AL-KHWARIZMI
INTERNATIONAL MATHEMATICAL AND INFORMATICAL OLYMPIAD

SCIENCE OLYMPIAD CENTER

2024

Problem 1. We have triangle ABC with area S. In one step we can move only one vertex at the time so that area of the triangle during movement remains constant. Prove that we can move this triangle into any other arbitrary triangle DEF with area S.

Proposed by Marek Maruin, Slovakia

Problem 2. For how many $x \in \{1, 2, 3, \ldots, 2024\}$ is it possible that *Bekhzod* summed 2024 non-negative consecutive integers, *Ozod* summed $2024 + x$ non-negative consecutive integers and they got the same result?

Proposed by Marek Maruin, Slovakia

Problem 3. Find all $x, y, z \in (0, \frac{1}{2})$ such that

$$\begin{cases} (3x^2 + y^2)\sqrt{1 - 4z^2} \geq z; \\ (3y^2 + z^2)\sqrt{1 - 4x^2} \geq x; \\ (3z^2 + x^2)\sqrt{1 - 4y^2} \geq y. \end{cases}$$

Proposed by Ngo Van Trang, Vietnam

Problem 4. We call a permutation of the set of real numbers $\{a_1, \cdots, a_n\}$, $n \in \mathbb{N}$ *average increasing* if the arithmetic mean of its first k elements for $k = 1, \cdots, n$ form a strictly increasing sequence.

1. Depending on n, determine the smallest number that can be the last term of some average increasing permutation of the numbers $\{1, \cdots, n\}$;
2. Depending on n, determine the lowest position (in some general order) that the number n can be achieved in some average increasing permutation of the numbers $\{1, \cdots, n\}$.

Proposed by David Hruska, Czech Republic

Time: 4 hours.
Each problem is worth 10 points.

AL-KHWARIZMI
INTERNATIONAL MATHEMATICAL AND INFORMATICAL OLYMPIAD

SCIENCE OLYMPIAD CENTER

Problem 1. At a party, every guest is a friend of exactly fourteen other guests (not including him or her). Every two friends have exactly six other attending friends in common, whereas every pair of non-friends has only two friends in common. How many guests are at the party? Please explain your answer with proof.

Proposed by Alexander Slavik, Czech Republic

Problem 2. Let a, b, c be distinct real numbers such that $a+b+c=0$ and

$$a^2-b=b^2-c=c^2-a.$$

Evaluate all the possible values of $ab+ac+bc$. Please explain your answer with proof.

Proposed by Nguyen Anh Vu, Vietnam

Problem 3. Two circles with centers O_1 and O_2 intersect at P and Q. Let ω be the circumcircle of the triangle PO_1O_2; the circle ω intersect the circles centered at O_1 and O_2 at points A and B, respectively. The point Q is inside triangle PAB and PQ intersects ω at M. The point E on ω is such that $PQ=QE$. Let ME and AB meet at L, prove that $\angle QLA = \angle MLA$.

Proposed by Amir Parsa Hoseini Nayeri, Iran

Problem 4. Three positive integers are written on the board. In every minute, instead of the numbers a, b, c, Elbek writes $a+\gcd(b,c), b+\gcd(a,c), c+\gcd(a,b)$. Prove that there will be two numbers on the board after some minutes, such that one is divisible by the other.

Note. $\gcd(x,y)$ – *Greatest common divisor of numbers* x *and* y

Proposed by Sergey Berlov, Russia

Time: 4 hours.
Each problem is worth 10 points.