

Homework 1

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Monte Carlo method

We aim to find the area of the figure bounded by these curves:

$$y = 2|x| - \pi; \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad y = |\sin(2x)|; \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

It can be done using Monte Carlo method.

From the equations (or a picture) we know that $x_{min} = -\frac{\pi}{2}$, $x_{max} = \frac{\pi}{2}$, $y_{min} = -\pi$, $y_{max} = 1$

The algorithm is the following:

- 1) Generate a dot with random (x, y) coordinates within the $[x_{min}; x_{max}] \times [y_{min}; y_{max}]$ rectangle;
- 2) Check whether or not these coordinates are within the given figure;
- 3) Count the number of dots (k) than pass this criteria;
- 4) Repeat N times
- 5) Calculate the area of the figure using this formula:

$$S = \frac{k}{N} S_0$$

where S_0 is the area of the $[x_{min}; x_{max}] \times [y_{min}; y_{max}]$ rectangle

```
N <- 100000
k <- 0

for (i in 1:N) {
  x <- runif(1, min = -pi/2, max = pi/2)
  y <- runif(1, min = -pi, max = 1)
  if (y >= 2*abs(x) - pi && y <= abs(sin(2*x))) {
    k <- k + 1
  }
}

cat('k = ', k)
```

```
## k = 53321
```

```
S_0 <- pi*(1+pi)
S <- k/N*S_0
```

```
cat('Area of the rectangle: S_0 = ', S_0)
```

```
## Area of the rectangle: S_0 = 13.0112
```

```
cat('Area of the figure S = ', S)
```

```
## Area of the figure S = 6.9377
```