第 1 讲: 测试

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评阅: ______ 评分: ______

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这是适用于问题求解作业的 Typst 模板,
也可用于其他类型的作业与报告等。
该模板仍在进行测试中,
请谨慎使用。

1 作业 (必做部分)

Problem 1 (AC 1.2-3)

Solution:

1 Lorem ipsum dolor.

a) when $\min(\|\boldsymbol{x}\|_2)$, $\boldsymbol{x} = \boldsymbol{x}^*$ is the solution to the problem, which is $x^* = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

b) We have a matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$, the projection operator is

$$\boldsymbol{P} = \boldsymbol{A}(\boldsymbol{A}^T A)^{-1}\boldsymbol{A}^T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$\boldsymbol{x}^* = \boldsymbol{P}\boldsymbol{v} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}.$$

2 Lorem ipsum dolor sit amet.

c) We have a matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{pmatrix}$, the projection operator is

$$\boldsymbol{P} = \boldsymbol{A}(\boldsymbol{A}^T A)^{-1}\boldsymbol{A}^T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$\boldsymbol{x}^* = \boldsymbol{P}\boldsymbol{v} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}.$$

Problem 2

Solution:

1. we know that:

$$\operatorname{prox}_{\varphi}(z) = \arg\min_{x\in\mathbb{R}}\left\{\frac{1}{2}\|x - z\|^2 + \phi(x - c)\right\}.$$

let $x' = x - c$

$$\operatorname{prox}_{\varphi}(z) = \arg\min_{x\in\mathbb{R}}\left\{\frac{1}{2}\|x' - (z-c)\|^2 + \phi(x'+c-c)\right\} + c = \operatorname{prox}_{\phi}(z-c) + c.$$

2. if we want to $f(x) = \frac{1}{2}\|x - z\|^2 + \phi(x)$ to be minimized, we need to find the x that makes the derivative of the function equal to zero.

   we know

$$\partial f(x) = \begin{cases} x - z + \lambda \text{ when } \ x > 0 \\ [x - z - \lambda, x - z + \lambda] \text{ when } x = 0 \\ x - z - \lambda \text{ when } x < 0 \end{cases}$$

   . Hence, let

$$\partial f(x) = 0$$

   , we have

$$\operatorname{prox}_{\phi(z)} = x^* = \begin{cases} z - \lambda \text{ when } \ z > \lambda \\ [z - \lambda, z + \lambda] \text{ when } z \in [-\lambda, \lambda]. \\ z + \lambda \text{ when } z < -\lambda \end{cases}$$

3. if $\varphi(x) = \lambda|x - c|$, where $c \in \mathbb{R}$ and $\lambda > 0$. Use the result from part a.

$$\operatorname{prox}_{\varphi(z)} = \operatorname{prox}_{\phi(z-c)} + c = \begin{cases} z - \lambda \text{ when } \ z > \lambda + c \\ [z - \lambda, z + \lambda] \text{ when } z \in [-\lambda + c, \lambda + c] \\ z + \lambda \text{ when } z < -\lambda + c \end{cases}$$

Problem 3

Solution:

1. If we take the derivative of $\frac{1}{2}\left\|\boldsymbol{x} - \boldsymbol{x}^{t-1}\right\|^2 + \gamma g(\boldsymbol{x})$, we have

$$\boldsymbol{x^t} = \operatorname{prox}_{\gamma g}(\boldsymbol{x}^{t-1}) = \boldsymbol{x}^{t-1} - \gamma\nabla g(\boldsymbol{x^t})$$

2. By the convexity of g, we know that $g(\boldsymbol{x}^t) + \nabla g(\boldsymbol{x^t})^T(\boldsymbol{x}^{t-1} - \boldsymbol{x^t}) \le g(\boldsymbol{x^{t-1}})$. Hence, we have

$$g(\boldsymbol{x}^t) \le g(\boldsymbol{x^{t-1}}) - \nabla g(\boldsymbol{x^t})^T(\boldsymbol{x}^{t-1} - \boldsymbol{x^t}) = g(\boldsymbol{x^{t-1}}) - \gamma\nabla\left\|g(\boldsymbol{x^t})\right\|_2^2$$

3. because $\boldsymbol{x^t} = \boldsymbol{x}^{t-1} - \gamma\nabla g(\boldsymbol{x^t})$ which is a gradient descent method, so

$$-\infty < g(\boldsymbol{x}^t) \le g(\boldsymbol{x}^{t-1})$$

   and we have

$$g(\boldsymbol{x}^t) \le g(\boldsymbol{x^{t-1}}) - \gamma\nabla\left\|g(\boldsymbol{x^t})\right\|_2^2$$

   hence

$$0 \le \gamma\nabla\left\|g(\boldsymbol{x^t})\right\|_2^2 \le 0$$

if

$$t \to +\infty$$

Problem 4 (ST 5.5-5)

Proof:

1. because

$$\partial f(\boldsymbol{x}) = \{\boldsymbol{v} \in \mathbb{R}^n : f(\boldsymbol{y}) \geq f(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\}$$

if $g(\boldsymbol{x}) = \theta f(\boldsymbol{x})$,

$$\partial g(\boldsymbol{x}) = \{\boldsymbol{v} \in \mathbb{R}^n : g(\boldsymbol{y}) \geq g(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\}$$

$$\partial g(\boldsymbol{x}) = \{\boldsymbol{v} \in \mathbb{R}^n : \theta f(\boldsymbol{y}) \geq \theta f(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\}$$

$$\partial g(\boldsymbol{x}) = \left\{\boldsymbol{v} \in \mathbb{R}^n : f(\boldsymbol{y}) \geq f(\boldsymbol{x}) + \frac{\boldsymbol{v}^T}{\theta}(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\right\}$$

$$\partial g(\boldsymbol{x}) = \theta\{\boldsymbol{v} \in \mathbb{R}^n : f(\boldsymbol{y}) \geq f(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\} = \theta \partial f(\boldsymbol{x})$$

2.

$$\partial h(\boldsymbol{x}) = \{\boldsymbol{v} \in \mathbb{R}^n : f(\boldsymbol{y}) + g(\boldsymbol{y}) \geq f(\boldsymbol{x}) + g(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n\}$$

all of the elements that satisfy

$$f(\boldsymbol{y}) \geq f(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n$$

and

$$g(\boldsymbol{y}) \geq g(\boldsymbol{x}) + \boldsymbol{v}^T(\boldsymbol{y} - \boldsymbol{x}), \forall \boldsymbol{y} \in \mathbb{R}^n$$

are in the set

$$\partial h(\boldsymbol{x})$$

hence

$$\partial f(\boldsymbol{x}) + \partial g(\boldsymbol{x}) \subseteq \partial h(\boldsymbol{x})$$

3. we know that

$$\partial \|x\|_1 = \begin{cases} 1 \text{ when } \ x > 0 \\ [-1, 1] \text{ when } x = 0 \\ -1 \text{ when } x < 0 \end{cases}$$

.

hence $\mathrm{sgn}(x) \in \partial \|x\|_1$.

Problem 5

Solution:

中文排印测试：

Here’s a test sentence, “I can eat glass, it does not hurt me.”

这是一条测试语句：“我能吞下玻璃而不伤身体。”

這是一條測試語句：「我能吞下玻璃而不傷身體。」

默認使用 “Noto Serif”, “IBM Plex Serif” 字形， 並且設置語言為 “zh”， 地區為 “cn”。

目前的效果是，當引號"兩邊有 CJK 字符，引號將以半角顯示“，否則正常顯示英文引號。

測試：“中文引號”, “quotation marks”.

Problem 6

Solution:

This a test for code blocks.

For rust:

```rust
pub fn main() {
    println!("Hello, world!");
}
```

For haskell:

```haskell
zipWith' :: (a → b → c) → [a] → [b] → [c]
zipWith' _ [] _ = []
zipWith' _ _ [] = []
zipWith' f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

Select only a range of lines to show:

```python
def fibonaci(n):
  if n ≤ 1:
    return n
  else:
    return(fibonaci(n-1) + fibonaci(n-2))
```

Disable line numbers:

```cpp
int main() {
  cout << "Hello, World!"; // 你好，世界
  return 0;
}
```

Then pseudocodes.

Algorithm 1: The Euclidean algorithm

input: integers a and b
output: greatest common divisor of a and b

1 **while** $a \neq b$ **do**
2   **if** $a > b$ **then**
3     $a \leftarrow a - b$
4   **else**
5     $b \leftarrow b - a$ ▷ comment test
6   **end** ▷ another comment test
7 **end**
8 **return** a

Algorithm 1

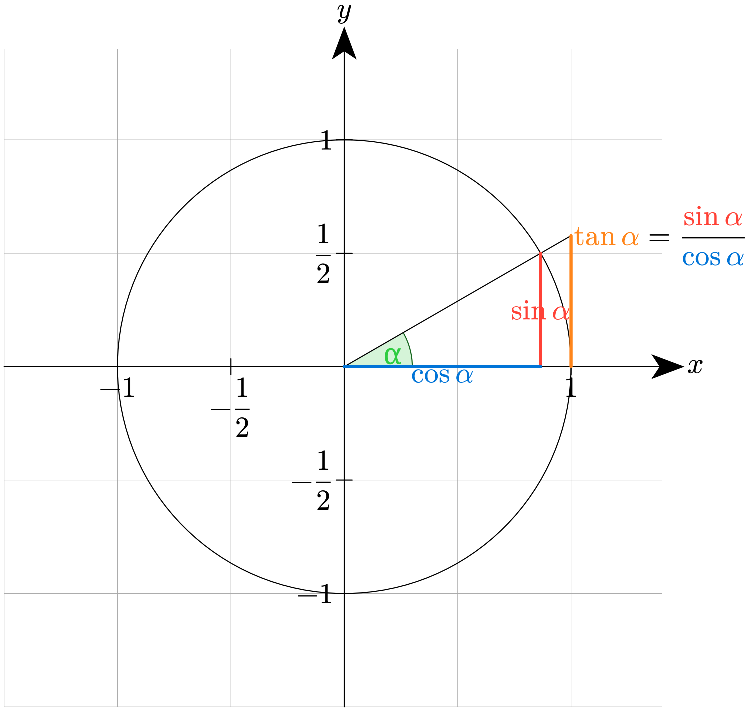
In Line 1, we have a while loop.

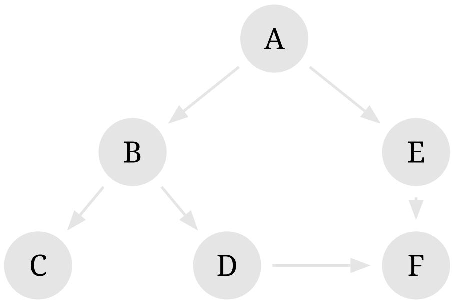
The algorithm figure’s breakable.

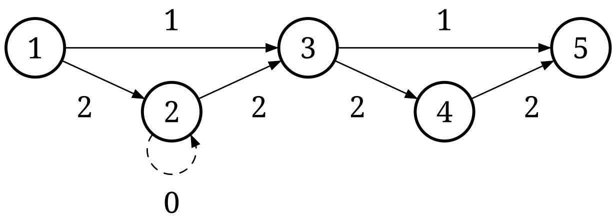
Problem 7 ()

This is a test for CeTZ.

Solution:







2 作业 (选做部分)

Problem 1 (EoSD 9961)

How to pass 「レッドマジック」?

Solution:

Practice more.