Mathematical Thinking.
Test Flight Assignment Solutions.

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Question 4

Proposition 1. *Every odd natural number is one of the forms* $4n + 1$ *or* $4n + 3$ *where* n *is an Integer.*

Proof. (By Cases)
Let m be any odd natural number. From the definition of an odd natural number, there exists an integer k, such that $m = 2k + 1$.

Case 1: (k is even)
If k is even, then from the definition of an even integer, there exists an integer n, such that $k = 2n$. It follows that

$$\begin{aligned} m &= 2k + 1 \\ &= 2(2n) + 1 \\ &= 4n + 1 \end{aligned} .$$

Case 2: (k is odd)
If k is odd, then from the definition of an odd integer there exists an integer n, such that $k = 2n + 1$. It follows that

$$\begin{aligned} m &= 2k + 1 \\ &= 2(2n + 1) + 1 \\ &= 4n + 3 \end{aligned} .$$

Hence every odd natural number, m can be written as $m = 4n + 1$ or $m = 4n + 3$ for some integer n. □