AL-KHWARIZMI
INTERNATIONAL MATHEMATICS AND INFORMATICS OLYMPIAD

Friday, 9 May 2025 ***Tashkent, Uzbekistan***

Day 1

Problem 1. Determine, with proof, the largest integer c for which the following statement holds: there exists at least one triple (x, y, z) of integers such that

$$x^2 + 4(y + z) = y^2 + 4(z + x) = z^2 + 4(x + y) = c$$

and all triples (x, y, z) of real numbers, satisfying the equations, are such that x, y, z are integers.

Proposed by Marek Maruin, Slovakia

Problem 2. Let $ABCD$ be a convex quadrilateral with

$$\angle ADC = 90^\circ, \quad \angle BCD = \angle ABC > 90^\circ, \text{ and } AB = 2CD.$$

The line through C, parallel to AD, intersects the external angle bisector of $\angle ABC$ at point T. Prove that the angles $\angle ATB$, $\angle TBC$, $\angle BCD$, $\angle CDA$, $\angle DAT$ can be divided into two groups, so that the angles in each group have a sum of 270°.

Proposed by Miroslav Marinov, Bulgaria

Problem 3. On a circle are arranged 100 baskets, each containing at least one candy. The total number of candies is 780. Asad and Sevinch make moves alternatingly, with Asad going first. On one move, Asad can take all the candies from 9 consecutive non-empty baskets, while Sevinch can take all the candies from a single non-empty basket that has at least one empty neighboring basket. Prove that Asad can take overall at least 700 candies, regardless of the initial distribution of candies and Sevinch's actions.

Proposed by Shubin Yakov, Russia

Problem 4. For two sets of integers X and Y we define $X \cdot Y$ as the set of all products of an element of X and an element of Y. For example, if $X = \{1, 2, 4\}$ and $Y = \{3, 4, 6\}$ then $X \cdot Y = \{3, 4, 6, 8, 12, 16, 24\}$. We call a set S of positive integers *good* if there do not exist sets A, B of positive integers, each with at least two elements and such that the sets $A \cdot B$ and S are the same. Prove that the set of all perfect powers greater than or equal to 2025 is good.

(*In any of the sets A, B, $A \cdot B$ no two elements are equal, but any two or three of these sets may have common elements. A perfect power is an integer of the form n^k, where $n > 1$ and $k > 1$ are integers.*)

Proposed by Lajos Hajdu and András Sarkozy, Hungary

Language: English

Time: 4 hours and 30 minutes.
Each problem is worth 10 points.

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Saturday, 10 May 2025 ***Tashkent, Uzbekistan***

Day 2

Problem 5. Sevara writes in red 8 distinct positive integers and then writes in blue the 28 sums of each two red numbers. At most, how many of the blue numbers can be prime? Justify your answer.

Problem 6. Let a, b, c be real numbers such that

$$ab^2 + bc^2 + ca^2 = 6\sqrt{3} + ac^2 + cb^2 + ba^2.$$

Find the smallest possible value of $a^2 + b^2 + c^2$. Justify your answer.

Problem 7. Let $ABCD$ be a cyclic quadrilateral with circumcenter O, such that CD is not a diameter of its circumcircle. The lines AD and BC intersect at point P, so that A lies between D and P, and B lies between C and P. Suppose triangle PCD is acute and let H be its orthocenter. The points E and F on the lines BC and AD, respectively, are such that $BD \parallel HE$ and $AC \parallel HF$. The line through E, perpendicular to BC, intersects AD at L, and the line through F, perpendicular to AD, intersects BC at K. Prove that the points K, L, O are collinear.

Problem 8. There are 100 cards on a table, flipped face down. Madina knows that on each card a single number is written and that the numbers are different integers from 1 to 100. In a move, Madina is allowed to choose any 3 cards, and she is told a number that is written on one of the chosen cards, but not which specific card it is on. After several moves, Madina must determine the written numbers on as many cards as possible. What is the maximum number of cards Madina can ensure to determine? Justify your answer.

Language: English

Time: 4 hours and 30 minutes.
Each problem is worth 10 points.