Mathematical Thinking.
Test Flight Assignment Solutions.

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Question 7

Proposition 1. *For any natural number n, $2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$.*

Proof. (By Induction)
Let n be any natural number, $\sum_{r=1}^{n} 2^r := 2 + 2^2 + 2^3 + \ldots + 2^n$ and $P(n) := (\sum_{r=1}^{n} 2^r = 2^{n+1} - 2)$.

Base Case: $(n = 0)$

$$\begin{aligned}\sum_{r=1}^{0} 2^r &= 0 \\ &= 2 - 2 \\ &= 2^{0+1} - 2\end{aligned}$$

Hence $P(0)$ is true.

Inductive Step: Assume $P(n)$ is true (i.e $\sum_{r=1}^{n} 2^r = 2^{n+1} - 2$). Then:

$$\begin{aligned}\sum_{r=1}^{n+1} 2^r &= 2^{n+1} + \sum_{r=1}^{n} 2^r \\ &= 2^{n+1} + (2^{n+1} - 2) && \textit{(Inductive Hypothesis)} \\ &= (2^{n+1} + 2^{n+1}) - 2 \\ &= 2(2^{n+1}) - 2 \\ &= 2^{(n+1)+1} - 2\end{aligned}$$

Hence $P(n+1)$ is true i.e $(\sum_{r=1}^{n+1} 2^r = 2^{(n+1)+1} - 2)$.

Hence, $(\forall n \in \mathbb{N})(2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2)$. $\square$