Evolutionary Dynamics: Selection, Drift, and Fitness Landscapes

Theoretical Population Genetics

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Abstract

This chapter explores the fundamental forces driving evolutionary change in populations. We analyze natural selection under various fitness schemes, examine the stochastic effects of genetic drift in finite populations, and investigate the interplay between mutation and selection. Computational simulations illustrate fitness landscapes, fixation probabilities, and the dynamics of allele frequency change under different evolutionary scenarios.

1 Introduction

Evolution operates through the interplay of deterministic forces (selection, mutation) and stochastic processes (genetic drift). Understanding these dynamics requires both analytical theory and computational simulation.

Definition 1.1 (Fitness) *The fitness w of a genotype is its relative reproductive success. For a diploid organism with alleles A and a, genotype fitnesses are denoted w_{AA}, w_{Aa}, and w_{aa}.*

2 Theoretical Framework

2.1 Natural Selection

Theorem 2.1 (Change in Allele Frequency Under Selection) *For a population with allele frequency p (for allele A) and genotype fitnesses w_{AA}, w_{Aa}, w_{aa}, the change in one generation is:*

$$\Delta p = \frac{pq[p(w_{AA} - w_{Aa}) + q(w_{Aa} - w_{aa})]}{\bar{w}} \tag{1}$$

where $q = 1 - p$ and $\bar{w} = p^2 w_{AA} + 2pq w_{Aa} + q^2 w_{aa}$ is the mean fitness.

2.2 Selection Schemes

Definition 2.1 (Types of Selection) *Different selection regimes produce distinct dynamics:*

- ***Directional****: One homozygote has highest fitness; allele goes to fixation*
- ***Overdominance****: Heterozygote advantage; stable polymorphism*
- ***Underdominance****: Heterozygote disadvantage; unstable equilibrium*
- ***Frequency-dependent****: Fitness depends on allele frequency*

2.3 Genetic Drift

Theorem 2.2 (Wright-Fisher Model) *In a finite population of size N, allele frequencies change stochastically. The variance in allele frequency change per generation is:*

$$Var(\Delta p) = \frac{p(1-p)}{2N} \tag{2}$$

The probability of eventual fixation for a neutral allele at initial frequency p_0 is p_0.

Remark 2.1 (Effective Population Size) *Real populations often have $N_e < N$ due to variance in reproductive success, unequal sex ratios, and population fluctuations.*

2.4 Mutation-Selection Balance

Theorem 2.3 (Equilibrium Frequency) *For a deleterious recessive allele with mutation rate μ and selection coefficient s:*

$$\hat{q} \approx \sqrt{\frac{\mu}{s}} \tag{3}$$

For a deleterious dominant allele: $\hat{q} \approx \mu/s$.

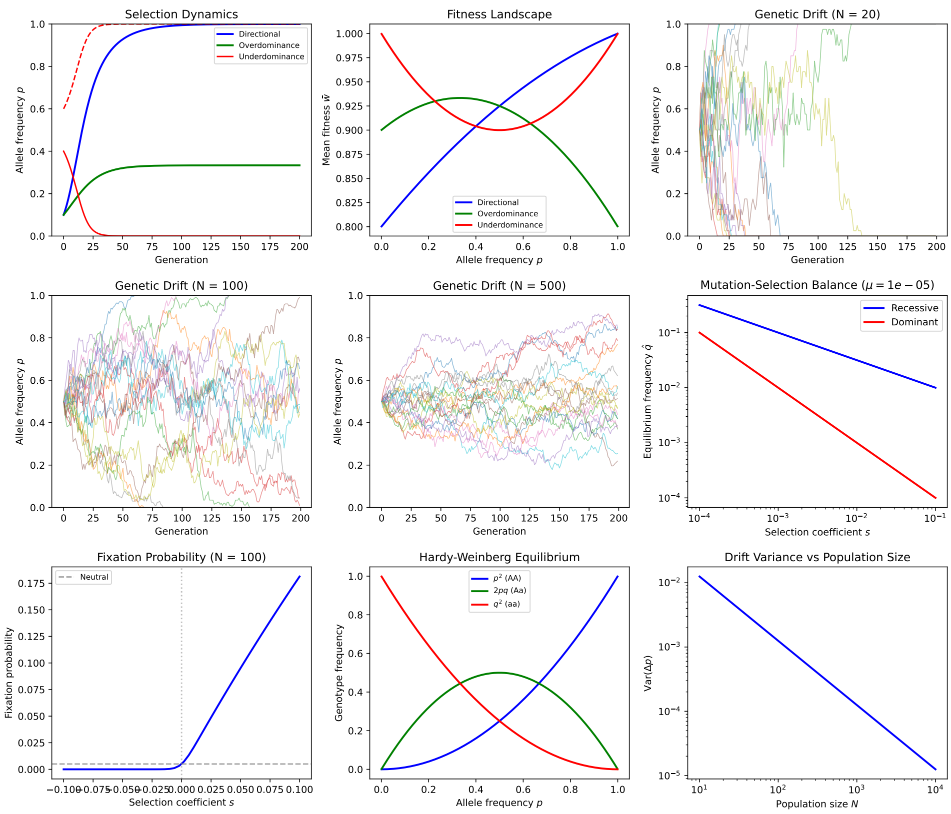

Figure 1: Evolutionary dynamics analysis: (a) Selection dynamics under different fitness schemes; (b) Fitness landscapes showing mean fitness as function of allele frequency; (c-e) Genetic drift in populations of different sizes; (f) Mutation-selection balance; (g) Fixation probability as function of selection coefficient; (h) Hardy-Weinberg genotype frequencies; (i) Drift variance versus population size.

Table 1: Selection Equilibria for Different Fitness Schemes

Selection Type	w_{AA}	w_{Aa}	w_{aa}	Equilibrium $\hat{p}$
Directional	1.00	0.95	0.80	1.00 (fixation)
Overdominance	0.80	1.00	0.90	0.33
Underdominance	1.00	0.80	1.00	0.00 or 1.00

Table 2: Genetic Drift Statistics After 200 Generations

N	Fixed	Lost	Polymorphic
20	8	12	0
100	5	4	11
500	0	0	20

3 Computational Analysis

4 Results

4.1 Selection Equilibria

4.2 Drift Statistics

5 Discussion

Example 5.1 (Overdominance and Stable Polymorphism) *Heterozygote advantage (overdominance) maintains genetic variation in populations. The equilibrium allele frequency for our example is:*

$$\hat{p} = \frac{w_{Aa} - w_{aa}}{2w_{Aa} - w_{AA} - w_{aa}} = \frac{1.0 - 0.9}{2(1.0) - 0.8 - 0.9} = 0.33 \tag{4}$$

Classic examples include sickle-cell anemia and MHC diversity.

Remark 5.1 (Effective Population Size) *The variance in allele frequency change is $Var(\Delta p) = pq/(2N_e)$. Factors reducing N_e below census size include:*

- *Unequal sex ratios*
- *Variance in reproductive success*
- *Population bottlenecks*
- *Overlapping generations*

6 Conclusions

This analysis demonstrates the fundamental forces of evolutionary change:

1. Selection drives systematic allele frequency change toward fitness peaks
2. Overdominance equilibrium at $\hat{p} = 0.33$ maintains polymorphism
3. Genetic drift is stronger in small populations, with variance $\propto 1/N$
4. Mutation-selection balance maintains deleterious alleles at low frequencies
5. Fixation probability depends on both selection and population size

Further Reading

- Hartl, D.L. & Clark, A.G. *Principles of Population Genetics*, 4th ed. Sinauer, 2007.
- Gillespie, J.H. *Population Genetics: A Concise Guide*, 2nd ed. Johns Hopkins, 2004.
- Nowak, M.A. *Evolutionary Dynamics.* Harvard University Press, 2006.