Mathematical Thinking.
Test Flight Assignment Solutions.

Okon Samuel

Question 1

Lemma 1. $(\forall m \in \mathbb{N})(3m + 5 \neq 12)$

Proof. (By Cases)

Case 1: $(m = 1)$
$3(1) + 5 \neq 12$.
Hence $3m + 5 \neq 12$.

Case 2: $(m = 2)$
$3(2) + 5 = 6 + 5 \neq 12$.
Hence $3m + 5 \neq 12$.

Case 3: $(m = 3)$
$3m \geq 9$.
$3m + 5 \geq 14 \neq 12$.
Hence $3m + 5 \neq 12$.

In conclusion $(\forall m \in \mathbb{N})(3m + 5 \neq 12)$. $\qquad\square$

Lemma 2. $(\forall m \in \mathbb{N})(3m + 10 \neq 12)$.

Proof. Let m be any natural number.
$\Rightarrow m \geq 1$. (All natural numbers are ≥ 1).
$\Rightarrow 3m \geq 3$.
$\Rightarrow 3m + 10 \geq 13 \neq 12$.
Hence $(\forall m \in \mathbb{N})(3m + 10 \neq 12)$ $\qquad\square$

Lemma 3. $(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(3m + 5n \neq 12)$

Proof. (By Cases)
Let m and n be any natural numbers.

Case 1: $(n = 1)$
$3m + 5n = 3m + 5 \neq 12$. (By Lemma 1).

Case 2: $(n = 2)$
$3m + 5n = 3m + 10 \neq 12$. (By Lemma 2)

Case 3: $(n \geq 3)$
$5n \geq 15$.
$3m + 5n \geq 15 + 3m > 15$.
Hence $3m + 5n \neq 12$.

In conclusion $(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(3m + 5n \neq 12)$. $\qquad \square$

Proposition 1. *It is not the case that* $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

Proof. $\neg(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$
$\Leftrightarrow (\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(3m + 5n \neq 12)$, which is true by Lemma 3. $\qquad \square$