Mathematical Thinking.
Test Flight Assignment Solutions.

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Question 5

Proposition 1. *For any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.*

Proof. (By Cases)
Let n be any Integer. Then by the Integer division theorem there exist unique integers q and r such that $n = 3q + r$ and $0 \leqslant r < 3$ (i.e $r \in \{0, 1, 2\}$).

Case 1: $(r = 0)$
If $r = 0$ then, $n = 3q$.
$\Rightarrow n$ is divisible by 3.

Case 2: $(r = 1)$
If $r = 1$ then, $n = 3q + 1$.

$$\begin{aligned} \Rightarrow n + 2 &= 3q + 1 + 2 \\ &= 3q + 3 \\ &= 3(q + 1). \end{aligned}$$

Hence $n + 2$ is divisible by 3.

Case 3: $(r = 2)$
If $r = 2$ then $n = 3q + 2$.

$$\begin{aligned} \Rightarrow n + 4 &= 3q + 2 + 4 \\ &= 3q + 6 \\ &= 3(q + 2). \end{aligned}$$

Hence $n + 4$ is divisible by 3.

Hence for any integer n n, at least one of n, $n + 2$, $n + 4$ is divisible by 3. $\square$