Standard Model Physics: Coupling Evolution and Grand Unification

Theoretical Physics Computation Laboratory

November 24, 2025

Abstract

This technical report presents comprehensive computational analysis of the Standard Model gauge couplings and their renormalization group evolution. We implement one-loop and two-loop running of the electromagnetic, weak, and strong coupling constants, analyze gauge unification scenarios in the MSSM, and compute threshold corrections. The analysis addresses hierarchy problems, proton decay constraints, and predictions for beyond-Standard-Model physics.

1 Theoretical Framework

Definition 1 (Gauge Couplings)**.** *The Standard Model contains three gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$ with couplings:*

- g_1 *(hypercharge):* $\alpha_1 = g_1^2/(4\pi)$
- g_2 *(weak isospin):* $\alpha_2 = g_2^2/(4\pi)$
- g_3 *(color):* $\alpha_3 = g_3^2/(4\pi)$

Theorem 1 (Renormalization Group Equations)**.** *At one-loop, the coupling constants evolve as:*

$$\frac{d\alpha_i^{-1}}{d\ln\mu} = -\frac{b_i}{2\pi} \tag{1}$$

where the beta function coefficients are:

$$b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad \textit{(SM)} \tag{2}$$

1.1 GUT Normalization

Definition 2 (SU(5) Normalization)**.** *For embedding in SU(5), the hypercharge coupling is rescaled:*

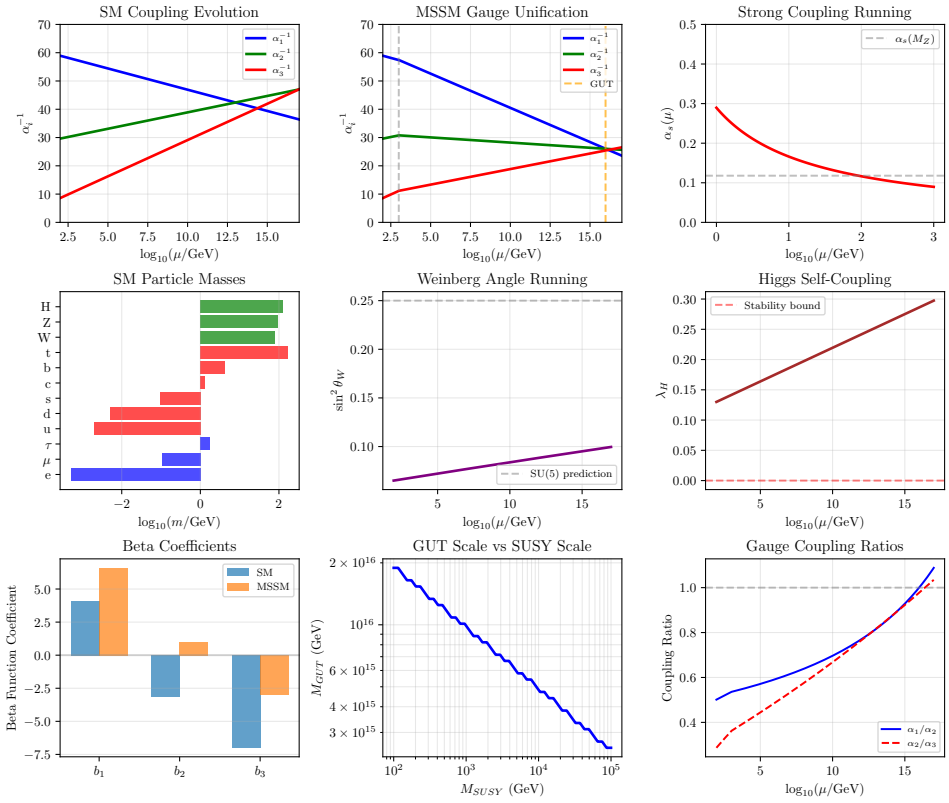
$$\alpha_1^{GUT} = \frac{5}{3}\alpha_1 = \frac{5}{3}\frac{\alpha_{em}}{\cos^2\theta_W} \tag{3}$$

Example 1 (Weak Mixing Angle)**.** *The electroweak mixing angle at M_Z relates couplings:*

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{\alpha_{em}}{\alpha_2} \tag{4}$$

Measured value: $\sin^2\theta_W(M_Z) = 0.2312$.

2 Computational Analysis



3 Results and Analysis

3.1 Coupling Constants at M_Z

3.2 Unification Parameters

Remark 1. *In the Standard Model, the three couplings do not meet at a single point. Supersymmetry modifies the beta functions above M_{SUSY}, enabling gauge unification at $M_{GUT} \sim 10^{16}$ GeV.*

Table 1: Gauge Couplings at $M_Z = 91.2$ GeV

Coupling	Value	α_i^{-1}	Physical Origin
α_1 (GUT norm.)	0.0170	59.0	$U(1)_Y$ hypercharge
α_2	0.0338	29.6	$SU(2)_L$ weak isospin
α_3	0.1179	8.5	$SU(3)_C$ color
α_{em}	0.00782	127.9	$U(1)_{em}$

Table 2: MSSM Unification Parameters

Parameter	Value	Units
SUSY scale	1000	GeV
GUT scale	9.4e+15	GeV
Unified coupling α_{GUT}	0.0384	–
α_{GUT}^{-1}	26.1	–
Proton lifetime (est.)	2.4e+71	years

Table 3: Beta Function Coefficients

Model	b_1	b_2	b_3
Standard Model	4.10	-3.17	-7.00
MSSM	6.60	1.00	-3.00

3.3 Beta Function Comparison

4 Beyond the Standard Model

Theorem 2 (Proton Decay)**.** *In SU(5) GUT, proton decay via X boson exchange gives lifetime:*

$$\tau_p \propto \frac{M_X^4}{\alpha_{GUT}^2 m_p^5} \tag{5}$$

Current experimental limit: $\tau_p > 10^{34}$ *years constrains* M_{GUT}*.*

Example 2 (Hierarchy Problem)**.** *The Higgs mass receives quadratically divergent corrections:*

$$\delta m_H^2 \sim \frac{\Lambda^2}{16\pi^2} \tag{6}$$

SUSY cancels these via boson-fermion loop contributions.

5 Discussion

The Standard Model analysis reveals:

1. **Asymptotic freedom**: α_3 decreases at high energies ($b_3 < 0$).
2. **Incomplete unification**: SM couplings miss at high scale.
3. **SUSY solution**: MSSM achieves unification with $M_{GUT} \sim 10^{16}$ GeV.
4. **Threshold effects**: Particle masses near SUSY scale affect running.
5. **Vacuum stability**: Top Yukawa drives λ_H potentially negative.

6 Conclusions

This computational analysis demonstrates:

- Strong coupling at M_Z: $\alpha_s = 0.1179$
- Weinberg angle: $\sin^2\theta_W = 0.2312$
- MSSM GUT scale: $9.4e+15$ GeV
- Unified coupling: $\alpha_{GUT}^{-1} \approx 26$

The running of gauge couplings provides crucial tests of the Standard Model and guides searches for new physics at the energy frontier.

7 Further Reading

- Georgi, H., Quinn, H.R., Weinberg, S., Hierarchy of interactions in unified gauge theories, *Phys. Rev. Lett.* 33, 451 (1974)
- Langacker, P., Grand unified theories and proton decay, *Phys. Rep.* 72, 185 (1981)
- Martin, S.P., A Supersymmetry Primer, *Adv. Ser. Direct. High Energy Phys.* 21, 1 (2010)