Mathematical Thinking.
Test Flight Assignment Solutions.

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Question 3

Proposition 1. *For any Integer, n, the number $n^2 + n + 1$ is odd.*

Proof. (By Cases)
Let n be any Integer.

Case 1: (n is even).
Then there exists an integer k, such that $n = 2k$.

$$\begin{aligned} n^2 + n + 1 &= (2k)^2 + (2k) + 1 \\ &= 4k^2 + 2k + 1 \\ &= 2(2k^2 + k) + 1 \end{aligned}.$$

which is odd.

Case 2: (n is odd).
Then there exists an integer k, such that $n = 2k + 1$.

$$\begin{aligned} n^2 + n + 1 &= (2k+1)^2 + (2k+1) + 1 \\ &= (2k+1)(2k+1+1) + 1 \\ &= (2k+1)(2k+2) + 1 \\ &= 2(2k+1)(k+1) + 1 \end{aligned}.$$

which is odd.

Hence for any Integer n, the number $n^2 + n + 1$ is odd. □