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Spatial organization in 2D segmented images: Representation and recognition of primitive spatial relations ☆

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Abstract

Object recognition and scene analysis tasks can be greatly enhanced when information about spatial organization in an image is available. Moreover, for recognition of complex objects a suitable representation of spatial relations between objects' components taking into account shape, size, orientation, etc., is required. This cannot be accomplished by reducing a region to one or a few representative points; instead the region as a whole must be treated.

This paper presents a fuzzy logic approach to the representation and recognition of spatial relations between regions in a 2D image. The main source of information on spatial relations is the geometry of the regions in question and we argue that this is complex enough to cause ambiguity in spatial relations, and hence to warrant a fuzzy logic approach. The basic idea is to calculate the angles between the line connecting two points (one in each region) and the horizontal line, to construct a histogram of these angles, and then upon an interpretation of the histogram as a fuzzy set to match it with the fuzzy sets representing a vocabulary of spatial relations. Other expressions of the spatial information which may be context dependent can be easily obtained by adding context knowledge. Several examples are used to illustrate our approach.

Key words: Pattern recognition; Spatial relations; Compatibility; Converse truth qualification

1. Introduction

Object recognition is an important task of machine vision. Model based methods for this task make use of knowledge about the objects to be recognized, typically in the form of an object model which may describe the object in terms of its constituent components. Production rules, semantic networks and frame-based knowledge representation have been used to express the object model. Components have attributes whose values can be obtained from image processing, such as shape, texture and spectrum. Then objects are recognized by matching extracted image features with the object model.

Relational graphs provide a way of representing the object model such that constraints on the

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location of the components are also included. A node of the graph represents a component and its features. A link between nodes represents a relation between the corresponding components. The use of relational graphs allows for a better representation of the topological relations and semantic constraints among the components of the object. However, relational graphs are limited in capturing the 3D structure of the object and context dependent spatial relations (including distance and direction of the objects).

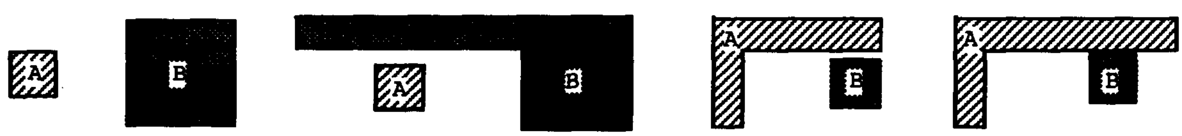
Spatial relations between objects, or components of objects provide important information regarding the content of the image, in particular for recognition. In fact, this study is motivated by our previous work in object recognition [8], in which we showed that information (hand coded) on spatial relations could be used to improve recognition results. A natural step is then the design of methods which can automatically evaluate the spatial relations between regions. To capture the ambiguity inherent in spatial relations we propose to set the problem of representation of spatial relations between two regions in the framework of fuzzy logic. More precisely, geometrical properties (such as shape, distance) of the regions in question are taken into account to obtain a context sensitive description and recognition method.

From this point onwards, this paper is organized as follows: in Section 2 we will discuss the need to capture and express the ambiguity of spatial relations; Sections 3 and 4 present the issues of representation and recognition of spatial relations between crisp regions; Section 5 extends the results of Section 3 to the case of fuzzy subsets of the image; Finally, Section 6 presents the experimental results for both the crisp and fuzzy case.

2. Ambiguity of spatial relations

Information on spatial organization in an image is both useful and at the same time difficult to obtain. The method of representation of this information greatly affects the kind of results obtained. In connection with this problem Marr [7] noted that "The necessity for representing spatial relations, with its attendant complexities of how much should be made explicit and how much can safely be left implicit, raises problems that are typical of and rather special to vision". We also note that the purpose for which the spatial information is needed will affect the choice of a method. If, as in this study, the purpose is image understanding, including object recognition and scene analysis, the method is different than if the purpose were spatial reasoning for a navigational aid system.

An object model may contain information about the relative location of some components, such as "A is right of B" or "C is below of D", but it is difficult to determine and recognize such spatial relations. For illustration purposes let us consider the spatial relations "left of" between the two objects of Fig. 1(a)–(d). What exactly does it mean to say that "A is left of B"? Should all the points in A be left to all the points in B as is the case in (a) and (b)? Should the relation hold between their centers of gravity (which is the case in all of (a)–(d))? But if the latter is sufficient it would follow that the relation "left of" holds to the same degree in all the examples of Fig. 1, which is certainly not the case. It seems reasonable to think that an integration of various criteria will provide an answer which would be more sensitive to the particular image considered. The question is then which such criteria should be used. Taking into consideration this discussion we could define the spatial relation "A is



(a) (b) (c) (d)

Fig. 1. Ambiguity of "A is left of B".

left of B" for Fig. 1 as follows:

(a) the center of gravity of A is left to the center of gravity of B.

(b) the rightmost point of A is left to the rightmost point of B [11].

Note that the shapes in Fig. 1 are simple, in the sense that they are well defined geometric shapes (or composed of such). For these shapes geometric characteristics can be extracted easily and therefore can be used to define the spatial relations between objects. Yet we can imagine situations in which the shapes are not of this type, but more complex and for which the same characteristics may be more difficult to extract.

It is also easy to see that the criteria (a) and (b) above are still not satisfactory. As an example, let us consider the two cases of Fig. 2: the centers of gravity as well as the shapes of the two objects are exactly the same in Fig. 2(a) and (b), yet the perception of the spatial relation "A is lower left of B" is different in (a) and (b). In (b) the degree to which A is lower than B appears higher than in (a).

The main point illustrated by Figs. 1 and 2 is that even in the case of simple or very simple objects an adequate representation of spatial relations allowing for context dependency is necessary, yet difficult. Other aspects increase this difficulty even more; the distance between regions affects the influence of size on the perception of spatial relations; the farther the regions are from each other, the less their spatial relations are affected by their shapes. By contrast, when the regions are close, their shapes and sizes will strongly affect their spatial relation.

As already indicated above, we have chosen to represent spatial relations as fuzzy sets. This choice is very natural when we consider the ambiguity of spatial relations, when we think of how gradual changes of the relative position of two regions should result in gradual changes in the perception of spatial relation between them. Qualitative descriptions of spatial relations have been presented previously: in [13], spatial relations are based on the potential model between two points in a 2D world; Abella [1] measured geometrical features such as width, height, center of gravity, principle axis of the objects and defined "NEAR", "FAR", "ABOVE", "BELOW" etc. However, the approach makes it impossible to distinguish between different cases of different shape regions whose representative points are the same (Fig. 3).

The use of fuzzy sets as position, or spatial relation descriptors has also been adopted previously: in geographical information systems [10, 5] and others. The work closer to ours is that of Keller and Sztandera [4] and more recently of Krishnapuram et al. [6]. Both [4] and [6] start with the assumption that the result of the image processing are fuzzy regions of the image plane (these regions are obtained from a fuzzy image segmentation method such as fuzzy clustering for example). This assumption justifies then the use of the fuzzy logic approach. For crisp regions the approach in [4] and [6] reduces to the spatial relations between the centers of gravity of the regions in question.

In the current work we give an alternative approach to problems considered in [4] and [6]. We concentrate on the representation and description of relative spatial relations between regions of arbitrary shapes. An important aspect of our study is that fuzzy sets are the suitable representation even

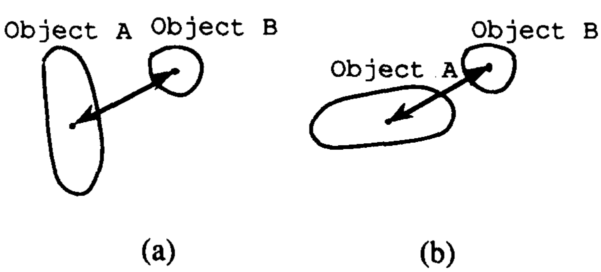


Fig. 2. Example of different linguistic descriptions according to orientation.

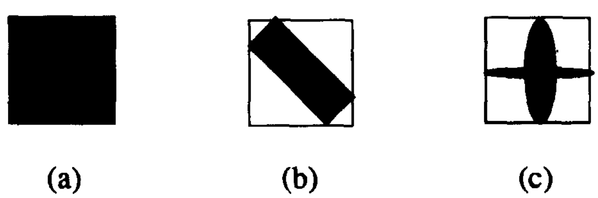


Fig. 3. Example of different shapes whose width, height, and centers of gravity are the same.

when the regions in question are crisp. In other words, the geometry of the crisp regions offers sufficient complexity to justify the use of fuzzy set for representation of spatial relations. Our method can however be applied to fuzzy regions as well. Our approach is strongly dependent on the assumption of the image processing and is based on taking into account the whole regions which are finite collections of crisp data (pixels).

3. Representation of spatial relations between crisp regions

Throughout this paper we are concerned with statements of the type "$A\ R\ B$" where R denotes a spatial relation. In this statement we call A the argument and B the referent. The relation R can take values in a set of linguistic labels which correspond to primitive spatial relations [9]: {"right of", "left off", "below", "above"}. Each A and B can be points or regions, and in the latter case crisp and/or fuzzy regions are allowed. The fuzzy case is presented as a straight generalization of the crisp case. Capital letters are used to indicate regions (sets) while lower case letters are used to indicate points in regions (members of sets).

3.1. Spatial relations between points

We start from the simplest case when both the referent and argument are points, say p and q as shown in Fig. 4.

From classical mathematical considerations we know that the relative position between points can

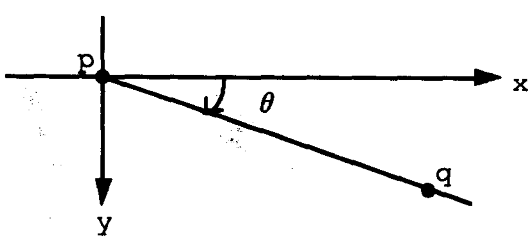


Fig. 4. Spatial relation between two points in xy coordinate system.

be expressed accurately as a function of the angle made by the line passing through these points and the x-axis of a system of coordinates in which these points are represented. We know that an angle opened towards the positive x-axis indicates a "right-of" relation: the sharper the angle the more the relation holds. A similar argument holds for other directions, associated with the opening of the angle towards positive or negative y-axis or negative x-axis.

Assumption 1: The spatial relations between the points p and q are determined by the angle θ made by the line passing through the two points and the x-axis.

Assumption 2: The spatial relations between the points p and q are fuzzy sets whose membership functions are given by trigonometric functions of θ.

In our approach, we use $\cos^2\theta$ and $\sin^2\theta$, as originally done in [9] to indicate the degree of "lateral" and "vertical" spatial relations. The trigonometric functions are a natural choice. However, here our motivation goes beyond this being the usual choice. Practitioners of fuzzy logic approach usually favor piecewise linear functions and in this view we must justify why this is not done here. Let us consider defining the spatial relation "right of": any function $f(\theta)$ which would satisfy the requirements: $f(\theta) = 0$, for $\theta = \pm\pi/2$, $f(\theta) = 1$ at $\theta = 0$, and equal to 0 otherwise seems suitable. If this function is linear it implies a change in the spatial relation proportional with the change in θ. If, however, we want to model a different kind of change – for example that as θ takes values from 0 to $\pi/2$ the change of spatial relation reflects the fact that initially the dominant spatial relation is "right-of", while later the dominant spatial relation is "above" passing through the point $\theta = \pi/4$ where both relations are equally perceived – a different choice is necessary. Based on this we decided that even powers of the trigonometric functions, that is $\cos^{2k}\theta$ and $\sin^{2k}\theta$, are best candidates for modeling the membership functions of spatial relations. The choice of k allows us to control the rate with which the relation changes. In this study we choose $k = 1$. This value gives the only membership functions which show correct behavior at the values of θ where two spatial relations are equally detected: $\cos^2(\pi/4) = \sin^2(\pi/4) = \frac{1}{2}$.

Thus the membership functions for the primitive spatial relations are as follows (Fig. 5):

$$\mu_{\text{right}}(\theta) = \begin{cases} \cos^2\theta & \text{if } -\pi/2 \leqslant \theta \leqslant \pi/2, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

$$\mu_{\text{below}}(\theta) = \begin{cases} \sin^2\theta & \text{if } 0 \leqslant \theta \leqslant \pi, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

$$\mu_{\text{above}}(\theta) = \begin{cases} \sin^2\theta & \text{if } -\pi \leqslant \theta \leqslant 0, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

$$\mu_{\text{left}}(\theta) = \begin{cases} \cos^2\theta & \text{if } -\pi \leqslant \theta \leqslant -\pi/2, \pi/2 \leqslant \theta \leqslant \pi, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

3.2. Spatial relations between regions

We consider directly the case when both the argument and the referent are sets of points. For the beginning we assume that both A and B are crisp sets (lump regions). Since both A and B are results of image processing they are both finite sets: $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_m\}$ with n and m not necessarily equal. Thus we reduce this case at considering $m \times n$ pairs of points (a_i, b_j), $i = 1, \ldots, n, j = 1, \ldots, m$, to which Section 3.1 applies. Let Θ denote the collection of angles $\theta_{ij} = \angle(a_i, b_j)$, $a_i \in A$ and $b_j \in B$. Obviously Θ is a multiset, as different pairs of points may result in the same angle θ.

For each $\theta \in \Theta$ we define f_θ to be the number of pairs (a_i, b_j) for which $\angle(a_i, b_j) = \theta$, that is, f_θ is the frequency of θ in Θ. Then $H_\Theta(A,B) = \{(\theta, f_\theta)\}$ defines the histogram associated to Θ. Obviously for every θ, $(\theta, f_\theta) \in H_\Theta$, $1 \leqslant f_\theta \leqslant m \times n$. From this the histogram of relative frequencies can be obtained, $f_\theta = f_\theta/(m \times n)$, which can be further normalized (dividing by the largest frequency).

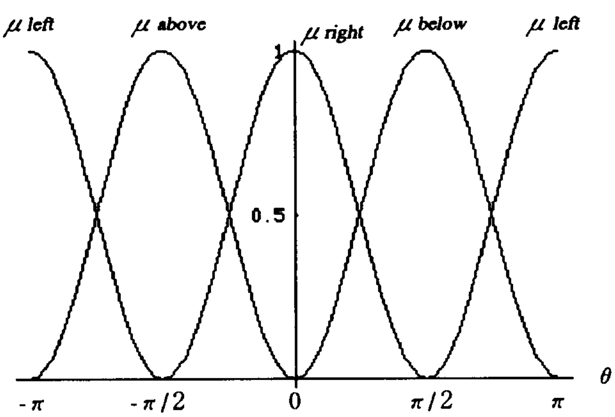


Fig. 5. Fuzzy sets which represent degree of spatial relation for four directions.

Several properties of the histogram recommend it for use in our problem:

(1) *Quasi invariance with respect to position*:

$$H_\Theta(A, B) = H_\Theta(B, A) + \pi$$

where

$$H_\Theta(B, A) + \pi = \{(\theta + \pi, f)/(\theta, f) \in H_\Theta(B, A)\}$$

(2) *Quasi invariance with respect to change of the system of coordinates*: If $H^{S1}\Theta^{(A,R)}$, and $H^{S2}\Theta^{(A,R)}$ denote the histograms associated with the angles corresponding to the regions A and B in two systems of coordinates S_1 and S_2 (both positioned in A). Then $H^{S1}\Theta^{(A,R)} = H^{S2}\Theta^{(A,R)} + \tau$ where τ is the angle between the coordinate systems S_1 and S_2.

(3) *Dependence on shape of regions*: The shape of a region is determined by the location of each of its points. Thus if the shape changes the angles and/or the frequencies recorded in the histogram change (Fig. 6).

Also, if we imagine for example, a region from which points are eliminated successively, the frequencies of the corresponding angles decrease and hence the histogram changes.

(4) *Dependence on the distance between regions*: As the distance between regions changes the shape and the location of the histogram change. To see this, it is enough to consider the special case when the referent region is reduced to one point $\{b\}$ and the argument region is reduced to two points $\{a_1, a_2\}$. Let θ_1, and θ_2 be the corresponding angles. As the argument region is translated along the positive x-axis, away from b, the angles θ_1, and θ_2 decrease since the x-coordinates of a_1 and a_2 increase, while their y-coordinates remain constant; the same argument holds for translation in both directions along each axis. Moreover, it should be noted that it is impossible to affect the distance between the regions (here point and region) such that all angles will remain constant. Thus in an arbitrary movement of the argument the histogram

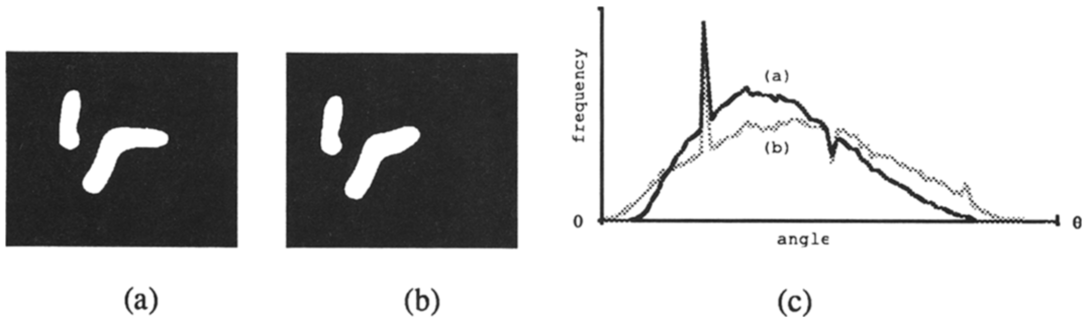


Fig. 6. Example of shape effect on the histogram.

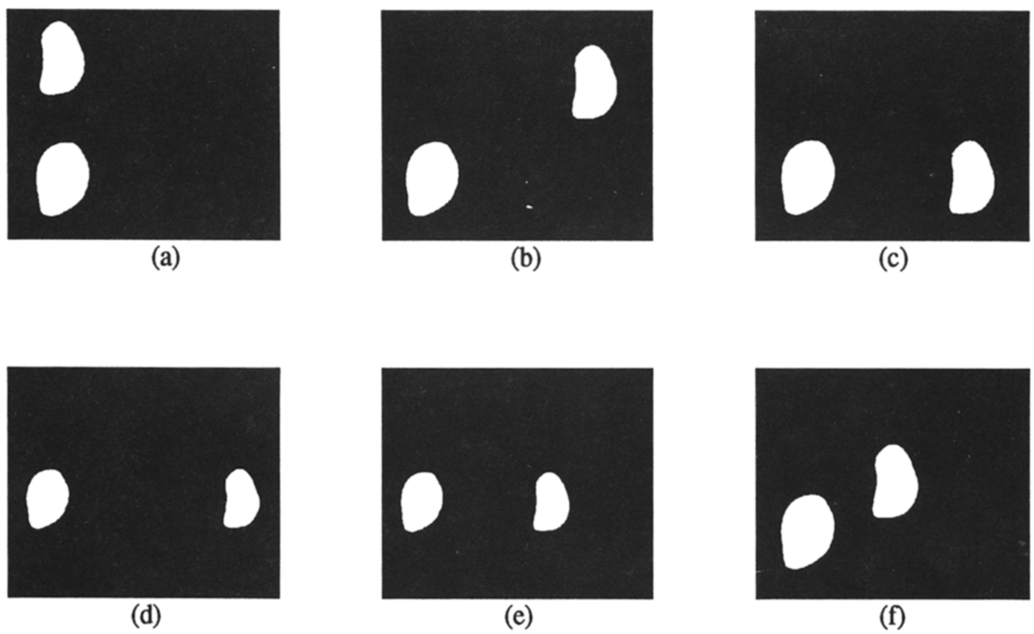


Fig. 7. Pairs of regions with varying distances between them.

will be affected as in the vertical or horizontal translating according to the cumulative effect of the movement. Figs. 7 and 8 illustrate the above discussion for the general case of two regions.

Properties (1)–(4) form the motivation for our use of the histogram to capture and evaluate spatial relations between regions.

Before we continue we should point out that since the histogram is a summarization of data, certain kinds of information about regions cannot be recovered from the histogram; all inverse problems in which we provide the referent, the histogram with the argument, and we want to recover the argument are difficult to solve, as many candidate regions are possible. Anyway, these kinds of problems appear more in the framework of spatial reasoning which is not considered in this paper.

4. Recognition of spatial relations between crisp regions

So far we have defined spatial relations as trigonometric functions and we obtained the

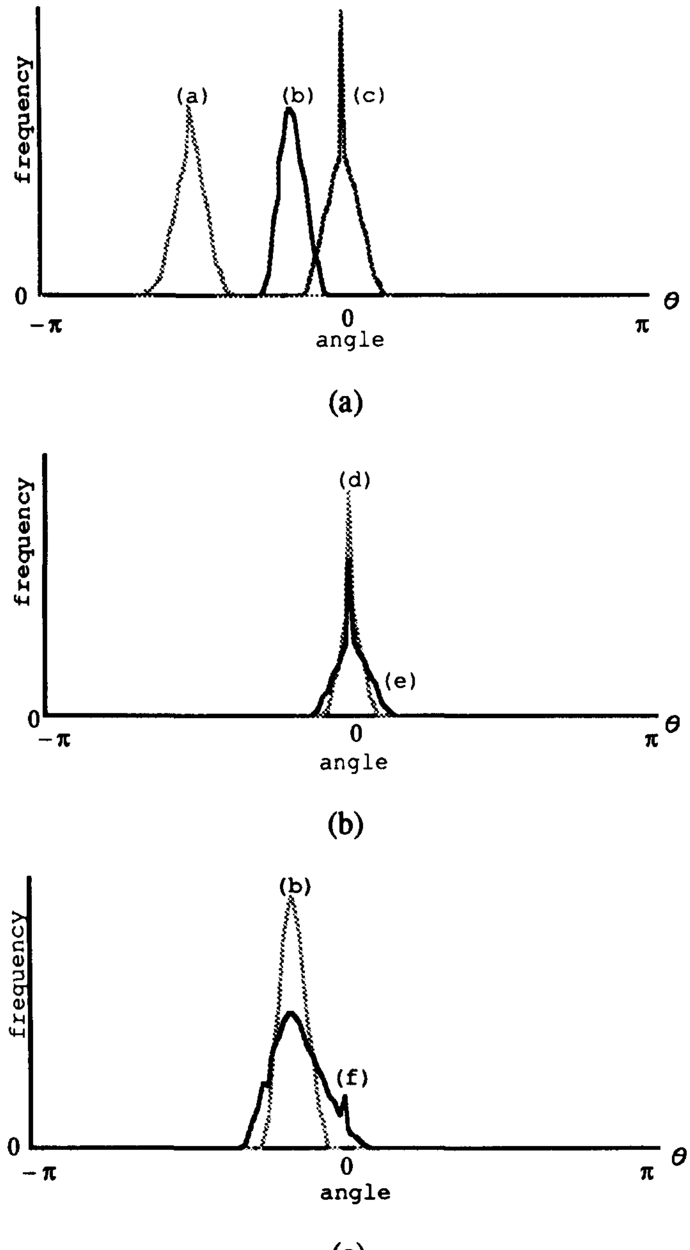


Fig. 8. The histograms of angles for the example regions of Fig. 6. The histograms are very sensitive to the distance between regions: as the distance increases the histograms approach the value corresponding to the centers of gravity.

histogram associated to the angles corresponding to two regions A and B. The question is how to use the histogram in order to evaluate the degree to which a spatial relation might hold, the degree of recognition. The answer to this question depends on the interpretation of the histogram. Strictly speaking the histogram is a frequency distribution. However, this meaning is departed from as operations of normalization, approximations and/or smoothing are applied to the histogram. In this case the histogram, H, an unlabeled fuzzy set, can be interpreted as "the spatial relation between A and B". Given the fuzzy sets "right of", "left of", "above", and "below" we ask to what extent is H each of these spatial relations? We treat this as a problem of compatibility of fuzzy sets.

In general, according to [2] the compatibility between two fuzzy sets F, G is the extension of evaluation of a fuzzy set in a point to a fuzzy set; the result is a fuzzy set $CP(F; G)$ whose membership function $\mu_{CP(F;G)}: [0, 1] \to [0, 1]$ obtained by the extension principle is defined as

$$\mu_{CP(F;G)}(v) = \sup_{s,\, v = \mu_F(s)} \mu_G(s)$$
$$= 0 \quad \text{if } \mu_F^{-1}(v) = \emptyset. \tag{5}$$

In our case F is the histogram and G is the fuzzy set R, associated to a spatial relation, as defined in (1)-(4) in Section 3. The final degree to which a spatial relation holds is obtained as the center of gravity of the compatibility fuzzy set, $CP(H, R)$.

5. Spatial relations between fuzzy regions

As we already mentioned earlier in this paper our main goal was to show the suitability and advantages of the fuzzy logic approach to the representation of spatial relations between crisp regions, that is in the classical framework of computer vision, where the results of image processing are exact, lump regions.

In this section we show that our approach can be extended in a straightforward manner to the fuzzy case. Let the regions A and B fuzzy subsets of the image plane: $A = \{(a_i, \mu_A(a_i));\ i = 1, \ldots, n\}$ and $B = \{(b_j, \mu_B(b_j));\ j = 1, \ldots, m\}$. We calculate again $\theta_{ij} = \angle(a_i, b_j)$ for all possible pairs (a_i, b_j), and we call Θ the collection of θ_{ij}. Θ is now a fuzzy multiset: a given value of θ may appear more than once; for each θ_{ij} in Θ we associate a membership value $\mu_\Theta(\theta_{ij})$, obtained from the corresponding $\mu_A(a_i)$ and $\mu_B(b_j)$. A natural choice for combining these is a t-norm, as we say that θ belongs to Θ if it is an angle between points in A and B, and the degree to which these points are in A *and* B respectively gives the degree to which θ is in Θ. Here we define $\mu_\Theta(\theta_{ij}) = \min(\mu_A(a_i), \mu_B(b_j))$. The histogram associated to Θ is constructed making use of the notions of fuzzy count and relative fuzzy count [2] which can be defined both as crisp or fuzzy quantities. For illustration purposes here we use the definitions which result in crisp quantities: we have that the cardinality of Θ is $|\Theta_\theta| = \sum \mu_\Theta(\theta_{ij})$; $i = 1, \ldots, n, j = 1, \ldots, m$. Next, for θ in Θ we define its frequency as $f_\theta = |\Theta_\theta|$ where $|\Theta_\theta| = \sum \mu_\Theta(\theta_{ij})$; $\theta_{ij} = \theta$. The relative frequency is defined as $f_\theta / |\Theta_\theta|$.

The histogram associated to Θ is then defined identically as in the crisp case, that is the set of pairs (θ, f_θ), where f_θ is the (relative) frequency of θ in Θ, the only difference being in how f_θ is calculated. It is easy to see that the above approach is consistent with the crisp case.

6. Experimental results

In this section, we show the experimental results obtained by applying our method to two groups of regions: crisp and fuzzy subsets of the image plane.

6.1. Experimental results for crisp regions

We applied this method to the recognition of the spatial relation "A is left of B" for the regions A and B of Fig. 1(a)-(d) where A and B denote the argument and referent regions respectively. Fig. 9 shows the regions which are used in this experiment. For evaluation of results we compared them with those obtained using only the spatial relation between the centers of gravity of A and B. The latter are obtained by substituting the angle θ between the line connecting two centers of gravity of the two regions and the horizontal line in the formulae (1)-(4). The results are shown in Table 1. Note that we have not normalized the recognition results. That is the degrees with which individual spatial relations are detected for each case do not add up to 1. If desired such a normalization can be easily done. However, previous, informal experiments showed us that the perception of spatial relations may not necessarily correspond to normalized values.

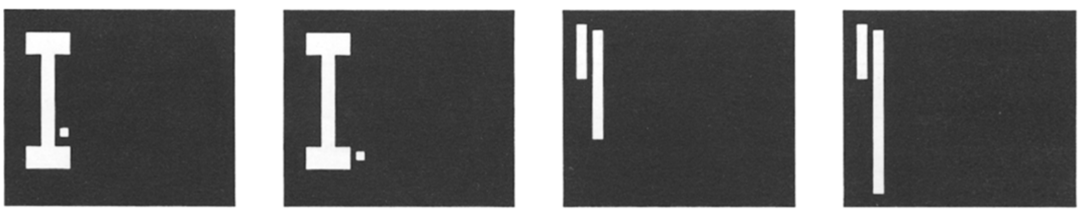
As Table 1 indicates, the method using centers of gravity tends to give recognition results very close to crisp values, one relation is recognized with

(a) (b) (c) (d)

Fig. 9. Geometric figures used in the experiment; these reproduce the examples of Fig. 1.

Table 1
Results of degree of location between reference and argument objects for Fig. 9

	Centers of gravity				Our method			
	right	left	above	below	right	left	above	below
(a)	0.000	1.000	0.000	0.000	0.000	0.962	0.038	0.032
(b)	0.000	0.986	0.000	0.014	0.383	0.739	0.044	0.261
(c)	0.000	0.935	0.065	0.000	0.089	0.608	0.392	0.084
(d)	0.000	0.908	0.092	0.000	0.261	0.608	0.392	0.084



(a) (b) (c) (d)

Fig. 10. Cases not suitable for definitions using centers of gravity only.

degree close to 1, the remaining with degrees 0 or close to 0.

In Fig. 10 we show typical examples where the use of the center of gravity gives particularly unsatisfactory results: In Fig. 10(a) and (b), the arguments are the small squares situated in the lower right side of the referents; in Fig 10(c) and (d), the arguments are the longer rectangles situated to the right of the shorter ones. As Table 2 indicates, in each case the results of degree of "right of" obtained by the method using centers of gravity are much lower than the ones obtained by our method.

Finally, we applied our method to Winston's [12] examples, Fig. 11, who suggested that the spatial relations depend on the extent of regions in both directions. In Fig. 11 the referents are the bigger rectangles. The results are contained in Table 3: the degree of "above" obtained using the centers of gravity are much lower than the ones obtained by our method.

6.2. Experimental results for fuzzy regions

We present next the experimental results obtained for fuzzy subsets of the image (Fig. 12). Once again

Table 2
Results of degree of location between reference and argument objects for Fig. 8

	Centers of gravity				Our method			
	right	left	above	below	right	left	above	below
(a)	0.219	0.000	0.000	0.781	0.351	0.006	0.466	0.672
(b)	0.243	0.000	0.000	0.757	0.427	0.000	0.104	0.576
(c)	0.200	0.000	0.000	0.800	0.340	0.000	0.305	0.661
(d)	0.069	0.000	0.000	0.931	0.309	0.000	0.190	0.692

Fig. 11. Extent dependent spatial relations between identical pairs of regions.

Table 3
Results of degree of location between reference and argument objects for Fig. 11

	Centers of gravity				Our method			
	right	left	above	below	right	left	above	below
(b)	0.000	0.849	0.151	0.000	0.000	0.719	0.281	0.000
(b)	0.000	0.654	0.346	0.000	0.140	0.598	0.402	0.000

the results are compared to those obtained from using the centers of gravity. As it can be seen from Table 4 this method cannot distinguish between (a), (b) and (c), (d). By contrast our method is much more sensitive and it detects correctly the differences between (a) and (b) on one hand and (c) and (d) on the other.

Likewise a consistent behavior of our method is illustrated in Fig. 13 and the associated Table 5.

A word about the computational aspect of the approach of evaluating spatial relations introduced in this paper: to calculate the histogram we must consider all the pairs of points from the regions considered, that is $|A| \times |B|$ pairs, where $|A|$ and $|B|$ denote the size of the region A and B respectively. This is the most expensive step of the approach presented. The number of pairs can be reduced, for instance, by sampling, while the subsequent treatment remains unchanged.

7. Conclusions

We have shown in this paper that fuzzy sets can be used to represent spatial relations. This work argues that, even in the classical framework of computer vision in which the results of the image processing are exact, the spatial relations are ambiguous. Our method takes into account each region globally, via the histogram of all possible

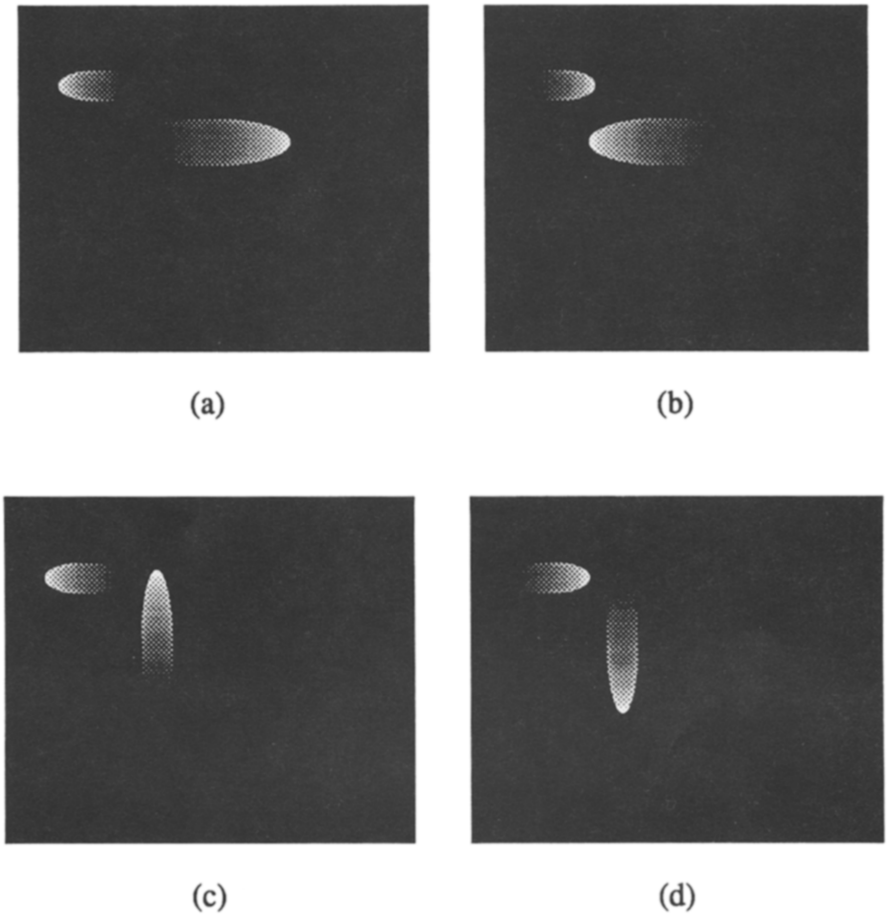
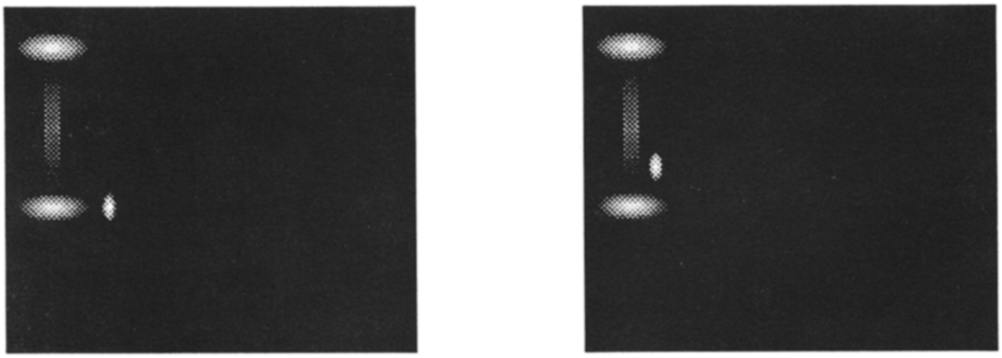


Fig. 12. Fuzzy subsets of images used in the experiment.

Table 4
Results of degree of location between reference and argument objects for Fig. 12

	Centers of gravity				Our method			
	right	left	above	below	right	left	above	below
(a)	0.870	0.000	0.000	0.130	0.780	0.000	0.000	0.220
(b)	0.676	0.000	0.000	0.324	0.636	0.000	0.000	0.364
(c)	0.768	0.000	0.000	0.232	0.625	0.000	0.001	0.375
(d)	0.360	0.000	0.000	0.640	0.446	0.000	0.003	0.554

angles between pairs of points in the regions considered. This is possible because the regions are finite collections of points. The histogram is interpreted as the fuzzy set describing the spatial relation between regions. The compatibility between the histogram and the fuzzy sets associated to a vocabulary of spatial relation gives the recognition results. The extension of the method from crisp sets



(a) (b)

Fig. 13. Fuzzy subsets of images used in the experiment.

Table 5
Results of degree of location between reference and argument objects for Fig. 13

	Centers of gravity				Our method			
	right	left	above	below	right	left	above	below
(a)	0.282	0.000	0.000	0.718	0.399	0.000	0.030	0.601
(b)	0.208	0.000	0.000	0.792	0.316	0.006	0.498	0.710

to fuzzy sets has been illustrated as well. Other issues, such as the definition of higher level spatial relations, and interpretations can be addressed in the framework proposed and remain to be discussed.

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