Machine Learning–Based Preconditioners for Dirac Operator Inversion in Lattice QCD

Lattice Quantum Chromodynamics (QCD) provides a non-perturbative framework for understanding the interactions of quarks and gluons. A central computational challenge in lattice QCD simulations is the inversion of the Dirac operator, which corresponds to solving large, sparse linear systems. The cost of this step dominates the overall simulation time, particularly as the lattice volume and physical accuracy increase. Conventional iterative solvers, such as conjugate gradient methods, often rely on carefully designed preconditioners to accelerate convergence. Techniques like multigrid approaches (Lüscher 2007; Frommer et al. 2014) and deflation strategies (Neff et al. 2001) have proven successful, but the construction of effective preconditioners for the Dirac operator remains a highly non-trivial task.

This project aims to investigate the use of machine learning methods to develop preconditioners that can reduce the cost of Dirac operator inversion. The idea is to train models capable of capturing structural properties of the operator and of generating approximate solutions or improved variable transformations that make the inversion process more efficient. While the choice of machine learning approach is flexible, particular attention will be given to real-valued non-volume preserving (realNVP) normalizing flows (Dinh et al. 2016). These provide an efficient framework for learning invertible transformations of variables, which could be leveraged to construct mappings that improve conditioning of the system or generate informed initial guesses for iterative solvers.

References

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