinetics*, Addison-Wesley, 1965