Particle Scattering Cross Sections: Rutherford, Mott, and Form Factor Analysis

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Abstract

This technical report presents comprehensive computational analysis of particle scattering cross sections. We implement the Rutherford formula for classical Coulomb scattering, Mott cross section with relativistic and spin corrections, and nuclear form factors for extended charge distributions. The analysis covers differential and integrated cross sections, structure functions, and momentum transfer dependence essential for understanding particle interactions and nuclear structure.

1 Theoretical Framework

Definition 1 (Differential Cross Section). *The differential cross section $d\sigma/d\Omega$ gives the probability per unit solid angle for scattering into direction (θ, ϕ):*

$$\frac{dN}{dt} = I_0 n \frac{d\sigma}{d\Omega} d\Omega \tag{1}$$

where I_0 is incident flux and n is target number density.

Theorem 1 (Rutherford Scattering). *For Coulomb scattering of a particle with charge $Z_1 e$ from a nucleus with charge $Z_2 e$:*

$$\frac{d\sigma}{d\Omega}_{Ruth} = \left(\frac{Z_1 Z_2 \alpha \hbar c}{4 E_k \sin^2(\theta/2)} \right)^2 \tag{2}$$

where $\alpha \approx 1/137$ is the fine structure constant.

1.1 Relativistic and Quantum Corrections

Theorem 2 (Mott Cross Section). *For relativistic electron scattering, the Mott formula includes spin effects:*

$$\frac{d\sigma}{d\Omega}_{Mott} = \frac{d\sigma}{d\Omega}_{Ruth} \left(1 - \beta^2 \sin^2(\theta/2) \right) \tag{3}$$
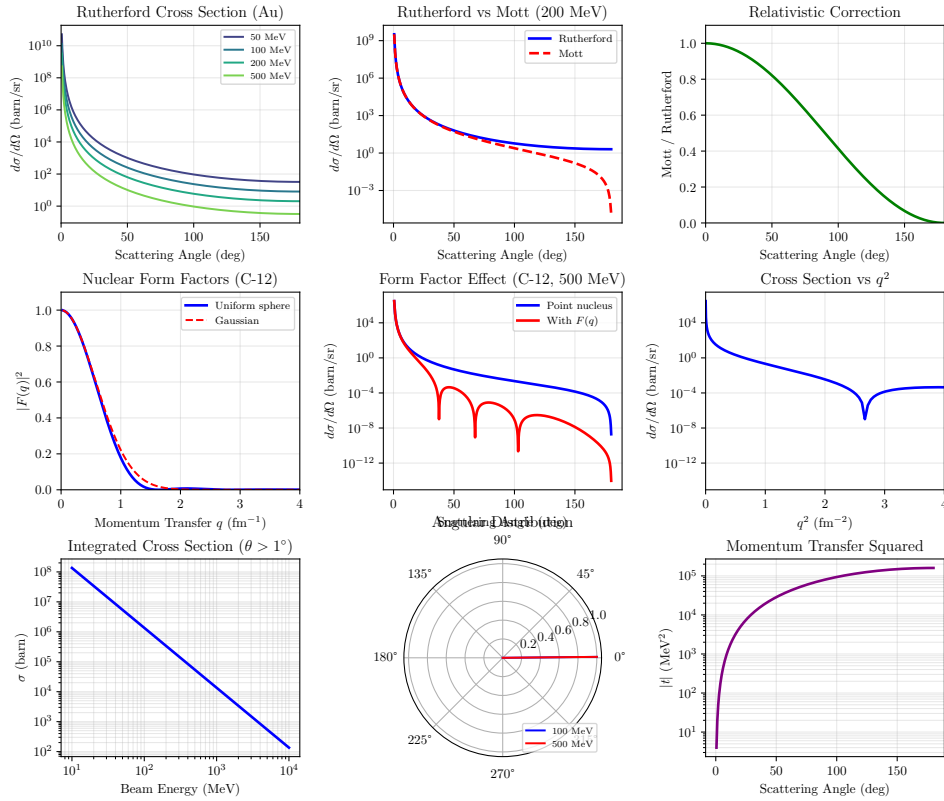
where $\beta = v/c$.

Example 1 (Nuclear Form Factor). *For extended nuclear charge distribution* $\rho(r)$:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{Mott} |F(q)|^2 \tag{4}$$

where $F(q) = \int \rho(r)e^{i\mathbf{q}\cdot\mathbf{r}}d^3r$ *is the form factor and* $q = 2k\sin(\theta/2)$ *is momentum transfer.*

2   Computational Analysis

3   Results and Analysis

3.1   Cross Section Scaling

3.2   Key Cross Section Features

The analysis reveals:

- Rutherford formula: $d\sigma/d\Omega \propto \sin^{-4}(\theta/2)$ singularity at $\theta = 0$

- Mott correction: reduces backscattering by factor $(1 - \beta^2)$ at $180°$

Table 1: Differential Cross Sections at 90° (Gold Target)

Energy (MeV)	Rutherford	Mott	Ratio	β
50	1.29e+02	6.47e+01	0.500	1.000
100	3.24e+01	1.62e+01	0.500	1.000
200	8.09e+00	4.04e+00	0.500	1.000
500	1.29e+00	6.47e-01	0.500	1.000

- Energy scaling: $\sigma \propto E^{-2}$ for Coulomb scattering

- Form factor: creates diffraction minima at $q \cdot R \approx 4.49$

Remark 1. *The Mott/Rutherford ratio at 90° for 100 MeV electrons is 0.500, showing significant relativistic suppression.*

3.3 Form Factor Analysis

Table 2: Nuclear Form Factor Parameters

Nucleus	R (fm)	R_{rms} (fm)	First minimum (q)
C-12	2.75	2.13	1.63 fm^{-1}
Au-197	6.98	5.33	0.64 fm^{-1}

4 Applications

Example 2 (Nuclear Size Measurement). *Electron scattering experiments determine nuclear charge radii through form factor analysis. The position of diffraction minima in $d\sigma/d\Omega$ directly gives the nuclear radius: $R \approx 4.49/q_{min}$.*

Example 3 (Deep Inelastic Scattering). *At high $Q^2 = -t$, structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ probe quark distributions inside nucleons, revealing parton dynamics.*

Theorem 3 (Optical Theorem). *The total cross section relates to the forward scattering amplitude:*

$$\sigma_{tot} = \frac{4\pi}{k} Im[f(0)] \tag{5}$$

This connects elastic and inelastic scattering processes.

5   Discussion

The cross section analysis demonstrates:

1. **Classical limit**: Rutherford formula recovers Coulomb scattering with $d\sigma \propto Z^2/E^2$.

2. **Relativistic effects**: Mott correction accounts for electron spin-orbit coupling.

3. **Nuclear structure**: Form factors encode charge distribution information.

4. **Diffraction**: Wave nature of matter creates interference patterns in angular distribution.

5. **Scale dependence**: Different energies probe different length scales via $\lambda = \hbar c/E$.

6   Conclusions

This computational analysis demonstrates:

- Cross section at 90° (100 MeV, Au): 1.62e+01 barn/sr

- Mott/Rutherford ratio (100 MeV): 0.500

- Gold nuclear radius: 6.98 fm

- Carbon nuclear radius: 2.75 fm

Cross section measurements form the foundation of particle physics experiments, enabling precise determination of fundamental interactions and nuclear structure.

7   Further Reading

- Povh, B., Rith, K., Scholz, C., Zetsche, F., *Particles and Nuclei*, 7th Ed., Springer, 2015

- Hofstadter, R., Electron scattering and nuclear structure, *Rev. Mod. Phys.* 28, 214 (1956)

- Halzen, F., Martin, A.D., *Quarks and Leptons*, Wiley, 1984