Optics: Optical Fiber Mode Analysis

Computational Science Templates

November 24, 2025

1 Introduction

Optical fibers guide light through total internal reflection. The number and characteristics of guided modes depend on the fiber parameters including core radius, refractive indices, and wavelength. This analysis examines single-mode and multi-mode fiber behavior, mode field characteristics, dispersion properties, and bend loss effects.

2 Mathematical Framework

2.1 Normalized Frequency (V-number)

The V-number determines the number of modes:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} \cdot NA \tag{1}$$

For step-index fiber, the number of modes is approximately:

$$N \approx \frac{V^2}{2} \tag{2}$$

Single-mode condition: $V < 2.405$ (LP_{01} mode only)

2.2 LP Mode Characteristic Equation

The characteristic equation for LP_{lm} modes:

$$u\frac{J_{l\pm1}(u)}{J_l(u)} = \pm w\frac{K_{l\pm1}(w)}{K_l(w)} \tag{3}$$

where $u = a\sqrt{k_0^2 n_1^2 - \beta^2}$, $w = a\sqrt{\beta^2 - k_0^2 n_2^2}$, and $u^2 + w^2 = V^2$.

2.3 Mode Field Diameter

The Marcuse approximation for mode field diameter:

$$\frac{w}{a} = 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} \tag{4}$$

2.4 Dispersion

Total dispersion in single-mode fiber:

$$D_{total} = D_M + D_W = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2} \tag{5}$$

3 Environment Setup

4 V-Number and Mode Count Analysis

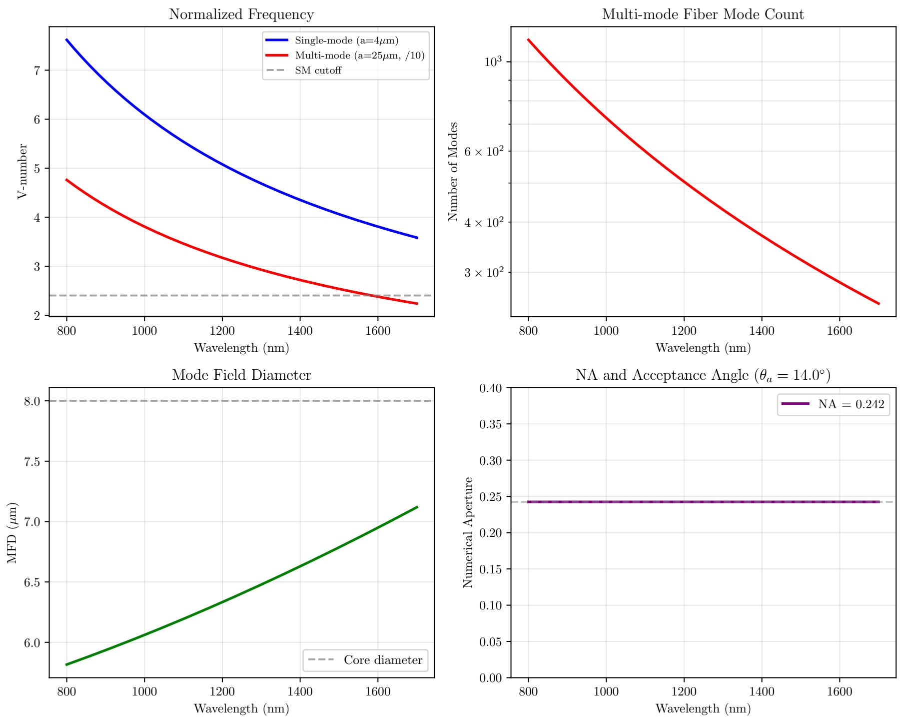

Figure 1: V-number, mode count, and MFD analysis for step-index fibers.

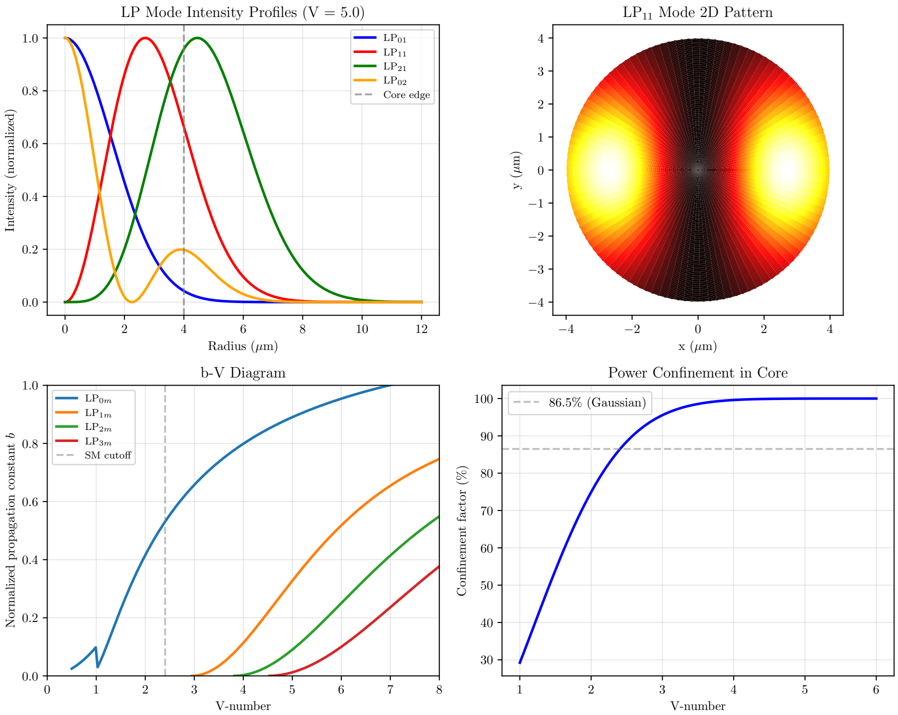

Figure 2: LP mode profiles, b-V diagram, and confinement analysis.

Table 1: Summary of Optical Fiber Parameters

Parameter	Symbol	1310 nm	1550 nm
V-number (SM)	V	4.65	3.93
Mode field diameter	MFD	6.5 μm	6.9 μm
Attenuation	α	0.26 dB/km	0.14 dB/km
Dispersion	D	–	21.8 ps/(nm·km)
Numerical aperture	NA	0.242	
Cutoff wavelength	λ_c	2534 nm	
Zero-dispersion	λ_0	1274 nm	
MM fiber modes	N	$\approx$ 302	

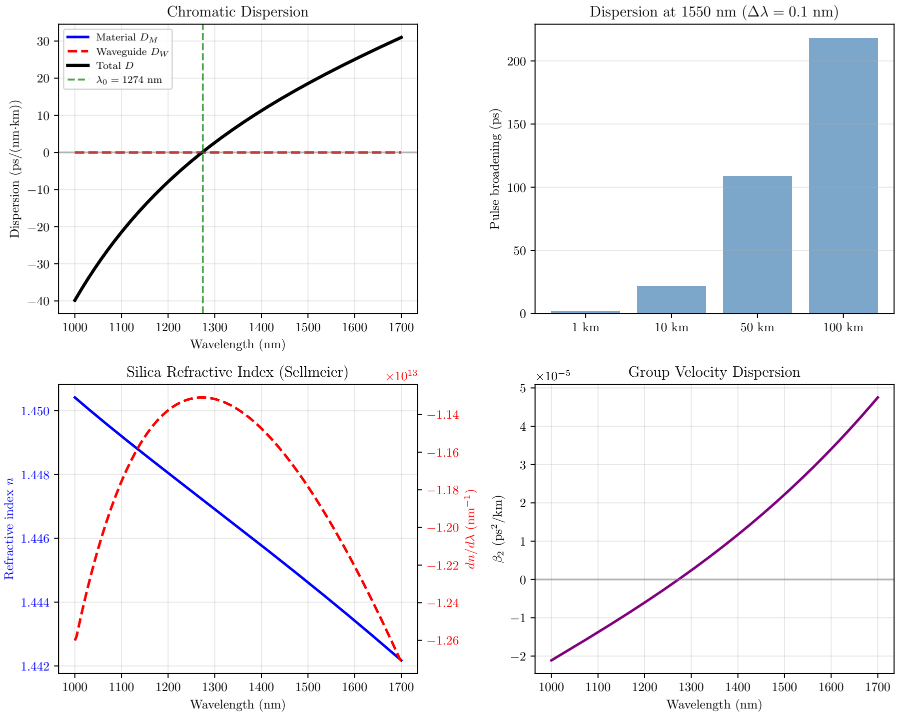

Figure 3: Chromatic dispersion analysis and pulse broadening effects.

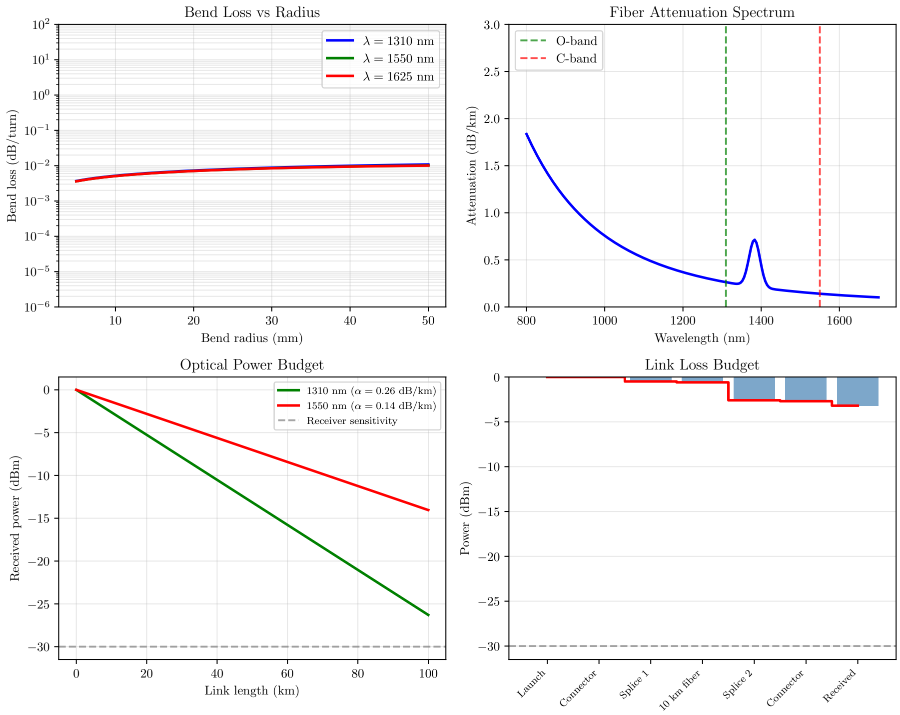

Figure 4: Bend loss, attenuation spectrum, and link power budget.

5 LP Mode Profiles and Characteristic Equation

6 Dispersion Analysis

7 Bend Loss and Attenuation

8 Results Summary

9 Statistical Summary

- **Core refractive index**: $n_1 = 1.48$
- **Cladding refractive index**: $n_2 = 1.46$
- **Core radius (SM)**: $a = 4$ μm
- **Core radius (MM)**: $a = 25$ μm
- **Relative index difference**: $\Delta = 1.35\%$
- **Acceptance angle**: $\theta_a = 14.0°$
- **Minimum attenuation at**: 1700 nm

10 Conclusion

Single-mode fibers require $V < 2.405$, achieved through small core diameters around 8-10 μm. The mode field extends beyond the core into the cladding, with the MFD increasing with wavelength. Chromatic dispersion arises from both material and waveguide contributions, with a zero-dispersion wavelength around 1310 nm for standard fiber. Multi-mode fibers support many modes that travel different paths, causing modal dispersion. The 1550 nm C-band offers minimum attenuation (0.2 dB/km) and is preferred for long-haul telecommunications, while dispersion-shifted fibers move the zero-dispersion wavelength to 1550 nm for optimized performance.