Mathematical Thinking.
Test Flight Assignment Solutions.

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Question 8

Proposition 1. *If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.*

Proof. Let $\{a_n\}_{n=1}^{\infty}$ be any sequence and M be any real number greater than 0. Suppose that $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$ and is denoted by $\lim_{n\to\infty} a_n = L$.
Then from the definition of a limit of a sequence, for every $\epsilon > 0$, there exists a $N \in \mathbb{Z}^+$ such that, $n > N$

$$\begin{aligned}
&\Rightarrow |a_n - L| < \epsilon \\
&\Rightarrow M|a_n - L| < M\epsilon \\
&\Rightarrow |Ma_n - ML| < M\epsilon
\end{aligned}$$

Hence $(\forall \epsilon \in \mathbb{R}^+)(\exists N \in \mathbb{Z}^+)(|Ma_n - ML| < M\epsilon)$.
Let $\epsilon_o = M\epsilon$ (i.e $\epsilon = \frac{\epsilon_o}{M}$), then $\epsilon_o > 0$ since ($\epsilon > 0$ and $M > 0$).
Hence $\mathbb{R}^+ = \{\epsilon : \epsilon \in \mathbb{R} \wedge \epsilon > 0\} = \{\epsilon_o : \epsilon_o \in \mathbb{R} \wedge \epsilon_o > 0\}$.
It follows that, $(\forall \epsilon_o \in \mathbb{R}^+)(\exists N \in \mathbb{Z}^+)(|Ma_n - ML| < M\epsilon_o)$.
Hence the limit of the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the ML as $n \to \infty$ for any real number $M > 0$. □