Binary Phase Diagrams: Computational Analysis

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Abstract

This report presents computational analysis of binary phase diagrams in materials science. We examine isomorphous and eutectic systems, the lever rule for phase fraction calculations, cooling curve analysis, and Gibbs phase rule applications. Python-based computations provide quantitative analysis with dynamic visualization.

Contents

1 Introduction to Phase Diagrams

Phase diagrams are graphical representations of the equilibrium phases present in a material system as a function of temperature, pressure, and composition. They are essential for:

- Predicting microstructure evolution during solidification
- Designing heat treatment processes
- Understanding alloy behavior and properties
- Optimizing casting and welding procedures

2 Gibbs Phase Rule

The Gibbs phase rule determines the degrees of freedom in a system:

$$F = C - P + 2 \tag{1}$$

where F is degrees of freedom, C is number of components, and P is number of phases.

For a binary system ($C = 2$) at constant pressure:

$$F = 3 - P \tag{2}$$

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Figure 1: Gibbs phase rule showing degrees of freedom for different numbers of phases.

3 Isomorphous System

An isomorphous system exhibits complete solid solubility. The Cu-Ni system is a classic example.

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Figure 2: Cu-Ni isomorphous phase diagram with tie line construction at $T = 1250°$C.

4 The Lever Rule

The lever rule calculates phase fractions in a two-phase region:

$$W_\alpha = \frac{C_L - C_0}{C_L - C_\alpha} \quad \text{and} \quad W_L = \frac{C_0 - C_\alpha}{C_L - C_\alpha} \tag{3}$$

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Figure 3: Lever rule analysis: phase fractions during cooling and graphical representation.

5 Eutectic System

A eutectic system has limited solid solubility. The Pb-Sn system is a classic example.

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Figure 4: Pb-Sn eutectic phase diagram showing liquid, solid, and two-phase regions.

6 Cooling Curves

Cooling curves reveal phase transformations through thermal arrests.

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Figure 5: Cooling curves for hypoeutectic (10%, 40%), eutectic (61.9%), and hypereutectic (80%) Pb-Sn alloys.

7 Microstructure Evolution

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Figure 6: Microstructure evolution during cooling of 40% Sn alloy.

8 Phase Fraction Calculations

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Figure 7: Phase fractions and microconstituent analysis for Pb-Sn system.

9 Summary Table

Table 1: Key Phase Diagram Parameters for Pb-Sn System

Parameter	Value	Units
Eutectic temperature	??	°C
Eutectic composition	??	wt% Sn
Max solubility in α	??	wt% Sn
Max solubility in β	??	wt% Sn
Melting point of Pb	??	°C
Melting point of Sn	??	°C

10 Conclusions

This analysis demonstrates key aspects of binary phase diagrams:

1. The Gibbs phase rule determines degrees of freedom in multi-phase systems
2. Isomorphous systems show complete solid solubility with smooth liquidus/solidus curves
3. The lever rule enables quantitative calculation of phase fractions
4. Eutectic systems exhibit invariant reactions at fixed temperature and composition
5. Cooling curves reveal thermal arrests during phase transformations
6. Microstructure depends on both equilibrium phases and solidification path