Al Khwarizmi international

junior mathematical Olympiad

Second day

Thursday, 1. June 2023

Part 1: each problem is worth 1.9 points

1. In rebus $UZBEK + IS + TAN$ different letters represent different numbers and same letters represent same numbers. Find the largest possible value of that sum $UZBEK + IS + TAN$?

A) 99631 B) 99190 C) 99387 D) 99423

2. Babur has stones weighing 1 kg, 2 kg, ..., 16 kg and three boxes A, B, C. He put two stones in each box so that the total weight of the stones in each box was M kg. Find the number of all possible values of M.

A) 21 B) 24 C) 19 D) 15

3. Anora wrote 5 integer numbers on the paper. From these numbers, Anora calculated the arithmetic mean of all groups with 4 numbers and got the numbers 37, 44, 25, 46 and 68 as a result. Find the largest number Anora wrote on the paper.

A) 147 B) 120 C) 68 D) 95

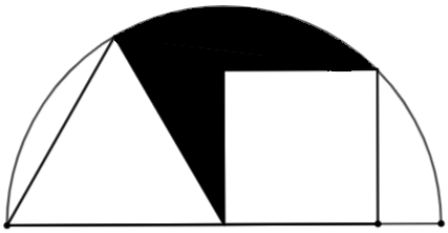
4. $1 \cdot 2 \cdot 3 \cdot \ldots \cdot 29 \cdot 30$ at least how many of the multipliers can be deleted to form a perfect square?

A) 7 B) 6 C)5 D) 4

5. Numbers from 1 to 2023 are written on the board. At each step, two arbitrary numbers are erased and their difference is written instead. This is done until there is only one number left at the end of the work. Which of the following cannot be the remaining number at the end?

A) 16 B) 1024 C) 2023 D) 2048

6. An equilateral triangle and a square are drawn inside a semicircle with a diameter of 10 cm as shown in the picture. What is the area of the painted part in cm^2?



A) $\frac{25\pi-150}{24}$ B) $\frac{25\pi-150}{12}$ C) $\frac{125\pi-150}{24}$ D) $\frac{45\pi-50}{8}$

7. Find the 1st digit of the smallest number which is divisible by 11 and whose sum of digits is 2023.

A) 9 B) 8 C) 7 D) 6

8. Given 5 consecutive positive integers. If the LCM of the first three numbers is less than twice the LCM of the last two numbers, find the largest value of the sum of these 5 consecutive numbers.

A) 30 B) 40 C) 45 D) 50

9. For the 4-digit number $\overline{abcd}$ function is defined and $f(\overline{abcd}) = a * b * c + d$. For example $f(6542) = 6 * 5 * 4 + 2 = 122$. What is the result of $f(2023) - f(2022) + f(2021) - f(2020) + \cdots + f(1003) - f(1002)$?

A) 509 B) 495 C) 517 D) 511

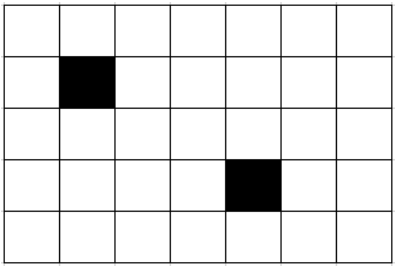
10. Find the smallest positive integer n such that 2023n has exactly 66 positive integer divisors.

A) 2^{33} B) 9216 C) 216 D) 1024

Part 2: each problem is worth 3.1 points

11. There is one rectangular piece of paper on the table. At each step, Olim chooses one of the largest pieces of paper on the table, divides it into two equal pieces and puts them back on the table. After 2023 steps, if the area of the smallest paper on the table is 1 cm^2, what is the area of the initial rectangular paper?

12. How many rectangles are there in the following figure whose sides lie on the lines of this figure and contain at least 1 black square?



13. 1000 straight lines are given and the intersection point of any 2 of them is marked. Find the maximum number of these marked points which can lie on one circle?

14. Given a parallelogram ABCD. The midpoint of AD is M. The projection of point B intersects the side CM at the point P. If ∠APB=61°, what is the angle ∠PAB?

15. Sardor created one number by writing even numbers from 23 to 2023 in a row. Then he deleted all the odd digits in the decimal notation of this number. How many digits are left?

16. Find the number of integer triples (a, b, c) satisfying the following conditions:

$2 \leq a, b, c \leq 8$ and $1 \leq d \leq 12$, $a + b + c + d = 30$.

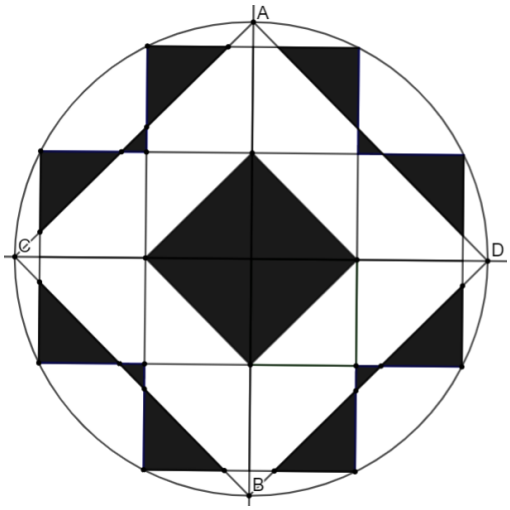
17. The following is appropriate for $\{A_n\}$ sequence of integer numbers:

$$A_{10} < 10 \text{ and } 1 \leq A_1 \leq 2023$$

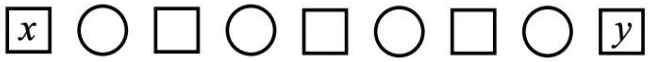
$$A_{n+1} = \begin{cases} \frac{A_n}{2}, & \text{if } A_n \text{ is even number} \\ A_n^2 + 1, & \text{if } A_n \text{ is odd number} \end{cases}$$

Find the number of all possible values of A_1.

18. 12 small squares whose sides are parallel to the diameters AB and CD of the circle (AB and CD are perpendicular to each other) are drawn as shown in the picture. AC, AD, BC, BD are the chords of the circle. Find the area of the painted area.



19. Anvar wrote the numbers 1, 2, 4, 5, 6, 9, 10, 11, 13 in these circles and squares. Each number must be written exactly once. In this case, the number in each circle is equal to the sum of the numbers in its two neighboring squares. If x and y are arranged as follows, find the largest value of x+y.



20. 10 people came to the ticket office, 5 of them have $5 and 5 have $10 each, but the ticket seller has no money. One ticket costs $5. How many ways can people line up in several different ways so that everyone who needs change (the remaining part of their money) can get it immediately after purchasing the ticket?

Language: English