STU FEI
ÚSTAV ROBOTIKY A KYBERNETIKY

Kolekcia učebných textov

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KUT003

O základných vlastnostiach lineárnych systémov

1 Riaditeľnosť

1.1 Diskrétny systém

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{B}\mathbf{u}_n \tag{1}$$

$$\mathbf{x}_1 = \mathbf{A}\mathbf{x}_0 + \mathbf{B}\mathbf{u}_0 \tag{2a}$$

$$\mathbf{x}_2 = \mathbf{A}^2\mathbf{x}_0 + \mathbf{A}\mathbf{B}\mathbf{u}_0 + \mathbf{B}\mathbf{u}_1 \tag{2b}$$

$$\vdots$$

$$\mathbf{x}_n = \mathbf{A}^n\mathbf{x}_0 + \mathbf{A}^{n-1}\mathbf{B}\mathbf{u}_0 + \cdots + \mathbf{A}\mathbf{B}\mathbf{u}_{n-2} + \mathbf{B}\mathbf{u}_{n-1} \tag{2c}$$

$$\mathbf{x}_n - \mathbf{A}^n\mathbf{x}_0 = \underbrace{\begin{bmatrix}\mathbf{B} & \mathbf{A}\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}}_{\mathcal{C}} \begin{bmatrix}\mathbf{u}_{n-1} \\ \mathbf{u}_{n-2} \\ \vdots \\ \mathbf{u}_0\end{bmatrix} \tag{3}$$

kde $\mathcal{C}$ označuje maticu riaditeľnosti. Matica $\mathcal{C}$ má rozmery $n \times nr$, kde r je počet vstupov (rozmer vektora $\mathbf{u}$).

Veta 1.1. Diskrétny lineárny systém (1) je riaditeľný pokiaľ matica riaditeľnosti má plnú hodnosť:

$$\operatorname{rank}\{\mathcal{C}\} = \operatorname{rank}\left\{\begin{bmatrix}\mathbf{B} & \mathbf{A}\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}\right\} = n \tag{4}$$

1.2 Spojitý systém

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{5}$$

$$\mathbf{x}(t) = \exp\left(\mathbf{A}(t - t_i)\right) + \int_{t_i}^{t} \exp(\mathbf{A}(t-\tau))\mathbf{B}\mathbf{u}(\tau)d\tau \tag{6}$$

$$\exp(-\mathbf{A}t)\mathbf{x}(t) - \exp(-\mathbf{A}t_i)\mathbf{x}(t_i) = \int_{t_i}^{t} \exp(-\mathbf{A}\tau)\mathbf{B}\mathbf{u}(\tau)d\tau \tag{7}$$

$$\exp(\mathbf{A}\tau) = \sum_{n=0}^{\infty} \frac{1}{n!}\mathbf{A}^n\tau^n \tag{8}$$

$$\det\{\mathbf{A} - \lambda I\} = 0 \tag{9}$$

$$\lambda^n + \alpha_{n-1}\lambda^{n-1} + \cdots + \alpha_1\lambda + \alpha_0 = 0 \tag{10}$$

$$\mathbf{A}^n + \alpha_{n-1}\mathbf{A}^{n-1} + \cdots + \alpha_1\mathbf{A} + \alpha_0\mathbf{I} = 0 \tag{11}$$

$$\mathbf{A}^n = \sum_{i=0}^{n-1} \alpha_i \mathbf{A}^i \tag{12}$$

$$\exp(\mathbf{A}\tau) = \sum_{n=0}^{n-1} \alpha_i(\tau)\mathbf{A}^i \tag{13}$$

$$\exp(-\mathbf{A}t)\mathbf{x}(t) - \exp(-\mathbf{A}t_i)\mathbf{x}(t_i) = \sum_{n=0}^{n-1} \left(\mathbf{A}^i\mathbf{B} \int_{t_i}^{t} \alpha_i(\tau)\mathbf{u}(\tau)d\tau \right) \tag{14}$$

$$\begin{aligned} &\exp(-\mathbf{A}t)\mathbf{x}(t) - \exp(-\mathbf{A}t_i)\mathbf{x}(t_i) \\ &\qquad = \underbrace{\begin{bmatrix} \mathbf{B} & \mathbf{AB} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} \int_{t_i}^{t} \alpha_0(\tau)\mathbf{u}(\tau)d\tau \\ \int_{t_i}^{t} \alpha_1(\tau)\mathbf{u}(\tau)d\tau \\ \vdots \\ \int_{t_i}^{t} \alpha_{n-1}(\tau)\mathbf{u}(\tau)d\tau \end{bmatrix} \end{aligned} \tag{15}$$

Veta 1.2. Spojitý lineárny systém (5) je riaditeľný pokiaľ matica riaditeľnosti $\mathcal{C}$ má plnú hodnosť:

$$\operatorname{rank}\{\mathcal{C}\} = \operatorname{rank}\left\{ \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix} \right\} = n \tag{16}$$