

第 1 讲: 测试

姓名: 张三 学号: 998244353
评阅: _____ 评分: _____

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这是适用于问题求解作业的 Typst 模板,
同时也可用于写作实验报告等。
但该模板仍在进行测试中,
请谨慎使用。

1 作业 (必做部分)

Problem 1 (AC 1.2-3)

Solution:

1 Lorem ipsum dolor.

a) when $\min(\|x\|_2)$, $x = x^*$ is the solution to the problem, which is $x^* = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

b) We have a matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$, the projection operator is

$$P = A(A^T A)^{-1} A^T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$x^* = P v = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}.$$

2 Lorem ipsum dolor sit amet.

c) We have a matrix $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{pmatrix}$, the projection operator is

$$P = A(A^T A)^{-1} A^T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$x^* = P v = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}.$$

Problem 2

Solution:

1. we know that:

$$\text{prox}_\varphi(z) = \arg \min_{x \in \mathbb{R}} \left\{ \frac{1}{2} \|x - z\|^2 + \phi(x - c) \right\}.$$

let $x' = x - c$

$$\text{prox}_\varphi(z) = \arg \min_{x \in \mathbb{R}} \left\{ \frac{1}{2} \|x' - (z - c)\|^2 + \phi(x' + c - c) \right\} + c = \text{prox}_\phi(z - c) + c.$$

2. if we want to $f(x) = \frac{1}{2} \|x - z\|^2 + \phi(x)$ to be minimized, we need to find the x that makes the derivative of the function equal to zero.

we know

$$\partial f(x) = \begin{cases} x - z + \lambda & \text{when } x > 0 \\ [x - z - \lambda, x - z + \lambda] & \text{when } x = 0 \\ x - z - \lambda & \text{when } x < 0 \end{cases}$$

. Hence, let

$$\partial f(x) = 0$$

, we have

$$\text{prox}_{\phi(z)} = x^* = \begin{cases} z - \lambda & \text{when } z > \lambda \\ [z - \lambda, z + \lambda] & \text{when } z \in [-\lambda, \lambda] \\ z + \lambda & \text{when } z < -\lambda \end{cases}$$

3. if $\varphi(x) = \lambda|x - c|$, where $c \in \mathbb{R}$ and $\lambda > 0$. Use the result from part a.

$$\text{prox}_{\varphi(z)} = \text{prox}_{\phi(z-c)} + c = \begin{cases} z - \lambda & \text{when } z > \lambda + c \\ [z - \lambda, z + \lambda] & \text{when } z \in [-\lambda + c, \lambda + c] \\ z + \lambda & \text{when } z < -\lambda + c \end{cases}$$

Problem 3

Solution:

1. If we take the derivative of $\frac{1}{2} \|x - x^{t-1}\|^2 + \gamma g(x)$, we have

$$x^t = \text{prox}_{\gamma g}(x^{t-1}) = x^{t-1} - \gamma \nabla g(x^t)$$

2. By the convexity of g , we know that $g(x^t) + \nabla g(x^t)^T (x^{t-1} - x^t) \leq g(x^{t-1})$. Hence, we have

$$g(x^t) \leq g(x^{t-1}) - \nabla g(x^t)^T (x^{t-1} - x^t) = g(x^{t-1}) - \gamma \nabla \|g(x^t)\|_2^2$$

3. because $x^t = x^{t-1} - \gamma \nabla g(x^t)$ which is a gradient descent method, so

$$-\infty < g(x^t) \leq g(x^{t-1})$$

and we have

$$g(x^t) \leq g(x^{t-1}) - \gamma \nabla \|g(x^t)\|_2^2$$

hence

$$0 \leq \gamma \nabla \|g(\mathbf{x}^t)\|_2^2 \leq 0$$

if

$$t \rightarrow +\infty$$

Problem 4 (ST 5.5-5)

Proof:

1. because

$$\partial f(\mathbf{x}) = \{\mathbf{v} \in \mathbb{R}^n : f(\mathbf{y}) \geq f(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n\}$$

if $g(\mathbf{x}) = \theta f(\mathbf{x})$,

$$\partial g(\mathbf{x}) = \{\mathbf{v} \in \mathbb{R}^n : g(\mathbf{y}) \geq g(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n\}$$

$$\partial g(\mathbf{x}) = \{\mathbf{v} \in \mathbb{R}^n : \theta f(\mathbf{y}) \geq \theta f(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n\}$$

$$\partial g(\mathbf{x}) = \left\{ \mathbf{v} \in \mathbb{R}^n : f(\mathbf{y}) \geq f(\mathbf{x}) + \frac{\mathbf{v}^T}{\theta}(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n \right\}$$

$$\partial g(\mathbf{x}) = \theta \{\mathbf{v} \in \mathbb{R}^n : f(\mathbf{y}) \geq f(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n\} = \theta \partial f(\mathbf{x})$$

2.

$$\partial h(\mathbf{x}) = \{\mathbf{v} \in \mathbb{R}^n : f(\mathbf{y}) + g(\mathbf{y}) \geq f(\mathbf{x}) + g(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n\}$$

all of the elements that satisfy

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n$$

and

$$g(\mathbf{y}) \geq g(\mathbf{x}) + \mathbf{v}^T(\mathbf{y} - \mathbf{x}), \forall \mathbf{y} \in \mathbb{R}^n$$

are in the set

$$\partial h(\mathbf{x})$$

hence

$$\partial f(\mathbf{x}) + \partial g(\mathbf{x}) \subseteq \partial h(\mathbf{x})$$

3. we know that

$$\partial \|x\|_1 = \begin{cases} 1 & \text{when } x > 0 \\ [-1, 1] & \text{when } x = 0 \\ -1 & \text{when } x < 0 \end{cases}$$

.

hence $\text{sgn}(x) \in \partial \|x\|_1$.

Problem 5

Solution:

中文排印测试:

Here's a test sentence, "I can eat glass, it does not hurt me."

这是一条测试语句: "我能吞下玻璃而不伤身体。"

這是一條測試語句: 「我能吞下玻璃而不傷身體。」

默認使用 "Noto Serif", "IBM Plex Serif" 字形, 並且設置語言為 "zh", 地區為 "cn".

目前的效果是, 當引號"兩邊有 CJK 字符, 引號將以半角顯示", 否則正常顯示英文引號。

測試: "中文引號", "quotation marks".

Problem 6

Solution:

This a test for code blocks.

For rust:

```
1 pub fn main() {  
2     println!("Hello, world!");  
3 }
```

rust

For haskell:

```
1 zipWith' :: (a -> b -> c) -> [a] -> [b] -> [c] haskell  
2 zipWith' _ [] _ = []  
3 zipWith' _ _ [] = []  
4 zipWith' f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

Select only a range of lines to show:

```
3 def fibonaci(n): python  
4     if n <= 1:  
5         return n  
6     else:  
7         return(fibonaci(n-1) + fibonaci(n-2))
```

Disable line numbers:

```
int main() {
    cout << "Hello, World!"; // 你好, 世界
    return 0;
}
```

cpp

Then pseudocodes.

Algorithm 1: The Euclidean algorithm

input: integers a and b

output: greatest common divisor of a and b

1 **while** $a \neq b$ **do**

2 **if** $a > b$ **then**

3 $a \leftarrow a - b$

4 **else**

5 $b \leftarrow b - a$

6 **end**

7 **end**

8 **return** a

▷ comment test

▷ another comment test

Algorithm 1

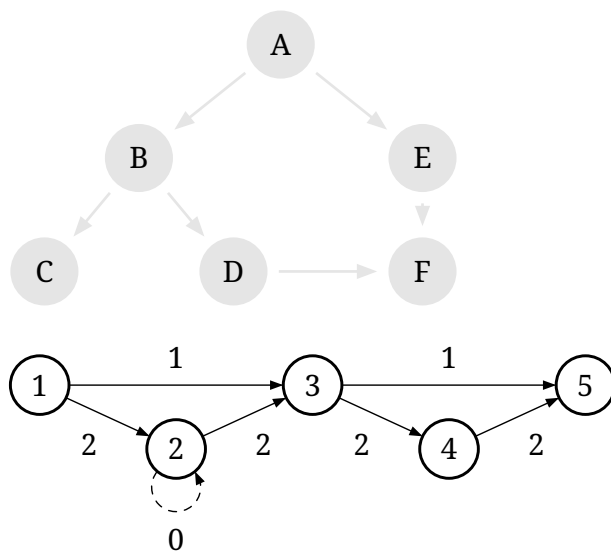
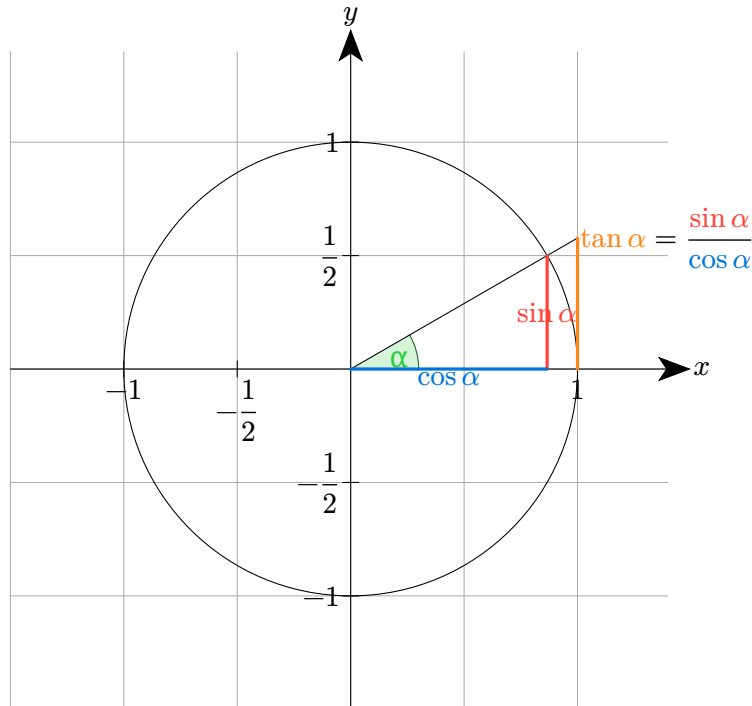
In Line 1, we have a while loop.

The algorithm figure's breakable.

Problem 7 ()

This is a test for CeTZ.

Solution:



2 作业 (选做部分)

Problem 1 (EoSD 9961)

How to pass 「レッドマジック」?

Solution:

Practice more.