Technology Choice and Firm Heterogeneity in an RBC Model with Endogenous Entry

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1 Introduction

The heterogeneity of producers is a defining feature of any economy. A key dimension of this heterogeneity is scalability: firms with highly scalable technologies exploit strong returns to scale and can grow to large sizes, whereas less scalable firms remain small. According to a report by Flachenecker et al. (2020) for the European Commission, high-growth enterprises accounted for only 11% of EU firms but generated 53% of net job creation between 2015 and 2016. Yet, the profit opportunities that make scalable ideas attractive also intensify competition, creating significant entry frictions for prospective entrepreneurs. Using U.S. Business Dynamics Statistics for 1979–2013, Sedláček and Sterk (2017) show that the probability of launching a successful business ranges from 0.2 % to 62 %, depending on the scalability of the underlying idea.

Heterogeneous scaling within startup cohorts also produces heterogeneous responses to macroeconomic shocks, giving rise to aggregate dynamics that standard models miss. Using census microdata, Smirnyagin (2023) shows that the entry rate of high returns-to-scale firms falls twice as much as for low returns-to-scale firms during recessions. Calligaris et al. (2023) find that employment losses in 2020 were significantly larger in low-telework sectors that could not scale rapidly in response to pandemic disruptions.

Hence, this thesis investigates how the transmission of shocks in a Real-Business-Cycle (RBC) model with endogenous entry is affected by the introduction of across-sector heterogeneity in returns-to-scale technology and entry-matching frictions. In particular, how do sector-specific scalability and matching frictions at birth alter the propagation of aggregate shocks, especially when compared to a baseline RBC framework? I also investigate the impact of an exogenous surge in business opportunities across the economy, and how firm-level and aggregate-level responses to this shock diverge.

To address these questions, I integrate the seminal RBC model with the endogenous entry framework found in Bilbiie et al. (2012) (henceforth BGM) with the technology-choice mechanism from Sedláček and Sterk (2017) in a single DSGE environment featuring multiple production sectors[1]. Additionally, a novel stock-of-ideas shock is introduced by modifying the composition shock found in the Sedláček-Sterk framework to capture booms in business viability in a micro-consistent manner. This approach allows the study of firm entry and heterogeneity dynamics inside a pre-existing and well-studied RBC framework and yields new insights for industrial policy that recognizes scaling differences across sectors.

The model shows that introducing heterogeneity in returns-to-scale coefficients amplifies the aggregate impact of a productivity shock relative to the BGM benchmark. This new magnitude comes from two new shock transmission channels: intersectoral labor reallocation and demand reallocation across sectors, both of which are not captured by the standard representative firm baseline. Additionally, I find a negative firm-level im-

[1] In my framework, each sector corresponds to one returns-to-scale parameter, such that a symmetric within-sector equilibrium makes all firms that belong to the same sector share equal prices and profits.

pact across different technology types of positive productivity and stock-of-ideas shocks, mainly caused by over-entry and a subsequent competition glut. This effect drives firm-level profits down despite sectoral aggregates going up from higher productivity and/or a higher number of incumbents.

Consistent with these mechanisms, the business cycle second moments and first-order autocorrelations (See Table 5.1) show that the expanded model still reproduces very persistent output and hours series and brings the volatility of consumption much closer to the data than both BGM and the canonical RBC benchmark found in King and Rebelo (1999). By contrast, investment and labor volatility become too smooth, as matching frictions dampen their response. On top of that, the model still inherits familiar RBC shortcomings, where the main variables move too perfectly with output and lack enough endogenous persistence.

As in BGM, each "producer" corresponds to a product line, acknowledging that incumbent firms can add varieties. With this in mind, analyzing steady-state firm behavior pins down interesting facts about the model's functioning. The relationship between firm-level output and labor demand, for instance, is highly connected to the returns-to-scale parameter. The larger the returns-to-scale parameter, the longer a firm can expand before encountering diminishing returns, so highly scalable sectors naturally host more product lines. On the competition front, any parameter change that depresses entry enlarges the slice of the market served by each surviving producer. A congestion channel also exists, such that prospective incumbents face decreasing "returns-to-entry", where success odds tumble in a non-linear way going from low- to high-scalability sectors. Finally, in the steady state all entry happens solely to rebuild the stock of varieties, such that entry and dividends work together much like depreciation and investment in a capital-based RBC model.

Recent work increasingly highlights firm heterogeneity as a driver of shock transmission and persistence. More foundational works like BGM and Ghironi and Melitz (2007) have given substantial attention to endogenizing entry dynamics as a way to explain the procyclicality of net entry in business cycles. Another strand of work focuses on technological heterogeneity and cohort effects as potential explanations for the persistent effects of recessions on startup cohorts (see e.g. Clementi and Palazzo, 2016; Sedláček and Sterk, 2017). Finally, more recent models base their dynamics solely on innovation/idea-driven mechanisms, exploring new transmission channels that arise from R&D activity (see e.g. Anzoategui et al., 2019; Comin and Gertler, 2006). This thesis bridges this literature by allowing technology choice to interact with variety creation within an endogenous-entry RBC framework.

The insights presented in this thesis are of value to the design of policies, such as startup subsidies, macro-prudential entry policies, industrial strategy, and others. For instance, facilitating conditions for early-stage entrepreneurs can have significant impacts on active firms, especially in highly attractive sectors prone to excessive entry and intense competition. Policymakers must calibrate such interventions to avoid harming large employers. Another important discussion comes from potential externalities in variety creation in the baseline BGM model, where a new entrant raises welfare through additional varieties but its private incentive equals the markup, not the social value of the variety. In my framework, it could be the case that across-sector returns-to-scale dispersion could generate allocative inefficiency, as is the case with markup dispersion (see e.g. Baqaee and Farhi, 2020), or amplify congestion externality[2] in highly scalable sectors.

The remainder of the thesis proceeds as follows. Section 2 surveys the broader literature on firm heterogeneity and shock transmission. Section 3 reviews BGM and Sedláček–Sterk in detail and presents the unified model. Section 4 describes the calibration and the algorithm used to compute the steady state and examines its properties, with a special focus on firm composition and entry. Section 5 analyzes impulse responses to a standard productivity shock and the stock-of-ideas shock, and reports the implied business-cycle moments. Section 6 concludes.

2 Related Literature

The two studies that most directly underpin the analysis presented in this thesis —Bilbiie et al. (2012) and Sedláček and Sterk (2017)—are analyzed in depth in Section 3, where their mechanisms can be better assessed alongside my framework. This section, therefore, opens with a survey of foundational work on incorporating endogenous firm dynamics in DSGE and RBC frameworks, and then progressively moves on to more recent contributions.

A first seminal contribution is given by Ghironi and Melitz (2007), which embeds a heterogeneous-firm trade model into a two-country dynamic GE setting. The authors show how the number of producers and the product variety become state variables affecting consumption and investment, establishing a baseline for incorporating firm entry in DSGE models. Subsequent research by Jaimovich and Floetotto (2008) shows that net business formation is strongly procyclical and can itself propagate shocks. The authors present a DSGE model where an expansion induces a surge in firm entry, which increases competition and drives markups countercyclically lower. This mechanism causes aggregate productivity to rise in booms, amplifying output fluctuations. Their findings underscore an important propagation channel that is similarly found in the model presented in this thesis.

However, Bilbiie et al. (2012) probably represents the most influential foundational work in this vein of literature, also starting a line of work that modifies their model to study optimal monetary policy with endogenous firm entry, weighing the benefits of

[2] See Bilbiie et al. (2014), where the authors show that Ramsey-optimal policy in the BGM framework departs from price stability precisely to offset the congestion/variety wedges, where more entrants raise marginal entry costs for all and lower existing profits, a negative pecuniary externality absent from firms' decisions.

variety creation against costs like entry investment and potentially inefficient boom-burst cycles of firm creation (see e.g. Bilbiie et al., 2014; Lewis and Poilly, 2012). Another influential line of work has opted for idiosyncratic firm productivity shocks alongside entry as their driver of firm heterogeneity within a DSGE structure. An important example is Clementi and Palazzo (2016), who build a quantitative RBC model with endogenous entry and exit, where entrants draw idiosyncratic productivity signals, invest, and may exit if staying is unprofitable. A key result presented by the authors that aligns with existing literature is that introducing firm heterogeneity greatly increases the internal propagation of aggregate shocks in the long run. Notably, earlier work by Samaniego (2008) had only found modest business-cycle effects of entry in a simpler setting with homogeneous firms. In short, these foundational studies established that endogenous firm entry and exit dynamics, coupled with firm heterogeneity, is a potent mechanism for shock propagation. It increases volatility, generates endogenous persistence, and introduces new welfare considerations via product variety.

A key extension in recent years is to model heterogeneity in firms' technologies or scale, often endogenously chosen, and study how this affects aggregate fluctuations. Excluding Sedláček and Sterk (2017), another interesting contribution to this modeling approach is given by Smirnyagin (2023). The author allows entrepreneurs to choose the returns-to-scale of their project upon entry. The resulting finding echoes the results in the Sedláček-Sterk framework, where high-returns-to-scale firms are less likely to be created in economic downturns. The explanation for this phenomenon, however, now comes from firm financing, with the author arguing that large-scale projects require more upfront investment or financing, so when profits or credit conditions are weak, entrepreneurs defer their ambitious ideas and only smaller-scale firms launch. The author also states that the absence of high-growth firms can hinder recovery in recessions, further corroborating the role of firm heterogeneity in amplifying the propagation of shocks.

Another branch of the literature incorporates endogenous innovation, R&D, and creative destruction into business-cycle models as both a source of firm heterogeneity and a new channel for technology shock propagation. Traditional RBC models take total factor productivity (TFP) as an exogenous process, but more recent models allow TFP to evolve via the firm sector's innovation-decision and diffusion of new technologies. This effectively brings Schumpeterian dynamics of innovation into the DSGE framework, enriching the set of propagation mechanisms. A prominent example is the work by Comin and Gertler (2006), and more recently Anzoategui et al. (2019). Anzoategui *et al.* develop a medium-scale DSGE model where firms improve the frontier technology by spending on R&D while also incuring costs to adopt existing innovations. This model is then applied to the 2008-09 recession. The authors find that the persistent productivity slowdown after the recession can, at large, be explained as an endogenous response to the drop in demand. In the model, the collapse in output and investment from the crisis led firms to cut R&D and delay the adoption of new technologies, causing TFP growth to falter.

On a similar note, other researchers have introduced idea production and knowledge spillovers into DSGE models. For instance, some DSGE models feature an "innovation sector" producing new blueprints or product lines, similar to Bilbiie et al. (2012), but incumbent firms now decide whether to adopt new technologies, effectively separating variety creation from firm dynamics. Kung and Schmid (2015) tie stock market valuations to R&D-driven growth in a production economy, producing Schumpeterian dynamics: positive innovation shocks can cause bursts of creative destruction that temporarily depress output but raise growth later. That happens because a wave of new firm entry and old firm exit is triggered by the innovation shock, leading to a reallocation of resources to a more optimal distribution. Conversely, and similarly to Sedláček and Sterk (2017), adverse shocks can have prolonged recessionary effects as the economy's idea pipeline dries up and fewer high-growth firms enter.

The frictions involved in creating new firms also play a fundamental role in dictating firm composition in an economy. There are many plausible sources of frictions worth considering. For instance, in the Sedláček-Sterk framework, a matching function dictates if entrants will find a matching business idea, capturing how easily a would-be founder finds a business partner, investor, or idea. Vardishvili et al. (2023) on the other hand, study entrepreneurs' option to delay entry, causing a "wait-and-see" friction. The authors argue that if entrepreneurs can pause plans until aggregate conditions improve, recessions will not only feature fewer entries but also many delayed projects, leading to a surge of entries at the subsequent recovery.

Finally, there are welfare and policy implications to consider. In models like BGM, there is a variety externality issue, where individual firms do not internalize that their entry adds consumer surplus by increasing variety. This might lead to under-entry in equilibrium, suggesting a possible socially inefficient equilibrium and a role for subsidies to startup formation. However, other potential distortions such as markups, fixed costs, and financial frictions might complicate the picture, making it uncertain whether more entry is always welfare-adding. Recent research by Bilbiie et al. (2019) tackles this issue by deriving optimal monetary and fiscal policy in these settings, finding that the planner typically wants to encourage firm entry, especially during downturns, to offset the tendency of recessions to reduce varieties and future supply. Baqaee and Farhi (2020) also discuss the possibility that frictions caused by markup dispersions or any friction to resource reallocation can potentially cause within-sector misallocations, creating an allocative-efficiency externality.

The integration of firm heterogeneity and endogenous entry into macroeconomics has opened up many questions. The fundamental debate with which I occupy myself in this thesis is: How important is firm heterogeneity for macroeconomic fluctuations? Although Bilbiie et al. (2012) show that pro-cyclical startup entry amplifies business-cycle volatility, they treat all entrants as technologically identical, while Sedláček and Sterk (2017) highlight how recessions skew the composition of entrants through matching frictions but do so outside a love-of-variety RBC setting and with a different question of firm employment in mind. By embedding Sedláček–Sterk's technology-choice and matching block inside the BGM framework, this thesis delivers a DSGE model in which sector-specific returns-to-scale heterogeneity, entry-stage matching frictions, and endogenous idea supply shocks jointly shape and enrich the classic RBC shock propagation mechanisms. The model delivers a more sophisticated picture of how aggregate shocks can affect different types of firms in the economy, and how this heterogeneity in turn affects aggregate outcomes through rich firm dynamics, not explored in the standard BGM framework.

3 Model

In this section, I first provide an overview of Bilbiie et al. (2012) and lay out the firm side in their baseline model with no capital and C.E.S. preferences in Section 3.1. Then, I move on to discussing Sedláček and Sterk (2017) and explaining their heterogeneity and matching friction mechanisms in Section 3.2. Finally, I fully characterize the main model used throughout this thesis and the departures made from the standard BGM framework in Section 3.3.

3.1 Baseline Bilbiie-Ghironi-Melitz Framework

3.1.1 Overview

BGM expand on the classic RBC model found in Campbell (1994) by introducing monopolistic competition and endogenous firm entry for both a C.E.S. and a translog preferences framework.

Firms incur a one-time sunk entry cost to enter and then start producing after one period with constant returns (time-to-build). This one-time sunk entry cost is interpreted in the model as an "investment" paid in effective labor units to create a new firm/variety. Consequently, the economy has two sectors. A production sector where incumbent firms produce output using labor and (as an additional exercise) physical capital, and an "entry" or variety-expanding sector, where labor can be used to create new firms/varieties. BGM find that introducing endogenous entry significantly improves the RBC model's ability to propagate shocks. The authors argue that this is due to the internal persistence mechanism generated by the slow build-up of new firms following a shock which the exogenous entry model lacks. Quantitatively, the model reproduces key second moments of macro data at least as well as the standard RBC model without the introduction of capital, and even better than the RBC model once capital is introduced.

An important aspect of the BGM framework is that each variety is interpreted as a new product line, instead of a new firm in itself. Therefore, whenever any reference to the number of varieties is made, it does not necessarily translate to the actual number of firms in the economy. The stock of varieties is a state variable that evolves with entry and exit, although in the baseline, the main driver of change is entry, since exit happens through a fixed probability "death shock" that affects all firms equally. Because of the sunken costs, entry is forward-looking. Prospective entrants base their entry decisions on the expected present value of profits, which in this framework represents the "stock-market price" of a new firm.

Aggregate conditions drive entry in the BGM model. In an economic expansion, higher demand and profits raise the value of starting a firm, which naturally induces more entry. However, due to the entry cost and, in particular, the time-to-build aspect, new firms enter sluggishly. This slow adjustment in the number of producers creates a novel propagation mechanism for shocks. A positive permanent productivity shock, for example, boosts output and profits immediately. However, it also sets off a gradual increase in firm entry, which persists even after the initial shock, given that the larger cohort of new firms continues to produce additional output, prolonging the boom. In essence, the increase in product variety acts like capital stock in the classic RBC model, building up slowly and sustaining production, enhancing persistence.

Another important feature is that an increase in firm entry (product variety) can affect profits at the firm level and even markups in their translog specification. Their work laid a foundation for many subsequent DSGE models with entry, including applications to monetary policy (see e.g. Bergin and Corsetti, 2008; Bilbiie et al., 2014; Etro and Rossi, 2015) and open economy/trade contexts (see e.g. Bergin and Corsetti, 2015; Epifani and Gancia, 2011).

3.1.2 Model Framework (C.E.S. Preferences)

In the original BGM framework, the key elements of the firm side are fully characterized by the production/labor decision and an entry decision pinned down by a free entry condition in equilibrium. At this point, I do not fully explore the consumer side of the framework for the sake of brevity, since it is analogous to the consumer side in the main model which will be explained in detail later. As described before in Section 2, each firm (or production line) produces a unique variety ω using labor as its single production factor, with the production function

$$y_t(\omega) = Z_t \cdot l_t(\omega),$$

where Z_t is the exogenous aggregate productivity level and $l_t(\omega)$ is the firm's labor input. Consumers have C.E.S. preferences over a continuum of varieties

$$C_t = \left[\int_{\omega \in \Omega_t} c_t(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right]^{\frac{\theta}{\theta-1}},$$

which implies a consumption-based price index and constant markups for the firm. Solving the firm's pricing problem, they arrive at the equilibrium real pricing condition of

$$\rho_t(\omega) = \frac{p_t(\omega)}{P_t} = \mu \cdot \frac{w_t}{Z_t},$$

with $\mu = \frac{\theta}{\theta-1}$ and marginal costs corresponding to $\frac{w_t}{Z_t}$, which are symmetrical across firms/varieties.

In the entry block, firms face a sunk entry cost $f_{E,t}$, expressed in effective labor units, and once firms have successfully entered, they survive with probability $(1-\delta)$, where $\delta \in (0,1)$ is a "death shock". The law of motion dictating the total number of producers is

$$N_t = (1-\delta)(N_{t-1} + N_{e,t-1}),$$

where $N_{e,t}$ is the measure of new entrants, which face a time-to-build lag before entering. In equilibrium, the free-entry condition equates the expected present value of a firm's future profit stream to the sunk cost in units of the consumption good, such that entry occurs until firm value is equalized with entry cost:

$$v_t(w) = \frac{w_t f_{E,t}}{Z_t}.$$

Prospective entrants compute their expected post-entry value $v_t(\omega)$ given the present discounted value of their expected stream of profits $\{d_s(\omega)\}_{s=t+1}^{\infty}$:

$$v_t(\omega) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-1} d_s(\omega).$$

In the BGM framework, this also represents the value of incumbent firms after production has happened, which is relevant for the household's investment decision.

3.2 Baseline Sedláček–Sterk Framework

3.2.1 Overview

Sedláček and Sterk (2017) introduce the other core framework from which I derive the firm heterogeneity and matching friction mechanisms used in this thesis. Sedláček and Sterk focus on how the composition of new firm cohorts varies over the business cycle and the effects of this on aggregate employment dynamics. Empirically, the authors document rich evidence using U.S. Business Dynamics Statistics for 1979-2013, finding that employment created by startup cohorts is highly volatile and procyclical, and these cohort differences persist for many years. Additionally, they find that total employment differences across cohorts are mostly driven by average firm size rather than by the number of startups.

To explain these facts, Sedláček and Sterk develop a DSGE firm dynamics model with heterogeneous startups and entry uncertainty. Firm heterogeneity in the model arises from differences in each startup's growth potential, modeled via the demand side. That is, new firms can choose to produce either in a low-scalability/"niche" environment or in a high-scalability/"mass-market" environment, which is captured by the heterogeneous returns-to-scale parameters found in each environment. Crucially, highly scalable startups have the incentive to invest aggressively in demand expansion, since there are larger returns-to-scale to begin with, whereas niche firms cannot profitably scale to as great an extent. The model also features an equilibrium entry condition to equalize profits (and thus entry) among different types and a friction resembling congestion in entry, where the stock of ideas is limited and firms have to compete for entry, especially for highly scalable types.

The key mechanism driving the firm composition dynamics in the Sedláček-Sterk framework is the aggregate conditions at the time of entry. A favorable initial aggregate state disproportionally benefits mass-market firms, which rely primarily on expansion. As a result, booms tend to induce a greater fraction of high-growth potential startups, whereas recessions see a higher share of niche, low-growth firms. This endogenous composition effect means that entry in booms is not only higher in quantity but also in quality, which translates to the fact that cohorts born during booms end up becoming much larger contributors to employment over time. Thus, initial conditions at birth have persistent effects on the path of aggregate employment/labor.

Sedláček and Sterk's model, when estimated on U.S. data, can explain several observations that are not fully addressed by other DSGE models. The model reproduces the fact that employment fluctuations of startups are strongly procyclical and persistent, which the authors attribute to the composition effect described above. In the aggregate, shocks to startup composition can create large, low-frequency movements in employment, thus establishing a novel propagation mechanism for business cycles: cohort effects. Ultimately, the Sedláček-Sterk framework builds a bridge between business cycle research and entrepreneurship literature, emphasizing that heterogeneity in startup quality is macro-relevant through theory and empirical work.

3.2.2 The Sedláček–Sterk Heterogeneity Mechanism

I abstract away from almost all other relevant model aspects presented by Sedláček and Sterk (2017) in this section to focus on two key mechanisms: the technology/returns-to-scale choice and the entry-stage matching friction. Before entry, each prospective entrepreneur/startup chooses a technology type $i \in \{1, \dots, I\}$, which pins down the constant returns-to-scale parameter $\alpha_i \in (0, 1)$. Once active, an incumbent of age a produces according to

$$y_{i,a,t} = A_t \, n_{i,a,t}^{\alpha_i},$$

where A_t is aggregate TFP and $n_{i,a,t}$ denotes employment. The heterogeneity in α_i implies heterogeneous optimal sizes for different startups, with size referring to the size of employment $n_{i,a,t}$. Low-α_i firms remain small due to smaller scalability, whereas high-α_i

firms can scale to thousands of workers over their life cycle, matching empirical cohort size distributions.

After a business idea/technology type is chosen, the firms face the entry-stage matching friction. Let $e_{i,t}$ be the measure of aspiring entrants that select technology i at time t, and let $\psi_{i,t}$ denote the stock of business opportunities of type i at time t (with $\sum_i \psi_{i,t} = \Psi$). Because multiple entrants may chase the same idea, only a fraction actually succeed. Successful startups $m_{i,t}$ are generated by a Cobb-Douglas matching function

$$m_{i,0,t} = e_{i,t}^{\phi} \psi_{i,t}^{1-\phi},$$

where $\phi \in (0,1)$ is the matching elasticity, such that the success probability is pinned down by:

$$P_{i,t} \;=\; \frac{m_{i,0,t}}{e_{i,t}} = e_{i,t}^{\phi-1} \psi_{i,t}^{1-\phi}.$$

After paying the sunk entry cost χ, a potential entrant earns expected value $P_{i,t}V_{i,0,t}(0, F_t)$, where $V_{i,0,t}$ is the value of a newborn firm at time t for technology type i. Free entry therefore requires

$$\chi \;=\; P_{i,t}V_{i,0,t}(0, F_t) \qquad \forall i, \tag{FE}$$

which pins down $P_{i,t}$ (and hence $m_{i,0,t}$) endogenously. In their quantitative implementation, the authors treat the probability $P_{i,t}$ as a fixed parameter due to data availability issues with $\psi_{i,t}$, which is a strategy also adopted by the main model presented later on.

The interaction between these two mechanisms creates rich firm dynamics. Because higher-α_i technologies yield larger $V_{i,0,t}$, condition (FE) forces their $P_{i,t}$ to fall—i.e. they face tougher matching—so shocks that shift the opportunity mix $\psi_{i,t}$ endogenously tilt the composition of entrants toward or away from high-growth firms. This joint mechanism of technology heterogeneity and matching frictions drives the persistent cohort-quality effects documented before in Section 2.

3.3 The Model

The model is a variant of the RBC model with endogenous firm entry and monopolistic competition found in Bilbiie et al. (2012), from which I make two departures based on the heterogeneity mechanism found in Sedláček and Sterk (2017): i) multiple sectors with heterogeneity in returns-to-scale parameters α_i; ii) matching frictions for aspiring entrants choosing and attempting to enter a sector.

I accommodate the technology choice mechanism by dividing the firm side of the economy into a finite number of sectors I indexed by $i \in \{1, 2, ..., I\}$. There is a continuum of unique firms/varieties indexed by $\omega \in \Omega$, with each one being fully defined by the unique product variety ω that it produces. I refer to firm ω that belongs to sector i as "firm ωi". I use Ω_i to denote the set that contains the indices of firms that belong to sector i, so that

$\bigcup_{i=1}^{I} \Omega_i = \Omega$. Its measure, here denoted M_i, gives the size of the sector in terms of firms (or production lines). I adopt the same interpretation as in BGM of understanding each firm/variety as a production line rather than a separate entity, which also matches well with the scalability logic introduced by a sectoral diversity in returns-to-scale parameters, as will be explored later in Section 4.

A. Households

The household framework is nearly identical to the one found in BGM, with the only difference being their preference specification. There is a unit mass of atomistic, identical households. All contracts are nominal. Prices are flexible and thus I only solve for real variables. As the composition of the consumption basket changes due to firm entry, I introduce money as a unit of account only, such that it plays no other role in the economy and no modeling of money demand is necessary, resorting to a cashless economy as in BGM and Woodford (2003).

The Household supplies L_t in a competitive labor market for W_t, and it maximizes $E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U(C_s, L_s)\right]$ where C is consumption and $\beta \in (0,1)$ is the subjective discount factor. The period utility function follows the form $U(C_t, L_t) = \ln C_t - \chi \frac{(L_t)^{1+1/\varphi}}{1+1/\varphi}, \quad \chi > 0$ where $\varphi \geq 0$ is the Frisch-elasticity of labor supply to wages and the intertemporal elasticity of substitution in labor supply. This choice of functional form makes sure that income and substitution effects of real wage variation on effort cancel out in steady state, as argued in BGM.

At time t, the household consumes the basket of goods C_t, defined over a continuum of goods Ω. At any time, only $\Omega_t \in \Omega$ is available. Given sector i, the continuum of goods stemming from that sector is $\Omega_{i,t}$, and the basket of goods is $C_{i,t}$. Let $p_{i,t}(\omega)$ denote the nominal price of a good $\omega \in \Omega_{i,t}$. Additionally, demand for an individual variety, $c_{i,t}(\omega)$, is obtained as $c_{i,t}(\omega)\, d\omega = C_{i,t}\, \partial P_{i,t} / \partial p_{i,t}(\omega)$.

As in BGM but adapted to the sectoral framework here presented, the symmetric price elasticity of demand in sector i ζ_i is a function of the number $M_{i,t}$ of goods/producers such that $\zeta(M_{i,t}) = (\partial c_{i,t}(\omega)/\partial p_{i,t}(\omega))(p_{i,t}(\omega)/c_{i,t}(\omega))$, for any symmetric variety ω. The benefit of additional product variety is here too described by the sectoral relative price $\rho_{i,t}(\omega) = \rho(M_{i,t}) \equiv p_{i,t}/P_{i,t}$, for any symmetric variety ω in sector i, or in elasticity form $\epsilon(M_{i,t}) \equiv \rho'(M_{i,t})\, M_{i,t}/\rho(M_{i,t})$. Together, $\zeta(M_{i,t})$, and $\rho(M_{i,t})$ completely characterize the effects of consumption preferences in this model. I define explicit functional forms for these objects later when specifying the functional form of preferences within and across sectors below.

B. Firms

The firm block is defined as a collection of I heterogeneous sectors with $M_{i,t}$ homogeneous firms each, facing two decisions: In which sector to enter and labor demand for each period after entering. There is a continuum of monopolistically competitive firms, each producing a different variety $\omega \in \Omega$, with one factor: labor. Aggregate labor productivity is indexed by Z_t (effectiveness of one unit of labor), which is exogenous, equal across sectors, and follows an $AR(1)$ process (in logarithms). For a firm that chooses sector i, the production function becomes $y_{i,t}(\omega) = Z_t \cdot l_{i,t}(\omega)^{\alpha_i}$. Similar to the Sedláček–Sterk framework, firms chose a technology type/business idea i that determines their returns-to-scale parameter α_i and thus directly influences their marginal product of labor.

Before entry, the firm pays $f_{E,t}$ effective labor units, equal to $\frac{w_t f_{E,t}}{Z_t}$ units of the consumption good. Firms that enter the economy produce every period until they get hit with a "death shock" (probability $\delta \in (0,1)$ in every period). Entrants at time t can only start producing at time t+1, such that they incur a one-period time-to-build lag. To produce a certain output level y, firms minimize their labor cost in a static minimization problem

$$\min_{l_{i,t}(\omega)} \; w_t \, l_{i,t}(\omega) \quad \text{subject to} \quad y = Z_t \cdot l_{i,t}^{\alpha_i}(\omega),$$

such that the optimal labor demand is then given at time t in sector i by

$$l_{i,t}^{*}(\omega) = \left(\frac{y}{Z_t}\right)^{\frac{1}{\alpha_i}}, \tag{3.1}$$

and therefore marginal costs are expressed as

$$\mathrm{MC}_{i,t} = \frac{w_t}{\alpha_i Z_t}\left(\frac{y_{i,t}(\omega)}{Z_t}\right)^{\frac{1}{\alpha_i}-1}, \tag{3.2}$$

which, differently from BGM, are sector-specific and depend directly on the returns-to-scale parameter α_i. However, the BGM marginal cost expression is retrievable by setting $\alpha_i = 1, \forall i \in \{1, 2, ..., I\}$. With C.E.S. preferences, firms set relative prices as constant markups over marginal costs

$$\rho_{i,t}(\omega) = \mu MC_{i,t}. \tag{3.3}$$

As in BGM, but at the sectoral level, I assume a conditional symmetric firm equilibrium, where firms from the same sector are symmetric over prices, quantities, and values. This implies that $p_{i,t}(\omega) = p_{i,t},\ \rho_{i,t}(\omega) = \rho_{i,t},\ l_{i,t}(\omega) = l_{i,t},\ y_{i,t}(\omega) = y_{i,t},\ d_{i,t}(\omega) = d_{i,t},\ v_{i,t}(\omega) = v_{i,t},\ \frac{p_{i,t}}{P_{i,t}} = \rho_{i,t} = \rho(M_{i,t})$. Therefore, profits/dividends for all firms in sector i are identical and given by

$$d_{i,t} = \left(1 - \tfrac{1}{\mu}\right)\frac{C_{i,t}}{M_{i,t}}, \tag{3.4}$$

where $y_{i,t} = \frac{C_{i,t}}{M_{i,t}}$, since all firms are symmetrical and supply equal amounts of demand in their sector. This is analogous to firm-level output in the BGM framework but adapted for the sectoral framework.

C. Firm Entry with Uncertainty

The matching friction mechanism found in Sedláček and Sterk (2017) is adapted to the RBC framework presented here, such that firms face matching frictions in entering a sector and not all potential entrants are successful. In every time period t and sector i, $\exists M_{i,t}$ mass of producing product lines and an unbounded mass of prospective entrants $e_{i,t}$. The entrants are forward-looking and correctly anticipate their future profits $d_{i,s}$ in every period $s \geq t+1$ as well as δ. Prospective entrants choose a technology type i and the corresponding sector. Let technology types reflect returns-to-scale in increasing order such that $i = 1$ is the lowest and $i = I$ is the highest. Let $e_{i,t}$ denote the measure of entrants choosing i at time t. Not every attempt to enter is successful. The Cobb-Douglas matching function is given by $m_{i,t} = e_{i,t}^{\phi}\, \psi_{i,t}^{1-\phi}$ such that $m_{i,t}$ is the measure of successful entrants, $\psi_{i,t}$ is the measure of business opportunities for type i at time t, and ϕ is a matching elasticity common across all types.

Unlike in the Sedláček–Sterk setup, the total mass of business ideas Ψ can fluctuate over time in this framework because it responds to an aggregate stock-of-ideas shock X_t; what stays fixed is the relative mix of ideas within that total. The motivation behind this change is that the original formulation investigates how the composition of cohorts influences employment, whereas this framework is interested in the aggregate effects of more available business ideas, not a shift in their distribution. To stay consistent with this new formulation, I specify the measure of business opportunities for technology i as

$$\psi_{i,t} = X_t^{\frac{1}{1-\phi}}\, \bar{\psi}_i, \tag{3.5}$$

where $\bar{\psi}_i$ is the steady-state measure of business opportunities for technology i and X_t follows an AR(1) process of the type $X_t = 1 - \rho_X + \rho_X X_{t-1} + \epsilon_t^X$, where ρ_X is a persistence parameter and ϵ_t^X are i.i.d innovations normally distributed with mean zero and variance σ_X. Consequently, the probability of a successful startup in sector i is then:

$$\Pi_{i,t} = \frac{m_{i,t}}{e_{i,t}} = e_{i,t}^{\phi-1}\, \psi_{i,t}^{1-\phi}. \tag{3.6}$$

Before entry, as in the BGM framework, prospective entrants compute their expected post-entry value by discounting the value of their expected stream of profits $\{d_{i,s}\}_{s=t+1}^{\infty}$ as

$$v_{i,t} = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_{i,s}}{C_{i,t}}\right)^{-1} d_{i,s}, \tag{3.7}$$

and thus entry occurs until firm value is equalized with entry cost, yielding the free entry condition for sector i

$$\Pi_{i,t} v_{i,t} = f_{E,t} w_t / Z_t, \tag{3.8}$$

for all $i \in \{1, 2, ..., I\}$. This entry framework yields the new sectoral law of motion that dictates the number of incumbents in sector i, given by the following expression

$$M_{i,t+1} = (1-\delta)(M_{i,t} + \Pi_{i,t} e_{i,t}). \tag{3.9}$$

An important observation is that, as demonstrated in BGM, there is no option value for waiting, since there remains no idiosyncratic/firm-specific uncertainty and exit constitutes an exogenous process. This means that uncertainty related to firm death is also aggregate. Despite the introduction of matching frictions, every potential entrant faces the same matching probabilities $\Pi_{i,t}$ for each sector i determined by aggregate conditions that are not firm-dependent[3].

D. Preference Specification and Markups

To capture heterogeneity across technology types, I adopt a two-layer aggregation process that nests the Dixit–Stiglitz framework at both the micro/sector and macro/aggregate levels, following Carvalho et al. (2021). For each sector i, consumption is an aggregate of the continuum of varieties $\omega \in \Omega_i$. Specifically, the type-i consumption bundle is defined as

$$C_{i,t} \;=\; \left[\int_{\omega\in\Omega_i} c_{i,t}(\omega)^{\frac{\theta-1}{\theta}}\, d\omega\right]^{\frac{\theta}{\theta-1}},$$

and the corresponding price index is given by

$$P_{i,t} \;=\; \left[\int_{\omega\in\Omega_i} p_{i,t}(\omega)^{1-\theta}\, d\omega\right]^{\frac{1}{1-\theta}},$$

where $\theta \geq 1$ is the elasticity of substitution among varieties within sectors. By construction, total expenditure on goods in sector i is $P_{i,t}C_{i,t}$. At the aggregate level, the type-specific bundles $\{C_{i,t}\}_{i=1}^{I}$ are aggregated into a final consumption basket. The final consumption aggregator is given by

$$C_t \;=\; \left[\sum_{i=1}^{I} (C_{i,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

with the overall price index defined as

$$P_t \;=\; \left[\sum_{i=1}^{I} (P_{i,t})^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

where $\eta \geq 1$ is the elasticity across sectors. This top-level aggregation treats each technology type as a distinct variety, such that consumers can also substitute between sectoral bundles.

It follows that the household's demand for each good and sectoral good is $c_{i,t}(\omega) = \left(\frac{p_{i,t}}{P_{i,t}}\right)^{-\theta} C_{i,t}$ and $C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} C_t$. Additionally, the markup and the within/across sector benefit of variety are independent of the number of goods, and related by $\epsilon = \mu - 1 = 1/(\theta-1)$. The C.E.S. expressions are then: $\mu(M_{i,t}) = \mu = \frac{\theta}{\theta-1}$, $\epsilon_{\text{within}}(M_{i,t}) =$

[3] See Section C in the Appendix for the expanded BGM proof for my framework with matching frictions.

$\mu - 1$, $\epsilon_{\text{across}}(N_t) = \eta - 1$ and most notably for the relative price:

$$\rho(M_{i,t}) = M_{i,t}^{\mu-1} = (M_{i,t})^{\frac{1}{\theta-1}}. \tag{3.10}$$

E. Household Problem

As stated before in Section 3.1, the household problem here is analogous to the one found in BGM, since the consumers are effectively "blind" to the sectoral diversity and consume a final aggregate basket of goods C_t. The only significant difference is regarding the heterogeneity of the firms whose shares the household buys, but consumers only observe the aggregate quantities. Therefore, I define the aggregate firm-level profit and value as

$$d_t = \frac{\sum_i M_{i,t} d_{i,t}}{\sum_i M_{i,t}} \quad \text{and} \quad v_t = \frac{\sum_i e_{i,t} v_{i,t}}{\sum_i e_{i,t}},$$

and N_t as the measure of Ω_t, representing the total number of incumbents in the economy at time t.

In this framework, there are 2 types of assets that the household can buy in period t. The first are shares in a mutual fund for firms x_t and the second are risk-free bonds B_t. Mutual funds pay a total profit in each period equal to the total profit of all firms that produce in t, namely $P_t N_t d_t$. During t, the household buys x_{t+1} shares in a mutual fund of $N_{H,t} = N_t + \mathcal{E}_t$ firms (those already operating at time t and new potential entrants $\mathcal{E}_t = \sum_i e_{i,t}$), but only $N_{t+1} = (1-\delta)(N_t + N_{E,t})$ (where $N_{E,t} = \sum_i m_{i,t}$) will produce and pay at time $t+1$, due to matching frictions and the death shock. The price of a claim to the future profit stream of the fund of $N_{H,t}$ firms at t is equal to the nominal price of claims to future firm profits, $P_t v_t$.

Given this setup, the period budget constraint for the representative household can be written as

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t = (1 + r_t) B_t + (d_t + v_t) N_t x_t + w_t L_t,$$

where the left side represents expenditures including the purchase of new assets and consumption, and the left side represents income including the dividends from firms, returns on bond stocks, and labor income.

From this framework, I derive the first-order conditions of the household for optimal behavior by solving the utility maximization problem, yielding the Euler equation for bonds

$$(C_t)^{-1} = \beta(1 + r_{t+1}) E_t \left[(C_{t+1})^{-1} \right], \tag{3.11}$$

the Euler equation for shareholdings in the fund of firms

$$v_t = \beta(1-\delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right], \tag{3.12}$$

which, if iterated forward without the presence of speculative bubbles, yields the aggregate version of the relationship expressed by the firm valuation equation (3.7), and finally the intratemporal first-order condition

$$\chi L_t^{1/\varphi} = \frac{w_t}{C_t}, \tag{3.13}$$

where φ represents the Frisch-elasticity of labor and χ is the disutility of labor, as before[4].

F. Aggregate Accounting, Equilibrium, and the Labor Market

Under equilibrium, the following conditions hold for market clearing: $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1\ \forall t$. Therefore, by applying these equilibrium conditions to the household budget constraint and disaggregating firm-level variables, I arrive at the aggregate accounting identity

$$C_t + v_t \mathcal{E}_t = w_t.L_t + d_t N_t \;\Leftrightarrow\; C_t + \sum_i v_{i,t} e_{i,t} = w_t.L_t + \sum_i d_{i,t} M_{i,t}, \tag{3.14}$$

which states that the expenditure in consumption and investment in new firms must be equal to the income derived from labor and firm dividends in equilibrium.

Similar to the dynamic in the BGM baseline model, and in contrast to the standard one-sector RBC model of Campbell (1994), the model presented here has two "macro-sectors", such that one employs part of the labor supply to produce consumption goods and the other sector employs the rest of the labor supply to establish new production-lines/firms. Consequently, the economy's GDP Y_t is simultaneously equal to total income $w_t.L_t + \sum_i d_{i,t} M_{i,t}$ and the total output of the economy $C_t + \sum_i v_{i,t} e_{i,t}$, and $v_{i,t}$ represents the relative price of investment in terms of consumption.

As a direct consequence of the two-sector nature of the model, the labor market equilibrium is characterized by $L_t^C + L_t^E = L_t$, that is, the total labor is equal to the labor used in production $L_t^C = \sum_i M_{i,t} \ell_{i,t}$ at time t plus the total labor used to build firms $L_t^E = \sum_i e_{i,t} f_{E,t}/Z_t$ at time t. Entry determines the amount of labor allocated to setting up new production lines, such that entry at time t affects labor demand in time $t+1$. Therefore, aggregate labor clearing can be expressed as:

$$L_t = \underbrace{\sum_i M_{i,t} l_{i,t}}_{\text{production}} + \underbrace{\sum_i e_{i,t} f_{E,t}/Z_t}_{\text{entry}}. \tag{3.15}$$

Matching frictions, however, introduce a dynamic where not all the labor spent in building new firms is fruitful. Labor is used to produce firms that might not survive the matching process, especially for highly scalable sectors that have lower probabilities of successful

[4] As in BGM, I omit the transversality conditions for bonds and shares that must be satisfied to ensure optimality. Additionally, the interest rate is determined residually since it appears only in the Euler equation for bonds and is pinned down once consumption is known.

entry.

G. Characterizing Equilibrium

Given predetermined variables $M_{i,t}$, $\psi_{i,t}$, r_t and exogenous shocks (Z_t, X_t)[5], the intra-period sequence in the model goes as follows: i) Given their preferences, households choose $\{C_t, L_t, B_{t+1}, x_{t+1}\}$; ii) Incumbent firms hire labor, set prices, produce and pay dividends in all sectors; iii) Potential entrants pay the sunk entry cost $f_{E,t}w_t/Z_t$. Matching occurs with probability $\Pi_{i,t}$ and successful entrants $m_{i,t}$ join $M_{i,t+1}$ in all sectors i (time-to-build); iv) Exogenous shocks (Z_{t+1}, X_{t+1}) realize and date $t{+}1$ begins.

Out of the 15 main equations highlighted above, many of the variables introduced are merely definitions and/or identities that add no new information to the system. For instance, the definition of firm-level output and labor demand (3.1) simply build $y_{i,t}$ and $l_{i,t}$ once $C_{i,t}$, $M_{i,t}$ are known. Additionally, the aggregate consumption index C_t is not included since it is mechanically defined once $C_{i,t}$ is known for each sector, adding no information to the system. Therefore, I do not include the aggregate goods clearing condition as an equilibrium condition, since it does not pin down any additional variable. Furthermore, nominal variables like $P_{i,t}$ and $p_{i,t}$ are not solved for, since, with C.E.S. preferences, blocks (3.3) and (3.10) solve for the real/relative sectoral price $\rho_{i,t}$.

Therefore, equilibrium can be characterized as follows: Given exogenous processes (Z_t, X_t), initial masses $\{M_{i,0}\}_{i=1,\dots,I}^{I}$, and a top-level price numéraire $P_t \equiv 1$ as a nominal anchor, I summarize equilibrium as a sequence

$$\left\{\, w_t^*, L_t^*, r_t^*, \{M_{i,t}^*, C_{i,t}^*, \rho_{i,t}^*, d_{i,t}^*, v_{i,t}^*, e_{i,t}^*, \psi_{i,t}^*, \Pi_{i,t}^*\}_{i=1,\dots,I}^{I} \right\},$$

of $3 + 8I$ endogenous variables, with 3 variables being predetermined at time t ($M_{i,t}$, r_t and $\psi_{i,t}$). This satisfies the simplified system of equilibrium conditions in Table 3.1 out of all the relevant equations explored in Section 3.3.

[5] Note: I treat $f_{E,t}$ as a parameter during quantitative exercises.

	Description	Equation(s) in Section 3	Dimension
1	Pricing Rule in sector i	$\rho_{i,t} = \mu \frac{w_t}{\alpha_i Z_t} \left(\frac{y_{i,t}}{Z_t}\right)^{1/\alpha_i - 1}$ combining (3.3) and (3.2)	I
2	Variety Effect in sector i	$\rho_{i,t} = M_{i,t}^{\frac{1}{\theta-1}}$ from (3.10)	I
3	Profits in sector i	$d_{i,t} = \left(1 - \frac{1}{\mu}\right) \frac{C_{i,t}}{M_{i,t}}$ from (3.4)	I
4	Firm valuation in sector i	$v_{i,t} = \beta(1-\delta)E_t\Big[\big(C_{i,t+1}/C_{i,t}\big)^{-1}\big(v_{i,t+1} + d_{i,t+1}\big)\Big]$ from iterating (3.7) backwards	I
5	Free entry in sector i	$\Pi_{i,t} = \frac{f_{E,t} w_t}{Z_t v_{i,t}}$ from (3.8)	I
6	Stock of ideas in sector i	$\psi_{i,t} = X_t^{\frac{1}{1-\phi}} \bar{\psi}_i$ from (3.5)	I
7	Matching definition	$e_{i,t} = \psi_{i,t}\big(\Pi_{i,t}\big)^{\frac{1}{\phi-1}}$ from (3.6)	I
8	Number of firms in sector i	$M_{i,t+1} = (1-\delta)\big(M_{i,t} + e_{i,t}\Pi_{i,t}\big)$ from (3.9)	I
9	Intra-temporal labor FOC	$\chi L_t^{1/\varphi} = \frac{w_t}{C_t}$ from (3.13)	1
10	Inter-temporal Euler (bonds)	$1 = \beta(1 + r_{t+1})E_t\left[\frac{C_t}{C_{t+1}}\right]$ from (3.11)	1
11	Aggregate Labor Identity	$L_t = \sum_i (M_{i,t}\ell_{i,t} + e_{i,t} f_{E,t}/Z_t)$ from (3.15)	1
		Total independent conditions	$3 + 8I$

Table 3.1: Equilibrium conditions and dimensionality

The household Euler equation for shares is left out of the list of equilibrium conditions as in BGM because it represents the same relationship as the firm valuation equation but at an "average" aggregate level. Since I am interested in the sectoral profit and firm value dynamics, I do not include it in the equilibrium conditions. Finally, when building the "$3 + 8I$" system, I keep only three unknowns per sector for the matching-entry block, $e_{i,t}, \psi_{i,t}$ and $\Pi_{i,t}$. The auxiliary object $m_{i,t}$ is left out of the list, precisely because it is mechanically pinned down once $(M_{i,t}, e_{i,t}, \psi_{i,t}, \Pi_{i,t})$ are known.

4 Model Solution and Properties

4.1 Sector–by–Sector Steady State

To derive the steady-state expressions for all variables, I divide the solution into I sectoral blocks and one aggregate block, to stay consistent with the formulation of equilibrium in

Section 3.3. I normalize aggregate productivity $Z = 1$ and set the stock of ideas shock $X = 1$ in steady state, but I keep the variables in the expressions for tractability. Every variable is constant over time and thus time subscripts are dropped. Otherwise, the notation matches the main model description in Section 3.3.

From the intertemporal Euler equation for bonds, the steady-state interest rate is already pinned down by $1 + r = \beta^{-1}$. I follow BGM and Campbell (1994) and make use of $1 + r = \beta^{-1}$ to treat r as a parameter in the solution. The firm valuation equation (3.7) together with the definition $\kappa = \frac{r+\delta}{1-\delta}$ yields $v_i = d_i/\kappa$, which captures a premium for the expected firm destruction, since a higher δ brings down the price of the investment good (*ceteris paribus*). Combining this expression with the free-entry condition (3.8) and substituting in the definition of profits (3.4), I arrive at

$$\frac{C_i}{M_i} = \frac{\kappa\, w f_E}{Z\Pi_i(1 - 1/\mu)}, \tag{4.1}$$

which pins down the ratio between sectoral consumption and the number of firms as a function of model parameters and the probability of successful entry in sector i, and also represents the definition of firm-level output y_i. This expression helps pin down in steady state all variables dependent on y_i such as ℓ_i with equation (3.1), MC_i with equation (3.2) and ρ_i with (3.3).

Expression (4.1) hints at the fundamental role that competition plays in pinning down firm-level behavior in the conditional symmetric firm equilibrium. Almost counterintuitively, an increase in the death rate, real wage, or sunk entry-cost all lead to an increase in the firm-level output and labor demand, given that all these changes decrease the incentives for entering sector i. This, in turn, means that active incumbents in sector i have fewer competing firms with whom to share the market, leading to a higher level of output and larger firm size despite the negative impact of an increase in wages.

The contrasting behavior of potential entrants and active incumbents comes from the fact that, after the forward-looking choice of entry, successful entrants only perform the myopic-static choice of optimal labor demand every period, and do not take into consideration their exogenous probability of exit in doing so. Additionally, the negative impact of a wage surge on the labor decision is counterbalanced by the positive impact of a decrease in incumbents due to rising entry costs on within-sector competition, hence the positive impact of wage on firm-level output and employment.

Finally, an increase in the probability of successful matching also drives up entry, since an increase in Π_i raises the expected payoff of entering into sector i. In steady state, wage rises one-to-one with productivity since the marginal product of labor (MPL) is $MPL_i = \alpha_i Z l_i^{\alpha_i - 1}$, such that only the ratio w/Z actually matters, and thus a momentary increase in Z has no direct impact on firm-level output in a steady-state static equilibrium. Markups also have an interesting impact on firm-level output and size, where an increase in markups drives firm-level profits-per-unit up, which makes entry appear more attractive to prospective entrepreneurs, raising M_i. On top of that, however, higher markups also drive household demand for goods down since they increase relative prices, lowering C_i. These two effects combined make rising markups lower firm-level output.

As for the sectoral entry-matching block, combining the law-of-motion of incumbents (3.9) and the matching definition (3.6) both in steady state yields

$$m_i = e_i \Pi_i = \frac{\delta}{1-\delta} M_i. \tag{4.2}$$

That is, the number of successful new entrants m_i makes up for the exogenous destruction of new firms, and the number of prospective new entrants e_i also compensates for the probability Π_i in each sector of successful matching taking place.

The aggregate accounting identity (3.14) in steady state is also particularly informative when it comes to understanding the general structure of the model. By substituting in the free entry condition (3.8), matching probability definition (3.6), and the law of motion of incumbents (3.9), all in steady state, the identity now becomes:

$$C + \frac{wf_E}{Z}\frac{\delta}{1-\delta}\sum_i M_i = w.L + \frac{wf_E}{Z}\kappa \sum_i \frac{M_i}{\Pi_i}. \tag{4.3}$$

The same general idea of matching expenditure (consumption plus investment) with income (labor income plus dividends) in a two "macro-sector" economy remains. However, investment and dividends can be broken down into further detail.

Investment, on the one hand, now captures the replacement principle seen in the classic RBC model of Campbell (1994) more clearly, where investment made in steady state acts as pure maintenance to keep the variety/firm stock from shrinking. Therefore, a higher death rate immediately requires more labor to be spent on producing new firms. Dividends, on the other hand, represent in steady state the profits required to finance this replacement. A higher death rate, through κ, makes households require higher payoffs to keep financing new entries. Better matching in the form of a higher Π_i also frees up profits that can go to consumption or lower labor effort, an effect that is masked in the non-static version (3.14) because $e_{i,t}$ simultaneously adjusts on the investment side.

The blocks are, as discussed previously, divided into sectoral and aggregate. Due to concerns introduced by the heterogeneous sectoral blocks, no one-size-fits-all closed-form steady-state solution can be found, mainly due to the real wage. For that matter, I solve for the steady-state values of all variables using an outer-inner loop dynamic explored in the following Section 4.2. Subsequently, I explore these results in Section 4.4, with a particular focus on the composition of the firm sector and entry dynamics.

4.2 Numerical Algorithm for Solving the Steady State

In practice, the steady-state routine is a two-tier guess-and-verify scheme, similar to the price-clearing bisection loop found in Khan and Thomas (2008), but simpler. First, I propose a trial real wage $w^{(0)}$ that the household and firms would face. Given this provisional wage, each sector solves its conditional problem, pinning down M_i initially, after which e_i and C_i follow algebraically, yielding firm-level output, labor, profits, and value. These quantities are then aggregated to compute total consumption $C(w^{(0)})$ and labor $L(w^{(0)})$, allowing the evaluation of the household FOC residual:

$$g\big(w^{(0)}\big) \;=\; \chi\, L(w^{(0)})^{1/\varphi} \;-\; \frac{w^{(0)}}{C\big(w^{(0)}\big)}. \tag{4.4}$$

The routine then updates the wage guess until $g(w) = 0$, at which point, the last sectoral solutions constitute the general equilibrium steady-state values for the sectoral variables, which are then aggregated to compute total quantities. The exogenous inputs used to pin down the steady-state quantities are summarized in Table 4.1. They are broken down based on the block of the model where they have a direct impact. As mentioned in Section 3.2, there is no empirical counterpart to the stock of ideas variable ψ_i, only to the matching probability Π_i. Therefore, as in Sedláček and Sterk (2017), I take backed-out steady-state values for Π_i from BDS firm data in the steady-state calculations performed here. The exact calibration used in steady-state calculations and the quantitative exercise is discussed in depth in Section 4.3.

Block	**Parameters**
Preferences	$\beta,\ \chi,\ \varphi \quad \big(r = \beta^{-1} - 1,\ \kappa = \dfrac{r+\delta}{1-\delta}\big)$
Technology	$\theta > 1,\ \eta > 1,\ Z\ (=1),\ \{\alpha_i\}_{i=1}^{I},\ \{\Pi_i\}_{i=1}^{I}$
Entry / matching	$f_E,\ \delta,\ \phi,\ X\ (=1)$
Markup constant	$\mu = \dfrac{\theta}{\theta - 1}$

Table 4.1: Exogenous inputs for the steady-state routine

The main variables that I am interested in are the $3 + 8I$ endogenous variables from equilibrium, but the routine also pins down the whole system including the other variable definitions. Every sectoral variable is expressed as a function of the single scalar w, the current real wage guess. By equating the cost-based relative price (3.3) with the love-for-variety expression (3.10) and plugging in (4.1), the result is a single scalar equation for M_i as a function of the model parameters, the real wage, and the matching probability:

$$M_i^{\frac{1}{\theta-1}} = \mu\, \frac{w}{Z\alpha_i} \left[\frac{\kappa\, w\, f_E}{Z(1 - 1/\mu)\, \Pi_i^*} \right]^{\frac{1}{\alpha_i} - 1}, \tag{4.5}$$

Solving (4.5) numerically given the current wage guess yields the steady-state number of varieties/firms for each sector. After this step, I pin down the sectoral consumption level C_i using (4.1) (and thus firm-level output y_i and employment l_i are also mechanically defined), the relative price ρ_i using (3.10), firm-level profits d_i using (3.4), firm value v_i using $v_i = d_i/\kappa$ and the number of prospective entrants e_i using (4.2) (which pins down both m_i and ψ_i by the matching probability definition (3.6)).

Subsequently, the aggregate final quantities such as C and L are mechanically determined. Consequently, the outer loop using the intratemporal household FOC can be closed by finding the unique w^* such that $g(w^*) = 0$. Since (4.4) is a one-dimensional monotone function (when w rises, labor demand $L(w)$ falls and consumption $C(w)$ rises, hence $g(w)$ is strictly decreasing), a unique root exists, and a bisection algorithm converges fast. At convergence, the set

$$\left\{ w^*, L^*, r^*, \left\{C_i^*, M_i^*, \rho_i^*, d_i^*, v_i^*, e_i^*, \psi_i^*, \Pi_i^*\right\}_{(i=1,\dots,I)}^{I} \right\},$$

satisfies all equilibrium conditions listed in Table 3.1, with all other variables being pinned from their definitions and the numéraire being $P \equiv 1$.

4.3 Calibration

In my baseline calibration, I adopt parameters from both Bilbiie et al. (2012) and Sedláček and Sterk (2017) and interpret periods as years. Therefore, I adapt quarterly parameters from BGM, such as the exogenous firm exit shock $\delta = 0.09631$, to a yearly basis. All calibration values and their targets can be found in Table A.1 in the Appendix.

For the C.E.S. preferences, I use the value of $\theta = 3.8$ from Bernard et al. (2003) adopted by BGM. I set $\beta = 0.96$ to match an annual real interest rate of 4%. I set productivity Z and the fixed entry cost f_E both to 1. Finally, I consider $\varphi = 0$ for the inelastic labor case, and $\varphi = 4$ for the elastic labor case, and I take $\chi = 0.924271$ from BGM directly for ease of comparison with their baseline results. For the across-sector C.E.S. parameter, I take the calibration value of $\eta = 2$ from Carvalho et al. (2021), due to the similar sectoral structure found in their model.

For the Sedláček-Sterk matching block, I set the elasticity in the entry function to $\phi = 0.3$ to match their calibration. Following their approach with the BDS data, I set the total number of technology types equal to the number of size groups available in the database, yielding $I = 9$ sectors/technology types. I also adopted their implied values for the returns-to-scale parameters and matching probabilities, which they back out directly from BDS data.

Finally, for the two shocks explored in Section 5.1, I take the exact same persistence and standard deviations used in the original BGM and Sedláček-Sterk exercises. The productivity shock has persistence $\rho_Z = 0.9$ and standard deviation $\sigma_Z = \log(1.01)^2$, whereas the stock-of-ideas shock has persistence $\rho_X = 0.415$ and variance $\sigma_X = 0.000009$.

4.4 Properties of the Steady State

In this section, I explore the resulting dynamics of the numeric steady-state solution achieved through the algorithm in the previous section, with a special focus on the differences in behavior between different technology types. Overall, I explore a total of $I = 9$

sectors that come directly from the BDS data categorization done by Sedláček and Sterk (2017).

Figure 4.1 shows three important dimensions for each sector, namely firm-level output, firm-level employment, and the number of varieties/firms in each sector, which is represented by the size of each circle both in the figure and the legend. Focusing first on the relationship between firm labor demand and output, the axes are logged to make the power law governing the relationship clearer, especially the ways in which the returns-to-scale coefficient affects this relationship. That is, by taking logs of equation (3.1) and rearranging, the relationship between labor and output at the firm-level becomes

$$\ln y_i = \alpha_i \ln l_i + \ln Z, \tag{4.6}$$

which puts in evidence the direct impact that α_i has, such that, holding everything else fixed, a one percent rise in labor raises output by α_i percent.

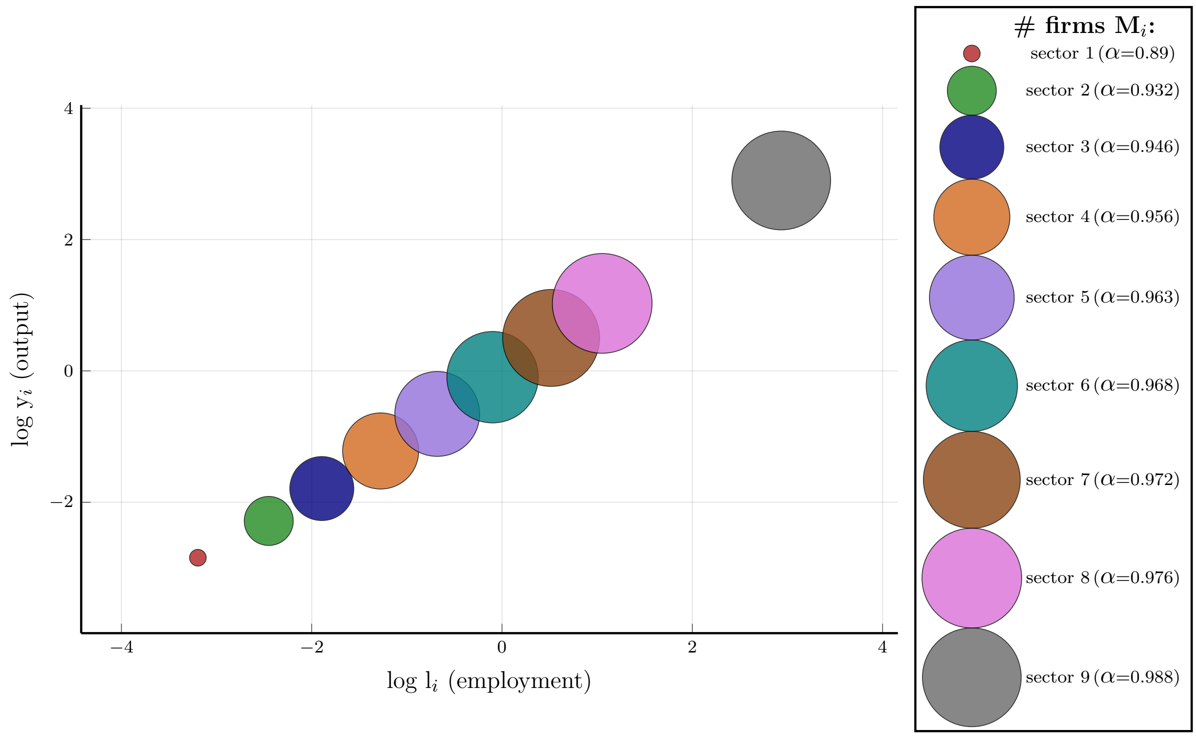

Figure 4.1: Steady-state relationship between firm-level output and employment (logged).

Therefore, the smaller α_i the more mild the response of output to an increase in employment, so low-scalability firms quickly run into diminishing returns and stay small (low employment). On the other hand, for high-scalability sectors, a one-percent increase in employment raises output by almost one percent. Because marginal product falls slowly, it takes longer for the firm to face diminishing returns to employment, and thus the firms' optimal size/employment is larger.

A real-life analogy could be the difference between a multi-plant manufacturer (highly scalable) and a specialized B2B producer (medium-small scalability). For the multi-plant manufacturer, despite their large fixed cost base, their standardized processes, efficient IT, or big capital stock lets each extra worker generate almost proportional extra output. On the other hand, for a specialized B2B producer output is tied closely to a small specialized crew, and extra workers might not contribute with equal extra output due to coordination/training costs[6].

An interesting case is sector 9, which significantly stands above the rest in both output and employment levels. This is due to its substantial returns-to-scale parameter, especially when compared to the preceding sectors 8 and 7. The increase in α from sector 7 to 8 is merely 0.004, whereas from sector 8 to 9 it is three times bigger at 0.012. This significantly impacts the size to which firms in sector 9 can grow when compared to other sectors, leading to bigger employment and output levels.

Moving on to the number of firms/varieties M_i, it is important to recall that, in steady state, the higher the returns-to-scale parameter in a sector, the smaller the matching probability Π_i due to a limited stock of ideas and excessive competition. Therefore, one might find the high number of firms in sectors 9, 8, or 7 contradictory, since entry is harder and firms are larger and thus the number of incumbents should be smaller. This would, however, be an incorrect interpretation, since M_i should be interpreted as the number of product lines (or varieties) rather than the number of stand-alone legal entities, as this is the interpretation favored in the baseline BGM model without sectoral heterogeneity.

In this reading, each incumbent variety corresponds to a point on the continuum ω, and the results displayed in Figure 4.1 can be made sense of. Highly scalable firms end up having a bigger number of product lines due to the easiness of scaling up the business, whereas low-scalability industries cannot expand their varieties/product lines so easily due to scaling issues. Therefore, for high-α technologies, marginal labor requirements to generate output are low, so once a core platform is in place (such as Amazon's fulfillment network or a cloud software data center), adding another feature or product line costs only the fixed entry cost[7]. By contrast, labor-intensive sectors such as restaurants, bespoke services, and others, have lower returns to scale. Even with abundant ideas, they cannot scale far, so their steady-state variety counts remain small due to the small number of prospective new product lines.

However, starting a successful product line in a high-α business is inherently harder, due to the high competition in entry and the limited stock of ideas. This is balanced out by the high expected profits that translate into a high net present value of v_i, making the free-entry condition (3.8) accommodate a vast flow of attempts, which in equilibrium yields a large stock of active varieties M_i (that might be all owned by the same firm, but the model abstracts away from this perspective). However, the difficulty of entry starts

[6] See Baqaee et al. (2024), where the authors show using Belgian data that plants with the highest value-added per worker systematically expand and capture market share. Sedláček and Sterk (2017) also document a similar dynamic where high-scalability firms invest more in demand expansion due to higher returns to scale.

[7] See DeStefano et al. (2023) for empirical evidence of this dynamic using data from cloud service firms in the UK.

to overpower the high scalability factor at some point. This is particularly true for sector 9, whose number of active varieties M_9 is slightly smaller than M_8 for sector 8, despite the significantly higher value of α_9, due to the extremely low steady-state probability of successful entry in Π_9 of around 0.2%[8].

Another interesting aspect of the steady-state solution is the friction of entry and the resulting relationship between prospective entrants and successful ones, depicted in Figure 4.2. The horizontal axis represents the steady-state number of prospective entrants in sector i, e_i, and the vertical axis represents the steady-state number of successful entrants in sector i, m_i. The α_i parameter for each sector is indicated next to each circle, and the steady-state matching probability Π_i for each sector is shown in the legend.

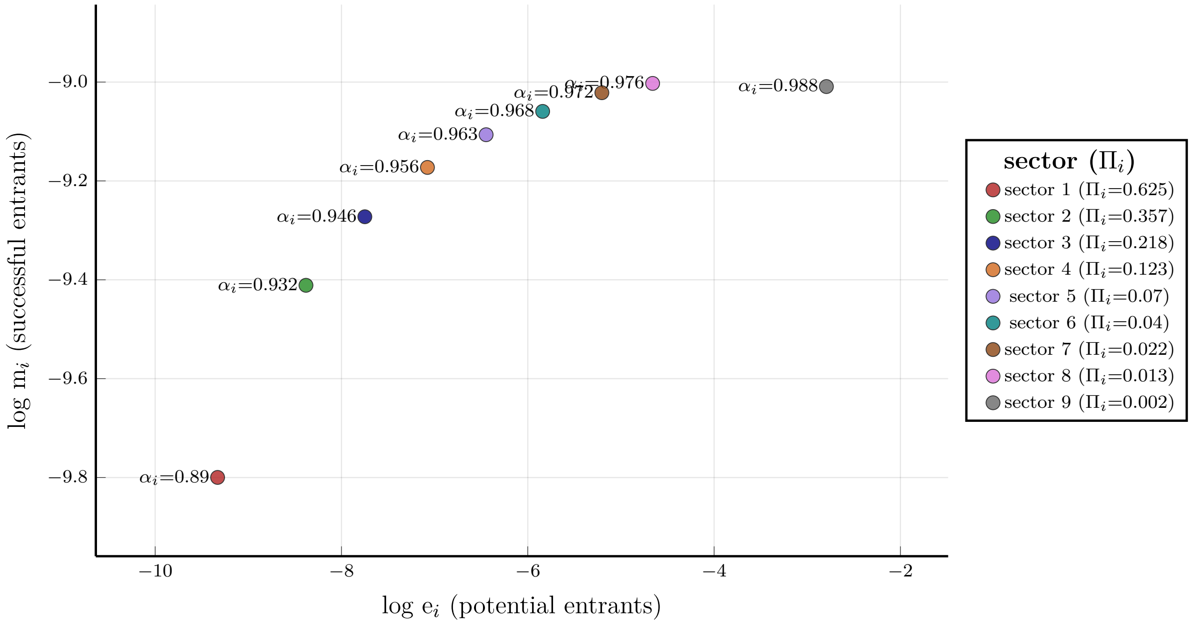

Figure 4.2: Steady-state relationship between prospective and successful entrants (logged).

As before, the axes are logged to reveal the elasticity between variables. Therefore, taking logs turns the relationship into a line whose slope is elasticity $\epsilon_i = \frac{\Delta \ln m_i}{\Delta \ln e_i}$. The figure puts the matching friction aspect of the model into the spotlight, showcasing the "decreasing-returns-to-entry" as one moves up the α_i values (which consequently moves down the matching probabilities Π_i). That is, on the double-log axes the curvature shows that the elasticity between potential entrants e_i and successful startups m_i falls steadily as the entry pool gets larger. In other words, each successive sector is fighting increasingly smaller returns-to-entry than the one to its left.

This concurs with the findings discussed in the previous section, where high-α sectors are more attractive despite the lower probability of success and low-α sectors face little congestion due to their unattractiveness, despite the large stock of ideas. By sector 9, the success rate is so minimal that the curve starts to invert on itself, such that, as explored

[8] For quick reference, Π_8 is 1.3% and Π_7 is 2.2%.

before for Figure 4.1, the low-matching-probability effect starts to dominate the high-net-present-value effect, beginning to showcase a "Laffer-like" (Laffer, 1974) relationship between e_i and m_i.

When interpreting the flow of prospective entrants e_i as investment outlay, another insight about Figure 4.2 appears. Namely, that it can be re-read almost one-for-one as a capital-production function $m_i = F(e_i)$. Therefore, the slope now represents the elasticity of new capital concerning investment. Because only a fraction Π_i is successful, the gap $e_i - m_i$ is akin to depreciation in transit, and the curved shape pictures the declining marginal product of piling ever more entry resources into a sector where profitable ideas are increasingly scarce. In sum, the introduction of matching frictions also enriches the "firm-entry-as-investment" dynamics of the original BGM framework, creating a more elaborate process of capital creation where not all investment translates directly to capital/firm formation.

5 Quantitative Implementation

In this section, I will focus on exploring the mechanisms of shock propagation in the model using a quantitative exercise. I compute the impulse responses to a productivity shock analogous to the one found in BGM, and to the supply-of-ideas shock which is, as explored in Section 3.3, based on the composition shock in the Sedláček-Sterk quantitative exercise. The results corroborate the intuition presented in the previous sections while adding novel insights about the model. In line with the BGM approach, I present the impulse response functions (IRFs) only for the inelastic labor case, with the IRFs for the elastic labor found in Section A of the Appendix. Finally, in Section 5.2 I report and explore the business cycle second moments of my model in comparison to the BGM framework, and the Real-Business-Cycle data and standard RBC model results both found in King and Rebelo (1999).

5.1 Impulse responses

5.1.1 Productivity Shock

In Figure 5.1 and Figure 5.2, the responses for the inelastic labor case to a transitory one percent productivity shock are displayed for both sectoral variables in 5.1 and aggregate ones in 5.2. As described before, periods are interpreted as years, and the number of periods after the shock is on the horizontal axis. I start by analyzing the sectoral responses and then move on to the aggregate ones.

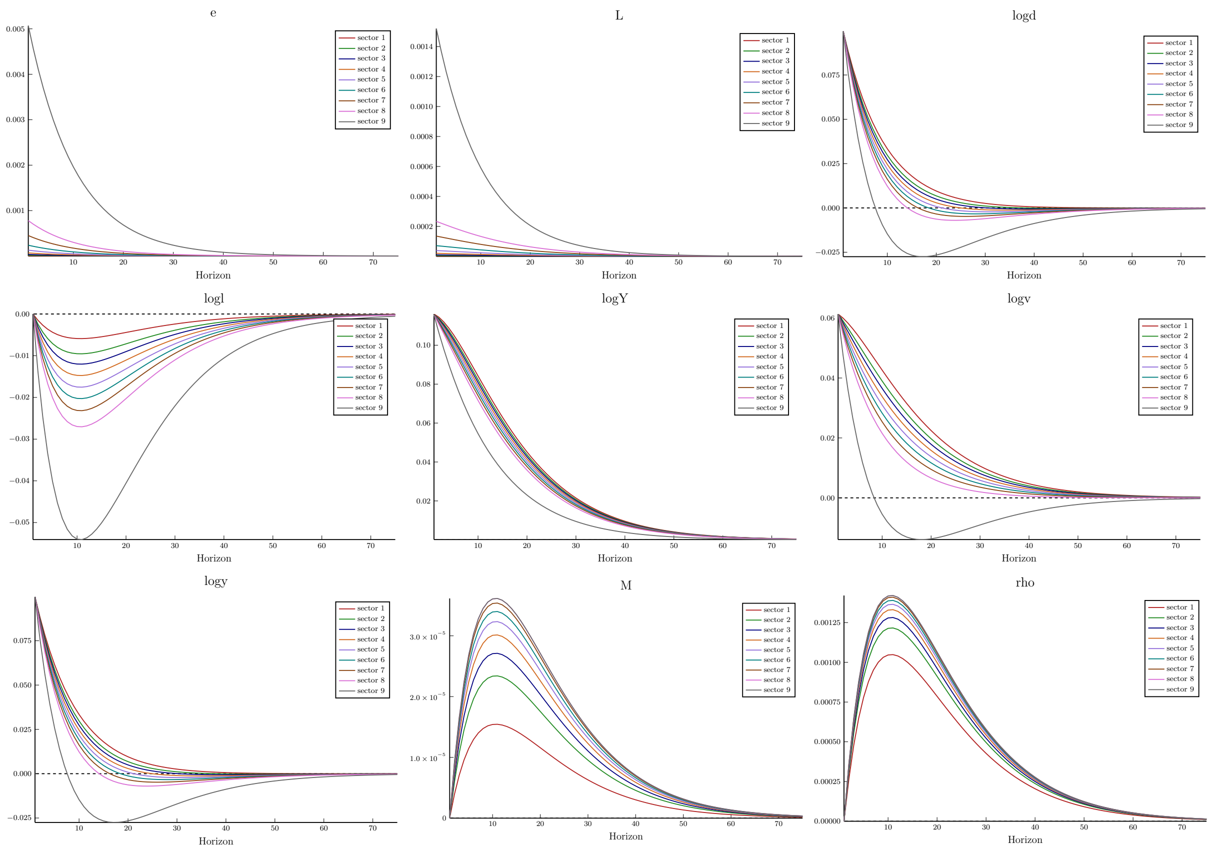

Figure 5.1: Responses to a transitory positive productivity shock for main sectoral variables.

In Figure 5.1, the response of sectoral flow variables such as $M_{i,t}$, $e_{i,t}$, and $L_{i,t}$ are kept in level deviation from steady state, to make the absolute quantities more clear for sectoral comparison. The variables are not normalized, however, so absolute values should not be interpreted literally. On the other hand, all other variables are logged, such that the response represents percentage deviations from steady state, to characterize the heterogeneous responses of different sectors. The only exception is the IRF for the real/relative price $\rho_{i,t}$, which is kept in levels to help clearly distinguish which sectors have higher and lower levels.

Initially, a temporary increase in TFP leads to a more attractive business environment, thus enticing entry in all sectors. In particular, sectors with higher α parameters face more entry because of higher MPLs, meaning that a rise in TFP yields higher returns. Due to the time-to-build lag, $M_{i,t}$ rises subsequently, but because of the matching frictions, the sectoral differences are more balanced. Consequently, the relative price rises, in particular in sectors that face more successful entry, due to the love-for-variety expression (3.10). Finally, a spike in labor used in building new product lines can be seen, especially in high-α sectors since prospective entry is higher, demanding more hours.

The most interesting dynamics, however, are the responses of the firm-level variables $(y_{i,t}, d_{i,t}, v_{i,t}, l_{i,t})$. In particular, the negative dip that profits, output, and value suffer in high-α sectors. Initially, the temporary positive increase in Z leads to bigger future profits and therefore bigger net present value of entering across all sectors[9], as can be seen in Figure 5.1. Due to time-to-build lag, firms pay the entry cost today but only start producing next period. Once this new cohort starts producing, the over-entry that happened, in particular in highly scalable sectors, leads to a competition glut (excessively high number of incumbents $M_{i,t}$).

This has two effects on firm-level variables. The first is the increase in the relative price $\rho_{i,t}$, which leads to consumers substituting away from the sectors with the highest prices, decreasing sectoral demand $C_{i,t}$, which forces firm-level output $y_{i,t}$ downwards. The second is the dilution of firm-level output and profits due to the conditional symmetric equilibrium conditions, where more firms now share the same demand. The higher the increase in $M_{i,t}$, the higher the impact of this mechanism, leading to a bigger impact on high-α sectors, as visible in the figure. This dynamic is well-portrayed in the firm-level labor demand $l_{i,t}$, which decreases substantially since firms now produce less output.

The impact of the competition glut alongside the increase in relative prices is what ultimately leads to the negative dip seen in some firm-level variables in Figure 5.1, in particular for sector 9, which, as explored before, represents a significant outlier and thus suffers from the competition glut the hardest. The recovery back to steady state happens as the exogenous firm exit shock thins out the bloated number of incumbents $M_{i,t}$. This, in turn, leads to less competition and lowers relative prices, bringing $y_{i,t}$ and $d_{i,t}$ back and therefore also $v_{i,t}$. Low-α sectors never attract enough entrants to flip their incumbents into the red, so they just glide back as Z dissipates.

Finally, the response of $M_{i,t}$ is also worth some exploration. $M_{i,t}$ is the key endogenous state in the model and, much like in BGM, the sunk cost dynamic and time-to-build lag makes $M_{i,t}$ behave very much like capital in the baseline RBC. Thus, $e_{i,t}$ represents consumer investment, and $v_{i,t}$ is the key price for household financing, with $e_{i,t}$ initially overshooting because $v_{i,t}$ is expected to temporarily increase in the future, especially in high α sectors. This makes it profitable for households to invest today to reap the temporary benefits of the positive productivity shock.

Moving on to Figure 5.2, the aggregate responses to the productivity shock are now of interest. The response of labor used in firm production (Le_t) is in line with the dynamics explored before, fuelling the substantial entry of new incumbents into the many sectors. Labor used in production (Lc_t) then rises to match the production of these new firms after they are built, hence the lag. Real wage rises to match the new productivity level through the increase in labor demand for both entry and production.

[9] The productivity shock affects all sectors with the same initial magnitude, and therefore, the deviation from steady state is the same across sector for firm-level variables.

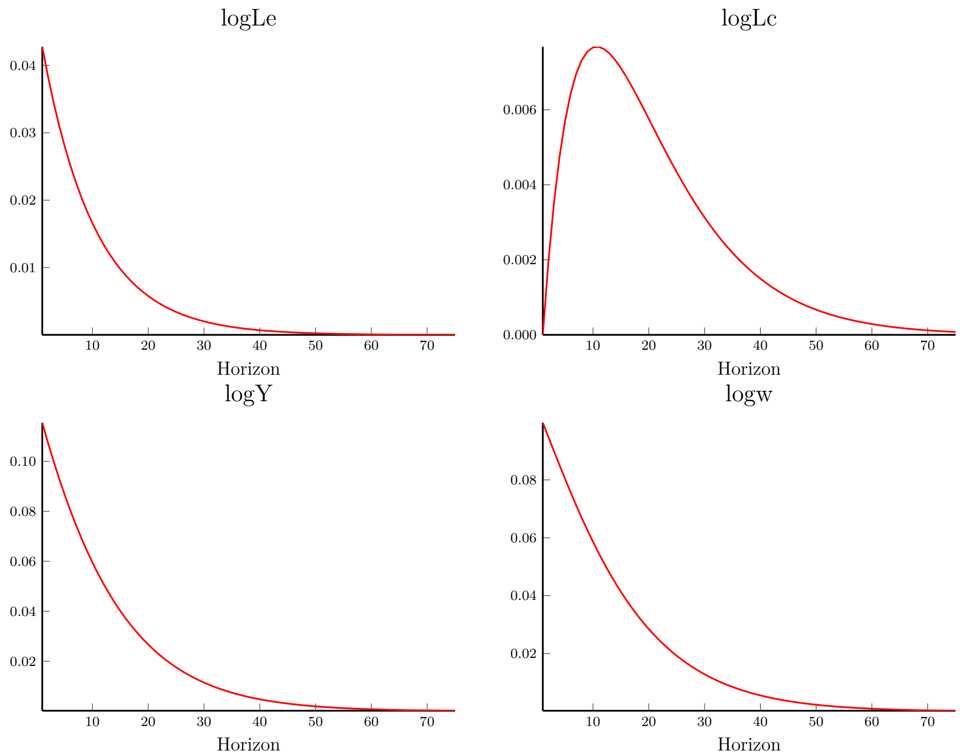

Figure 5.2: Responses to a transitory positive productivity shock for main aggregate variables.

The most interesting effect, however, consists of the response of aggregate output Y_t. In the baseline BGM framework, output reacts to the one-percent increase in Z with a response of around one to two percent. In Figure 5.2, however, the response far exceeds the one of BGM, leading to an increase of around eleven percent. This happens through the love-for-variety effect found in BGM – which drives an increase in output due to the increase in the number of varieties – but also due to two novel channels. The first is the possibility of cross-sector C.E.S. substitution, where households redirect demand towards sectors whose relative price rise the least[10]. Because the elasticity parameter η is bigger than one, the quantity response is elastic, such that the demand shift itself raises Y_t. This channel is sensitive to the calibration of η and thus its effect should be taken with a grain of salt.

The second novel channel constitutes the labor reallocation between sectors after a productivity shock. Since the effect of an increase in TFP on the MPL depends directly on the returns-to-scale parameter α, the shock induces reallocation from low-α to high-α sectors. This, in turn, raises output, since labor is now used more efficiently across the economy for production. The two novel channels represent extensions to the intensive composition margin in the model, going beyond the love-for-variety transmission mechanism found in BGM. Thus, the introduction of heterogeneous firm sectors into the BGM framework not only enriches the pre-existing dynamics but introduces new transmission mechanisms that enrich the propagation and effect of productivity shocks in the model.

[10] See Gouel and Jean (2023), where the same dynamic in a multi-sector C.E.S. framework is found.

As found in the original BGM quantitative exercise with productivity shocks, the response in the elastic labor case is qualitatively similar for almost all variables. Figures A.1 and A.2 in the Appendix present the IRFs for both sectoral and aggregate variables, respectively. All sectoral variables follow the exact same pattern as in the inelastic labor case, albeit with stronger responses. That is because the household now has an additional margin of adjustment in the face of shocks, thus enhancing the model's propagation mechanism and amplifying the responses of most endogenous variables, since households adjust their hours worked.

As for the aggregate responses in Figure A.2, all variables stay the same excluding labor that goes into production (Lc_t). For this variable, there is an initial decline to levels lower than steady state, followed by a recovery and a brief period of above steady-state levels of work hours. The initial drop is not present in the inelastic labor case because now hours are free to adjust, so the income effect of the positive productivity shock makes households enjoy more leisure now, leading to a fall in production labor even though the marginal product of labor is higher. The subsequent rise comes from the newly built product lines demanding labor to produce goods, as in the inelastic labor scenario.

5.1.2 Stock-of-Ideas Shock

In Figure 5.3 and Figure 5.4, the responses for the inelastic labor case to a transitory stock-of-ideas shock are displayed for both sectoral variables in 5.3 and aggregate ones in 5.4. As described before, periods are interpreted as years, and the number of periods after the shock is on the horizontal axis. I start by analyzing the sectoral responses and then move on to the aggregate ones. The stock-of-ideas shock follows the same magnitude as the composition shock found in Sedláček and Sterk (2017). That is, the shock is persistent ($\rho_X = 0.415$) but minuscule, with annual innovation volatility of $\sigma_X = 0.000009$ (less than one-hundredth of a percent). This explains the minimal responses of most endogenous variables, especially when compared to the responses to the productivity shock explored above.

As briefly discussed before in Section 3.3, I choose to modify the original composition shock to have it affect the stock of ideas of all sectors equally. That is because my goal is the study of aggregate RBC dynamics rather than cohort-by-year composition. This makes sure that the shock now has a larger macro impact, functioning as a facilitator of business activity across the board, from high- to low-scalability types. Empirical analogues could be a patent-law change, the arrival of an innovation, or a broad surge in entrepreneurial funding. The bottom line is that the shock now endogenously models an aggregate event that facilitates starting businesses by affecting the actual aggregate stock of ideas Ψ, preserving the microfoundations of the model. This differs from the literature that models entry-rate shocks as exogenous "animal-spirits" phenomena (see e.g. Leduc and Liu, 2016), simply hardcoding a higher $e_{i,t}$.

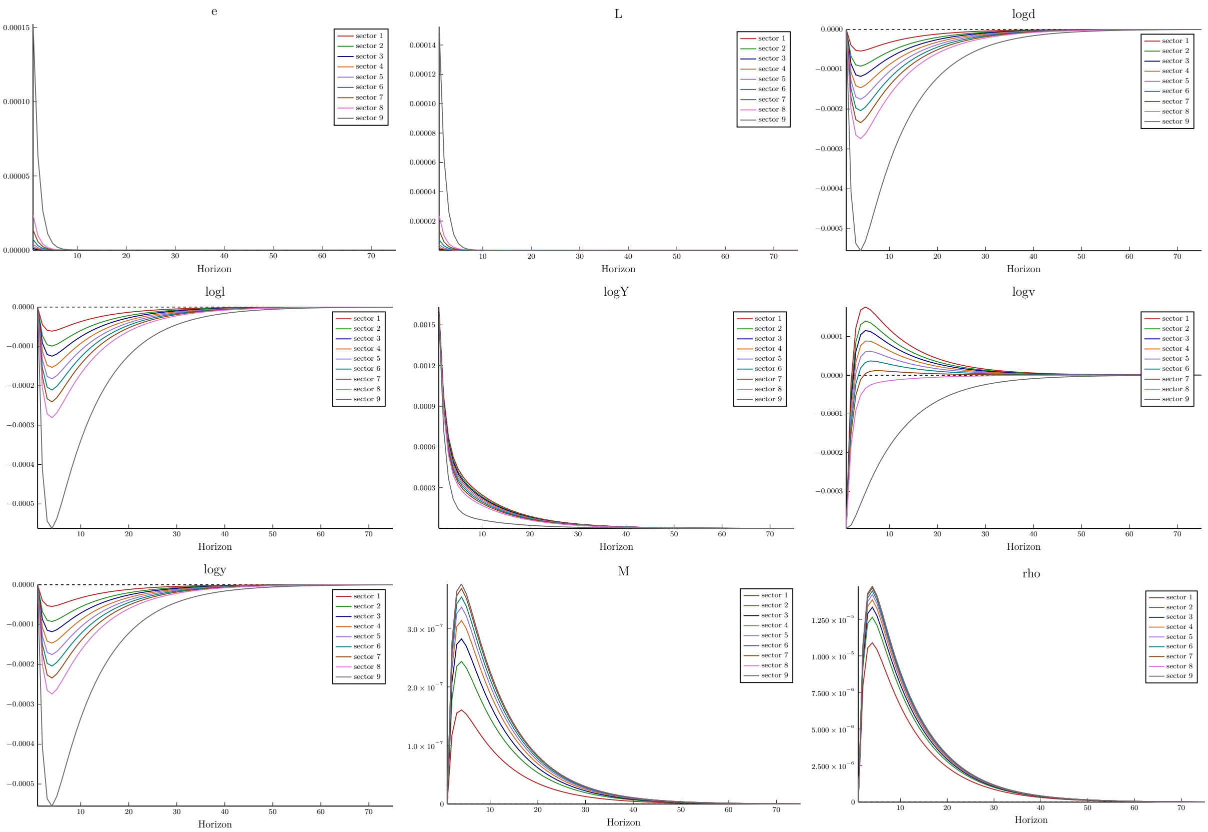

Figure 5.3: Responses to a transitory positive stock-of-ideas shock for main sectoral variables.

Despite the shock multiplying the stock of ideas for each sector equally, entry still differs due to different returns-to-scale parameters and matching probabilities, as visible in Figure 5.3. Prospective entry in all nine sectors jumps on impact due to a higher number of ideas being available, a dynamic similar to an increase in the number of lottery tickets for a given prize (successful entry in this scenario). As before, sector nine sees the highest surge in entrants by far, given the attractiveness of its high-α technology. Due to time-to-build and sunk entry costs, the number of product lines $M_{i,t}$ in each sector adapts with a lag to the new entrants, behaving again as physical capital in the textbook RBC model. A stark increase in sectoral-level labor L to build new firms is once again present, matching the spike in prospective entrants.

Echoing the previous results for the productivity shocks, the firm-level variable dynamics are plagued by the same curse of over-entry and competition, but without the positive effect of a temporary rise in productivity to compensate. Therefore, due to the over-entry mechanism and the subsequent competition glut explored before, firm-level output $(y_{i,t})$, labor $(l_{i,t})$, and profits $(d_{i,t})$ all fall with a lag after the time-to-build lag. Evidently, high-α sectors suffer the biggest impact, with over-entry biting their profits the hardest. Firm value, however, exhibits a different behavior from the three other variables. It instantly falls when the shock happens, since investors take the future increase in competitors and thus lower profits into consideration. Interestingly, however, $v_{i,t}$ overshoots above steady-state levels for low-α sectors only, since these sectors face less future competition and therefore have the fastest profit recovery of all sectors. Ultimately, it is the combination of the initial expected profit impact of entry and the speed of the expected relief that dictates the firm valuation dynamics.

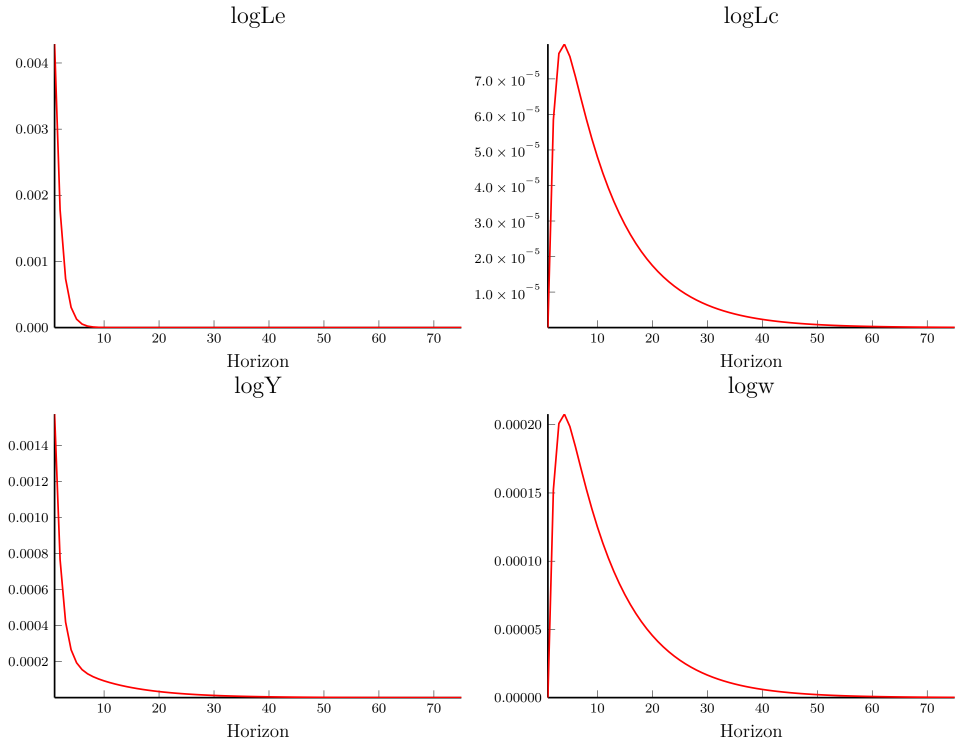

Figure 5.4: Responses to a transitory positive stock-of-ideas shock for main aggregate variables.

Moving on to aggregate variables in Figure 5.4, a similar dynamic as in Figure 5.2 can be seen for both production and entry labor. Labor used in entry (Le_t) spikes initially to support the new entrants, with the labor used in production (Lc_t) following suit to support production after the time-to-build lag. Output rises due to a sheer increase in the total number of producers, which, due to the love-for-variety effect, fosters higher demand. Some reallocation of demand across sectors also happens, since the shock also causes changes in the relative price $\rho_{i,t}$, but less than the response to the productivity shock. Finally, the real wage now rises with a lag since labor demand increases in production only after firms are built, and the stock-of-ideas shock does not directly influence MPL on impact and thus the real wage does not immediately rise as with the TFP shock.

Once more, the responses for the elastic labor case can be found in Figures A.3 and A.4 in the Appendix. For the aggregate responses, a similar dynamic to the one found in Figure A.2 can be seen, with labor used in production (Lc_t) facing the initial dip as before, due to households now adjusting their labor supply in response to the income effect from higher labor demand in entry (Le_t).

On the other hand, sectoral-level responses for firm-level variables present a new dynamic not seen in the inelastic case. Namely, the dip in the firm-level variables of employment ($l_{i,t}$), output ($y_{i,t}$), profits/dividends ($d_{i,t}$), and valuation ($v_{i,t}$) happens immediately after the shock, without lagging. This is also due to households now being able to adjust their total labor supply. Now, households can use fewer hours in production and more in entry since labor demand there is higher, as can be seen by the significantly higher levels of entry in Figure A.3 when compared to the inelastic case. Naturally, this leads to lower output, profits, and overall firm value, in practice bringing the over-entry/competition glut effect forward in time. The same effect is absent from the response in Figure A.1, simply because the productivity shock also raises the MPL in production, forcing households to keep their hours used in producing goods.

5.2 Business Cycle Moments

To further explore the new heterogeneous-sector framework presented here, I compute and report the implied second moments of the artificial economy for some key macroeconomic variables. I then compare these to real data and data produced both by the baseline BGM framework and by the benchmark RBC model from King and Rebelo (1999). As in BGM, I focus on random shocks to productivity Z_t as the source of business cycle fluctuations. I adopt the same productivity process as in King and Rebelo (1999) and BGM, which has persistence .979 and standard deviation of innovation .0072.

Following BGM, I now change $\varphi = 2$ to match the benchmark calibration in the RBC baseline. In Table 5.1, the first column (bold) in each subtable represents the empirical moment implied by the U.S. data as reported by King and Rebelo (1999), the second column (standard typeface) represents the moments implied by the BGM model, the third column (italics) displays the moments generated by King and Rebelo's baseline RBC model. Finally, the fourth column (underlined) presents the moments generated by the model here presented. Analogous to BGM, I compute model-implied second moments for HP-filtered variables for consistency, and I measure investment in my model as the real value of household investment in new prospective entrants $v_R\mathcal{E}$. To get empirically relevant variables (Y_R, C_R, and v_R), I make use of the sectoral relative price ρ_i to deflate the corresponding sectoral variables, and then merely aggregate them as $Y_R = \sum_{i=1}^{I}(Y_i/\rho_i)$, $C_R = \left[\sum_{i=1}^{I}(C_i/\rho_i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, and $v_R\mathcal{E} = \sum_{i=1}^{I}\frac{v_i}{\rho_i}e_i$.

(a) Volatility σ_X

X	**Data**	BGM	*RBC*	Model
Y_R	**1.81**	1.34	*1.39*	0.99
C_R	**1.35**	0.65	*0.61*	0.83
$v_R\mathcal{E}$	**5.30**	5.23	*4.09*	1.57
L	**1.79**	0.63	*0.67*	0.25

(b) Relative volatility σ_X/σ_{Y_R}

X	**Data**	BGM	*RBC*	Model
Y_R	**1.00**	1.00	*1.00*	1.00
C_R	**0.74**	0.48	*0.44*	0.84
$v_R\mathcal{E}$	**2.93**	3.90	*2.95*	1.59
L	**0.99**	0.47	*0.48*	0.25

(c) Persistence $E[X_t X_{t-1}]$

X	**Data**	BGM	*RBC*	Model
Y_R	**0.84**	0.70	*0.72*	0.73
C_R	**0.80**	0.75	*0.79*	0.74
$v_R\mathcal{E}$	**0.87**	0.69	*0.71*	0.71
L	**0.88**	0.69	*0.71*	0.74

(d) Contemporaneous corr. with Y_R

X	**Data**	BGM	*RBC*	Model
Y_R	**1.00**	1.00	*1.00*	1.00
C_R	**0.88**	0.97	*0.94*	0.99
$v_R\mathcal{E}$	**0.80**	0.99	*0.99*	0.97
L	**0.88**	0.98	*0.97*	0.99

Table 5.1: Second-moment statistics from **Data**, Bilbiie et al. (2012), *Baseline RBC*, and My Model

Volatility-wise in Panel a), the model underperforms when compared to the alternatives, being distant from both the benchmark RBC and BGM specifications and the real data, especially regarding investment and hours. This could potentially be explained by the ways in which the cross-sectoral averaging of variables dampens total variance since the shocks are aggregate and hit all sectors equally. Another potential source of dampening, especially for investment and labor, is the low calibration value for the matching elasticity parameter ϕ. A low ϕ caps the elasticity of entry to profits, such that entry reacts less, investment becomes far smoother than in the BGM baseline, and so does labor used in producing new firms.

This dynamic of overly smooth investment and hours is also present in the relative volatilities in Panel b), where the value for investment is half of the value in all other specification, and the same can be said for labor. The relative volatility of consumption, however, displays a value closer to the data than the overly smooth BGM and RBC frameworks. A potential cause could stem from consumers now facing sector-relative prices ρ_i, so inter-sectoral substitution takes place, making the response of consumption to variations in output stronger.

Moving on to the persistence in Panel c), the values are at least as good as the ones from the other two models when compared to the data. The persistence of labor, output, and investment are closer to empirical observations than the ones presented by BGM, albeit by a small margin. The addition of matching frictions on top of the time-to-build lag keeps entry reacting slowly, improving upon the already existing internal propagation mechanism that resembles physical capital accumulation in the benchmark RBC.

Finally, in Panel d) the high correlations found in BGM are preserved in my model, despite the addition of sectoral heterogeneity. Consumption moves too perfectly with output, however, which could be corrected by the introduction of preference shocks or sticky prices (see e.g. Smets and Wouters, 2007). Investment displays slightly lower procyclicality when compared to BGM or the RBC benchmark, but is still far off the 0.8 empirical baseline.

Despite overly-smooth entry, mainly due to calibration, hindering volatilities, the model captures persistence and correlations relatively well when compared to the other three baselines presented and displays adequate volatility of consumption with regard to output. Nonetheless, the model still faces the same well-known challenges as the standard RBC model: insufficient endogenous persistence and all variables are too procyclical in comparison to the data.

6 Conclusion

In this thesis, I sought to understand how sectoral firm heterogeneity in returns to scale and entry frictions alter RBC shock propagation, and how entry dynamics affect sectors differently. To that intent, I embeded the Sedláček-Sterk technology choice and matching into the Bilbiie-Ghironi-Melitz RBC with endogenous entry framework and explored responses to both a TFP and stock-of-ideas shock. Doing this amplified the transmission of an aggregate positive TFP shock roughly five-fold and uncovered a competition-induced negative impact on firm-level profits of an increase in entry, which greatly varied by sector.

The key findings can be separated into aggregate results, firm-level/sectoral results, and mechanistic insights. As for the aggregate results, the main finding is the new magnitude of the response of aggregate output to the temporary TFP shock. Output increases around 11% (1-2% in BGM) due to the love-for-variety effect from BGM, and due to two new channels: cross-sector C.E.S. substitution and labor reallocation toward high-α sectors. The stock-of-ideas shock also drives a modest increase in GDP due to an increase in varieties, but real wages lag until firms start producing. Turning to business cycle moments, the heterogeneous-sector specification reproduces persistence and co-movements found in BGM and brings the consumption-to-output volatility ratio closer to the data, but it severely understates the absolute and relative volatility of investment and hours, potentially due to overly-smooth entry from low matching elasticity ϕ and cross-sector averaging.

Moving on to firm-level/sectoral results, the findings suggest that over-entry and competition glut push profits, output, and employment per firm temporarily below steady state after a positive TFP or stock-of-ideas shock, with the effect being the strongest in high-α sectors, due to higher expected profits. Sectoral heterogeneity also influences steady-state properties, where highly scalable sectors showcase higher labor demand and output and overall number of varieties than their less scalable counterparts.

Finally, the findings echo the original BGM interpretation that entry behaves like capital investment in a classic RBC, such that the total number of incumbents evolves with time-to-build and depreciation-like exit, strengthening persistence. On top of that, however, the expanded model allows different investment responses for different sectors and adds a new potential source of investment loss through the matching frictions, which further bolsters persistence in the model. Steady-state entry dynamics also show decreasing "returns-to-entry", where each successive sector is facing steeper diminishing chances of success than the previous one, raising losses at entry.

Policy interventions should therefore be more granular in their incentives. For instance, startup support that varies by scalability class helps avoid the blanket subsidy pitfall, where incentives can exacerbate the competition glut identified in highly scalable (high-α) sectors. Steering some of those incentives toward less-scalable industries would spread entrepreneurial effort more evenly, easing matching frictions and the losses they create. Downturn subsidies should also factor in heterogeneous variety externalities and sectoral misallocation, nudging entrants to internalize the consumer surplus gains that new varieties generate based on their sector.

Some potential caveats qualify the main conclusions, however. First, calibration relies heavily on BDS-implied data and the sectoral splits made by Sedláček and Sterk (2017), such that results could be sensitive to alternative splits or new data altogether. Second, constant markups mute potential endogenous markup/price dispersion channels that naturally play an important role in firm dynamics. Third, all firms, no matter their scalability, face the same uniform exogenous exit probability, biasing competitive pressure and persistence. Fourth, the lack of capital shuts down capital-adjustment dynamics that usually amplify RBC persistence. Finally, ignoring household heterogeneity and portfolio risk may understate the distributional effects, particularly when entry reallocates labor across sectors.

Therefore, there are many potential avenues for future research. The first and most obvious one would be experimenting with endogenous markups generated by translog preferences, and implementing capital into the model. Both expansions are done in BGM, where the authors find a significant impact of translog preferences through endogenous markups on shock transmission, and that the addition of capital improves RBC second-moments significantly. Another potential idea is exploring more in-depth the welfare impacts of variety and entry externalities in a sectoral firm heterogeneity setup. As discussed before, this could yield interesting and novel policy insights not found in the baseline BGM framework. Additionally, comparing the framework with frameworks that have different sources of heterogeneity, such as markups or productivity parameters, could also be fruitful.

Overall, recognizing firm heterogeneity in scalability and matching frictions enhances how shock transmission is understood in RBC models and can guide smarter policy during downturns that respect the heterogeneous responses of firms.

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Appendices

A Additional Figures

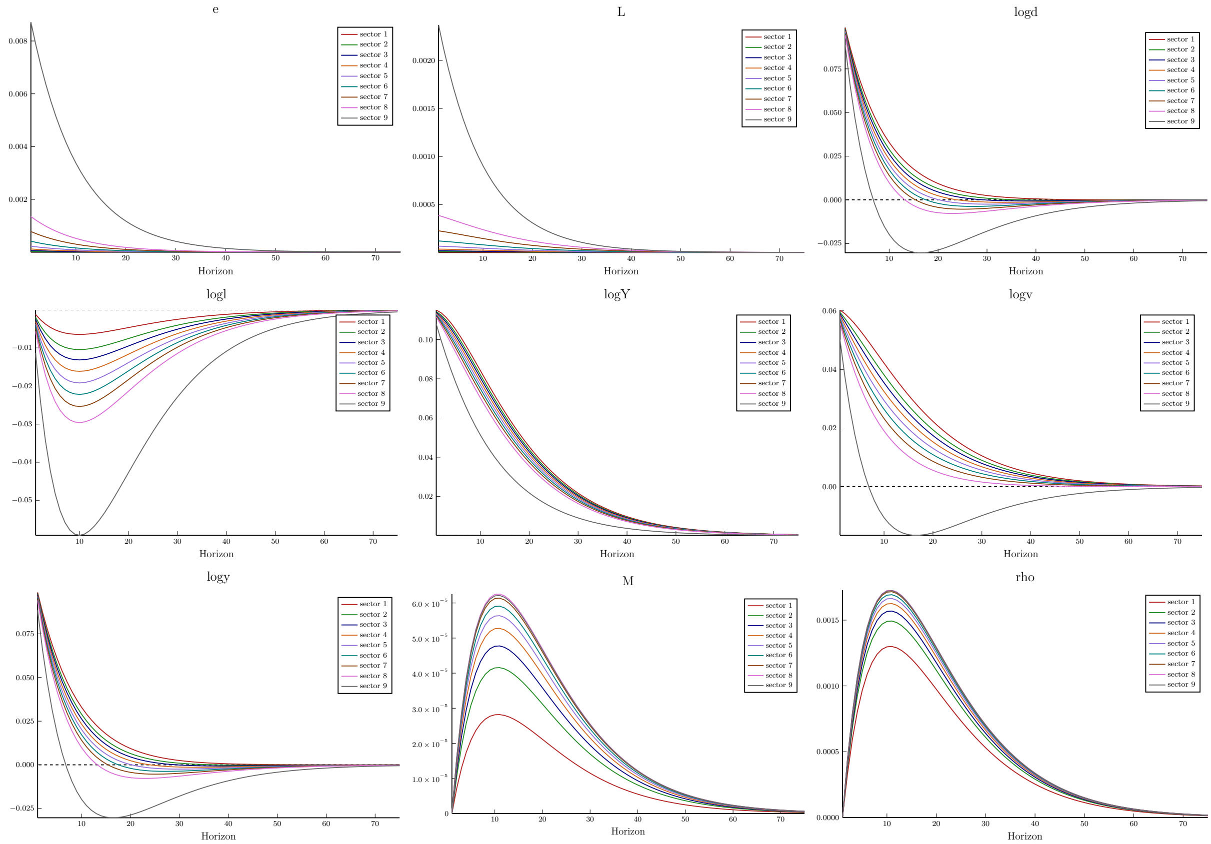

Figure A.1: Responses to a transitory positive productivity shock for main sectoral variables (Elastic Labor).

Table A.1: Calibrated parameters

Parameter	Value	Target
β discount factor to match the 4% annual interest rate	0.96	Sedláček and Sterk (2017)
δ exogenous firm exit shock (yearly)	0.09631	Bilbiie et al. (2012)
θ C.E.S. within-sector parameter	3.8	Bilbiie et al. (2012)
η C.E.S. across-sector parameter	2	Carvalho et al. (2021)
f_E steady-state entry cost	1	Bilbiie et al. (2012)
Z initial productivity	1	Bilbiie et al. (2012)
χ disutility of labor	0.924271	Bilbiie et al. (2012)
φ elasticity of labor supply	4 (2 for moment calculation)	Bilbiie et al. (2012)
ϕ elasticity in the entry function	0.3	Sedláček and Sterk (2017)
ρ_X stock of Ideas shock persistence	0.415	Sedláček and Sterk (2017)
σ_X stock of Ideas shock standard deviation	0.000009	Sedláček and Sterk (2017)
ρ_Z productivity shock persistence	0.9	Bilbiie et al. (2012)
σ_Z productivity shock standard deviation	$\log(1.01)^2$	Bilbiie et al. (2012)
α_i returns to scale parameters for each of the 9 sectors	$[0.890, 0.932, 0.946, 0.956, 0.963, 0.968, 0.972, 0.976, 0.988]$	average size in BDS
$\Pi_i = \left(\psi_i/e_i\right)^{1-\varphi}$ probability of successful entry for each of the 9 sectors	$[0.625, 0.357, 0.218, 0.123, 0.070, 0.040, 0.022, 0.013, 0.002]$	firm shares in BDS

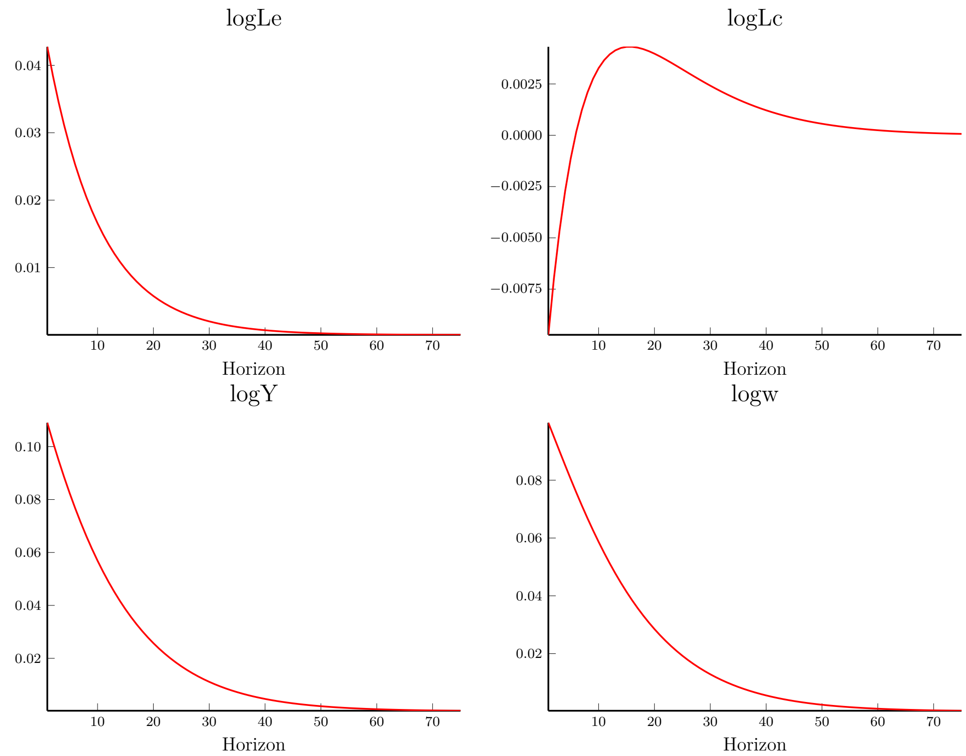

Figure A.2: Responses to a transitory positive productivity shock for main aggregate variables (Elastic Labor).

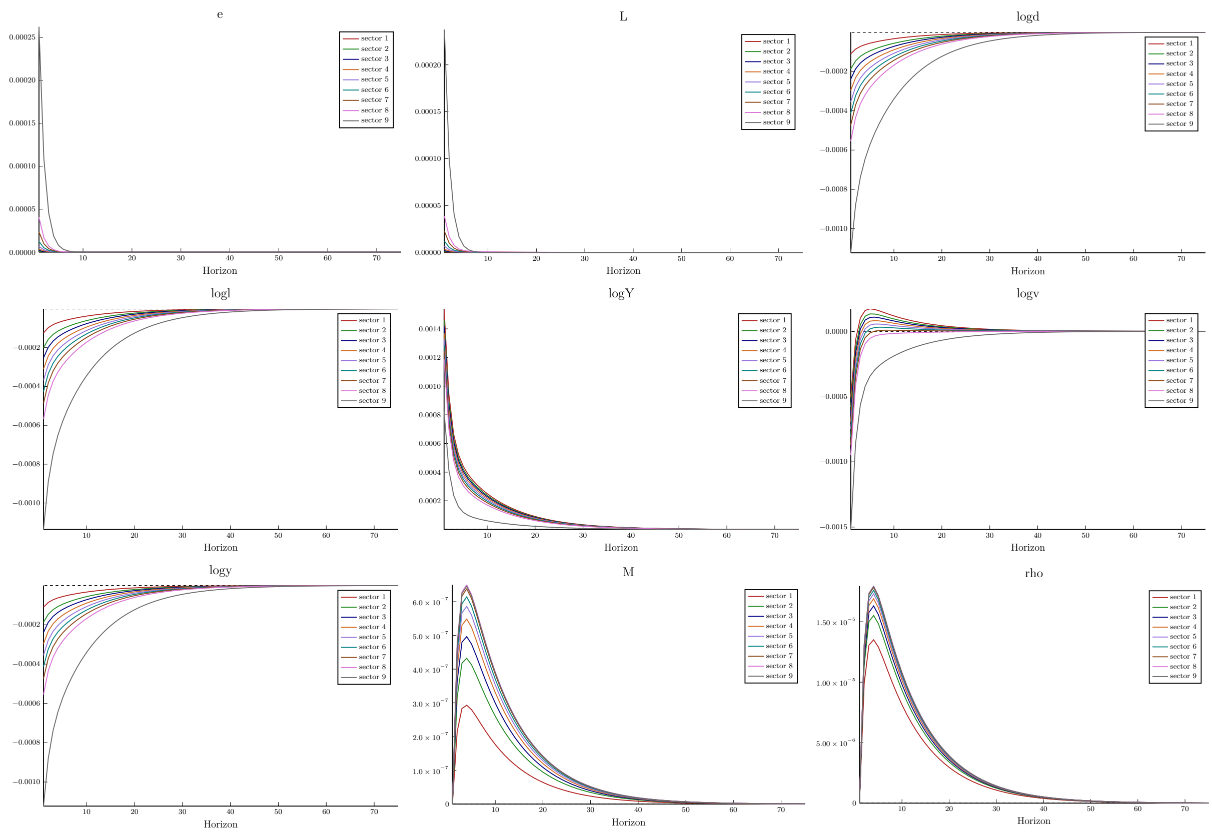

Figure A.3: Responses to a transitory positive stock-of-ideas shock for main sectoral variables (Elastic Labor).

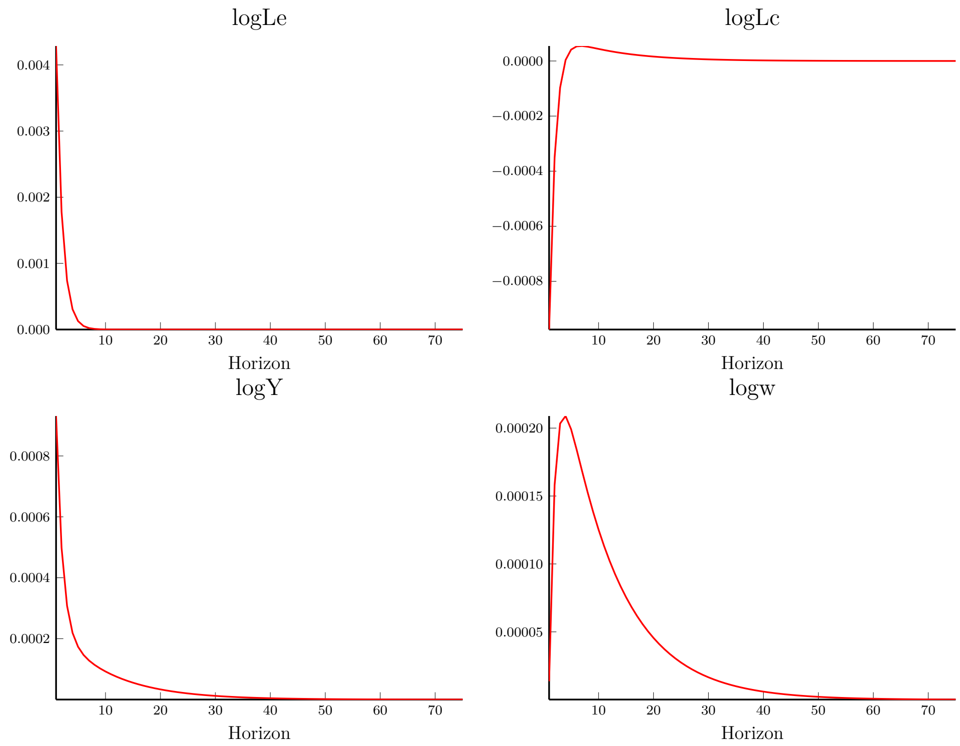

Figure A.4: Responses to a transitory positive stock-of-ideas shock for main aggregate variables (Elastic Labor).

B Computational Implementation

The computational Appendix provides a guide to the code repository accompanying this thesis. All scripts and data files are available at the GitHub repository:

https://github.com/Okim343/MyHeteroBilbiie_Thesis.git

B.1 Setup

The code is written in Julia (v1.9.4) and uses the `Dynare.jl` package for solving and simulating the RBC model for both the inelastic and elastic labor specifications. To install the correct Julia version and dependencies:

1. Install Julia 1.9.4 via `juliaup`.

2. Open a Julia REPL in the project root and run:

```
] activate .
(MyHeteroBilbiie) pkg> instantiate
```

B.2 Folder Structure

- `src/`: Contains the main Julia scripts that solve the model, compute impulse responses, and generate all figures.

- `helper_functions/`: Utility routines for computing steady states (`steady_state.jl`), drawing figures (`steady_state_figures.jl`), and plotting IRFs (`plot_irfs.jl`).

- `DynareModFiles/`: Dynare model files (`calibration.mod`, `declarations.mod`, `model.mod`, etc.) for both elastic and inelastic labor specifications.

- `Thesis/`: The LaTeX source of the thesis, including the compiled PDF and bibliography.

B.3 Scripts

To reproduce the results presented in the thesis, execute the following in order:

`analyze_steadystate.jl` : Solves and visualizes the steady state of the heterogeneous-firm RBC model under both elastic (for $\varphi = 4$ and $\varphi = 2$ for the business cycle moments calculations) and inelastic labor. Prints steady-state values for all key variables.

`run_model.jl` : Loads the calibrated Dynare model with elastic labor, runs stochastic simulations, computes impulse response functions for aggregate shocks, and exports all sectoral and aggregate IRF plots.

`run_model_ineL.jl` : Analogous to `run_model.jl`, but for the inelastic labor specification.

B.4 Reproducing Figures

After obtaining steady-state values from `analyze_steadystate.jl`, these must be copied into the Dynare mod files (`main.mod`, `main_ineL.mod`, `main_bcycle.mod`, `calibration.mod`, `calibration_ineL.mod`, and `calibration_bcycle.mod`) because `Dynare.jl` does not yet support dynamic parameter passing. Then:

1. Run `run_model.jl` to generate IRFs for the elastic labor case.

2. Run `run_model_ineL.jl` to generate IRFs for the inelastic labor case.

3. The resulting figures are saved under `Images/`, organized by specification (elastic/inelastic) and shock type (XShock, ZShock).

4. To get the Business Cycle Moments, one has to run `model_bcycle.mod` directly on a MATLAB terminal using the MATLAB VSCode extension (or MATLAB itself) by typing:

   ```
   >> dynare main_bcycle
   ```

5. This will automatically report all the coefficients of interest, with the relative standard deviation being retrieved by dividing the reported standard deviations by the standard deviation of real output Y_R.

C No Option Value for Waiting to Enter with Matching Frictions

The following proof is very similar to the one found in BGM, with the only major difference being the addition of $\Pi_{i,t}$, the matching probability at time t for sector i, and that firms are now characterized by both i and ω. Let the option value of waiting to enter for firm ωi be $\mathcal{W}_{i,t}(\omega) \geq 0$. In all periods t,

$$\mathcal{W}_{i,t}(\omega) = \max\Big[v_{i,t}(\omega) - \frac{w_t\, f_{E,t}}{\Pi_{i,t} Z_t},\ \beta\, \mathcal{W}_{i,t+1}(\omega)\Big],$$

where the first term is the payoff of undertaking the investment and the second term is the discounted payoff of waiting. If firms are identical (there is no idiosyncratic uncertainty) and exit is exogenous (uncertainty related to firm death is also aggregate), this becomes:

$$\mathcal{W}_{i,t} = \max\Big[v_{i,t} - \frac{w_t\, f_{E,t}}{\Pi_{i,t} Z_t},\ \beta\, \mathcal{W}_{i,t+1}\Big].$$

Because of free entry, the first term is always zero, so the option value obeys:

$$\mathcal{W}_{i,t} = \beta\, \mathcal{W}_{i,t+1} \quad \forall i \in \{1, 2, ..., I\}.$$

This is a contraction mapping because of discounting, and by forward iteration, under the assumption

$$\lim_{T\to\infty} \beta^T\, \mathcal{W}_{i,t+T} = 0$$

(i.e., there is a zero value of waiting when reaching the terminal period), the only stable solution for the option value is

$$\mathcal{W}_{i,t} = 0 \quad \forall i \in \{1, 2, ..., I\}.$$

Appendix References

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Statement of Authorship

I hereby confirm that the work presented has been performed and interpreted solely by myself except for where I explicitly identified the contrary. I assure that this work has not been presented in any other form for the fulfillment of any other degree or qualification. Ideas taken from other works in letter and in spirit are identified in every single case.

Enrico Truzzi
Bonn, the 22nd July 2025