

# 第 1 讲: 测试

姓名: 张三          学号: 998244353  
评分:                  评阅:

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这是适用于问题求解作业的 Typst 模板,  
同时也可用于写作实验报告等。  
但该模板仍在进行测试中,  
**请谨慎使用。**

# 1 作业 (必做部分)

## Problem 1 (AC 1.2-3)

**Solution:**

a) when  $\min(\|x\|_2)$ ,  $x = x^*$  is the solution to the problem, which is  $x^* = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

b) We have a matrix  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$ , the projection operator is

$$P = A(A^T A)^{-1} A^T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$x^* = P v = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}.$$

c) We have a matrix  $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{pmatrix}$ , the projection operator is

$$P = A(A^T A)^{-1} A^T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$x^* = P v = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}.$$

## Problem 2

**Solution:**

1. we know that:

$$\text{prox}_\varphi(z) = \arg \min_{x \in \mathbb{R}} \left\{ \frac{1}{2} \|x - z\|^2 + \phi(x - c) \right\}.$$

let  $x' = x - c$

$$\text{prox}_\varphi(z) = \arg \min_{x \in \mathbb{R}} \left\{ \frac{1}{2} \|x' - (z - c)\|^2 + \phi(x' + c - c) \right\} + c = \text{prox}_\phi(z - c) + c.$$

2. if we want to  $f(x) = \frac{1}{2}\|x - z\|^2 + \phi(x)$  to be minimized, we need to find the  $x$  that makes the derivative of the function equal to zero.

we know

$$\partial f(x) = \begin{cases} x - z + \lambda & \text{when } x > 0 \\ [x - z - \lambda, x - z + \lambda] & \text{when } x = 0 \\ x - z - \lambda & \text{when } x < 0 \end{cases}$$

. Hence, let

$$\partial f(x) = 0$$

, we have

$$\text{prox}_{\phi(z)} = x^* = \begin{cases} z - \lambda & \text{when } z > \lambda \\ [z - \lambda, z + \lambda] & \text{when } z \in [-\lambda, \lambda] \\ z + \lambda & \text{when } z < -\lambda \end{cases}$$

3. if  $\varphi(x) = \lambda|x - c|$ , where  $c \in \mathbb{R}$  and  $\lambda > 0$ . Use the result from part a.

$$\text{prox}_{\varphi(z)} = \text{prox}_{\phi(z-c)} + c = \begin{cases} z - \lambda & \text{when } z > \lambda + c \\ [z - \lambda, z + \lambda] & \text{when } z \in [-\lambda + c, \lambda + c] \\ z + \lambda & \text{when } z < -\lambda + c \end{cases}$$

### Problem 3

**Solution:**

1. If we take the derivative of  $\frac{1}{2}\|x - x^{t-1}\|^2 + \gamma g(x)$ , we have

$$x^t = \text{prox}_{\gamma g}(x^{t-1}) = x^{t-1} - \gamma \nabla g(x^t)$$

2. By the convexity of  $g$ , we know that  $g(x^t) + \nabla g(x^t)^T(x^{t-1} - x^t) \leq g(x^{t-1})$ .  
Hence, we have

$$g(x^t) \leq g(x^{t-1}) - \nabla g(x^t)^T(x^{t-1} - x^t) = g(x^{t-1}) - \gamma \nabla \|g(x^t)\|_2^2$$

3. because  $x^t = x^{t-1} - \gamma \nabla g(x^t)$  which is a gradient descent method, so

$$-\infty < g(x^t) \leq g(x^{t-1})$$

and we have

$$g(x^t) \leq g(x^{t-1}) - \gamma \nabla \|g(x^t)\|_2^2$$

hence

$$0 \leq \gamma \nabla \|g(x^t)\|_2^2 \leq 0$$

if

$$t \rightarrow +\infty$$

## Problem 4

**Solution:**

1. because

$$\partial f(x) = \{v \in \mathbb{R}^n : f(y) \geq f(x) + v^T(y - x), \forall y \in \mathbb{R}^n\}$$

if  $g(x) = \theta f(x)$ ,

$$\partial g(x) = \{v \in \mathbb{R}^n : g(y) \geq g(x) + v^T(y - x), \forall y \in \mathbb{R}^n\}$$

$$\partial g(x) = \{v \in \mathbb{R}^n : \theta f(y) \geq \theta f(x) + v^T(y - x), \forall y \in \mathbb{R}^n\}$$

$$\partial g(x) = \left\{ v \in \mathbb{R}^n : f(y) \geq f(x) + \frac{v^T}{\theta}(y - x), \forall y \in \mathbb{R}^n \right\}$$

$$\partial g(x) = \theta \{v \in \mathbb{R}^n : f(y) \geq f(x) + v^T(y - x), \forall y \in \mathbb{R}^n\} = \theta \partial f(x)$$

2.

$$\partial h(x) = \{v \in \mathbb{R}^n : f(y) + g(y) \geq f(x) + g(x) + v^T(y - x), \forall y \in \mathbb{R}^n\}$$

all of the elements that satisfy

$$f(y) \geq f(x) + v^T(y - x), \forall y \in \mathbb{R}^n$$

and

$$g(y) \geq g(x) + v^T(y - x), \forall y \in \mathbb{R}^n$$

are in the set

$$\partial h(x)$$

hence

$$\partial f(x) + \partial g(x) \subseteq \partial h(x)$$

3. we know that

$$\partial \|x\|_1 = \begin{cases} 1 & \text{when } x > 0 \\ [-1, 1] & \text{when } x = 0 \\ -1 & \text{when } x < 0 \end{cases}$$

.

hence  $\text{sgn}(x) \in \partial \|x\|_1$ .

## Problem 5

**Solution:**

This a test for code blocks.

For rust:

```
1 pub fn main() {
2     println!("Hello, world!");
3 }
```

rust

For haskell:

```
1 zipWith' :: (a → b → c) → [a] → [b] → [c]
2 zipWith' _ [] _ = []
3 zipWith' _ _ [] = []
4 zipWith' f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

haskell

Select only a range of lines to show:

```
3 def fibonaci(n):
4     if n ≤ 1:
5         return n
6     else:
7         return(fibonaci(n-1) + fibonaci(n-2))
```

python

Disable line numbers:

```
int main() {
    cout << "Hello, World!"; // 你好, 世界
    return 0;
}
```

cpp

Then pseudocodes.

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### Algorithm 1: The Euclidean algorithm

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**input:** integers  $a$  and  $b$

**output:** greatest common divisor of  $a$  and  $b$

```
1 while  $a \neq b$  do
2     if  $a > b$  then
3          $a \leftarrow a - b$ 
4     else
5          $b \leftarrow b - a$ 
6     end
7 end
8 return  $a$ 
```

▷ comment test

▷ another comment test

In Line 1, we have a while loop.

The algorithm figure' s breakable.

## 2 作业 (选做部分)

### Problem 1 (EoSD 9961)

How to pass 「レッドマジック」?

**Solution:**

Practice more.

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